DSAIUebung-002 -- AussagenLogik

1 - Using Truth Tables

1 a)

To **prove the equivalence** $P \equiv Q$ *where*:

- $P \equiv \neg(A \lor B)$
- $Q \equiv \neg A \land \neg B$

> *we will construct a truth table for both P & Q and compare their truth values in all cases.*
> *If the truth values for P & Q match in every case, then $P \equiv Q$.*

- Build the truth table

The truth table will *include columns* for A, B, $A \lor B$, $\neg(A \lor B)$, $\neg A$, $\neg B$, $\neg A \land \neg B$.
We need to compute the truth values for both formulas based on all possible truth assignments.

A	B	$A \lor B$	$\neg(A \lor B)$	$\neg A$	$\neg B$	$\neg A \land \neg B$
T	T	T	**F**	F	F	**F**
T	F	T	**F**	F	T	**F**
F	T	T	**F**	T	F	**F**
F	F	F	**T**	T	T	**T**

- Compare

- $P \equiv \neg(A \lor B)$
- $Q \equiv \neg A \land \neg B$

***From the truth table*:**

> - $P \equiv \neg(A \lor B)$ *gives the column* $\neg(A \lor B)$.
> - $Q \equiv \neg A \land \neg B$ *gives the column* $\neg A \land \neg B$.
> - *The* **truth values of** P **&** Q **match in every row** *of the truth table.*

- Conclusion

Since the truth values for P & Q are identical in all cases, we conclude that:

$$P \equiv Q$$

therefor: $\neg(A \lor B) \equiv \neg A \land \neg B$.

> *This is an example of De Morgan's Law*

1 b)

To **prove the equivalence** $P \equiv A$ *where*:

- $P \equiv A \implies \neg B$
- $Q \equiv \neg(A \land B)$

> *To show that $P \equiv Q$, we'll construct a truth table and compare the truth values for both P & Q.*
> *If the truth values match for all combinations of A & B, then $P \equiv Q$.*

- Build the truth table

A	B	$A \land B$	$\neg(A \land B)$	$\neg B$	$A \implies \neg B$
T	T	T	**F**	F	**F**
T	F	F	**T**	T	**T**
F	T	F	**T**	F	**T**
F	F	F	**T**	T	**T**

- Compare

- $P \equiv A \implies \neg B$
- $Q \equiv \neg(A \land B)$

From the truth table:

> - *The* **truth values of** $P \equiv A \implies \neg B$ **match the columns of** $Q \equiv \neg(A \land B)$ **in every row**.

- Conclusion

Since the truth values for P & Q are identical in all cases, we conclude that:

$$P \equiv Q$$

therefor: $A \implies \neg B \equiv \neg(A \land B)$.

> *This equivalence also follows from the properties of implication and De Morgan's laws.*

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2 - De Morgan's laws

Take a close look at De Morgan's laws.

De Morgan's laws are two important logical equivalences that express the negation of conjunctions and disjunctions:

> In simple terms:
>
> The **negation of "A or B"** is *equivalent to* **"not A and not B**."
> The **negation of "A and B"** is *equivalent to* **"not A or not B**."

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2 a)

Negation of Disjunction:

$\neg(A \lor B) \equiv \neg A \land \neg B$

> **Inspired by Diogenes' encounter with Alexander the Great, where he asked Alexander to step aside as he blocked the sun.**

- A = "You are a king."
- B = "You have the power to control the sun."
- $\neg(A \lor B)$ = "It is not the case that you are a king or you have the power to control the sun."
- $\neg A \land \neg B$ = "You are not a king and you do not have the power to control the sun."

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2 b)

Negation of Conjunction:

$\neg(A \land B) \equiv \neg A \lor \neg B$

> **Inspired by Diogenes' statement: "I am a citizen of the world."**

- A = "I am a citizen of Greece."
- B = "I am a citizen of another country."
- $\neg(A \land B)$ = "It is not the case that I am a citizen of Greece and I am a citizen of another country."
- $\neg A \lor \neg B$ = "Either I am not a citizen of Greece or I am not a citizen of another country."

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3 - From Propositions to Truth Tables

You have the following propositions:

- A = It's cold.
- B = It's breezy.

Use these propositions to recreate the following sentences as propositional formulas and create a truth table for each one!

- It's cold and breezy.

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

- It's cold but not breezy.

A	B	¬A	¬B	¬A ∧ ¬B
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

- It's not cold and not breezy.

A	B	¬A	¬B	¬A ∧ ¬B
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

- It's either cold or breezy (or both).

A	**B**	**A ∨ B**
T	T	T
T	F	T
F	T	T
F	F	F

- It's cold or breezy, but it's not breezy when it's cold.

A	**B**	**A → B**	**¬(A → B)**	**(A ∨ B) ∧ ¬(A → B)**
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

- When it's breezy, it's cold.

A	**B**	**¬B**	**¬B ∨ A**
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

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4. Translate Complex Sentences

- ***Im Donauparkstadion trinke ich gerne einen Radler oder esse eine Bosna - aber nur wenn sie vegane Würstchen haben! Manchmal konsumiere ich auch beides.****

Aussagenvariablen:

> - `R` = Ich trinke einen Radler im Donauparkstadion.
> - `B` = Ich esse eine Bosna im Donauparkstadion.
> - `V` = Die Bosna hat vegane Würstchen.

Logische Formel: `(R ∨ B) ↔ V`

- ***Mittwochs gehen wir zur Schule und lernen DSAI oder C#, aber nur, wenn es kein Feiertag ist. Wenn es aber Feiertag ist, sind wir unglücklich.***

Aussagenvariablen:

> - `M` = Es ist Mittwoch.
> - `S` = Wir gehen zur Schule.
> - `D` = Wir lernen DSAI.
> - `C` = Wir lernen C#.
> - `F` = Es ist Feiertag.
> - `U` = Wir sind unglücklich.

Logische Formel: `((M → (S ∧ (D ∨ C))) ∧ (¬F → (S ∧ (D ∨ C)))) ∧ (F → U)`
