2-cyclic Laurent polynomial algebras and their Categorifications

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Abstract

In this paper, we explore the 2-cyclic quiver Laurent polynomial algebra and its categorification. Inspired by the framework of cluster algebras, the study begins by introducing the concept of 2-cyclic quivers and their mutation. We define 2-cyclic Laurent polynomial algebra is established and prove that the Laurent monomial phenomenon behaved on 2-cyclic Laurent polynomial algebra. We further classify 2-cyclic Laurent polynomial algebras in the finite type case and extends essential concepts from cluster algebras, such as c-vectors, g-vectors, and F-polynomials, to the 2-cyclic Laurent polynomial algebra framework, as a consequence, We prove the sign-coherence of c-vectors of 2-cyclic Laurent polynomial algebra.. Moreover, we establish the connections between 2-cyclic Laurent polynomial algebras and cluster algebras.

We also expand the Derksen-Weyman-Zelevinsky theory of quivers with potential to the 2-cyclic quivers. A mutation theory for 2-cyclic quivers with potentials is developed, and we prove that the mutation is an involution and that finite-dimensionality is a mutation invariant. Additionally, we show the existence of a well-behaved Jacobian-finite potential for the $\mathbb{A}_n$-type 2-cyclic quiver, and furthermore, proves that the potential is also non-degenerate when $n \leq 3$.

In the final part of this thesis, combining the theory of generalized cluster categories, the thesis investigates the generalized cluster category of a 2-cyclic quiver with potential of $\mathbb{A}_n$-type. By constructing the corresponding covering functor over the $\mathbb{Z}_3$-cover of the 2-cyclic quiver with potential, the reachable cluster tilting objects are identified. In addition, we introduce the definition of 2-cyclic Caldero-Chapoton (CC) formula:

$$CCX = X^{-\mathfrak{g}_X} \sum_{e \in \mathbb{N}^n} \chi\big(\mathrm{Gr}_e H^0 X\big) y^e, \tag{1}$$

and it is proven that this map categorifies the g-vectors of the $\mathbb{A}_n$-type 2-cyclic Laurent polynomial algebra, ensuring that the $\mathbb{A}_n$-type generalized cluster category is cluster-tilting finite. For the categorification of the F-polynomials of 2-cyclic Laurent polynomial algebras, we prove that the F-polynomials of indecomposable τ-rigid modules for $\mathbb{A}_n$-type Jacobian algebras correspond to the F-polynomials of the associated 2-cyclic quiver when $n \leq 3$.

1 Introduction

Foundational aspects of cluster algebras can be found in [1].

References

[1] Sergey Fomin and Andrei Zelevinsky. “Cluster algebras II: Finite type classification”. In: *Inventiones Mathematicae* 154.1 (), pp. 63–121. ISSN: 00209910. DOI: 10.1007/s00222-003-0302-y.