

Simulation of ADCS for TIROS I Mission

Introduction:

This report will detail the work done to create a simulation of ADCS for a reference LEO mission. This ADC system was developed using Simulink. All knowledge and theory used to construct this simulation was pulled from lecture material and the textbook. The simulation contains five parts: Dynamics, Actuators, Attitude Controller, Attitude Determination, and Sensors. Artificial noise was injected into the simulation at magnitudes described by the equipment chosen. The satellite orbits about and points toward the Earth with the use of 3 orthogonal reaction wheels. The primary force action on the satellite is gravity, but two perturbations were simulated: Gravity Gradient & Magnetic Torque. One orbit about the Earth was simulated with orbital elements matching that of the reference mission.

Reference Mission:

The reference mission for this simulation is the TIROS I satellite (launched in 1960). The inclination for this orbit was 48.4° , and the orbit was near circular with a radius of 700km. The inclination used in the simulation was rounded to 45° for simplicity in the model. The satellite was cylindrical with a diameter of 1.07m, a height of .56m, and a mass of 122.5kg. This gives the satellite an approximate inertia matrix of:

$$J_b = \begin{bmatrix} 17.5300 & 0 & 0 \\ 0 & 11.9700 & 0 \\ 0 & 0 & 11.9700 \end{bmatrix}$$

The satellite is oblate. The x axis points normal to the circular face of the spacecraft while the other two axes are orthogonal to it. The x-axis of this spacecraft **pointed directly to the Earth** for weather observation.

The satellite had a notable x-axis magnetic moment of $896 \text{ dyne} \cdot \text{cm} \cdot \text{gauss}^{-1}$ ($.896 \text{ N} \cdot \text{m} \cdot \text{Tesla}^{-1}$) with which a magnetic torque was found. The magnetic moments of the other axes were negligible.

Dynamics Model:

These equations of motions were used to propagate the dynamics of the simulation:

$$\begin{aligned} {}^b\dot{\omega}_{b/i}^b(t) &= J_{cg}^{-1} \left(-\omega_{b/i}^b(t) \times (J_{cg}^b \omega_{b/i}^b(t)) - {}^i\dot{h}_w^b(t) \right) \\ \dot{q}_i^b(t) &= \frac{1}{2} \begin{bmatrix} \omega_{b/i}^b(t) \\ 0 \end{bmatrix} \otimes q_i^b(t) \end{aligned}$$

where ${}^b(\dot{\omega}_{b/i})^b$ is the rate of change of the angular velocity of the spacecraft with respect to the inertia frame in the body coordinates as seen from the body. The J_{cg} matrix is the inertia matrix of the body in body coordinates. The $\omega_{b/i}$ is the angular velocity of the spacecraft with respect to the inertial frame in body coordinates. The ${}^i\dot{h}_w$ is the rate of change of the angular momentum of the reaction wheels as seen from the inertial frame in body coordinates. The

\dot{q}_i^b is the rate of change of the inertial to body quaternion, and the q_i^b is the current inertial to body quaternion. The top equation describes the rate of change of the angular velocity of the spacecraft while the bottom equation describes the rate of change of the inertial to body quaternion (attitude) of the spacecraft.

I used this equation to simulate the effect of the gravity gradient on the dynamics of the spacecraft:

$$\mathbf{m}_{cm} = 3\omega_{orb}^2 \mathbf{d} \times (\mathbf{J}_{cm} \mathbf{d})$$

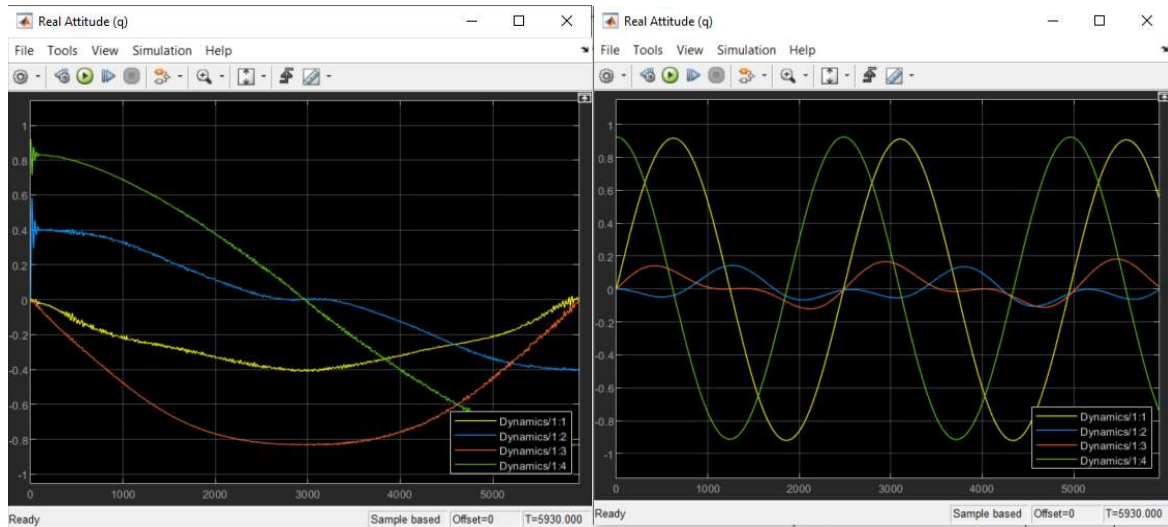
Where \mathbf{m}_{cm} is the moment about the center of mass caused by the gravity gradient, ω_{orb} is the angular frequency of the orbit, \mathbf{d} is the vector pointing towards the earth, and \mathbf{J}_{cm} is the inertia matrix for the satellite.

This equation was used to simulate the effect of the magnetic torque on the satellite:

$$\mathbf{m}_{mag} = \boldsymbol{\mu} \times \mathbf{B}$$

Where \mathbf{m}_{mag} is the moment caused by the magnetic torque, $\boldsymbol{\mu}$ is the magnetic moment of the spacecraft, and \mathbf{B} is the magnetic field of the Earth.

The graphs below show the inertial to body attitude quaternions. The graph on the left is under controlled rotation while the graph on the right is under uncontrolled rotation due to the torque perturbations.



As you can see, the controlled satellite completes one rotation per orbit. This allows it to always point at the Earth. The uncontrolled satellite completes multiple rotations.

Sensors and Actuators:

The actuators on board the spacecraft are 3 orthogonal reaction wheels manufactured by Rocket Lab. Their moments of inertia is .00381 kgm², and it has a maximum momentum of 1Nms ($\omega_{\max} = 262\text{rad/s}$). The precision of the angular velocity imparted on the system is $\pm 2\text{rad/s}$. This is the math model behind the reaction wheels:

$$\begin{aligned} {}^b\dot{\omega}_w^b(t) &= {}^b\dot{\omega}_{w,comm}^b(t) + \delta\alpha_w^b(t) \\ \omega_w^b(t) &= \int {}^b\dot{\omega}_w^b(t) dt \\ {}^b\dot{h}_w^b(t) &= C_w {}^b\dot{\omega}_w^b(t) \\ h_w^b(t) &= C_w \omega_w^b(t) \\ {}^i\dot{h}_w^b(t) &= {}^b\dot{h}_w^b(t) + \omega_{b/i}^b \times h_w^b(t) \end{aligned}$$

Where $\delta\alpha$ is the actuator error, C_w is the moment of inertia of the reaction wheels, and all other symbols are as previously stated. The change in angular momentum as seen by the inertial frame is calculated outside the actuator block and in the dynamics block.

The magnetometer used is the NSS Magnetometer. It has a noise power of $16\text{e-}18$. Here is the math model for the sensor:

$$\tilde{B}^b(t) = T_i^b(t) B^i(t) + \nu_{mag}(t)$$

Where ν_{mag} is the noise of the sensor, and everything else is as previously stated.

The Earth Horizon Sensor used is the Cubespace CubeStar which has a precision of $\pm 0.02^\circ$. This is the math model used for the sensor:

$$\tilde{d}^b(t) = T(\nu_{Hor}) T_i^b(t) \frac{-r^i(t)}{\|r^i(t)\|}$$

Where d is the vector pointing to the Earth, ν_{Hor} is the error of the sensor, and everything else is as previously stated.

The gyroscope used is the SAFRAN STIM277H Multi-axis Gyro Module. It has a bias of $0.3^\circ/h$ ($8.3e-5/s$), and a ARW of $.15^\circ/hour\sqrt{s}$ ($5.29e-13 rad^2/s^3$). Here is the math model used for the sensor:

$$\tilde{\omega}_{b/i}^b(t) = \omega_{b/i}^b(t) + b + \nu_{gyro}(t)$$

Where the b is the gyro bias, the ν_{gyro} is the Angular Random Walk, and everything else is as previously stated.

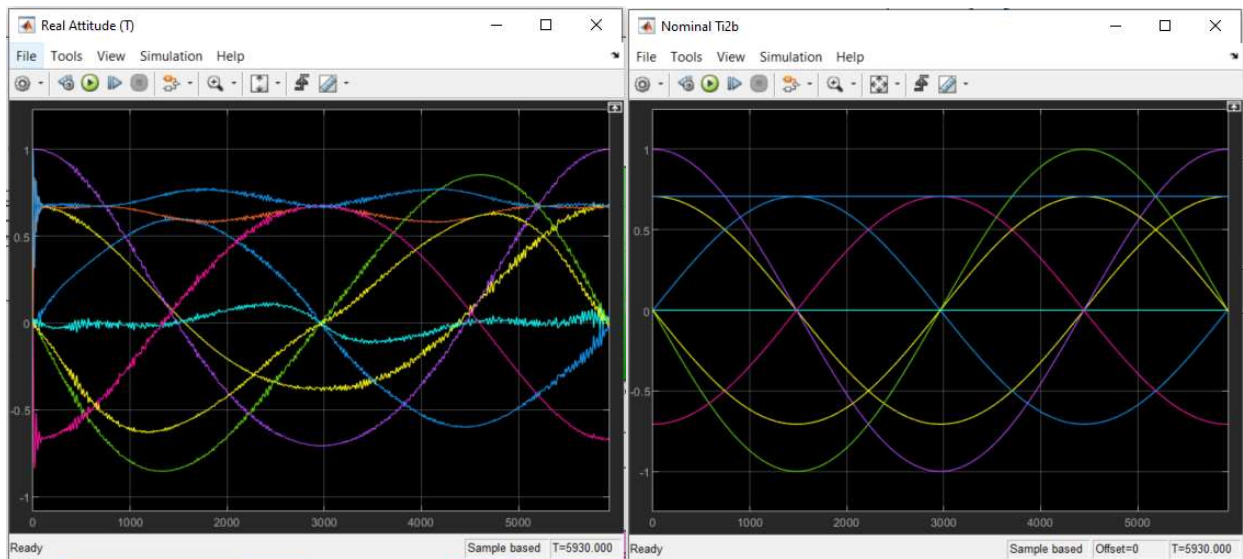
Attitude Determination and Control System:

The Attitude Determination system used in the ADCS was the TRIAD system. TRIAD requires two reference vectors that are known in both the inertial and body coordinates. The two vectors used for this were the direction of Earth's magnetic field (magnetometer) and the center of the Earth (Earth horizon sensor). From these two vectors a DCM from inertial to body was calculated and used as one representation of the attitude.

The controller used was a PD controller. The proportional gain was -0.05, and the derivative gain was -0.08. The controller determined the error of both the attitude of the spacecraft and the angular velocity of the spacecraft.

Simulation Results:

Below are two graphs containing the true attitude DCM and the nominal attitude DCM:



Within some error due to sensor and actuator imperfections, the nominal attitude is held throughout the duration of the orbit. The ADCS works as designed and intended.

Informal References:

- Reaction Wheels: <https://www.rocketlabusa.com/assets/Uploads/RL-RW-1.0-Data-Sheet.pdf>
- Horizon Sensor: <https://satsearch.co/products/cubespace-cube-sense-n>
- Magnetometer: <https://www.cubesatshop.com/product/nss-magnetometer/>
- Gyro: <https://sensor.azurewebsites.net/media/gwqdimq4/ts1672-r4-datasheet-stim277h.pdf>
- TIROS I Magnetic Moment: <https://ntrs.nasa.gov/api/citations/20150022969/downloads/20150022969.pdf>