Numerical Analysis

Iydon

2018 年 11 月 15 日

目录

第零章 Iterative Techniques in Matrix Algebra **5**

0.1 Norms of Vectors and Matrices 5

0.1.1 Matrix Norms and Distances 6

0.2 Eigenvalues and Eigenvectors 6

第零章 Iterative Techniques in Matrix Algebra

0.1 Norms of Vectors and Matrices

定义 0.1.1 *A vector norm on $\mathbb{R}^n$ is a function, $\|\cdot\|$, from $\mathbb{R}^n$ to $\mathbb{R}$ with the following properties.*

(i) $\|\mathbf{x}\| \geq \mathbf{0}$ *for all* $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$.

(ii) $\|\mathbf{x}\| = \mathbf{0}$ *if and only if* $\mathbf{x} = \mathbf{0}$.

(iii) $\|\alpha\mathbf{x}\| = |\alpha| \, \|\mathbf{x}\|$ *for all* $\alpha \in \mathbb{R}$ *and* $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$.

(iv) $\|\mathbf{x}+\mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$ *for all* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{\mathbf{n}}$.

定义 0.1.2 *The l_1, l_2, l_∞ norms for the vector* $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n})^{\mathbf{t}}$ *are defined by*

- $\|\mathbf{x}\|_{\mathbf{1}} = \sum_{i=1}^{n} |x_i|$
- $\|\mathbf{x}\|_{\mathbf{2}} = \left[\sum_{i=1}^{n} x_i^2\right]^{1/2}$
- $\|\mathbf{x}\|_{\infty} = \max_{1 \leq i \leq n} |x_i|$

定理 0.1.1 *The sequence of vectors* $\mathbf{x}^{(\mathbf{k})}$ *converges to* $\mathbf{x}$ *in* $\mathbb{R}^n$ *with respect to the* $l\infty$ *norm if and only if* $\lim_{k\to+\infty} x_i^{(k)} = x_i$, *for each*

$i = 1, 2, \ldots, n$.

0.1.1 Matrix Norms and Distances

定义 0.1.3 (Matrix Norms) *A matrix norm on the set of all $n \times n$ matrices s a real-valued function, $\|\cdot\|$, defined on this ser, satisfying for all $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ and all real numbers α.*

(i) $\|\mathbf{A}\| \geq \mathbf{0}$.

(ii) $\|\mathbf{A}\| = \mathbf{0}$ *if and only if* $\mathbf{A}$ *is* $\mathbf{0}$*, the matrix with all 0 entries.*

(iii) $\|\alpha\mathbf{A}\| = |\alpha|\,\|\mathbf{A}\|$.

(iv) $\|\mathbf{A} + \mathbf{B}\| \leq \|\mathbf{A}\| + \|\mathbf{B}\|$.

(v) $\|\mathbf{AB}\| \leq \|\mathbf{A}\|\|\mathbf{B}\|$.

定理 0.1.2 *If $\|\cdot\|$ is a vector norm on $\mathbb{R}$, then*

$$\|\mathbf{A}\| = \max_{\|\mathbf{x}\|=\mathbf{1}} \|\mathbf{Ax}\|$$

is a matrix norm.

定理 0.1.3 *If $\mathbf{A} = (\mathbf{a_{ij}})$ is on $n \times n$ matrix, then*

$$\|\mathbf{A}\|_\infty = \max_{\mathbf{1}\leq\mathbf{i}\leq\mathbf{n}} \sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{n}} |\mathbf{a_{ij}}|\,.$$

0.2 Eigenvalues and Eigenvectors