Daily Scheduling of a District Cooling System

Author:
Iyad Walwil

Supervisor:
Céline Gicquel

IORC Course - Project
Université Paris-Saclay
Department of Mathematics
M2 Optimization
October 30, 2021

0.1 Daily Scheduling: Winter Day

Decision Variables

- C_{ij} (Float): The produced cooling power by the standard chillers of level i for the period h

$$\forall i \in \{1, 2 = LS\} \;\&\; \forall j \in \{0, \ldots, 23 = h\}$$

- Y_{ij} (Boolean): The state of the standard chiller of level i for the period h whether it is off or on.

More precisely,

$$Y_{ij} = \begin{cases} 1 & \textit{If the standard chiller of level } i \textit{ for the period } h \textit{ is } \textbf{\textit{on}} \\ 0 & \textit{Otherwise} \end{cases}$$

Now, all what we aim to do is to minimize the consumed electricity costs according to how much we consume in each period and how much it costs for this period.

- The consumed electric power by the standard chiller i for the period j is given by:-

  $\forall i \in \{1, 2 = LS\} \;\&\; \forall j \in \{0, \ldots\ldots, 23 = h\}$

$$E_{ij} = a_i C_{ij} + b_i$$

  where,

  - a_i is the slope of the performance curve of the standard chiller of level i
  - b_i is the constant value of the performance curve of the standard chiller of level i
  - C_{ij} as defined above in the decision variables box 0.1

- In order to take into account the turned off and on chillers, I developed the "consumed electric power" defined above as follows:-

  $\forall i \in \{1, 2 = LS\}$ & $\forall j \in \{0, \ldots\ldots, 23 = h\}$

  $$E_{ij} = a_i C_{ij} + b_i Y_{ij}$$

- We denote the electricity price for each period j by EP_j $\forall j \in \{0, \ldots\ldots, 23 = h\}$

Objective Function

Thus, our objective function is:-

$$Min \sum_{j=0}^{h} \sum_{i=1}^{LS} E_{ij} EP_j$$

Subject to the following **constraints**:-

- The maximum output power of the standard chillers of level i for each period j if it's active.

  $$C_{ij} \leq Cmax_i Y_{ij} \quad \forall i \in \{1, 2 = LS\},\ \forall j \in \{0, .., 23 = h\}$$

- The minimum output power of the standard chillers of level i for each period j if it's active.

  $$C_{ij} \geq Cmin_i Y_{ij} \quad \forall i \in \{1, 2 = LS\},\ \forall j \in \{0, .., 23 = h\}$$

- The demand satisfaction: for each period j, the total cooling power produced should equal the demand.

  $$\sum_{i=1}^{LS} c_{ij} = D_j \qquad \forall j \in \{0, .., 23 = h\}$$

- The non-negative values of the decision variables C_{ij}

  $$C_{ij} \geq 0$$

Summery

$Min \ \sum_{j=0}^{h} \sum_{i=1}^{LS} (a_i C_{ij} + b_i Y_{ij}) EP_j$

Subject to:-

$$C_{ij} \leq Cmax_i Y_{ij} \quad \forall i \in \{1, 2 = LS\}, \ \forall j \in \{0, .., 23 = h\}$$

$$C_{ij} \geq Cmin_i Y_{ij} \quad \forall i \in \{1, 2 = LS\}, \ \forall j \in \{0, .., 23 = h\}$$

$$\sum_{i=1}^{LS} C_{ij} = D_j \qquad \forall j \in \{0, .., 23 = h\}$$

$$C_{ij} \geq 0$$

Optimal Solution

$$Z^* = 13367.5 \quad euros$$

$$\forall j \in \{0, \ldots\ldots, 23 = h\}$$

$$C_{1j}^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 500 \\ 3000 \\ 5000 \\ 0 \\ 0 \\ 0 \\ 3200 \\ 3200 \\ 4200 \\ 4200 \\ 4200 \\ 2200 \\ 500 \\ 5000 \\ 2000 \\ 600 \\ 0 \\ 0 \end{bmatrix} \quad C_{2j}^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 6000 \\ 6500 \\ 8000 \\ 8800 \\ 8800 \\ 8800 \\ 8800 \\ 8800 \\ 8800 \\ 8500 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad Y_{1j}^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad Y_{2j}^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

0.2 Daily Scheduling: Summer Day

Decision Variables

1. For the **standard chillers**.

$$\forall i \in \{1, 2 = LS\} \ \& \ \forall k \in \{0, \ldots, 23 = h\}$$

- c_{ik} (Float): The total amount of the produced cooling power by the standard chillers of the i^{th} level for the period k.
- n_{ik} (Integer): The number of active standard chillers of the i^{th} level for the period k.

2. For the **ice chillers**.

$$\forall k \in \{0, \ldots, 23 = h\}$$

- Γ_k^{Cold}(Float): The produced cooling power by the ice chiller for the period k
- Γ_k^{Ice}(Float): The produced ice by the ice chiller for the period k
- Γ_k^{Melted}(Float): The melted ice for the period k.
- Γ_k^{Rest}(Float): The ice amount in the inventory at the end of the period k.
- n_k^{Cold}(Integer): The number of ice chillers producing cooling power for the period k
- n_k^{Ice}(Integer): The number of ice chillers producing ice for the period k

Now, all what we aim to do is to minimize the consumed electricity costs according to how much we consume in each period and how much it costs for this period.

- The consumed electric power by the standard chillers of the i^{th} level for the period k is given by:-
  $\forall i \in \{1, 2 = LS\}$ & $\forall k \in \{0, \ldots\ldots, 23 = h\}$
  $$E_{ik} = a_i c_{ik} + b_i n_{ik}$$
- The consumed electric power by the ice chiller i producing cooling power for the period k is given by:-
  $\forall i \in \{1 = LI\}$ & $\forall k \in \{0, \ldots\ldots, 23 = h\}$
  $$E_{ik}^{Cold} = \alpha_i^{cold} \Gamma_k^{Cold} + \beta_i^{cold} n_k^{Cold}$$
- The consumed electric power by the ice chiller i producing ice for the period k is given by:-
  $\forall i \in \{1 = LI\}$ & $\forall k \in \{0, \ldots\ldots, 23 = h\}$
  $$E_{ik}^{Ice} = \alpha_i^{ice} \Gamma_k^{Ice} + \beta_i^{ice} n_k^{Ice}$$
- We denote the electricity price for each period h by EP_k
  $\forall k \in \{0, \ldots\ldots, 23 = h\}$

Objective Function

Thus, our objective function is:-

$$Min \ \sum_{i=1}^{LS} \sum_{k=0}^{h} E_{ik} EP_k + \sum_{i=1}^{LI} \sum_{k=0}^{h} E_{ik}^{Cold} EP_k + \sum_{i=1}^{LI} \sum_{k=0}^{h} E_{ik}^{Ice} EP_k$$

Subject to the following **constraints**:-

- The maximum output power of the standard chillers of the i^{th} level for each period k

$$c_{ik} \leq Cmax_i n_{ik} \quad \forall i \in \{1, 2 = LS\},\ \forall k \in \{0, .., 23 = h\}$$

- The minimum output power of the standard chillers of the i^{th} level for each period k

$$c_{ik} \geq Cmin_i n_{ik} \quad \forall i \in \{1, 2 = LS\},\ \forall k \in \{0, .., 23 = h\}$$

- For each period k, the number of active standard chillers of the i^{th} level shouldn't exceeds the total number of chillers for this level

$$n_{ik} \leq NS_i \quad \forall i \in \{1, 2 = LS\},\ \forall k \in \{0, \ldots, 23 = h\}$$

- The maximum output power of the ice chillers producing cooling power for each period k

$$\Gamma_k^{Cold} \leq \Gamma_{max_1}^{Cold} n_k^{Cold} \quad \forall k \in \{0, .., 23 = h\}$$

- The minimum output power of the ice chillers producing cooling power for each period k

$$\Gamma_k^{Cold} \geq \Gamma_{min_1}^{Cold} n_k^{Cold} \quad \forall k \in \{0, .., 23 = h\}$$

- The maximum output power of the ice chillers producing ice for each period k

$$\Gamma_k^{Ice} \leq \Gamma_{max_1}^{Ice} n_k^{Ice} \quad \forall k \in \{0, .., 23 = h\}$$

- The minimum output power of the ice chillers producing ice for each period k

$$\Gamma_k^{Ice} \geq \Gamma_{min_1}^{Ice} n_k^{Ice} \quad \forall k \in \{0, .., 23 = h\}$$

- For each period k, the ice chillers are either off or on and they produce ice or cooling power but not both. However, since we have just one ice chiller, then we can satisfy this property by the following constraint.

$$n_k^{Cold} + n_k^{Ice} \leq LI = 1 \quad \forall k \in \{0, \ldots, 23 = h\}$$

- The ice inventory at the beginning of the day ($i.e$ $h = 0$) is assumed to be equal to 0. Thus, there is no ice to melt

$$\Gamma_0^{Melted} = 0$$

- At the end of $h = 0$, the inventory will either contain the produced ice during $h = 0$ or nothing.

$$\Gamma_0^{Rest} = \Gamma_0^{Ice}$$

- For each period k, we can melt at most the amount of ice in the inventory.

$$\Gamma_k^{Melted} \leq \Gamma_{k-1}^{Rest} \qquad \forall k \in \{0, \ldots, 23 = h\}$$

- At the end of each period k, the amount of ice in the inventory is equal to the difference between the amount of ice we already had and the amount of melted ice plus the amount of the ice produced during k

$$\Gamma_k^{Rest} = \Gamma_k^{Ice} + \Gamma_{k-1}^{Rest} - \Gamma_k^{Melted} \quad \forall k \in \{0, \ldots, 23 = h\}$$

- For each period k, the ice amount in the inventory shouldn't exceed the inventory capacity.

$$\Gamma_k^{Rest} \leq IceStoCap \qquad \forall k \in \{0, \ldots, 23 = h\}$$

- The demand satisfaction: for each period k, the total cooling power produced by the standard and the ice chillers plus these produced by the melted ice should equal the demand.

$$\sum_{i=1}^{LS} c_{ik} + \Gamma_k^{Cold} + \Gamma_k^{Melted} = D_k \qquad \forall k \in \{0, .., 23 = h\}$$

- The non-negative values of the decision variables.

$$C_{ik}, \Gamma_k^{Cold}, \Gamma_k^{Ice}, \Gamma_k^{Melted}, \Gamma_k^{Rest} \geq 0$$

$$n_{ik}, n_k^{Cold}, n_k^{Ice} \geq 0$$

Summery

$$Min \ \sum_{i=1}^{LS} \sum_{k=0}^{h} E_{ik} EP_k + \sum_{i=1}^{LI} \sum_{k=0}^{h} E_{ik}^{Cold} EP_k + \sum_{i=1}^{LI} \sum_{k=0}^{h} E_{ik}^{Ice} EP_k$$

Subject to:-

$$c_{ik} \leq Cmax_i n_{ik} \quad \forall i \in \{1, 2 = LS\}, \ \forall k \in \{0, .., 23 = h\}$$

$$c_{ik} \geq Cmin_i n_{ik} \quad \forall i \in \{1, 2 = LS\}, \ \forall k \in \{0, .., 23 = h\}$$

$$n_{ik} \leq NS_i \quad \forall i \in \{1, 2 = LS\}, \ \forall k \in \{0, \ldots, 23 = h\}$$

$$\Gamma_k^{Cold} \leq \Gamma_{max_1}^{Cold} n_k^{Cold} \quad \forall k \in \{0, .., 23 = h\}$$

$$\Gamma_k^{Cold} \geq \Gamma_{min_1}^{Cold} n_k^{Cold} \quad \forall k \in \{0, .., 23 = h\}$$

$$\Gamma_k^{Ice} \leq \Gamma_{max_1}^{Ice} n_k^{Ice} \quad \forall k \in \{0, .., 23 = h\}$$

$$\Gamma_k^{Ice} \geq \Gamma_{min_1}^{Ice} n_k^{Ice} \quad \forall k \in \{0, .., 23 = h\}$$

$$n_k^{Cold} + n_k^{Ice} \leq LI \quad \forall k \in \{0, \ldots, 23 = h\}$$

$$\Gamma_0^{Melted} = 0$$

$$\Gamma_0^{Rest} = \Gamma_0^{Ice}$$

$$\Gamma_k^{Melted} \leq \Gamma_{k-1}^{Rest} \quad \forall k \in \{0, \ldots, 23 = h\}$$

$$\Gamma_k^{Rest} = \Gamma_k^{Ice} + \Gamma_{k-1}^{Rest} - \Gamma_k^{Melted} \quad \forall k \in \{0, \ldots, 23 = h\}$$

$$\Gamma_k^{Rest} \leq IceStoCap \quad \forall k \in \{0, \ldots, 23 = h\}$$

$$\sum_{i=1}^{LS} c_{ik} + \Gamma_k^{Cold} + \Gamma_k^{Melted} = D_k \quad \forall k \in \{0, .., 23 = h\}$$

$$C_{ik}, \Gamma_k^{Cold}, \Gamma_k^{Ice}, \Gamma_k^{Melted}, \Gamma_k^{Rest} \geq 0$$

$$n_{ik}, n_k^{Cold}, n_k^{Ice} \geq 0$$

Optimal Solution

$$Z^* = 81636 \quad euros$$

$$\forall j \in \{0, \ldots\ldots, 23 = h\}$$

$$c_{1j}^T = \begin{bmatrix} 3600 \\ 1600 \\ 7400 \\ 5000 \\ 4400 \\ 2400 \\ 10000 \\ 7400 \\ 3600 \\ 5600 \\ 4800 \\ 0 \\ 6000 \\ 5000 \\ 6000 \\ 6000 \\ 5000 \\ 0 \\ 5000 \\ 4700 \\ 0 \\ 0 \\ 0 \\ 5600 \end{bmatrix} \quad n_{1j}^T = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 1 \\ 2 \\ 1 \\ 0 \\ 2 \\ 1 \\ 2 \\ 2 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \quad c_{2j}^T = \begin{bmatrix} 26400 \\ 26400 \\ 17600 \\ 17600 \\ 17600 \\ 17600 \\ 8800 \\ 17600 \\ 26400 \\ 26400 \\ 35200 \\ 26400 \\ 44000 \\ 44000 \\ 44000 \\ 44000 \\ 35200 \\ 44000 \\ 35200 \\ 35200 \\ 35200 \\ 35200 \\ 35000 \\ 26400 \end{bmatrix} \quad n_{2j}^T = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \\ 2 \\ 2 \\ 1 \\ 2 \\ 3 \\ 3 \\ 4 \\ 3 \\ 5 \\ 5 \\ 5 \\ 5 \\ 4 \\ 5 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 3 \end{bmatrix}$$

$$\forall j \in \{0, \ldots\ldots, 23 = h\}$$

$$\Gamma_j^{Cold,T} = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \qquad n_j^{Cold,T} = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \qquad \Gamma_j^{Ice,T} = \begin{bmatrix} 3500\\3500\\3500\\3500\\3500\\3500\\3500\\0\\0\\0\\0\\0\\3500\\3500\\3500\\3500\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \qquad n_j^{Ice,T} = \begin{bmatrix} 1\\1\\1\\1\\1\\1\\1\\0\\0\\0\\0\\0\\1\\1\\1\\1\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}$$

$$\forall j \in \{0, \ldots\ldots, 23 = h\}$$

$$\Gamma_j^{Melted,T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 400 \\ 0 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 21600 \\ 0 \\ 0 \\ 0 \\ 0 \\ 7800 \\ 1000 \\ 1800 \\ 100 \\ 3800 \\ 1800 \\ 0 \\ 0 \end{bmatrix} \qquad \Gamma_j^{Rest,T} = \begin{bmatrix} 3500 \\ 7000 \\ 10500 \\ 13600 \\ 17100 \\ 20600 \\ 23900 \\ 23900 \\ 23900 \\ 23900 \\ 23900 \\ 2300 \\ 5800 \\ 9300 \\ 12800 \\ 16300 \\ 8500 \\ 7500 \\ 5700 \\ 5600 \\ 1800 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$