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3COE2

"Parameter Estimation"

Date
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Question 1 Random Sample $= x_1, x_2 - - x_n$ from ND (Normal
ans → distribution with mean θ_1 and Variance θ_2

Probability density func is-

$$f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(\frac{-(x-\theta_1)^2}{2\theta_2}\right)} \quad \text{—(i)}$$

likelihood function $L(\theta_1, \theta_2)$ is joint pdf of sample

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) \quad \text{—(ii)}$$

taking log of (ii), then (ii) become

$$\ln L(\theta_1, \theta_2) = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

differentiate wrt θ_1 and put $=0$

$$\frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum_{i=1}^{n} x_i = n\theta_1$$

$$\boxed{\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

So, MLE for θ_1 is Sample mean.

Also wrt θ_2,

$$\frac{\partial \ln L(\theta_1, \theta_2)}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\Rightarrow -n + \theta_2^{-1} \sum_{i=1}^{n} (x_i - \theta i)^2 = 0$$

$$n = \theta_2^{-1} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

MLE for θ_2 is Sample Variance

Question 2 ans → Pdf of binomial distribution $B(n,p) = {}^nC_x\, p^x (1-p)^{n-x}$

$n = m,\ p = \theta$

$\therefore f(x_i) = {}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$

likelihood fun^c $L(m,\theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} {}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$

$$L(m,\theta) = \left(\prod_{i=1}^{n} {}^mC_{x_i}\right) \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}$$

$$= \log L(m,x) = \log\left(\prod_{i=1}^{n} {}^mC_{x_i}\right) \sum_{i=1}^{n} x_i \log\theta \left(mn - \sum_{i=1}^{n} x_i\right) \log(1-\theta)$$

$$\frac{\partial \log L(m,\theta)}{\partial\theta} = \sum_{i=1}^{n} x_i \times \frac{1}{\theta} + \frac{1}{1-\theta}\left(\sum_{i=1}^{n} x_i - mn\right)$$

$$\frac{1}{1-\theta}\left(mn - \sum_{i=1}^{n} x_i\right) - \frac{1}{\theta}\left(\sum_{i=1}^{n} x_i\right) = 0$$

$$\frac{mn}{\sum x_i} - 1 = \frac{1-\theta}{\theta}$$

$$\boxed{\therefore \theta_{MLE} = \frac{\bar{x}}{m}}$$