CSCI 544
Applied Natural Language Processing

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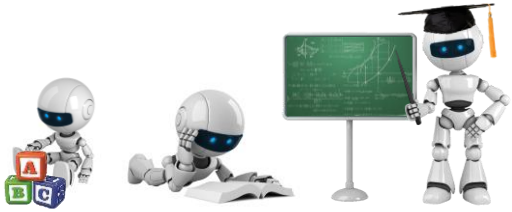


Logistical Notes

- HW1::
  - Report: explaining how you solve the problem along with the requested output + Jupyter Notebook with printed output + executable .py fond (different software versions are OK but specify the version clearly in your report)
- Quiz: the quiz will be done on Thursday if Tuesday is a holiday. Unless mentioned, we have quizzes every week.
- Project Group Formation Deadline: 9/13
  - We will form random assignment after this date
  - Check slack/Piazza and the Excel sheet to find groups
  - Meet weekly, helpful for both HW and project
- Proposal Deadline: 10/4
  - TA research interests and office hours
  - Written assignment
  - Last year projects: Youtube
  - Read papers from venues such as EMNLP, ACL, NAACL, etc. (often needed in industry as well)

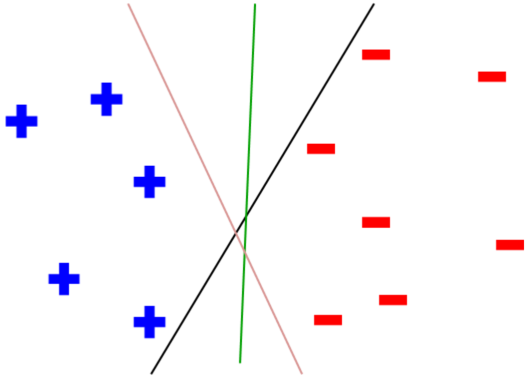
Linear models

- A linear function in *n*-dimensional space (i.e. we have *n* features) is define by *n*+1 weights:

$$Y = \sum_{i=0}^{n} \beta_i X_i$$

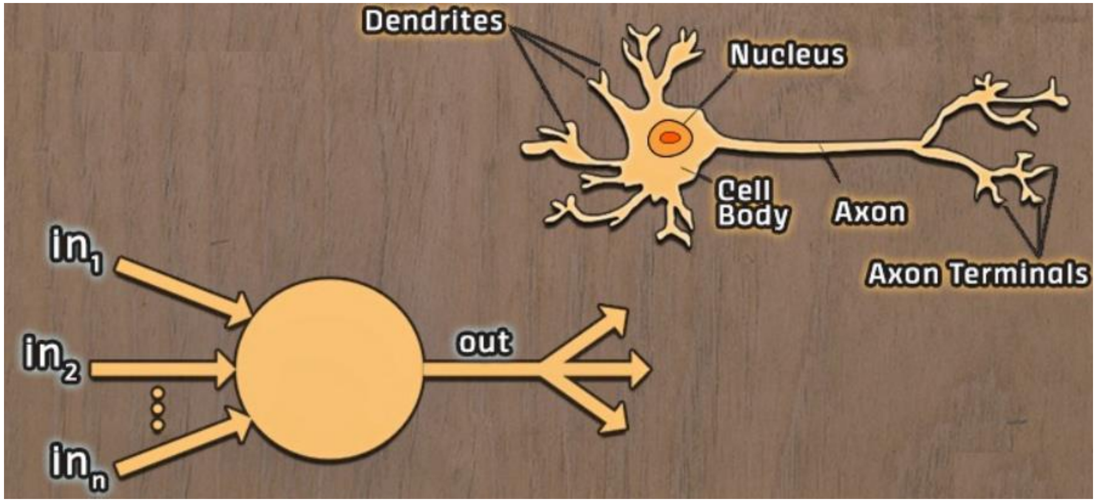
- We find the model weights such that the linear function acts as a good predictive model

- Is not necessarily unique!



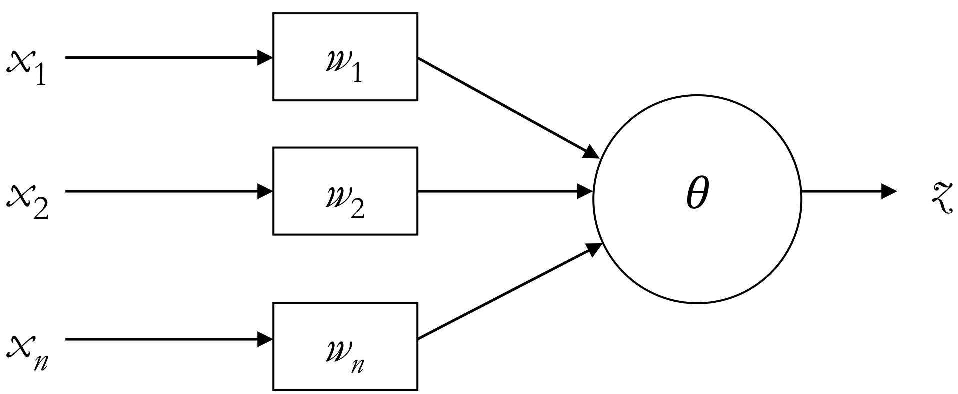
Perceptron

- Invented by Frank Rosenblatt in 1969
- Inspired by the nervous system
- Unit-based: analogous to a neural cell
- Model: neural activity is modeled by mathematical operations



Perceptron

- First neural network learning model in the 1960's
- Simple and limited (single layer models)
- Basic concepts are similar for multi-layer models so this is a good learning tool
- Still used in current applications (modems, etc.)

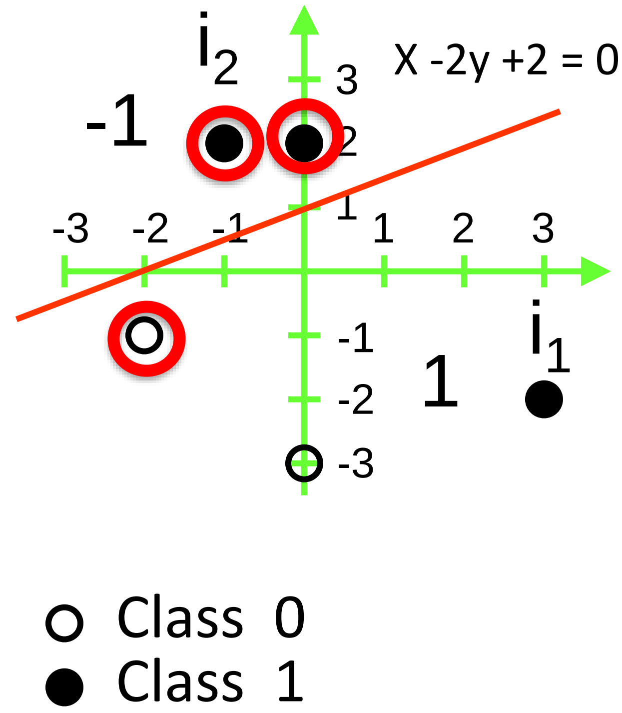


$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

Perceptron Learning Algorithm

- An iterative algorithm:
- we initialize weights with random values
- we do several pass on the whole training dataset one by one and update the weights

- Least perturbation principle
  - Only change weights if there is an error
  - Use small *learning rate (l)* to change weights sufficiently to make the current pattern correct
- Each iteration through the training set is an *epoch*



Perceptron Learning Algorithm

- Let w_i be the weights vector at iteration, *l* be a constant (the learning rate), *y* be the label for the current iteration, *z* be the current **thresholded** output, and *x* be the input

$$\beta_{i+1} = \beta_i + \Delta\beta$$
$$\Delta\beta = l(y - z)x$$

y	z	$y - z$
0	0	0
0	1	−1
1	0	1
1	1	0

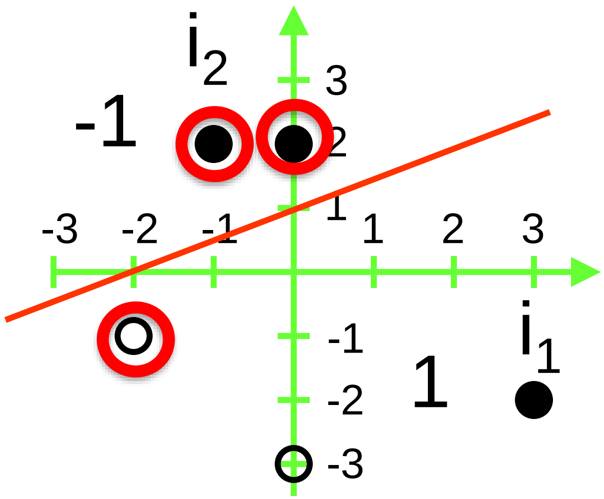
- Continue training until total training set error ceases to improve
- Perceptron Convergence Theorem: Guaranteed to find a solution in finite time if a solution exists

Perceptron Learning Example

We would like a perceptron to correctly classify the five 2-dimensional data points below.

Let the random initial weight vector $\boldsymbol{\beta}_0 = (1, \text{-}2, 2)^T$, why? (ax+by+c = 0 or [a b c][x y 1] ^{T} =0)

Then the dividing line crosses at $(0, 1)^T$ and $(\text{-}2, 0)^T$.



○ Class 0
● Class 1

Let us pick the misclassified point $(\text{-}2, \text{-}1)^T$ for learning:

$\mathbf{x}_1 = [\text{-}2, \text{-}1, 1]^T$ (include offset 1)

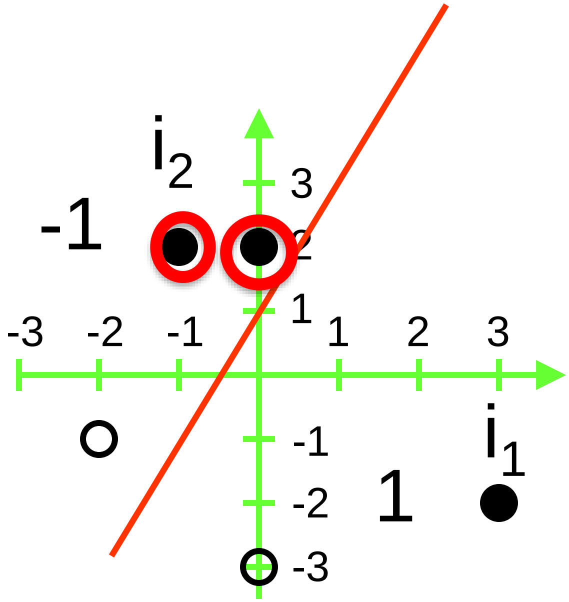
$\mathbf{z}_1 = [1,\text{-}2,2]\cdot(\text{-}2, \text{-}1, 1)^T = 2 \text{ -> } 1$ ($\mathbf{x}_1$ is in class 0) $(\mathbf{y}_1 - \mathbf{z}_1 = \text{-}1)$

$\boldsymbol{\beta}_1 = \boldsymbol{\beta}_0 - \mathbf{x}_1$ (let us set $\eta = 1$ for simplicity)

$\boldsymbol{\beta}_1 = (1, \text{-}2, 2)^T - (\text{-}2, \text{-}1, 1)^T = (3, \text{-}1, 1)^T$

The new dividing line crosses at $(0, 1)^T$ and $(\text{-}1/3, 0)^T$.

Perceptron Learning Example



○ Class 0
● Class 1

Let us pick the misclassified point $(-1, 2)^{\mathrm{T}}$ for learning:

$\mathbf{x}_2 = [-1, 2,1]^{\mathrm{T}}$ (include offset 1)

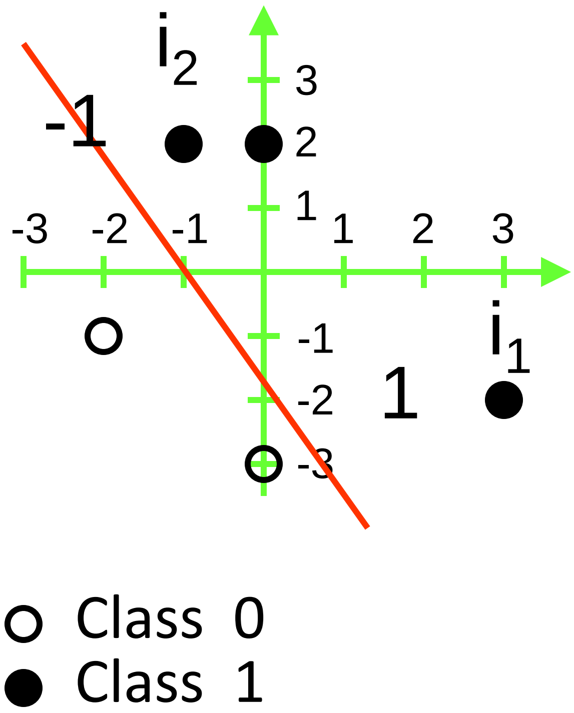
$\mathbf{z}_2 = [3,-1,1]\cdot(-1, 2, 1)^{\mathrm{T}} = -4 \to 0$ ($\mathbf{x}_2$ is in class 1) ($\mathbf{y}_2 - \mathbf{z}_2 = 1$)

$\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1 + \mathbf{x}_2$ (let us set $\eta = 1$ for simplicity)

$\boldsymbol{\beta}_2 = (3, -1, 1)^{\mathrm{T}} + (-1, 2, 1)^{\mathrm{T}} = (2, 1, 2)^{\mathrm{T}}$

The new dividing line crosses at $(0, -2)^{\mathrm{T}}$ and $(-1, 0)^{\mathrm{T}}$.

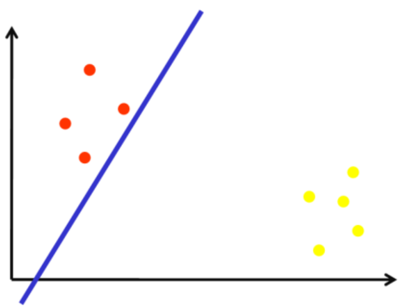
Perceptron Learning Example



- The algorithm has converged in one epoch because all the points are classified correctly
- In general, we may need several epochs
- Learning rate value is important for fast convergence
- At each epoch, we may use a different random order on the data points
- Neural networks, including the state-of-the-art deep networks are extensions of perceptron (potentially using different learning algorithms)

Optimal Boundary

- The perceptron learning algorithm is guaranteed to find a solution if the data points are linearly separable
- But are perceptron solutions optimal?



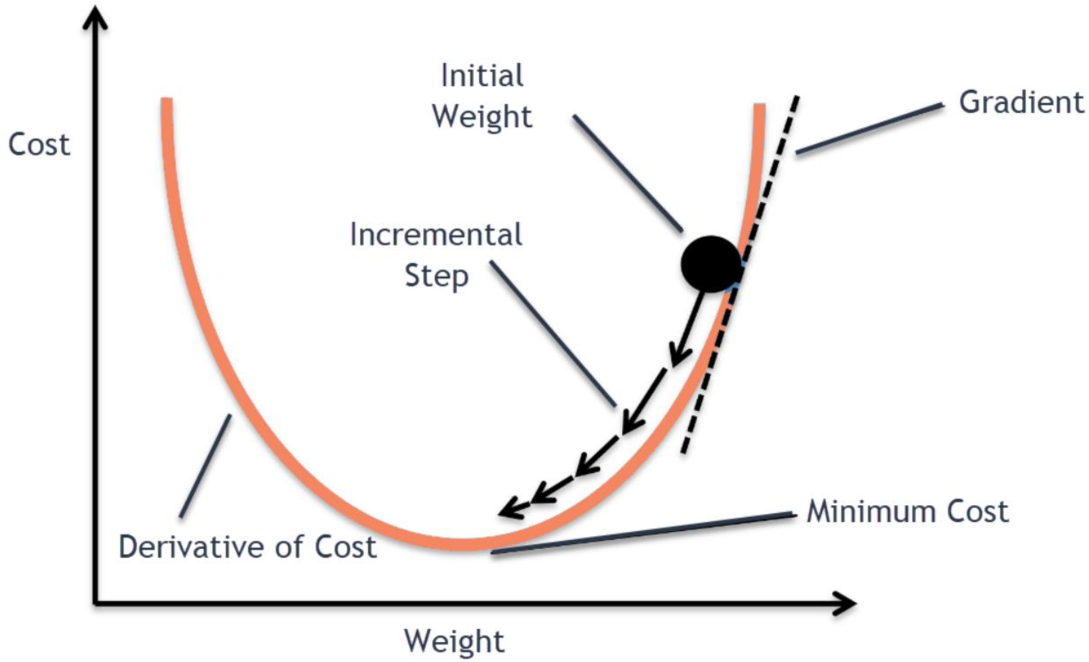
Perfect classification of training samples but may not generalize well to new (untrained) samples.

• Idea: compute a continuous, differentiable error function between input and desired output

- Define an objective function to measure quality of a model
- Find the weights for which the objective function is minimal.
- With a differential function, we can use the gradient descent techniques to find a good solution (at least locally optimal)

Gradient decent

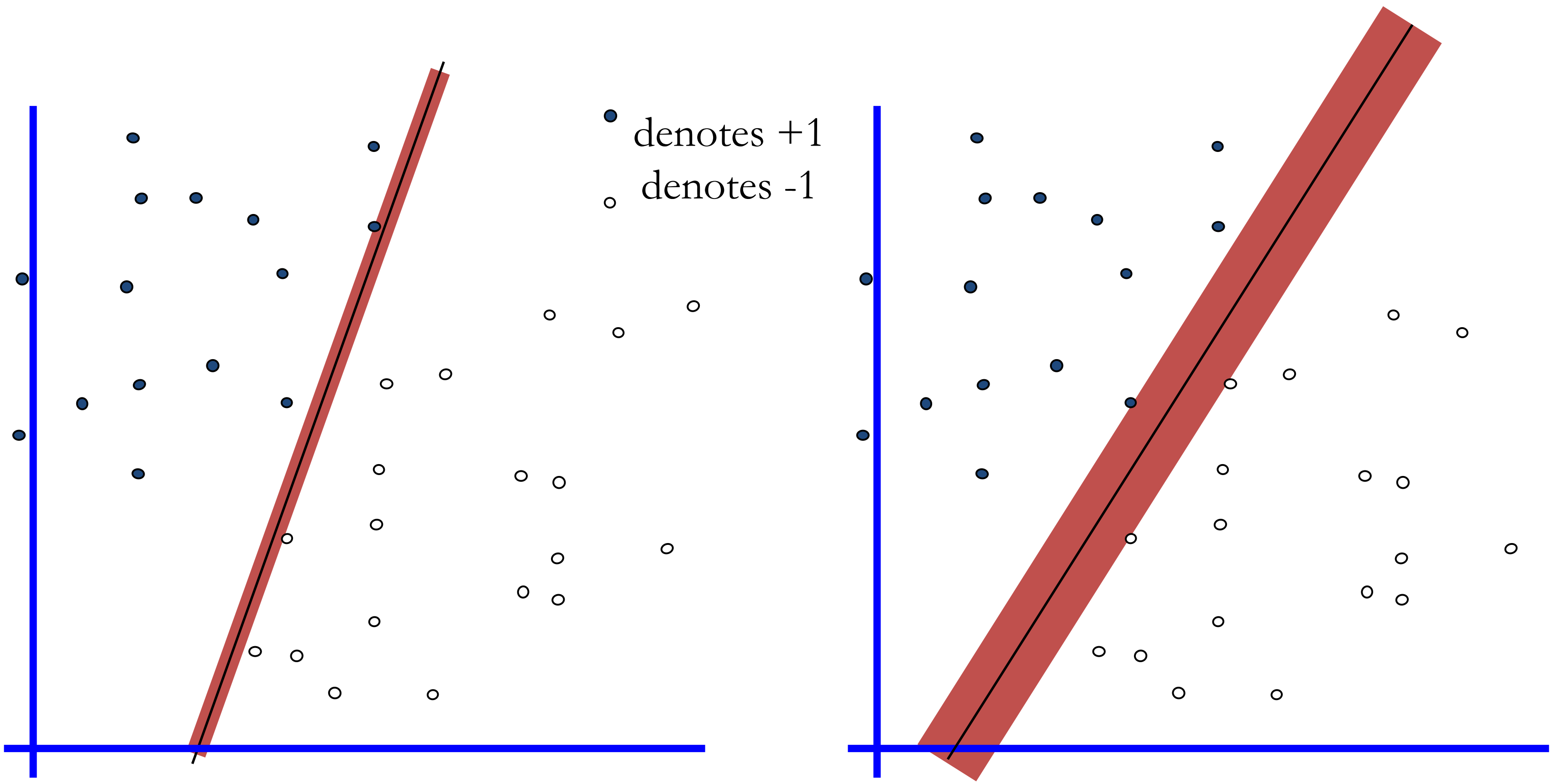
- Suitable for finding minimums of convex differentiable functions (cost functions)



$$\theta_j = \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

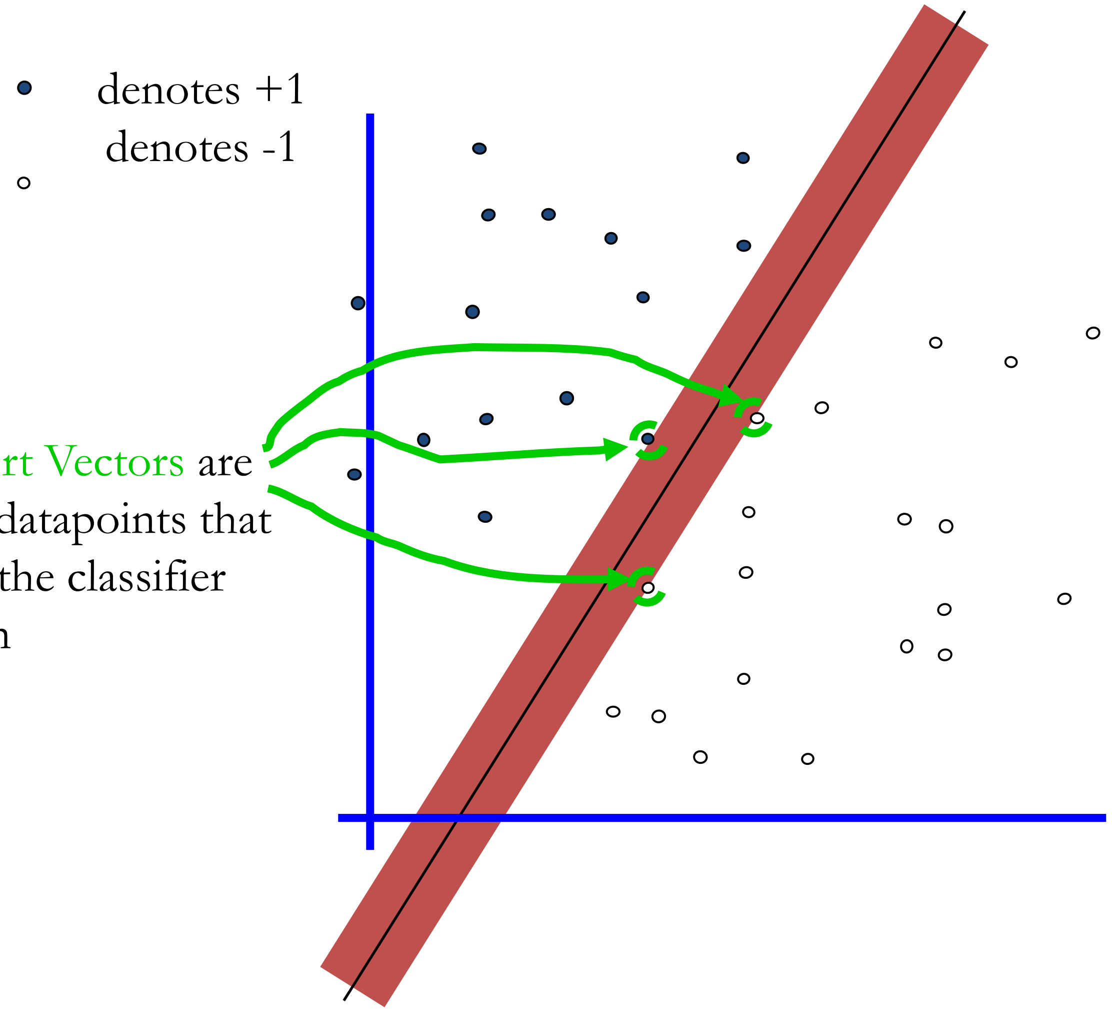
Support Vector Machine

The margin of a linear classifier as the width that the boundary could be increased by before hitting a datapoint.



Support Vector Machine (SVM): the maximum margin linear classifier

Support Vector Machine



Support Vectors are those datapoints that lie on the classifier margin

Computing the Margin Width

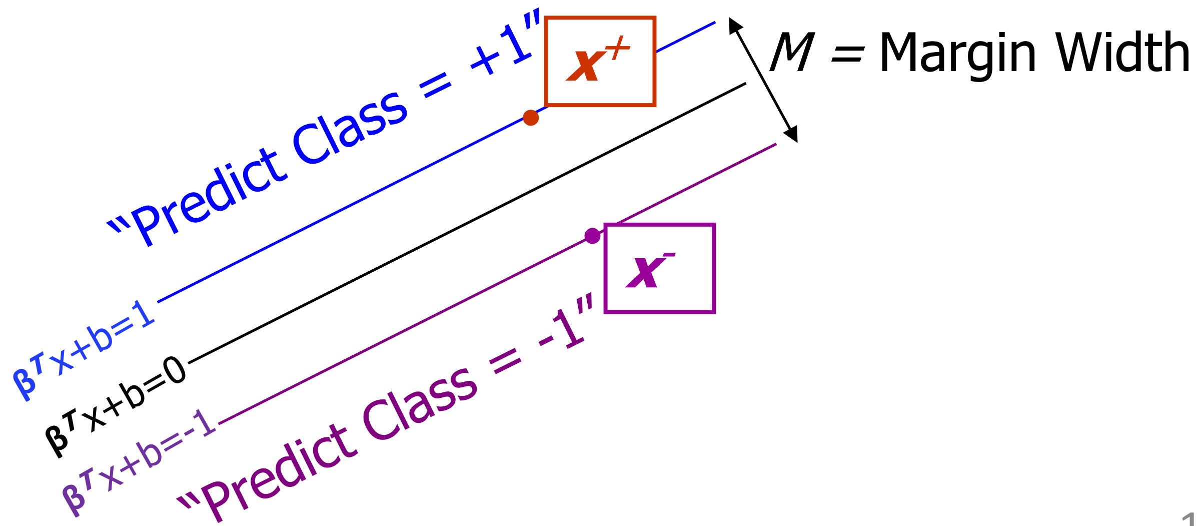
- Plus-plane = $\{ \boldsymbol{x} : \boldsymbol{\beta}^T . \boldsymbol{x} + b = +1 \}$
- Minus-plane = $\{ \boldsymbol{x} : \boldsymbol{\beta}^T . \boldsymbol{x} + b = -1 \}$
- Let $\boldsymbol{x}^-$ be any point on the minus plane
- Let $\boldsymbol{x}^+$ be the closest plus-plane-point to $\boldsymbol{x}^-$.
- We can deduce:

$$M = | \boldsymbol{x}^+ - \boldsymbol{x}^- | = | \lambda \boldsymbol{\beta} |$$

$$\begin{array}{l} \boldsymbol{\beta}^T . \boldsymbol{x}^+ + b = +1 \\ \boldsymbol{\beta}^T . \boldsymbol{x}^- + b = -1 \end{array} \longrightarrow \boldsymbol{\beta}^T . (\boldsymbol{x}^+ - \boldsymbol{x}^-) = 2 \longrightarrow \boldsymbol{\beta}^T . (\lambda \boldsymbol{\beta}) = 2 \longrightarrow \lambda = \frac{2}{\boldsymbol{\beta}^T . \boldsymbol{\beta}}$$

$$M = | \boldsymbol{x}^+ - \boldsymbol{x}^- |$$

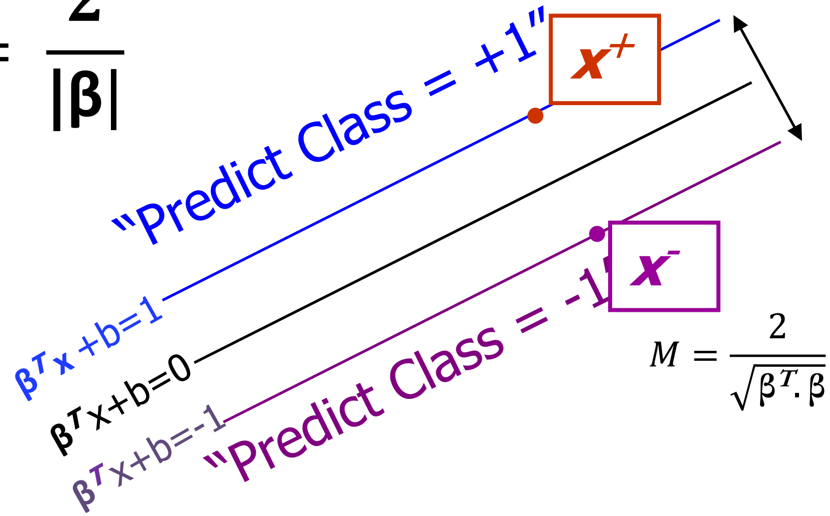
$$\boldsymbol{x}^+ = \boldsymbol{x}^- + \lambda \boldsymbol{\beta}$$



Adopted from slides by Andrew Moore

Computing the margin width

$$M = |x^{+} - x^{-}| = |\lambda \boldsymbol{\beta}| = \left|\frac{2}{\boldsymbol{\beta}^T.\boldsymbol{\beta}}\boldsymbol{\beta}\right| = \frac{2}{|\boldsymbol{\beta}|}$$



Given **β** and *b* (a boundary), we can:

- Classify all data points (only one boundary is optimal)
- Compute the width of the margin

Now we need to write an optimization problem to search for the optimal **β** and *b* with the widest margin that classifies all the training data points correctly

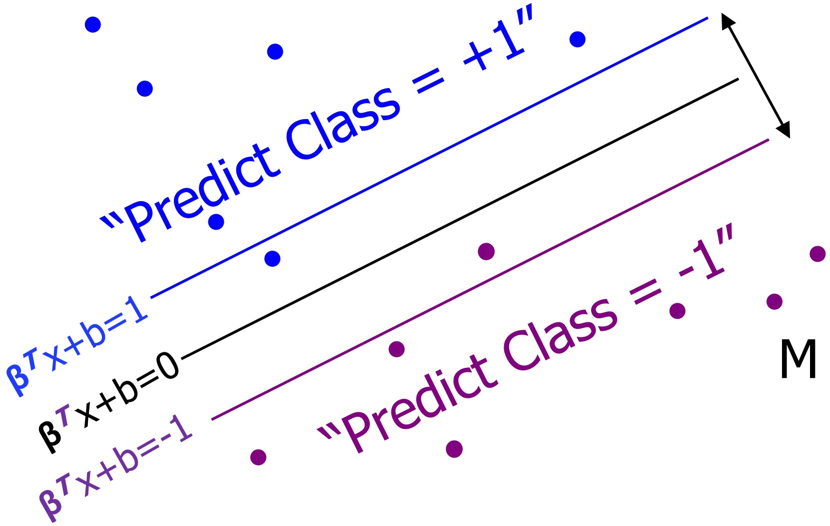
Minimize $|\boldsymbol{\beta}|$ Subject to

$\boldsymbol{\beta}^T . \boldsymbol{x} + b > 1$; when in class +1

$\boldsymbol{\beta}^T . \boldsymbol{x} + b < -1$; when in class -1

Adopted from slides by Andrew Moore

Learning the Maximum Margin Classifier



Minimize $\boldsymbol{\beta}^T . \boldsymbol{\beta}$ Subject to

$(\boldsymbol{\beta}^T . \mathbf{x}_k + b)\, y_k >= 1$ for all $(\mathbf{x}_k, y_k)$

$$M = \frac{2}{|\beta|}$$

Incorporating soft constraints

Min $\frac{1}{2}\beta^T.\beta + C\sum_{k=1}^{R}\varepsilon_k$ s.t $(\boldsymbol{\beta}^T . \mathbf{x}_k + b)\, y_k >= 1-\varepsilon_k$ for all $(\mathbf{x}_k, y_k)$

$0 <= \varepsilon_k <= 1$

Solving the above problem using quadratic programming is a straightforward task

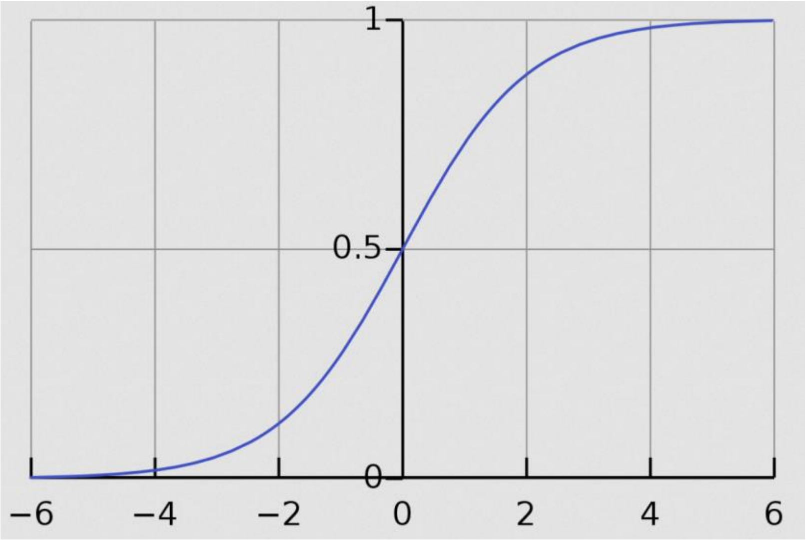
Logistic Regression

- Maximum Likelihood Estimation:

$$\hat{Y} = \arg\max P(Y|X)$$

- We assume that the likelihood function is a logistic function of a linear relationship

$$P(Y=1|X) = \frac{1}{1+e^{-\beta^T X}}$$



Logistic Regression

- Log-likelihood Inference:

Analogous to using Entropy loss

$$
\begin{aligned}
\hat{\beta} = &\arg\max_{\beta} \log(\Pi_i P(Y_i|X_i)) = \\
&\arg\max_{\beta} \Sigma_i \log(P(Y_i|X_i)) = \\
&\arg\max_{\beta} \Sigma_i Y_i \log(P(Y_i = 1|X_i)) + (1 - Y_i) \log(P(Y_i = 0|X_i)) = \\
&\arg\max_{\beta} \Sigma_i Y_i(-\log(1+e^{\beta^T X_i})+(1-Y_i)(-\beta^T X_i)-\log(1+e^{\beta^T X_i}) = \\
&\arg\min_{\beta} \Sigma_i (1 - Y_i)\beta^T X_i + \log(1 + e^{\beta^T X_i})
\end{aligned}
$$

- There is no closed form solution:

- We can use numerical optimization, e.g., Newton method
- We can approximate the logit term

Model Evaluation Process

- We use a training dataset for model selection
- A **good** parametric model along with a **suitable** training algorithm guarantees training a model that works well on the training data
- We need to validate that trained models **generalize** well on unseen data instances
- We need a second testing dataset which is fully independent of the training dataset
- We randomly split the annotated dataset into testing and training splits (sometimes, a validation set is generated as well)

Evaluation Metrics

- **Accuracy**: proportion of correctly classified items
- Accuracy can be dominated by **true negatives** (items correctly classified as not in a class).
- Sensitive with respect to imbalance

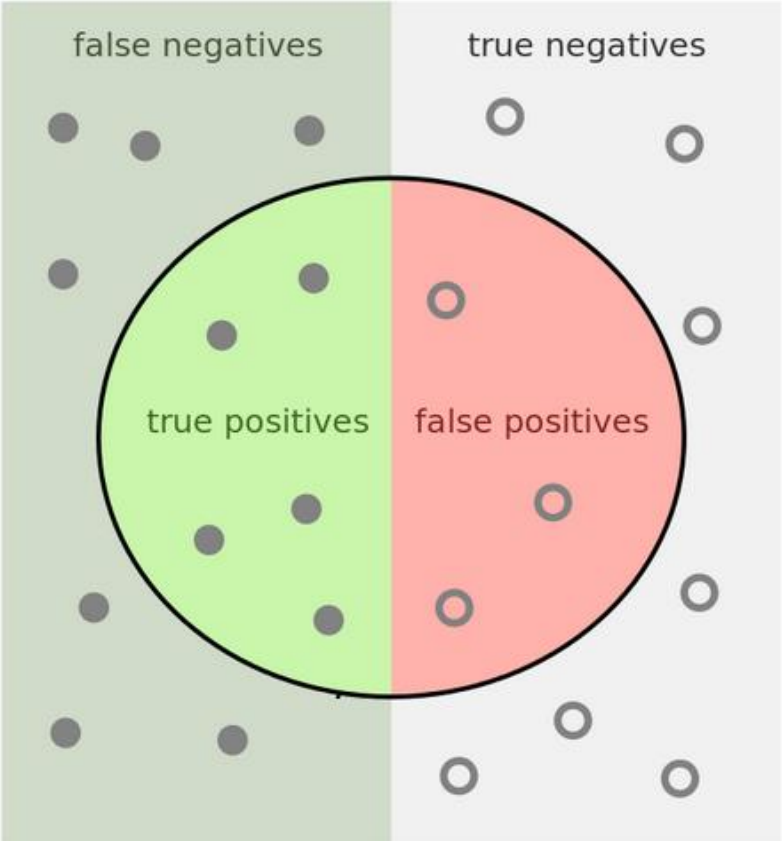
- Precision: $\frac{\text{True Positives}}{\text{True Positives+False Positive}}$

- Also called positive predictive value

- Recall: $\frac{\text{True Positives}}{\text{True Positives+False negative}}$

- Also called sensitivity

- Precision and recall are not useful metrics when used in isolation?
- We want our model to have good performance with respect to both metrics
- Implemented in sklearn



Evaluation Metrics

- Why having one measure is helpful?
- $\text{F1} = \frac{2\,\text{Precision Recall}}{\text{Precision} + \text{Recall}}$
- F1 is biased towards the lower of precision and recall:

- harmonic mean < geometric mean < arithmetic mean
- F1=0 when Precision=0 or Recall=0

- Generalized F score:

$$F_\beta = (1+\beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}}.$$