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Transformadas de Laplace - Izabela da Silva Neves

• Nise - Cap 02

① ⓑ sen(ωt) u(t)

$$\mathcal{L}(\text{sen}(\omega t)u(t)) = \int_0^\infty \text{sen}(\omega t)u(t)e^{-st}dt$$
$$= \frac{e^{-st}}{s^2+\omega^2}(s\,\text{sen}(\omega t) - \omega\cos(\omega t))\Big|_0^\infty$$
$$= \frac{\omega}{s^2+\omega^2}$$

② ⓑ $e^{-at}\cos\omega t\, u(t)$

$$\mathcal{L}(e^{-at}\cos(\omega t)u(t)) = \frac{s+a}{(s+a)^2+\omega^2}$$

Usando $\mathcal{L}[e^{-at}f(t)u(t)] = F(s+a)$, aqui trocamos cada s por s+a

⑧ função de Transferência: $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$

Assumindo condição inicial é zero:

$$s^3Y(s) + 3s^2Y(s) + 5sY(s) + Y(s) = s^3X(s) + 4s^2X(s) + 6sX(s) + 8X(s)$$
$$Y(s)[s^3+3s^2+5s+1] = X(s)[s^3+4s^2+6s+8]$$
$$\frac{X(s)}{Y(s)} = \frac{s^3+3s^2+5s+1}{s^3+4s^2+6s+8}$$

⑨ equação diferencial

ⓒ $\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$

$$X(s)[s^3+11s^2+12s+18] = F(s)[s+3]$$
$$s^3X(s) + 11s^2X(s) + 12sX(s) + 18X(s) = sF(s) + 3F(s)$$
$$\frac{d^3x}{dt^3} + \frac{11d^2x}{dt^2} + 12\frac{dx}{dt} + 18x = \frac{df}{dt} + 3f$$

Atividade 2 - Franklin

3.2) b, d, e (transformada de Laplace)

(b) $f(t) = 3 + 7t + t^2 + \delta(t)$

$\mathcal{L}(f(t)) = \frac{3}{s} + 7\frac{1}{s^2} + \frac{2}{s^{2+1}} + \frac{1}{s} = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + \frac{\delta}{s^2}$

(d) $f(t) = (t+1)^2 = t^2 + 2t + 1$

$\mathcal{L}(f(t)) = \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$

(e) $f(t) = \text{sen}(h t)$

$\mathcal{L}(f(t)) = \frac{h}{s^2+h^2}$

3.3) (a) $f(t) = 3\cos(6t)$

$\mathcal{L}(f(t)) = \frac{3s}{s^2+6^2} = \frac{3s}{s^2+36}$

(b) $f(t) = \text{sen}\,2t + 2\cos 2t + e^{-t}\,\text{sen}\,2t$

$\mathcal{L}(f(t)) = \mathcal{L}(\text{sen}(2t)) + \mathcal{L}(2\cos(2t)) + \mathcal{L}(e^{-t}\,\text{sen}\,2t)$

$= \frac{2}{s^2+4} + \frac{2s}{s^2+4} + \frac{2}{(s+1)^2+4}$

$\left\{ \mathcal{L}(\text{sen}(2t))_{s \to s-(-1)} \right.$

$\frac{2}{s^2+4}\Big|_{s \to s+1}$

$\frac{2}{(s+1)^2+4}$

(c) $f(t) = t^2 + e^{-2t}\,\text{sen}\,t$

$\mathcal{L}(f(t)) = \frac{2}{s^3} + \frac{1}{(s+2)^2+1}$

$\left\{ \mathcal{L}(\text{sen}\,t)_{s \to s-(-2)} \right.$

$\frac{1}{s^2+1}\Big|_{s \to s+2} = \frac{1}{(s+2)^2+1}$

3.5) a,

(a) $f(t) = \text{sen}\,t\ \text{sen}\,3t$

$\mathcal{L}(f(t)) = \frac{1}{s^2+1} \cdot \frac{3}{s^2+9} = \frac{3}{(s^2+1)(s^2+9)}$

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3.5) c) $f(t) = \frac{\operatorname{sen} t}{t} =$

$\mathcal{L}\{\operatorname{sen} t\} = \frac{1}{s^2+1}$

$\int_s^\infty \frac{1}{u^2+1}\,du = \frac{\pi}{2} - \arctan(s)$

3.7) a) $F(s) = \frac{2}{s(s+2)}$

$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s(s+2)}\right\} = \frac{A}{s} + \frac{2B}{s+2} \to 2 = A(s+2) + (2s)B$

- $s = -2$ $\quad 2 = 2(-2)B \quad B = \frac{-1}{2}$
- $s = 0$ $\quad 2 = 2A \quad A = 1$
- $\mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{-1}{s+2}\right\} = u(t) - e^{-2t}$

e) $F(s) = \frac{1}{s^2+4}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} = \frac{1}{2}\operatorname{sen}(2t)$

f) $F(s) = \frac{2(s+2)}{(s+1)(s^2+4)}$

- $\mathcal{L}^{-1}\{F(s)\} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4} \Rightarrow 2s+4 = A(s^2+4) + (Bs+C)(s+1)$
- $2s+4 = As^2 + Bs^2 + Bs + Cs + 4A + C$
- $\begin{cases} A+B=0 \\ B+C=2 \\ 4A+C=4 \end{cases}$ $\quad A = \frac{2}{5},\ B = -\frac{2}{5},\ C = \frac{12}{5}$ $\quad \bullet\ \frac{2}{5(s+1)} + \frac{-2s+12}{5(s^2+4)}$
- $\mathcal{L}^{-1}\left\{\frac{2}{5(s+1)} - \frac{2s}{5(s^2+4)} + \frac{12}{5(s^2+4)}\right\} = \frac{2}{5}e^{-t} - \frac{2}{5}\cos(2t) + \frac{12}{5}\cdot\frac{1}{2}\operatorname{sen}(2t)$
- $\mathcal{L}^{-1}\{F(t)\} = \frac{2}{5}e^{-t} - \frac{2}{5}\cos(2t) + \frac{6}{5}\operatorname{sen}(2t)$

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