ROBERT E. HENRY[1]
HANS K. FAUSKE
Argonne National Laboratory,
Argonne, Ill.

The Two-Phase Critical Flow of One-Component Mixtures in Nozzles, Orifices, and Short Tubes

The critical flow of one-component, two-phase mixtures through convergent nozzles is investigated and discussed including considerations of the interphase heat, mass, and momentum transfer rates. Based on the experimental results of previous investigators, credible assumptions are made to approximate these interphase processes which lead to a transcendental expression for the critical pressure ratio as a function of the stagnation pressure and quality. A solution to this expression also yields a prediction for the critical flow rate. Based on the experimental results of single-phase compressible flow through orifices and short tubes, the two-phase model is extended to include such geometries. The models are compared with steam-water, cryogenic, and alkali-metal experimental data.

Introduction

THE TWO-PHASE critical flow of one-component mixtures has been the subject of many analytical and experimental investigations because of its importance in (1) safety analyses of pressurized water, boiling water, and liquid-metal-cooled nuclear reactors [1–15],[2] (2) the flow of refrigerants and cryogens [16–19], and (3) the operation of turbines within the two-phase region [20]. With the exception of reference [1], the analytical models which resulted from these studies were either thermodynamic equilibrium or frozen (no mass transfer) models. In addition, most of these models require a knowledge of the throat pressure which is generally unknown. Those which can be based on stagnation conditions, such as the equilibrium model proposed by Moody [13], considerably overestimate the flow rates in nozzles [19, 21]. Therefore, the purpose of this paper is to develop a model which requires only a knowledge of the stagnation conditions and at the same time accounts for the nonequilibrium nature of the flow.

[1] Formerly, Lewis Research Center, Cleveland, Ohio.

Contributed by the Heat Transfer Division and presented at the Winter Annual Meeting, New York, N. Y., November 29–December 3, 1970, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Heat Transfer Division April 28, 1969; revised manuscript received June 12, 1970. Paper No. 70-WA/HT-5.

[2] Numbers in brackets designate References at end of paper.

Nomenclature

A = cross-sectional area
C = discharge coefficient
c = specific heat
F = viscous forces
f = function
G = flow rate per unit area
H = enthalpy
h = enthalpy
k = velocity ratio, u_g/u_l
N = experimental parameter
n = polytropic exponent
P = pressure
s = entropy
T = temperature
u = velocity
v = specific volume
W = flow rate
x = quality, $W_g/(W_g + W_l)$
z = axial length
α = void fraction, A_g/A_t
γ = isentropic exponent
η = critical pressure ratio, P_t/P_0
τ = time

Subscripts

B = back pressure
c = critical condition
E = equilibrium (corresponding to local static pressure)
F = frozen
g = vapor phase
H = homogeneous (equal phase velocities)
l = liquid phase
0 = stagnation
p = constant pressure
R = reduced pressure
TP = two-phase
t = throat
v = constant volume
w = wall

Analysis

The steady-state, one-dimensional continuity and momentum equations for one-component, two-phase flow can be written as

Liquid Continuity

$$W_l v_l = A_l u_l \tag{1}$$

Vapor Continuity

$$W_g v_g = A_g u_g \tag{2}$$

Momentum

$$-AdP = d(W_g u_g + W_l u_l) + dF_w \tag{3}$$

For the high-velocity flows in a converging nozzle, such as that shown in Fig. 1, the wall shear forces are negligible compared to the momentum and pressure gradient terms. Therefore, equation (3) can be approximated by

$$-dP = Gd[xu_g + (1 - x)u_l] \tag{4}$$

It is assumed that, for fixed stagnation conditions, x, v_g, v_l, u_g, and u_l are either constant or composite functions of P and z $(f[P(z)])$ for fixed upstream conditions. Hence, equation (4) shows that

$$G_t^{-1} = -\left\{\frac{d[xu_g + (1 - x)u_l]}{dP}\right\}_t \tag{5}$$

(The subscript t indicates that all the enclosed quantities are evaluated at the throat.) At critical flow, the mass flow rate exhibits a maximum with respect to the throat pressure.

$$\left.\frac{dG_c}{dP}\right|_t = 0 \tag{6}$$

$G = \dfrac{kv_L}{v_v x + v_L(1-x)k}$

To get $-G^{-1} = \dfrac{d}{dP}\left\{\dfrac{G}{k}[v_v x + v_L(1-x)k][1 + x(k-1)]\right\}$

NOW Equation (6) can be applied to equation (5) to give an expression for the critical flow rate

note $dG/dP = 0$

$$G_c^2 = \left\{-\frac{d}{dP}\left[\frac{xk + (1 - x)}{k}\,[(1 - x)kv_l + xv_g]\right]\right\}_t^{-1} \tag{7}$$

where k is defined by $k = u_g/u_l$. Equation (7) can be expanded to

$$G_c^2 = -\left\{k\left[[1 + x(k - 1)]x\frac{dv_g}{dP} + [v_g\{1 + 2x(k - 1) + kv_l 2(x - 1) + k(1 - 2x)\}]\frac{dx}{dP} + k[1 + x(k - 2) - x^2(k - 1)]\frac{dv_l}{dP} + x(1 - x)\left(kv_l - \frac{v_g}{k}\right)\frac{dk}{dP}\right]^{-1}\right\}_t \tag{8}$$

The rapid expansion of a one-component mixture through a converging nozzle is not expected to follow a thermodynamic equilibrium path, and, since the phases have different densities, the pressure gradient will also tend to accelerate the lighter vapor phase more than the liquid. These resulting temperature, free energy, and velocity differences cause the interphase transfer of heat, mass, and momentum. These interphase processes determine the thermodynamic paths followed by each phase in the expansion; thus, the variables v_g, v_l, x, and k are functions of the stagnation conditions and the path traced by the expansion. The local values of these quantities at the throat are indicative of the *amounts* of interphase heat, mass, and momentum transfer occurring in the expansion between the stagnation and throat regions. The derivatives $\left.\frac{dv_g}{dP}\right|_t$, $\left.\frac{dv_l}{dP}\right|_t$, $\left.\frac{dx}{dP}\right|_t$, and $\left.\frac{dk}{dP}\right|_t$, which are assumed to be functions of P and z, can be expressed as

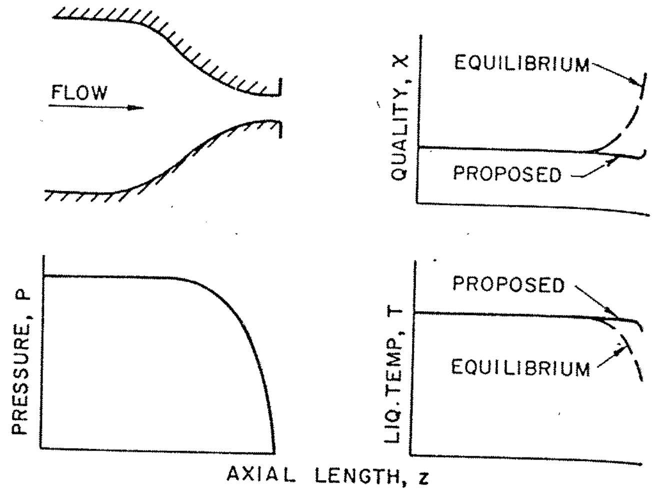

Fig. 1 Comparison of equilibrium and proposed heat and mass transfer processes for critical flow in a nozzle

$$\left.\frac{dL}{dP}\right|_t = \left.\frac{dL}{dz}\right|_t \bigg/ \left.\frac{dP}{dz}\right|_t \tag{9}$$

where L_t denotes v_{gt}, v_{lt}, x_t, or k_t. If the above derivatives are viewed from a Lagrangian system, such that $z = u_t\tau$ and $\frac{dz}{d\tau} = u_t$, equation (9) shows that

$$\left.\frac{dL}{dP}\right|_t = \left.\frac{dL}{d\tau}\right|_t \bigg/ \left.\frac{dP}{d\tau}\right|_t \tag{10}$$

Hence, these quantities describe the local *rates* of interphase heat, mass, and momentum transfer occurring at the throat.

In a converging nozzle, the acceleration and the accompanying steep pressure gradients essentially occur between the upstream location which has a diameter twice that of the throat and the throat itself as shown in Fig. 1. Therefore, in normal nozzle configurations, there is little time for mass transfer to take place, and it is reasonable to assume that the amount of mass transferred in the expansion is negligible.

$$x_t \approx x_0 \tag{11}$$

This is illustrated by the solid line in Fig. 1 as compared to the dotted line which is representative of the thermodynamic equilibrium behavior commensurate with the pressure profile.

An analogous argument can be applied to the transfer of heat between the phases, which is illustrated by the essentially constant liquid temperature shown in Fig. 1.

$$T_{lt} \approx T_{l0} \tag{12}$$

This is in agreement with the two-component measurements of Smith et al. [22].

There is little experimental data available to evaluate the velocity ratios in nozzles as a function of pressure. The measured void fractions in reference [1] indicate that for a throat pressure of 50 psia, the velocity ratios in long constant-area ducts are between 1.0 and 1.5. The interphase velocity differences result from density differences and are thus suppressed by increased pressures. Since many of the above applications involve rather high levels of reduced pressure $P_R > 0.05$, it is assumed that the phase velocities are equal.

$$u_g = u_l = u \tag{13}$$

The validity of this approximation increases with increased pressure.

The lack of interphase heat and mass transfer during the expansion generates temperature and free energy differences within the mixture. To evaluate such conditions it is assumed equilibrium thermodynamic relations can be used to approximate the behavior of each phase.

Since wall shear, heat exchange with the environment, and interfacial viscous terms are neglected, the system entropy during the expansion can be assumed constant

$$ds_0 = d[(1 - x)s_l + xs_g] = 0 \tag{14}$$

This result along with the assumptions stating negligible amounts of interphase heat and mass transfer imply that each phase expands isentropically.

$$s_{g0} = s_{gt} \quad \text{and} \quad s_{l0} = s_{lt} \tag{15}$$

$$\boxed{P_0 v_{g0}{}^\gamma = P_t v_{gt}{}^\gamma} \tag{16}$$

$$\boxed{v_{l0} = v_{lt}} \tag{17}$$

The negligible interphase heat transfer during the expansion results in a large temperature difference between the phases at the throat, which in turn indicates that the local rate of heat transfer can be large. The temperature data reported by Smith et al. [22] for two-phase, air-water critical flow in a venturi show that large heat transfer *rates* are in evidence at the throat. Due to these large heat transfer rates, it is not reasonable to evaluate the derivative $\left.\dfrac{dv_g}{dP}\right|_t$ in an adiabatic manner. A description of the actual heat transfer process requires a detailed knowledge of the flow configuration which is unknown. Therefore, as a compromise between simplicity and the real process, it is assumed that the vapor behavior at the throat can be described by a polytropic process such that

$$\left.\frac{dv_g}{dP}\right|_t = \left.\frac{v_g}{nP}\right|_t \tag{18}$$

where n is the thermal equilibrium polytropic exponent derived by Tangren et al. [23] and given by

$$n = \frac{(1 - x)c_l/c_{pg} + 1}{(1 - x)c_l/c_{pg} + 1/\gamma} \tag{19}$$

This exponent reflects a significant heat transfer rate at the throat.

The liquid compressibility is generally a very small part of that characterizing the two-phase system. Hence, it is assumed herein that the liquid phase can be considered incompressible.

$$\frac{dv_l}{dP} = 0 \tag{20}$$

The term $\left.\dfrac{dk}{dP}\right|_t$, which is indicative of the momentum transfer rate, is difficult to evaluate and appears to be significant [1, 24]. According to the approximation of equation (11), a one-component mixture essentially expands in a two-component manner. Vogrin [25] determined axial velocity ratio profiles for low-quality, air-water critical flows in a converging-diverging nozzle. Since k was assumed to be $f[P(z)]$,

$$\left.\frac{dk}{dP}\right|_t = \frac{dk/dz|_t}{dP/dz|_t} \tag{21}$$

Several of the axial pressure and velocity ratio profiles reported in reference [25] are shown in Fig. 2. Under critical flow conditions the velocity ratio appears to exhibit a minimum at the throat, but the pressure gradient is not zero. Thus,

$$\left.\frac{dk}{dP}\right|_t = 0 \tag{22}$$

It is assumed that this expression also applies to one-component critical flows in converging nozzles.

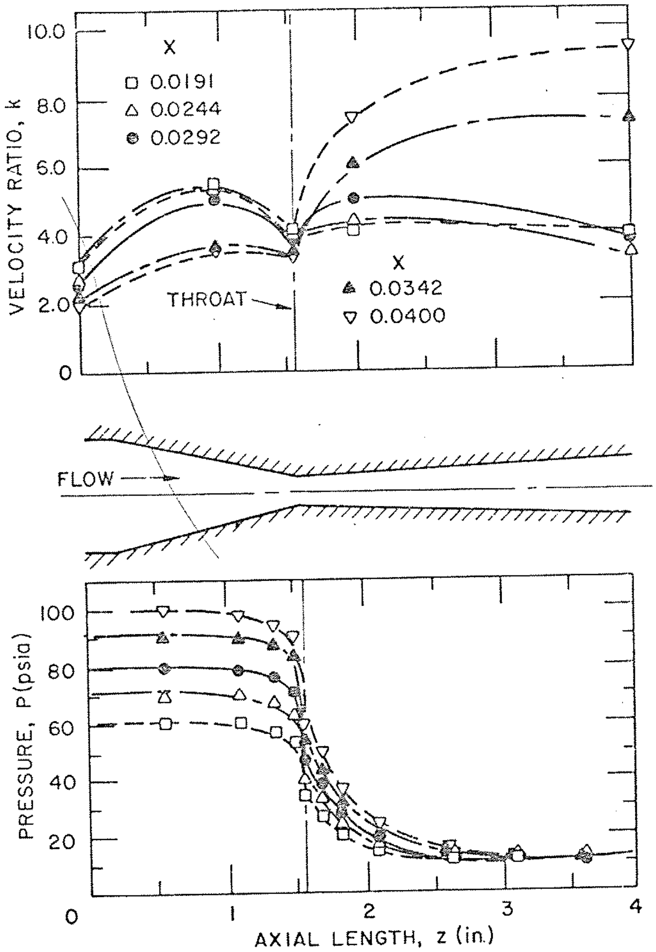

Fig. 2 Axial velocity ratio and pressure profiles for converging-diverging nozzle as reported in reference [25]

Like the local heat transfer rate, the rate of mass transfer at the throat can be appreciable. In reference [1] it was shown that, if an equilibrium quality is defined as

$$x_E = \frac{s_0 - s_{lE}}{s_{gE} - s_{lE}} \tag{23}$$

the exit plane mass transfer rate for steam-water critical flows in constant-area ducts can be correlated by

$$\left.\frac{dx}{dP}\right|_t = -\left[\frac{(1 - x_0)\dfrac{ds_l}{dP} + x_0\dfrac{ds_g}{dP}}{s_{g0} - s_{l0}}\right]_t = N\left.\frac{dx_E}{dP}\right|_t \tag{24}$$

where

$$N = N(x_E) \tag{25}$$

In the low-quality region for which this formulation was intended, it can be shown numerically that the mass transfer is dominated by the behavior of the liquid phase. Therefore, the correlation essentially describes the flashing of the liquid, or since $(1 - x_0) \approx 1$ at low qualities,

$$\left[\frac{1}{s_{g0} - s_{l0}}\frac{ds_l}{dP}\right]_t = N\left[\frac{1}{s_{gE} - s_{lE}}\frac{ds_{lE}}{dP}\right]_t \tag{26}$$

The experimental measurements of reference [1] show that N can be represented by $N = 20x_{Et}$. However, for the nozzle flows considered herein, the flow regimes and throat pressure gradients will differ from those of constant-area ducts. Therefore, it is assumed that the formulation of equation (26) is ap-

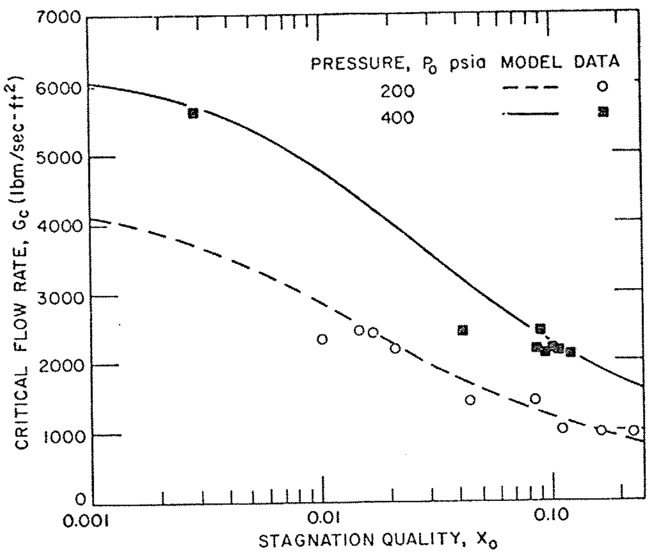

Fig. 3 Comparison between proposed model and experimental data of references [26, 27]

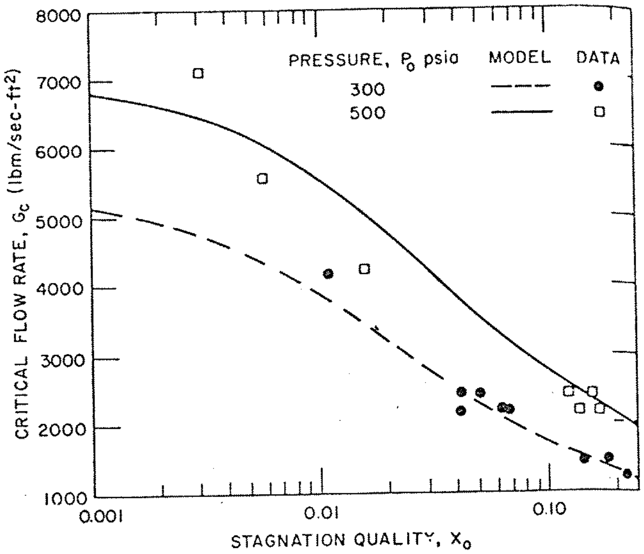

Fig. 4 Comparison between proposed model and experimental data of references [26, 27]

plicable to nozzle flows and that N can be represented by $N = C_1 x_{Et}$, but the constant C_1 will have a value other than 20.

The derivative $\left.\dfrac{ds_g}{dP}\right|_t$ can be determined from the expression

$$T_g ds_g = dh_g - v_g dP \tag{27}$$

If it is assumed that the vapor behaves as a real gas following the polytropic process described in equation (18), it can be shown that

$$\left.\frac{ds_g}{dP}\right|_t = -\frac{c_{pg}}{P_t}\left[\frac{1}{n} - \frac{1}{\gamma}\right] \tag{28}$$

The above approximations for x_t, $\left.\dfrac{dx}{dP}\right|_t$, k_t, and $\left.\dfrac{dk}{dP}\right|_t$ simplify the critical flow rate expression to

HF?

$$G_c^2 = \left[\frac{x_0 v_g}{nP} + (v_g - v_{l0})\left\{\frac{(1-x_0)N}{s_{gE} - s_{lE}}\frac{ds_{lE}}{dP} - \frac{x_0 c_{pg}(1/n - 1/\gamma)}{P(s_{g0} - s_{l0})}\right\}\right]_t^{-1} \tag{29}$$

If N equals unity, the prediction of equation (29) is close to that of the homogeneous equilibrium model, and if N equals zero the solution is approximately the homogeneous frozen model. (These two solutions are discussed later in the paper.) Therefore, the quantity N describes the partial phase change occurring at the throat. The experimental results of reference [11] indicate that the critical flow rates are in relatively good agreement with the homogeneous equilibrium model for stagnation qualities greater than 0.10. For qualities less than this value, the equilibrium model underestimates the data. Hence, since N describes the deviation from equilibrium mass transfer, N is set equal to unity when $x_0 = 0.10$. As was discussed earlier, the term N is correlated as a function of throat equilibrium quality, x_{Et}. For the operating conditions reported in reference [11], a stagnation quality of 0.10 corresponds to throat equilibrium qualities ranging from 0.125 to 0.155 depending on the pressure level. For the evaluations given herein, an average value of 0.14 was chosen, thus

$$N = x_{Et}/0.14 \tag{30}$$

For throat equilibrium qualities greater than 0.14, N is set equal to unity. This numerical evaluation for N is common to all fluids and geometries considered herein.

The equation for the critical flow rate is coupled with the momentum equation describing the overall pressure history to obtain a solution in terms of the stagnation conditions. The two-phase momentum equation (4), under the restrictions listed above, can be written as

$$-[(1 - x_0)v_{l0} + x_0 v_g]dP = d\left(\frac{u^2}{2}\right) \tag{31}$$

This expression can be integrated between the stagnation and throat locations to give

$$(1 - x_0)v_{l0}(P_0 - P_t) + \frac{x_0\gamma}{\gamma - 1}[P_0 v_{g0} - P_t v_{gt}] = \frac{[(1 - x_0)v_{l0} + x_0 v_{gt}]^2}{2} G_c^2 \tag{32}$$

Substitution of equation (29) for the critical flow rate enables one to rearrange equation (32) and express it more compactly as

$$\eta = \left\{\frac{\dfrac{1 - \alpha_0}{\alpha_0}(1 - \eta) + \dfrac{\gamma}{\gamma - 1}}{\dfrac{1}{2\beta\alpha_t^2} + \dfrac{\gamma}{\gamma - 1}}\right\}^{\frac{\gamma}{\gamma - 1}} \tag{33}$$

where

$$\eta = P_t/P_0 \tag{34}$$

$$\beta = \left[\frac{1}{n} + \left(1 - \frac{v_{l0}}{v_{gt}}\right)\left(\frac{(1 - x_0)NP_t}{x_0(s_{gE} - s_{lE})_t}\frac{ds_{lE}}{dP}\right)_t - \frac{c_{pg}(1/n - 1/\gamma)}{(s_{g0} - s_{l0})}\right] \tag{35}$$

$$\alpha_0 = \frac{x_0 v_{g0}}{(1 - x_0)v_{l0} + x_0 v_{g0}} \tag{36}$$

$$\alpha_t = \frac{x_0 v_{gt}}{(1 - x_0)v_{l0} + x_0 v_{gt}} \tag{37}$$

and

$$v_{gt} = v_{g0}\left(\eta^{-\frac{1}{\gamma}}\right) \tag{38}$$

For given stagnation conditions of P_0 and x_0, the transcen-

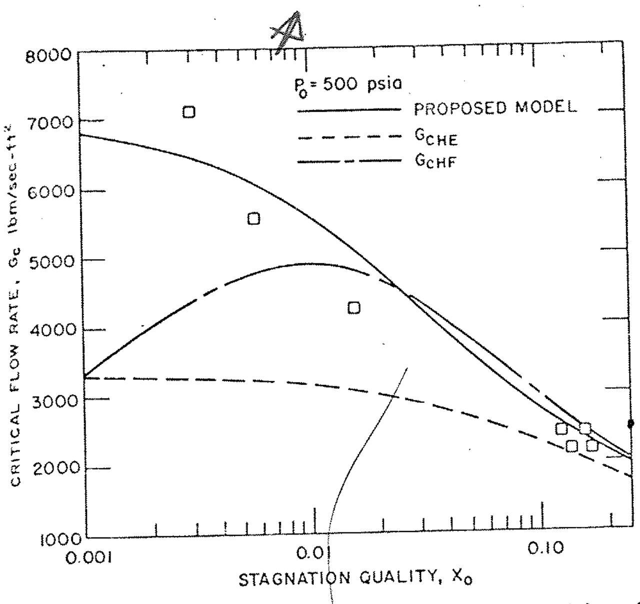

Fig. 5 Comparison of critical flow predictions and experimental data of references [26, 27]

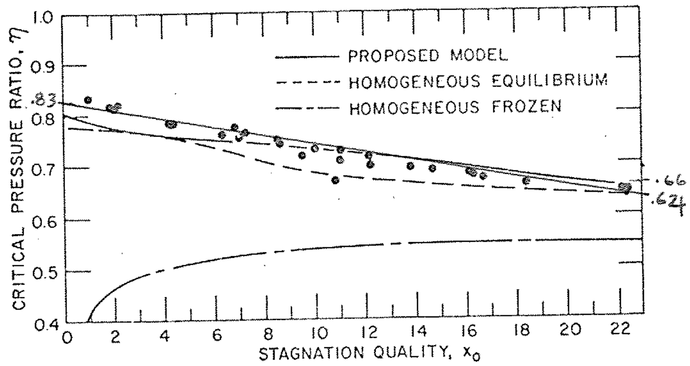

Fig. 6 Comparison of critical pressure ratio predictions for P_0 = 500 psia and experimental data of reference [27]

dental expression for the critical pressure ratio, equation (33), can be solved. This solution implicitly involves the critical flow rate as shown by equation (32). Therefore, a solution of equation (33) yields predictions of both the critical pressure ratio and flow rate.

In summary, a model is presented to describe the two-phase critical flow of one-component, liquid-vapor mixtures through convergent nozzles. The salient feature of the model is that it requires only a knowledge of the stagnation conditions. The model was formulated by examining pertinent high-velocity, two-phase flow data and extracting from these results reasonable approximations for the amounts and rates of interphase heat, mass, and momentum transfer. These approximations were used to generate a nonequilibrium critical flow model which was then combined with the overall momentum equation to yield a solution in terms of the stagnation properties.

Comparison With Experimental Results

The proposed model is compared with the experimental steam-water results of references [26, 27] in Figs. 3 and 4 for a range of stagnation pressures. The good agreement between the theory and the data throughout the quality range investigated is apparent.

One of the standard approaches used in the literature is the homogeneous equilibrium model. This model is described in reference [11] and is based on the following assumptions:

1 The average velocities for the phases are equal.
2 Thermodynamic equilibrium exists between the phases.
3 The expansion is isentropic.
4 Properties correspond to those presented in the steam tables, reference [28]. The flow rate prediction resulting from these assumptions is

$$\boxed{G_{cHE} = \frac{\{2[H_0 - (1-x_E)h_{lE} - x_E h_{gE}]\}^{1/2}}{(1-x_E)v_{lE} + x_E v_{gE}}} \qquad (39)$$

The critical flow rate prediction of this model is computed by choosing successively lower downstream pressures until the flow rate exhibits a maximum.

Another model which has appeared frequently in the literature is the homogeneous frozen model which is based on the following assumptions:

1 The average velocities of the phase are equal.
2 No heat or mass transfer occurs between the phases.
3 The vapor expands isentropically as a perfect gas, i.e., Pv_g^γ is constant.
4 The kinetic energy is due solely to the vapor expansion.
5 The critical flow rate is defined by gas-dynamic principles.

These assumptions lead to an expression for the critical pressure ratio which is given by the following transcendental equation:

$$\frac{(1-x_0)v_{l0}}{x_0 v_{g0}}(1-\eta) + \frac{\gamma}{\gamma-1}\left[1-\eta^{\frac{\gamma-1}{\gamma}}\right] = \left[\frac{(1-x_0)v_{l0}}{x_0 v_{g0}} + \eta^{-\frac{1}{\gamma}}\right]^2 \frac{\gamma}{2}\,\eta^{\frac{\gamma+1}{\gamma}} \qquad (40)$$

When $\frac{(1-x_0)v_{l0}}{x_0 v_{g0}} \ll 1$, equation (40) can be simplified to

$$\eta = \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} \qquad (41)$$

Under the assumptions outlined above, the energy equation and the two-phase specific volume can be expressed as

$$H_0 - h_t = x_0 v_{g0} P_0 \left(\frac{\gamma}{\gamma-1}\right)\left[1-\eta^{\frac{\gamma-1}{\gamma}}\right] \qquad (42)$$

$$v = (1-x_0)v_{l0} + x_0 v_{g0}\eta^{\frac{1}{\gamma}} \qquad (43)$$

respectively, and the critical flow rate is given by

$$G_{cHF} = \frac{1}{v}\left\{2x_0 v_{g0} P_0 \left(\frac{\gamma}{\gamma-1}\right)\left[1-\eta^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \qquad (44)$$

The predictions of the three models are compared to the experimental data in Fig. 5 for a stagnation pressure of 500 psia. The model proposed in this study exhibits the best agreement throughout the quality range of interest.

The critical pressure ratio predictions of the three models discussed above are compared to the experimental data in Fig. 6. Again, it is seen that the formulation developed herein is the best solution throughout the range. It should be noted here that there were discrepancies in the critical pressure ratio data reported in reference [27]. The two nozzles employed gave two different characteristic curves for the critical pressure ratio as a function of stagnation quality. As was discussed in reference [11], this difference could easily be a result of a small difference in the relative locations of the throat taps. In Fig. 6 the data of nozzle no. 2 was not used because it was found that these results gave critical pressure ratios which were less than those reported for two-component, air-water systems (reference [29]), which is not realistic. Therefore, the data shown were generated solely by nozzle no. 1.

By considering Figs. 5 and 6 together, the reader can discern

4

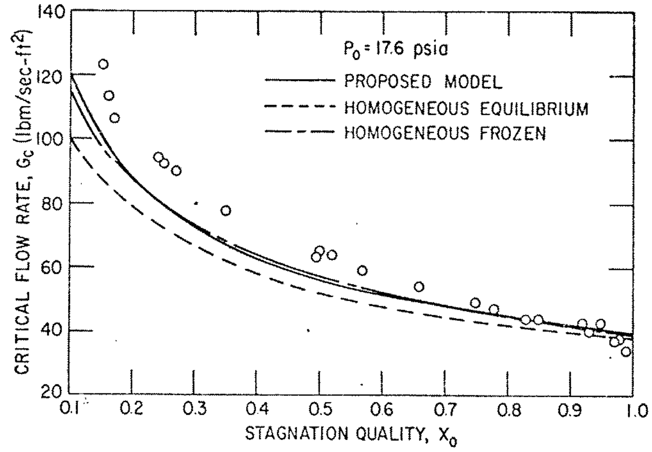

Fig. 7 Comparison of critical flow predictions and experimental data of reference [30]

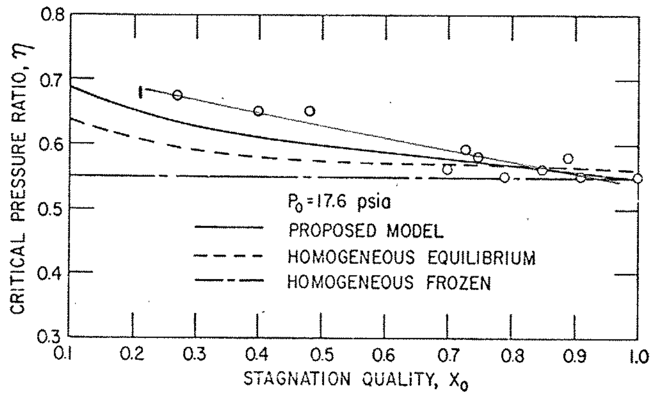

Fig. 8 Comparison of critical pressure ratio predictions and experimental data of reference [30]

the merits of the proposed model. The homogeneous frozen model yields a good prediction for the critical flow rate but considerably underestimates the critical pressure ratio. On the other hand, the homogeneous equilibrium model gives a good estimate of the critical pressure ratio but underestimates the flow rate. Therefore, neither of these solutions correctly describes the complete physical phenomenon. The proposed formulation gives accurate predictions for both the critical pressure ratio and flow rate and, thus, is more characteristic of the actual behavior. This is further verified by the recent low-pressure, steam-water data of Deich et al. [30] shown in Figs. 7 and 8. The stagnation pressure is so low that the applicability of the approximation given in equation (13) is weakened considerably. However, these results confirm the above discussion regarding various models.

Fig. 9 compares the proposed solution to the high-temperature, high-quality potassium data [31]. The tabular properties of Weatherford [32] were used in calculating the predictions. Table 1 shows the good agreement between the predictions of the proposed model and the recent high-quality, steam-water nozzle data of Carofano and McManus [33]. Figs. 10 and 11 compare the present analysis with the two-phase carbon dioxide nozzle reported by Hesson and Peck [16] and detailed in reference [17].

Saturated and Subcooled Conditions

The proposed solution applies to a saturated-vapor stagnation condition when $x_0 = 1$. The model can also be applied to cases where the stagnation condition is either saturated or subcooled liquid. For such cases $x_0 = 0$ and the critical flow expression is simplified to

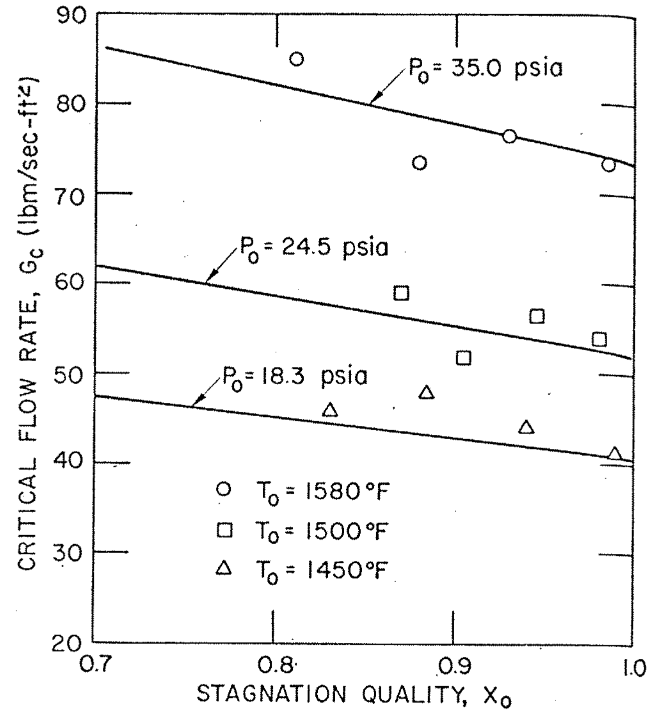

Fig. 9 Comparison between proposed model and potassium data of reference [31]

Table 1 Comparison between the proposed model and data of reference [33]

Run	P_0(psia)	X_0	Flow rate (lbm/sec-ft²) Experimental	Predicted
1	21.1	0.788	54.8	53.7
2	19.6	0.727	54.7	52.2
3	26.9	0.871	64.1	64.7
4	26.3	0.890	61.4	62.7
5	27.5	0.834	68.1	67.5
6	26.0	0.873	61.3	62.6
7	26.8	0.795	67.4	67.4
8	23.0	0.680	66.0	62.7
9	19.5	0.641	59.1	55.2

Subcooled

$$\boxed{G_c^2 = \left[(v_{gE} - v_{l0}) \frac{N}{s_{gE} - s_{lE}} \frac{ds_{lE}}{dP} \right]_t^{-1}} \quad (45)$$

where N is given by equation (30). Since no vapor is formed until the throat is reached, an additional assumption is necessary to estimate the vapor specific volume at the throat. As shown in equation (45), it is assumed that the vapor which is formed is saturated at the local pressure.

The critical pressure ratio relationship for such flows is greatly simplified.

$$\eta = 1 - \frac{v_{l0} G_c^2}{2P_0} \quad (46)$$

Equation (45) can be substituted into equation (46) to give a transcendental expression for the critical pressure ratio. A solution to this expression gives predictions for both the critical pressure ratio and flow rate.

In Fig. 12 the predictions for initially saturated and subcooled water are compared to the data of references [8, 34, 35, 36, 37, 38]. The model describes the general behavior of the experimental results. The agreement is better for higher pressures and subcoolings. The model is also compared to recent subcooled liquid nitrogen data [39] in Fig. 13. The agreement is excellent throughout the reported ranges of pressures and sub-

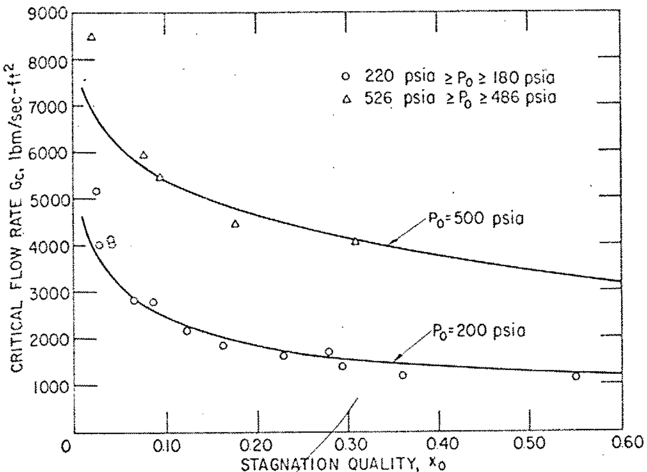

FIG. 10

Fig. 10 Comparison between proposed model and carbon dioxide data of reference [17]

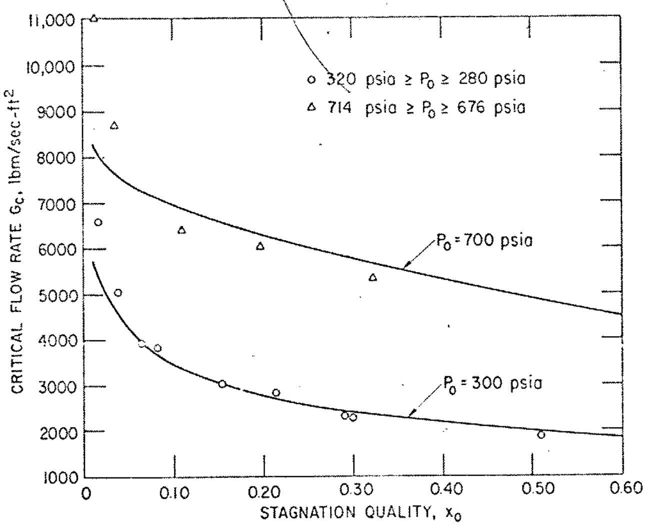

Fig. 11 Comparison between proposed model and carbon dioxide data of reference [17]

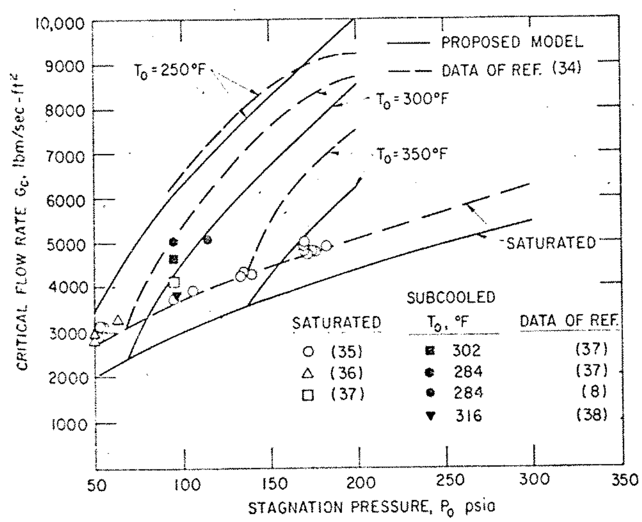

Fig. 12 Comparison between proposed model and saturated and subcooled water data of references [8, 34–38]

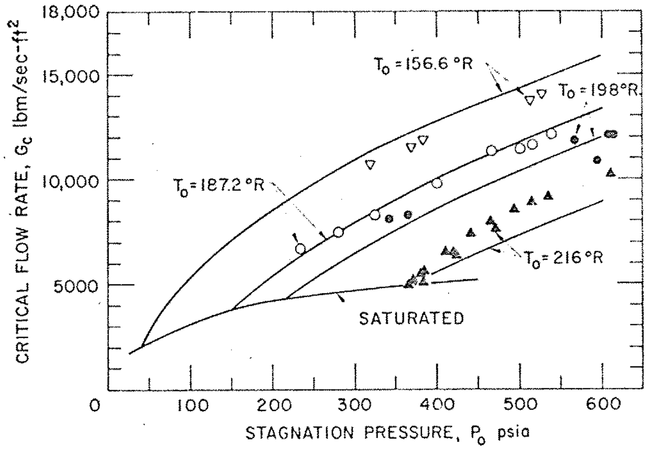

Fig. 13 Comparison between proposed model and subcooled liquid nitrogen data of reference [39]

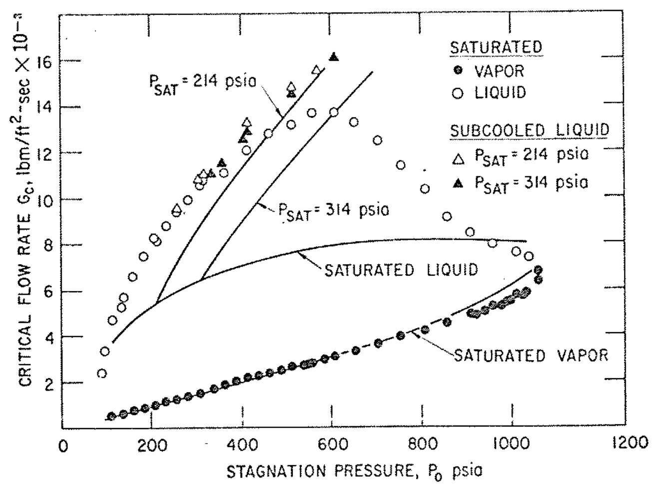

Fig. 14 Comparison between proposed model and subcooled and saturated liquid and saturated vapor data of reference [17]

coolings. Finally, Fig. 14 compares the model with the data of Hesson [17] for stagnation conditions of subcooled and saturated liquid and saturated vapor.

Orifices and Short Tubes

The model derived above, and in particular the mass transfer correlation given in equation (26), is for a thoroughly dispersed mixture. It has been shown by numerous investigators [3, 8, 16, 40–43] that the discharge of initially subcooled or saturated liquid through an orifice or short tube (short, constant-area duct with a sharp-edged entrance) has a unique separated flow pattern. However, if the upstream stagnation condition is a thoroughly dispersed mixture, the flow will remain dispersed throughout the expansion. Thus, the model can be extended to such flows.

As shown by Perry [44], compressible flows through sharp-edged orifices do not choke; however, they do asymptotically approach a maximum flow rate. For flows operating well into the compressible range ($P_B/P_0 < 0.3$ where P_B is the downstream receiver pressure), a compressible discharge coefficient for fixed stagnation conditions can be defined as

$$C = \frac{\text{actual flow rate}}{\text{critical flow rate in ideal nozzle}}$$

This definition can be incorporated into equation (32) to give a

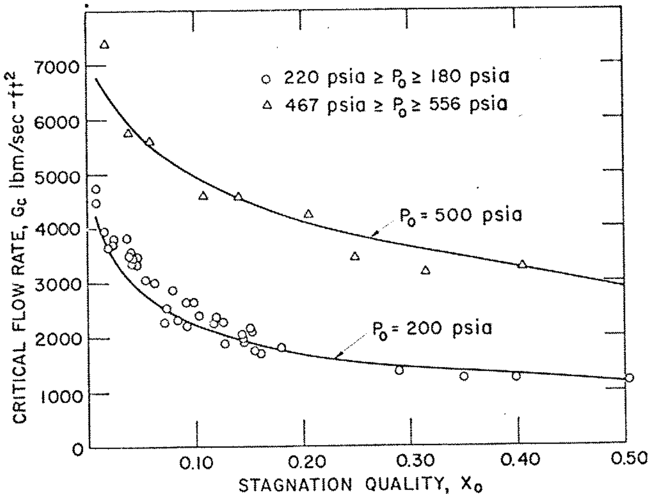

Fig. 15 Comparison between proposed model and two-phase orifice data of reference [17]

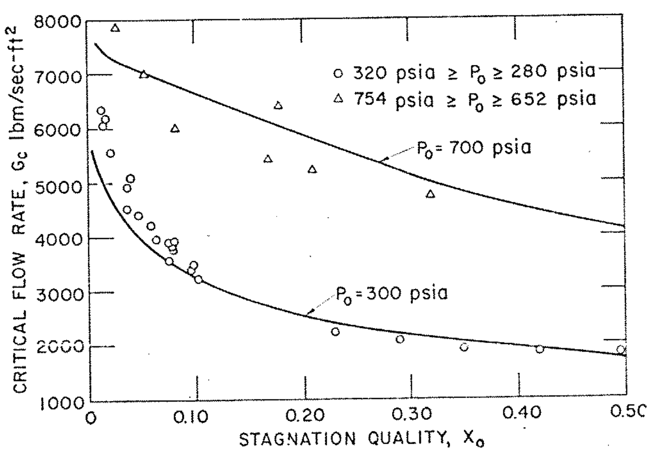

Fig. 16 Comparison between proposed model and two-phase orifice data of reference [17]

formulation for two-phase compressible flows through orifices. Equation (33) is then written as

$$\eta = \left\{ \frac{\dfrac{1 - \alpha_0}{\alpha_0}(1 - \eta) + \dfrac{\gamma}{\gamma - 1}}{\dfrac{1}{2C^2\beta\alpha_{t^2}} + \dfrac{\gamma}{\gamma - 1}} \right\}^{\frac{\gamma}{\gamma - 1}} \tag{47}$$

As before, this transcendental expression for the critical pressure ratio can be solved to give a prediction for the maximum flow rate.

The single-phase experiments of Perry show the compressible discharge coefficient for sharp-edged orifices is 0.84. This value is also representative of single-phase critical flows through short tubes [45]. It is assumed that this discharge coefficient is also applicable to one-component, two-phase flows through similar geometries. (As stated above, this only applies to systems which are two-phase mixtures in the upstream stagnation chamber.)

The predictions for compressible two-phase flow of carbon dioxide through a sharp-edged orifice are compared to the data of Hesson [17] in Figs. 15 and 16. The saturation properties of carbon dioxide were assembled from references [46, 47]. Fig. 17 compares the model to the two-phase nitrogen data of Bonnet [19] and Campbell and Overcamp [18]. To effect a common basis for comparison, the model and the data from both references were all evaluated from the saturation properties of

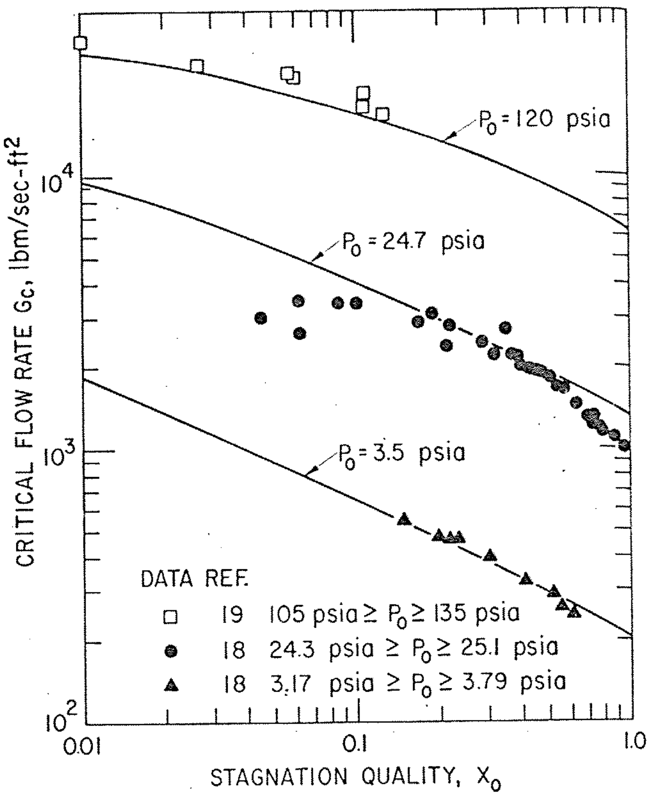

Fig. 17 Comparison between proposed model and two-phase nitrogen orifice data of references [18, 19]

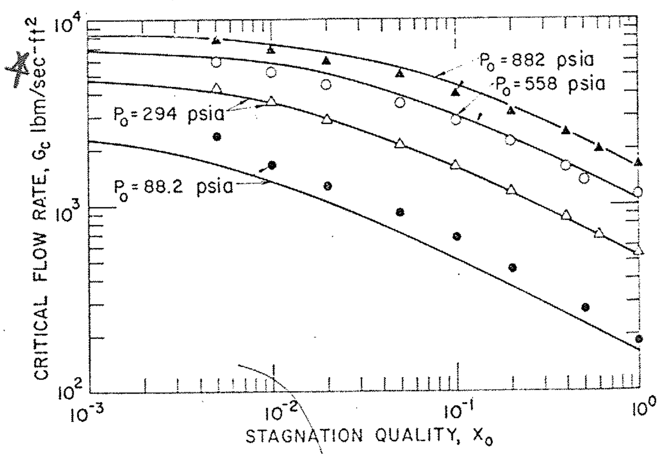

Fig. 18 Comparison between proposed model and short-tube, steam-water data of reference [49]

Strobridge [48]. In Fig. 18, the model is also compared to the short-tube, steam-water results of Friedrich [49]. The model is in good agreement for all the mixtures considered.

Summary and Conclusions

A model is developed for the two-phase critical discharge of one-component mixtures through convergent nozzles. The salient features of the model are:

1 The proposed solution is based on and requires a knowledge of only the geometry and the upstream stagnation conditions.

2 The stagnation conditions covered by the model include subcooled liquid, saturated liquid, two-phase mixtures, and saturated vapor.

3 The model assumes neither completely frozen nor complete equilibrium heat and mass transfer processes. Instead, the model uses the best available data to determine reasonable approximations for the heat transfer process and the best available correlation for the rate of interphase mass transfer at the throat.

Comparisons between the theoretical predictions and the available experimental results, which include water, nitrogen, potassium and carbon dioxide data, show that the model is in good agreement with the data over a wide range of stagnation conditions.

For the cases where the mixture is in a two-phase condition in the stagnation chamber, the compressible flow through orifices and short tubes can be related to nozzle flow in the same manner as single-phase flow. The predictions show good agreement with the experimental data for such geometries.

References

1 Henry, R. E., "A Study of One- and Two-Component, Two-Phase Critical Flows at Low Qualities," ANL-7430, Argonne National Laboratory, 1968.

2 Fauske, H. K., "Contribution to the Theory of Two-Phase, One-Component Critical Flow," ANL-6633, Argonne National Laboratory, 1962.

3 Fauske, H. K., "The Discharge of Saturated Water Through Tubes," *Chem. Eng. Prog. Symp. Ser.*, Vol. 61, 1965, p. 210.

4 Fauske, H. K., "Two-Phase, Two- and One-Component Critical Flow," *Proc. of Symp. on Two-Phase Flow*, University of Exeter, Devon, England, Vol. 3, SG101, 1965.

5 Isbin, H. S., Moy, J. E., and Cruz, A. J. R., "Two-Phase, Steam-Water Critical Flow," *AIChE Journal*, Vol. 3, 1957, p. 361.

6 Faletti, D. W., and Moulton, R. W., "Two-Phase Critical Flow of Steam-Water Mixtures," *AIChE Journal*, Vol. 9, 1963, p. 247.

7 Zaloudek, F. R., "The Low-Pressure Critical Discharge of Steam-Water Mixtures from Pipes," HW-68936, Hanford Works, 1961.

8 Zaloudek, F. R., "The Critical Flow of Hot Water Through Short Tubes," HW-77594, Hanford Works, 1963.

9 Zaloudek, F. R., "Steam-Water Critical Flow From High Pressure Systems," Interim Report, HW-80535, Hanford Works, 1964.

10 Zaloudek, F. R., "The Low Pressure Critical Discharge of Steam-Water Mixtures From Pipe Elbows and Tees," BNWL-34, Pacific Northwest Laboratory, 1965.

11 Starkman, E. S., Schrock, V. E., Neusen, K. F., and Maneely, D. J., "Expansion of a Very Low Quality Two-Phase Fluid Through a Convergent-Divergent Nozzle," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 86, No. 2, June 1964, pp. 247–256.

12 Levy, S., "Prediction of Two-Phase Critical Flow Rate," Journal of Heat Transfer, Trans. ASME, Series C, Vol. 87, No. 1, Feb. 1965, pp. 53–58.

13 Moody, F. J., "Maximum Flow Rate of a Single Component, Two-Phase Mixture," Journal of Heat Transfer, Trans. ASME, Series C, Vol. 87, No. 1, Feb. 1965, pp. 134–142.

14 Cruver, J. E., "Metastable Critical Flow of Steam-Water Mixtures," PhD thesis, University of Washington, 1963.

15 Klingebiel, W. J., "Critical Flow Slip Ratios of Steam-Water Mixtures," PhD thesis, University of Washington, 1964.

16 Hesson, J. C., and Peck, R. E., "Flow of Two-Phase Carbon Dioxide Through Orifices," *AIChE Journal*, Vol. 4, 1958, p. 207.

17 Hesson, J. C., "Flow of Two-Phase Carbon Dioxide Through Orifices," PhD thesis, Illinois Institute of Technology, 1957.

18 Campbell, H. M., and Overcamp, T. J., "Critical Flowrate of Two-Phase Nitrogen," NASA TM X-53492, 1966.

19 Bonnet, F. W., "Critical Two-Phase Flow of Nitrogen and Oxygen Through Orifices," *Advances in Cryogenic Engineering*, Vol. 12, 1966, p. 427.

20 Ryley, D. J., and Parker, G. J., "Two-Phase Critical Flow Through Suction Slots in Low Pressure Steam Turbine Blades," *Journal of Mechanical Engineering Science*, Vol. 10, 1968, p. 337.

21 Neusen, K. F., discussion of reference [13], Journal of Heat Transfer, Trans. ASME, Series C, Vol. 87, No. 1, Feb. 1965, pp. 141–142.

22 Smith, R. V., Cousins, L. B., and Hewitt, G. F., "Two-Phase Two-Component Critical Flow in a Venturi," AERE-R5736, 1968.

23 Tangren, R. F., Dodge, C. H., and Seifert, H. S., "Compressibility Effects in Two-Phase Flow," *Journal of Applied Physics*, Vol. 20, 1949, p. 736.

24 Henry, R. E., Grolmes, M. A., and Fauske, H. K., "Propagation Velocity of Pressure Waves in Gas-Liquid Mixtures," *Cocurrent Gas-Liquid Flow*, Vol. 1, Plenum Press, 1969.

25 Vogrin, J. A., "An Experimental Investigation of Two-Phase, Two-Component Flow in a Horizontal Converging-Diverging Nozzle," ANL-6754, Argonne National Laboratory, 1963.

26 Maneely, D. J., "A Study of the Expansion Process of Low Quality Steam Through a de Laval Nozzle," UCRL-6230, University of California Radiation Laboratory, 1962.

27 Neusen, K. F., "Optimizing of Flow Parameters for the Expansion of Very Low Quality Steam," UCRL-6152, University of California Radiation Laboratory, 1962.

28 Keenan, J. H., and Keyes, F. G., *Thermodynamic Properties of Steam*, John Wiley and Sons, N. Y., 1959.

29 Netzer, D. W., "Calculations of Flow Characteristics for Two-Phase Flow in Annular Converging-Diverging Nozzles," TM-62-3, Jet Prop. Center, Purdue University, Lafayette, Ind., 1962.

30 Deich, M. E., Danilin, V. S., Tsiklauri, G. V., and Shanin, U. K., "Investigation of the Flow of Wet Steam in Axisymmetric de Laval Nozzles Over a Wide Range of Moisture Content," *High Temperature*, Vol. 7, 1969, p. 294.

31 Rossbach, R. J., "Critical Flow of Potassium Vapor Through Instrumented Convergent-Diverging Nozzle," ASME Paper No. 65-GTP-22.

32 Weatherford, W. D., Jr., Tyler, J. C., and Ku, P. M., "Properties of Inorganic Energy-Conversion and Heat Transfer Fluids for Space Applications," WADD TR-61-96, Southwest Research Institute, 1961.

33 Carofano, G. C., and McManus, H. N., Jr., "An Analytical and Experimental Study of Air-Water and Steam-Water Mixtures in a Converging-Diverging Nozzle," *Progress in Heat and Mass Transfer*, ed. by Irvine et al., Vol. 2, 1969, p. 395.

34 Yarnall, D. R., discussion of "The Flow of Saturated Water Through Throttling Orifices," Trans. ASME, Vol. 63, 1941, p. 428.

35 Burnell, J. G., "Flow of Boiling Water Through Nozzles, Orifices, and Pipes," *Engineering*, Vol. 164, 1948, p. 572.

36 Bottomley, W. T., "The Flow of Boiling Water Through Orifices and Pipes," *Trans. of North East Coast Inst. of Engrs. and Shipbuilders*, Vol. 43, 1936, p. 65.

37 Friedrich, H., and Vetter, G., "Influence of Nozzle Shape on the Through Flow Behavior of Jets for Water at Various Thermodynamic States," *Energie*, Vol. 14, 1962, p. 3.

38 Danforth, J. L., "Flow of Hot Water Through A Rounded Orifice," MS thesis, M.I.T., 1941.

39 Henry, R. E., Henricks, R. C., Simoneau, R. J., and Watterson, R., "Two-Phase Critical Flow of Initially Subcooled Liquid Nitrogen," to be published as an NASA Technical Note.

40 Fauske, H. K., and Min, T. C., "A Study of the Flow of Saturated Freon-11 Through Apertures and Short Tubes," ANL-6667, Argonne National Laboratory, 1963.

41 Pasqua, P. F., "Metastable Flow of Freon-12," *Refrigerating Eng.*, Vol. 61, 1953, p. 1084A.

42 Uchida, H., and Nariai, H., "Discharge of Saturated Water Through Pipes and Orifices," *Proceedings of the Third International Heat Transfer Conference*, Vol. 5, 1966, p. 1.

43 Benjamin, M. W., and Miller, J. G., "The Flow of Saturated Water Through Throttling Orifices," Trans. ASME, Vol. 63, 1941, p. 419.

44 Perry, J. A., Jr., "Critical Flow Through Sharp-Edged Orifices," Trans. ASME, Vol. 71, 1949, p. 757.

45 Arnberg, B. T., "Review of Critical Flowmeters for Gas Flow Measurements," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 84, No. 4, Dec. 1962, pp. 447–460.

46 *Handbook of Chemistry and Physics*, 45th ed., The Chemical Rubber Co., Cleveland, Ohio, 1964.

47 Perry, J. H., *Chemical Engineers Handbook*, 3rd ed., McGraw-Hill, New York, 1950.

48 Strobridge, T. R., "The Thermodynamic Properties of Nitrogen from 64 to 300°K Between 0.1 and 200 Atmospheres," TN-129, U. S. Dept. of Commerce, National Bureau of Standards, 1962.

49 Friedrich, H., "Flow Through Single-Stage Nozzles with Different Thermodynamic States," *Energie*, Vol. 12, 1960, p. 3.