Numerical Analysis

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2018 年 11 月 1 日

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1 Hermite Interpolation

The osculating polynomial approximating a function $f \in C^m[a,b]$ at x_i for each $i = 0 : n$, of which the derivatives of order less than or equal to m_i, then the degree of this osculating polynomial is at most $M = \sum_{i=0}^{n} m_i + n$.

$$\frac{\mathrm{d}^k P(x_i)}{\mathrm{d}x^k} = \frac{\mathrm{d}^k f(x_i)}{\mathrm{d}x^k}, \quad \text{for each } i = 0 : n,\ k = 0 : m_i.$$

$$\begin{cases} n = 0 & m_0 \text{ Taylor polynomial for } f \text{ at } x_0 \\ m_i = 0 (\text{each } i) & n\text{th Lagrange polynomial} \end{cases}$$

1.1 Hermite Polynomials

定理 1.1 *$f \in C'[a,b]$ and $x_0, \cdots, x_n \in [a,b]$, the unique polynomial of least degree agreeing with f and f' at $x_0, \cdots, x_n$ is the Hermite polynomial of degree at most $2n+1$.*

$$H_{2n+1}(x) = \sum_{j=0}^{n} f(x_j) H_{n,j}(x) + \sum_{j=0}^{n} f'(x_j) \hat{H}_{n,j}(x)$$

$$\begin{cases} H_{n,j}(x) & = \left[1 - 2(x - x_j) L'_{n,j}(x_j)\right] L^2_{n,j}(x) \\ \hat{H}_{n,j}(x) & = (x - x_j) L^2_{n,j}(x) \end{cases}$$

Moreover, if $f \in C^{2n+2}[a,b]$, then

$$f(x) = H_{2n+1}(x) + \frac{(x-x_0)^2 \cdots (x-x_n)^2}{(2n+2)!} f^{(2n+2)}\left(\xi(x)\right).$$

[Proof]

$$\begin{aligned} H_{n,j}(x_i) &= \delta_{i,j} & \hat{H}_{n,j}(x_i) &= 0 \\ H'_{n,j}(x_i) &= 0 & \hat{H}'_{n,j}(x_i) &= \delta_{i,j} \end{aligned}$$

□

1.2 Hermite Polynomials Using Divided Differences

Suppose that the distinct numbers $x_0, \cdots, x_n$ are given together with values of f and f'. Define a new sequence $z_0, \cdots, z_{2n+1}$ by

$$z_{2i} = z_{2i+1} = x_i \quad \text{for } i = 0 : n.$$

We have $H_{2n+1}(x) = f[z_0] + \sum_{k=1}^{2n+1} f[z_0, \cdots, z_k](x - z_0) \cdots (x - z_{k-1})$.

z	$f(z)$	First divided differences	$\cdots$
$z_0 = x_0$	$f[z_0] = f(x_0)$	$f[z_0, z_1] = f'(x_0)$	$\vdots$
$z_1 = x_0$	$f[z_1] = f(x_0)$	$f[z_1, z_2] = \frac{f[z_2]-f[z_1]}{z_2-z_1}$	$\vdots$
$z_2 = x_1$	$f[z_2] = f(x_1)$	$f[z_2, z_3] = f'(x_1)$	$\vdots$
$z_3 = x_1$	$f[z_3] = f(x_1)$	$f[z_3, z_4] = \frac{f[z_3]-f[z_4]}{z_3-z_4}$	$\vdots$
$z_4 = x_2$	$f[z_4] = f(x_2)$	$f[z_4, z_5] = f'(x_2)$	
$z_5 = x_2$	$f[z_5] = f(x_2)$		