

2-cyclic Laurent Polynomial Algebras and their categorifications

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Abstract

In this paper, we explore the 2-cyclic quiver Laurent polynomial algebra and its categorification. Inspired by the framework of cluster algebras, the study begins by introducing the concept of 2-cyclic quivers and their mutation. We define 2-cyclic Laurent polynomial algebra is established and prove that the Laurent monomial phenomenon behaved on 2-cyclic Laurent polynomial algebra. We further classify 2-cyclic Laurent polynomial algebras in the finite type case and extends essential concepts from cluster algebras, such as c -vectors, g -vectors, and F -polynomials, to the 2-cyclic Laurent polynomial algebra framework, as a consequence, We prove the sign-coherence of c -vectors of 2-cyclic Laurent polynomial algebra. Moreover, we establish the connections between 2-cyclic Laurent polynomial algebras and cluster algebras.

We also expand the Derksen-Weyman-Zelevinsky theory of quivers with potential to the 2-cyclic quivers. A mutation theory for 2-cyclic quivers with potentials is developed, and we prove that the mutation is an involution and that finite-dimensionality is a mutation invariant. Additionally, we show the existence of a well-behaved Jacobian-finite potential for the \mathbb{A}_n -type 2-cyclic quiver, and furthermore, proves that the potential is also non-degenerate when $n \leq 3$.

In the final part of this thesis, combining the theory of generalized cluster categories, the thesis investigates the generalized cluster category of a 2-cyclic quiver with potential of \mathbb{A}_n -type. By constructing the corresponding covering functor over the \mathbb{Z}_3 -cover of the 2-cyclic quiver with potential, the reachable cluster tilting objects are identified. In addition, we introduce the definition of 2-cyclic Caldero-Chapoton (CC) formula:

$$CC(X) = X^{-\mathfrak{g}_X} \sum_{e \in \mathbb{N}^n} \chi(\text{Gr}_e(H^0(X)))y^e, \quad (0.1)$$

and it is proven that this map categorifies the g -vectors of the \mathbb{A}_n -type 2-cyclic Laurent polynomial algebra, ensuring that the \mathbb{A}_n -type generalized cluster category is cluster-tilting finite. For the categorification of the F -polynomials of 2-cyclic Laurent polynomial algebras, we prove that the F -polynomials of indecomposable τ -rigid modules for \mathbb{A}_n -type Jacobian algebras correspond to the \mathcal{F} -polynomials of the associated 2-cyclic quiver when $n \leq 3$.

1 Introduction

[1]

2 Preliminaries

- 2.1 Quivers and Quiver Representations
- 2.2 Cluster Algebras
- 2.3 Ginzburg dg Algebras and Generalized Cluster Categories
- 2.4 Covering Theory

3 2-cyclic Laurent Polynomial Algebras

- 3.1 2-cyclic Quivers and their Mutations
- 3.2 2-cyclic Laurent Polynomial Algebras
- 3.3 c -vectors, g -vectors and F -polynomials
- 3.4 Finite Type Classification
- 3.5 Relation with Cluster Algebras

4 2-cyclic Quivers with Potentials

- 4.1 2-cyclic Quivers with Potential and their Mutations
- 4.2 Finite-dimensionality and Non-degeneracy

Acknowledgements.

References

- [1] Sergey Fomin and Andrei Zelevinsky. "Cluster algebras IV: Coefficients". en. In: *Compositio Math.* 143.01 (Jan. 2007), pp. 112–164. ISSN: 0010-437X, 1570-5846. DOI: [10.1112/S0010437X06002521](https://doi.org/10.1112/S0010437X06002521). URL: http://www.journals.cambridge.org/abstract_S0010437X06002521 (visited on 10/15/2024) (cit. on p. 1).

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