1

1. Let $f(x) = \frac{6x^2-19x+20}{(x+1)(x-2)^2}$ be a rational fraction.

i) Split $f(x) = \frac{6x^2-19x+20}{(x+1)(x-2)^2}$ in to simple partial fractions.

ii) Evaluate $\int \frac{6x^2-19x+20}{(x+1)(x-2)^2} dx$ in its simplest form

iii) Also obtain the expansion of $f(x)$ in ascending power of x upto and including the term in x^3.

Sol?

(i) Let $f x = \frac{6x^2-19x+20}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

or, $\frac{6x^2-19x+20}{(x+1)(x-2)^2} = \frac{A(x-2)^2 + B(x+1)(x-2) + C(x+1)}{(x+1)(x-2)^2}$ ------(i)

or, $6x^2-19x+20 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

Putting $x=-1$, we get.

$A = 5$

Putting $x=2$, we get.

$C = 2$

Putting $x=0$, we get.

$B = 1$

So,

$\therefore f(x) = \frac{5}{x+1} + \frac{1}{x-2} + \frac{2}{(x-2)^2}$

Then,

(ii) $\int f(x) = \int \left(\frac{5}{x+1} + \frac{1}{x-2} + \frac{2}{(x-2)^2}\right) dx$

$= 5\int \frac{1}{x+1} dx + \int \frac{1}{x-2} dx + 2\int \frac{1}{(x-2)^2} dx$

$= 5\ln(x+1) + \ln(x-2) + 2\frac{(x-2)^{-2+1}}{-2+1} + C$

$= 5\ln(x+1) + \ln(x-2) - \frac{2}{(x-2)} + C$

(ii) We have, $f(x) = \frac{5}{x+1} + \frac{1}{x-2} + \frac{2}{(x-2)^2}$

for the first term,

$$\frac{5}{x+1} = 5(1+x)^{-1}$$
$$= 5\left(1 + \frac{(-1)}{1!}x + \left[\frac{(-1)(-1-1)}{2!}\right]x^2 + \left[\frac{(-1)(-1-1)(-1-2)}{3!}\right]x^3 + \cdots\right]$$
$$= 5\left[1 - x + x^2 - x^3 + \cdots\cdots\right]$$
$$= 5 - 5x + 5x^2 - 5x^3 + \cdots\cdots \infty$$

for second term,

$$\frac{1}{x-2} = (-2+x)^{-1}$$
$$= (-2)^{-1}\left(1 - x/2\right)^{-1}$$
$$= -\frac{1}{2}\left[1 + \frac{(-1)(-x/2)}{1!} + \left\{\frac{(-1)(-1-1)}{2}\right\}\left(\frac{-x}{2}\right)^2 + \left\{\frac{(-1)(-1-1)(-1-2)}{3!}\right\}\left(\frac{-x}{2}\right)^3 + \cdots\right.$$
$$= -\frac{1}{2}\left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} \cdots\cdots \infty\right.$$
$$= -\frac{1}{2} - \frac{x}{4} - \frac{x^2}{8} - \frac{x^3}{16} + \cdots\cdots \infty$$

for 3rd term,

$$\frac{2}{(x-2)^2} = 2(x-2)^{-2}$$
$$= 2(-2+x)^{-2}$$
$$= 2(-2)^2\left(1 - \frac{x}{2}\right)^{-2}$$
$$= \frac{2}{4}\left(1 - \frac{x}{2}\right)^{-2}$$
$$= \frac{1}{2}\left(1 - \frac{x}{2}\right)^{-2}$$
$$= \frac{1}{2}\left[1 + \frac{(-2)(-x/2)}{1!} + \frac{(-2)(-2-1)}{2!}\left(-x/2\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(\frac{-x}{2}\right)^3 + \cdots \infty\right]$$
$$= \frac{1}{2}\left[1 + x + \frac{3x^2}{4} + \frac{4x^3}{8} + \cdots\right]$$
$$= \frac{1}{2} + \frac{1}{2}x + \frac{3x^2}{8} + \frac{1}{4}x^3 + \cdots\cdots \infty$$

Now,

Adding all three terms,

$$f(x) = 5 - 5x + 5x^2 - 5x^3 - \cancel{\frac{1}{2}} - \frac{x}{4} - \frac{x^2}{8} - \frac{x^3}{16} + \cancel{\frac{1}{2}} + \frac{x}{2} + \frac{3x^2}{8} + \frac{1}{4}x^3 + \cdots$$

$$= 5 - \left[5x + \frac{x}{4} - \frac{x}{2}\right] + \left[5x^2 - \frac{x^2}{8} + \frac{3x^2}{8}\right] - \left[5x^3 + \frac{x^3}{16} - \frac{1}{4}x^3\right] + \cdots$$

$$= 5 - \left[\frac{20x + x - 2x}{4}\right] + \left[\frac{40x^2 - x^2 + 3x^2}{8}\right] - \left[\frac{80x^3 + x^3 - 4x^3}{16}\right] + \cdots$$

$$= 5 - \frac{19}{4}x + \frac{42}{8}x^2 - \frac{77}{16}x^3 + \cdots \infty$$

$$f(x) = 5 - \frac{19}{4}x + \frac{21}{4}x^2 - \frac{77}{16}x^3 + \cdots \infty$$

(2) Let $z = 1 + i\sqrt{3}$ be a complex number.

(i) form a quadratic equation whose one of the root is $z = 1 + i\sqrt{3}$.

(ii) Express the complex number $z = 1 + i\sqrt{3}$ in modulus argument form.

(iii) By using De moivre's theorem, find the cube roots of the complex number $z = 1 + i\sqrt{3}$.

(iv) Express all three cube roots of z in exponential form, i.e. $z = re^{i\theta}$; where θ measured in radian.

Sol?

(i) Given, $z = 1 + i\sqrt{3}$ be α.

If $1 + i\sqrt{3}$ be one root of quadratic equation then another root will be $\beta = 1 - \sqrt{3}$ [$\because$ Imaginary roots of quadratic equation always occurs in conjugate pair.

then,

Quadratic equation is given as,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

or, $x^2 - [1 + i\sqrt{3} + (1 - i\sqrt{3})]x + [(1 + i\sqrt{3})(1 - i\sqrt{3})] = 0$

or, $x^2 - 2x + [1 - i\sqrt{3} + i\sqrt{3} - (-3)] = 0$

or, $x^2 - 2x + 4 = 0$ ------ (i)

which is the required equation.

(ii) Sol?!

Given, $z = 1 + i\sqrt{3}$

Comparing with $z = x + iy$ then $x = 1, y = \sqrt{3}$.

$$\therefore r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3}$$

$$= \sqrt{4}$$

$$= 2$$

Hence, modulus $|z| = r = 2$

similarly,

Argument $(\theta) = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{1} = [60°, 120°]$

so,

Either, $z = 2[\cos 60° + i\sin 60°]$

OR, $z = 2[\cos 120° + i\sin 120°]$

Soln $z = 1 + i\sqrt{3}$

We have, $r = 2$, $\theta = 60^\circ$

Let, z be cube root of $1+i\sqrt{3}$ then,

$$z^3 = 2(\cos 60^\circ + i\sin 60^\circ)$$

or, $z^3 = 2\left[\cos(n.360^\circ + 60^\circ) + i\sin(n.360^\circ + 60^\circ)\right]$

or, $z = 2^{1/3}\left[\cos(n.360 + 60^\circ) + i\sin(n.360^\circ + 60^\circ)\right]^{1/3}$

or, $z = 2^{1/3}\left[\cos\left(\frac{n.360^\circ + 60^\circ}{3}\right) + i\sin\left(\frac{n.360^\circ + 60^\circ}{3}\right)\right]$

where $n = 0, 1, 2$

So,

when $n = 0$,

$$z_1 = 2^{1/3}\left[\cos\left(\frac{0.360^\circ + 60^\circ}{3}\right) + i\sin\left(\frac{0.360^\circ + 60^\circ}{3}\right)\right]$$

or, $z_1 = 2^{1/3}\left[\cos 20^\circ + i\sin 20^\circ\right]$

when $n = 1$,

$$z_2 = 2^{1/3}\left[\cos\left(\frac{1.360^\circ + 60^\circ}{3}\right) + i\sin\left(\frac{1.360^\circ + 60^\circ}{3}\right)\right]$$

or, $z_2 = 2^{1/3}\left[\cos 140^\circ + i\sin 140^\circ\right]$

when $n = 2$,

or, $z_3 = 2^{1/3}\left[\cos\left(\frac{2.360^\circ + 60^\circ}{3}\right) + i\sin\left(\frac{2.360^\circ + 60^\circ}{3}\right)\right]$

or, $z_3 = 2^{1/3}\left[\cos 260^\circ + i\sin 260^\circ\right]$

(11) Soln $z = re^{i\theta}$ where θ measured in radian.

Given,

$$z_1 = 2^{1/3}e^{i20^\circ} \Rightarrow 2^{1/3}e^{i\pi/9}$$

$$z_2 = 2^{1/3}e^{i140^\circ} \Rightarrow 2^{1/3}e^{i7\pi/9}$$

$$z_3 = 2^{1/3}e^{i260^\circ} \Rightarrow 2^{1/3}e^{i13\pi/9}$$

6. The diagram shows the curve $y = \frac{1}{..}(3x-1)^2$ th.. point

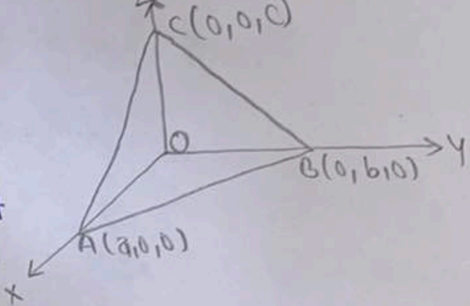
3 Define Plane.
A variable plane is at a constant distance 3p from the origin and meets the axes in the points A, B and C as shown in fig.

i) Prove that the locus of the centroid of triangle ABC is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}.$$

(ii) Also show that area of ΔABC is $\frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$ (Use vector method).

Soln



A plane is a surface such that the line joining any two points on it lies entirely on it.

(i) Soln:-

Let O be the origin and ABC be a plane with A = (a,0,0), B = (0,b,0) and C = (0,0,c) i.e. OA = a, OB = b and OC = C. Hence the equation of plane is given by;

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\text{or, } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0 \text{ ------- (i)}$$

where, a = x-intercept, b = y-intercept, c = z-intercept.

Since, the distance of the plane (i) from origin O is 3P so we have,

$$3P = \left| \frac{x/a + y/b + z/c - 1}{\sqrt{(1/a)^2 + (1/b)^2 + (1/c)^2}} \right|$$

when passes through origin so, $(x, y, z) = (0,0,0)$. then,

$$3P = \left| \frac{0/a + 0/b + 0/c - 1}{\sqrt{(1/a)^2 + (1/b)^2 + (1/c)^2}} \right|$$

6. The diagram shows the ...

4 i) Express ...

or, $3p = \left| \dfrac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right|$

squaring both sides, we get.

$$9p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

or, $\dfrac{1}{9p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}$ ------ (ii)

Let (x_0, y_0, z_0) be the centroid of ΔABC. then,

$x_0 = \dfrac{a+0+0}{3}$, $y_0 = \dfrac{0+b+0}{3}$, $z_0 = \dfrac{0+0+c}{3}$

or, $x_0 = \dfrac{a}{3}$, $y_0 = \dfrac{b}{3}$, $z_0 = \dfrac{c}{3}$

$\therefore a = 3x_0$, $b = 3y_0$, $c = 3z_0$

Substituting these values of a, b, c in eqn (ii), we get.

$$\frac{1}{9p^2} = \frac{1}{(3x_0)^2} + \frac{1}{(3y_0)^2} + \frac{1}{(3z_0)^2}$$

or, $\dfrac{1}{9p^2} = \dfrac{1}{9x_0^2} + \dfrac{1}{9y_0^2} + \dfrac{1}{9z_0^2}$

or, $\dfrac{1}{9p^2} = \dfrac{1}{9}\left(\dfrac{1}{x_0^2} + \dfrac{1}{y_0^2} + \dfrac{1}{z_0^2}\right)$

or, $\dfrac{1}{x_0^2} + \dfrac{1}{y_0^2} + \dfrac{1}{z_0^2} = \dfrac{1}{p^2}$

(ii

Hence, the locus of centroid (x_0, y_0, z_0) is,

$\therefore \dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2} = \dfrac{1}{p^2}$

proved

6. The diagram shows the curve $y = 1\,(2x \ldots)^2 \ldots$

Express $\frac{1}{\ldots}$ in partial fractions.

soln

Given, $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$.

$A(a,0,0) = \cancel{(a_1, b_1, c_1)} = \vec{OA}$

$B(0,b,0) = \cancel{(a_2, b_2, c_2)} = \vec{OB}$

$C(0,0,c) = \cancel{(a_3, b_3, c_3)} = \vec{OC}$

from vector law,

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= (0,b,0) - (a,0,0)$$
$$= (-a, b, 0)$$
$$= (a_1, a_2, a_3)$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$= (0,0,c) - (a,0,0)$$
$$= (-a, 0, c)$$
$$= (b_1, b_2, b_3)$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2}$$
$$= \sqrt{(b \times c - 0 \times 0)^2 + (0 \times -a - (-a) \times c)^2 + (-a \times 0 - b \times -a)^2}$$
$$= \sqrt{(bc)^2 + (ac)^2 + (ab)^2}$$
$$= \sqrt{b^2c^2 + c^2a^2 + a^2b^2}$$
$$= \sqrt{a^2b^2 + b^2c^2 + c^2a^2}$$

So,

$$\text{Area of } \Delta ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$
$$= \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$$

proved

6. The diagram shows the curve $y = 1\ (2x+1)^2$ ...

4 i) Express $\frac{1}{x(2x+3)}$ in partial fractions.

ii) the variables x and y satisfy the differential equations. $x(2x+3)\frac{dy}{dx} = y$ and it is given that $y=1$ when $x=1$.

ⓐ find the particular solutions of above differential equation, express y in term of x.

ⓑ Calculate the value of y when $x=9$. given your answer in two decimal places.

Soln

(i) let $\frac{1}{x(2x+3)} = \frac{A}{x} + \frac{B}{(2x+3)}$

or, $\frac{1}{x(2x+3)} = \frac{A(2x+3)+Bx}{x(2x+3)}$

or, $1 = A(2x+3) + Bx$ ------(i)

when $x=0$,

$1 = A(0+3) + 0$

or, $1 = 3A$

$\therefore A = 1/3$

when $x=1$,

$1 = \frac{1}{3}(2\times1+3) + B\times1$

or, $1 = 5/3 + B$

$\therefore B = -2/3$

Then,

$$\frac{1}{x(2x+3)} = \frac{1}{3x} + \frac{-2/3}{(2x+3)}$$

$$\therefore \frac{1}{x(2x+3)} = \frac{1}{3x} - \frac{2}{3(2x+3)}$$

(ii) Soln

Given, $x(2x+3)\frac{dy}{dx} = y$

or, $\frac{1}{y}dy = \frac{1}{x(2x+3)}dx$

or, $\frac{1}{y}dy = \left[\frac{1}{3x} - \frac{2}{3(2x+3)}\right]dx$

Integrating both sides,

or, $\int \frac{1}{y}\,dy = \int \left[\frac{1}{3x} - \frac{2}{3(2x+3)}\right] dx$

or, $\ln y = \frac{1}{3}\left[\int \frac{1}{x}\,dx - \int \frac{2}{2x+3}\,dx\right]$

or, $\ln y = \frac{1}{3}\left[\ln x - \ln(2x+3) + \ln C\right]$

or, $\ln y = \frac{1}{3}\left[\ln \frac{x}{2x+3}\right] + \frac{\ln C}{3}$

or $\ln y - \frac{\ln C}{3} = \frac{1}{3}\left[\ln \frac{x}{2x+3}\right]$ ------ (ii)

$\boxed{\cancel{\ln \frac{y}{C} = \frac{1}{3}\left[\ln \frac{x}{2x+3}\right]} \quad 3\ln\left(\frac{y}{C}\right) = \ln\left(\frac{x}{2x+3}\right)}$

When $x = 1,\ y = 1$ then, eqn (ii) becomes,

or $\ln 1 - \frac{\ln C}{3} = \frac{1}{3}\ln\left(\frac{1}{2\times1+3}\right)$

or, $0 - \frac{\ln C}{3} = \frac{1}{3}\ln\left(\frac{1}{5}\right)$

or $-\frac{\ln C}{3} = \frac{1}{3}\times -1.6094$

or, $-\ln C = \cancel{-0.536}\ -1.6094$

$\therefore \ln C = \cancel{0.536}\ 1.6094$

Substituting value of $\ln C$ in eqn (ii),

or, $\ln y - \frac{1.6094}{3} = \frac{1}{3}\left[\ln \frac{x}{2x+3}\right]$

or $\ln y = \frac{1}{3}\left[\ln \frac{x}{2x+3}\right] + 0.536$

$\therefore \ln y = \frac{1}{3}\ln\left(\frac{x}{2x+3}\right) + 0.536$ ------ (iv)

– which is the required particular solution.

(iii) Given, particular eqn,

$\ln y = \frac{1}{3}\ln\left(\frac{x}{2x+3}\right) + 0.536$

When $x = 9$,

$\ln y = \frac{1}{3}\ln\left(\frac{9}{2\times9+3}\right) + 0.536$

or, $\ln y = \frac{1}{3} \ln\left(\frac{9}{21}\right) + 0.536$

or, $\ln y = \frac{1}{3} \times -0.847 + 0.536$

or, $\ln y = -0.282 + 0.536$

or, $\ln y = 0.253$

$\therefore y = 1.290$

5 i) Define direction cosine of a line.
ii) find the direction cosines of the line AB, which is perpendicular to the lines with direction cosines porportional to 1, 2, 3 and -1, 3, 5.
iii) Also find the projection of line joining the points C(1, 3, 4) and D(4, 3, 7) on the line AB.

Soln:

(i) ⇒ Let α, β, γ be the angles which a directed line makes with positive x-axis, Y-axis and z-axis respectively then the cosines of the angles α, β, γ of the line are called the direction cosines of the line.

(ii) ⇒ Soln:

Let l, m, n be the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 1, 2, 3 and -1, 3, 5. Then, by the conditions of perpendicularity,

$$1\times l + 2\times m + 3\times n = 0$$

$$\text{or, } l + 2m + 3n = 0 \text{ - - - - - (1)}$$

$$-1\times l + 3\times m + 5\times n = 0$$

$$\text{or, } -l + 3m + 5n = 0 \text{ - - - - - (2)}$$

solving eqn (1) & (2) by rule of cross multiplication,

$$\text{or, } \frac{l}{2\times5 - 3\times3} = \frac{m}{-1\times3 - 1\times5} = \frac{n}{1\times3 - (-1)\times2}$$

$$\text{or, } \frac{l}{10 - 9} = \frac{m}{-3-5} = \frac{n}{3+2}$$

$$\text{or, } \frac{l}{1} = \frac{m}{-8} = \frac{n}{5} \Rightarrow \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(1)^2+(-8)^2+(5)^2}} = \frac{1}{\sqrt{90}}$$

$$\text{or, } l = \frac{m}{-8} = \frac{n}{5} = \frac{1}{\sqrt{90}}$$

$$\therefore l = \frac{1}{\sqrt{90}}, \quad m = \frac{-8}{\sqrt{90}}, \quad n = \frac{5}{\sqrt{90}}$$

$\therefore$ Direction cosines of line $(l, m, n) = \left(\frac{1}{\sqrt{90}}, \frac{-8}{\sqrt{90}}, \frac{5}{\sqrt{90}}\right)$

6. The diagram shows the curve $y = \frac{1}{16}(3x-1)^2$, the point $P(3,4)$ lies on the curve and the tangent and normal at $P(3,4)$ cuts the x-axis at A and B respectively.

Given,

$C(1,3,4)$ and $D(4,3,7)$ be $C(x_1,y_1,z_1)$ and $D(x_2,y_2,z_2)$ respectively.

Then,

Projection of line joining points C & D on the line AB is,

$$(x_2-x_1)l + (y_2-y_1)m + (z_2-z_1)n$$

$$= (4-1)\frac{1}{\sqrt{90}} + (3-3)\frac{-8}{\sqrt{90}} + (7-4)\frac{5}{\sqrt{90}}$$

$$= \frac{3}{\sqrt{90}} + \frac{0 \times -8}{\sqrt{90}} + \frac{3 \times 5}{\sqrt{90}}$$

$$= \frac{3}{\sqrt{90}}(1+0+5)$$

$$= \frac{3 \times 6}{\sqrt{90}}$$

$$= \frac{3 \times 6}{3\sqrt{10}}$$

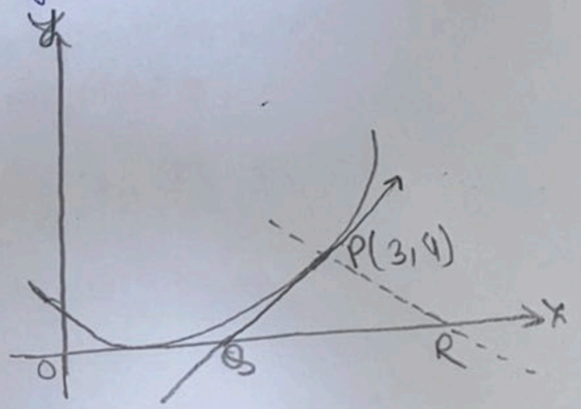
$$= \frac{6}{\sqrt{10}} //$$

6. The diagram shows the curve $y=\frac{1}{16}(3x-1)^2$, the point $P(3,4)$ lies on the curve and the tangent and normal at $P(3,4)$ cuts the x-axis at Q and R respectively.

(i) By using differentiation find the equation of tangent PQ and normal PR.

(ii) by using vector product of two vectors $\vec{QP}$ and $\vec{QR}$, find the area of the triangle PQR.

Soln



Given,

$$y=\frac{1}{16}(3x-1)^2$$

diff. w.r.t. x, we get.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{1}{16}(3x-1)^2\right]$$

$$=\frac{1}{16}\frac{d(3x-1)^2}{d(3x-1)}\times\frac{d(3x-1)}{dx}$$

$$=\frac{1\times 2(3x-1)^{2-1}}{16}\times 3$$

$$=\frac{6}{16}(3x-1)$$

$$=\frac{3}{8}(3x-1)$$

At $P(3,4)$, the slope of tangent is,

$$m=\frac{3}{8}(3\times 3-1)$$

$$=\frac{3}{8}\times 8$$

$$=3$$

So, Equation of tangent at $P(3,4)$ is,

$$y-4=3(x-3)$$

or $y-4=3x-9$

$\therefore 3x-y-5=0$ ------- (i)

let, Eqn of line perpendicular to the tangent be

$x+3y+k=0$ ------- (ii)

When eqn (ii) passes through $P(3,4)$, then

$3+3(4)+k=0$

or $3+12+k=0$

or, $15+k=0$

$\therefore k=-15$

Putting $k=-15$ in eqn (ii), we get the equation of normal PR is;

$\therefore x+3y-15=0$ ------(iii)

(iii) let, $Q(x_1,0)$ and $R(x_2,0)$ are points on tangent PQ and normal PR lying on x-axis respectively.

Then,

Q satisfy eqn (i) and R satisfy eqn (iii) so,

$3x_1-0-5=0$

$\therefore x_1=5/3$

$x_2+0-15=0$

$\therefore x_2=15$

so, co-ordinate of $Q(x_1,0)=(5/3,0)$ and $R(x_2,0)=(15,0)$

from vector product, $\vec{OP}=(3,4)$, $\vec{OQ}=(5/3,0)$, $\vec{OR}=(15,0)$

$\vec{QP}=\vec{OP}-\vec{OQ}$

$=(3,4)-(5/3-0)$

$=(3-5/3,\ 4+0)$

$=(4/3,\ 4)$

$=(a_2,b_2)$

$\vec{QR}=\vec{OR}-\vec{OQ}$

$=(15,0)-(5/3,0)$

$=(15-5/3,\ 0)$

$=(40/3,\ 0)$

$=(a_1,b_1)$

Then,

Area of $\Delta PQR=\frac{1}{2}|\vec{QR}\times\vec{QP}|$

$=\frac{1}{2}\left[\sqrt{a_1b_2-a_2b_1}\right]$

$=\frac{1}{2}\sqrt{\frac{40}{3}\times4-\frac{4}{3}\times0}$

$=\frac{1}{2}\sqrt{\frac{160}{3}}$

$=\frac{4}{2}\sqrt{\frac{10}{3}}$

$=2\sqrt{\frac{10}{3}}$ sq. units