$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi G \rho(x)$$

$$\varphi(0) = 5$$

$$\varphi(3) = 7$$

$$\rho(x) = \begin{cases} 1, & x \in (1,2) \\ 0, & x \notin (1,2) \end{cases}$$

$$\int_0^3 \varphi'' v \, dx = \int_0^3 4\pi G \rho(x) \, dx$$

$$\underbrace{[\varphi' v]_0^3}_{0} - \int_0^3 \varphi' v' \, dx = \int_0^3 4\pi G \rho(x) v \, dx$$

$$-\int_0^3 \varphi' v' \, dx = \int_0^3 4\pi G \rho(x) \cdot v \, dx$$

$$\varphi(x) = \bar{\varphi}(x) + \omega(x)$$

$$\bar{\varphi}(x) = 5 + \frac{2}{3} x$$

Stąd $\quad w(0) = w(3) = 0 \quad , \quad w(x) = \varphi(x) - \bar{\varphi}(x)$ :

$$-\int_0^3 w' v' \, dx = 4\pi G \int_0^3 v \cdot \rho(x) \, dx + \int_0^3 \tilde{\varphi} v' \, dx$$

$$B(w,v) = -\int_0^3 w' v' \, dx$$

$$L(v) = 4\pi G \int_1^2 v \, dx$$

$$\tilde{L}(v) = 4\pi G \int_1^2 v \, dx + \int_0^3 \hat{\varphi} v' \, dx$$

# Funkcje testowe

$$e_0(x) = \begin{cases} \frac{x_1 - x}{x_1 - x_0} & x \in [x_0, x_1) \\ 0 & x \in [x_1, x_2] \end{cases}$$

$$e_i(x) = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}}, & x \in [x_{i-1}, x_i) \\ \frac{x_{i+1} - x}{x_{i+1} - x_i}, & x \in [x_i, x_{i+1}] \end{cases} \quad i \in 1, \dots N-1$$

$$e_n(x) = \begin{cases} 0 & x \in (x_{n-2}, x_{n-1}] \\ \frac{x - x_{N-1}}{x_N - x_{N-1}} & x \in [x_{n-1}, x_n] \end{cases}$$

$$\begin{bmatrix} B(e_1, e_1) & \dots & B(e_{n-1}, e_1) \\ \vdots & \ddots & \\ B(e_1, e_{n-1}) & & B(e_{n-1}, e_{n-1}) \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} L(e_1) \\ \vdots \\ L(e_{n-1}) \end{bmatrix}$$

Przybliżenie:

$$\varphi(x) = \tilde{\varphi}(x) + \sum_{i=0}^{N} w_i e_i$$