

Learning Physics Constrained Dynamics Using Autoencoders

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Multifrequency media access control has been well understood in general wireless ad hoc networks, while in wireless sensor networks, researchers still focus on single frequency solutions. In wireless sensor networks, each device is typically equipped with a single radio transceiver and applications adopt much smaller packet sizes compared to those in general wireless ad hoc networks. Hence, the multifrequency MAC protocols proposed for general wireless ad hoc networks are not suitable for wireless sensor network applications, which we further demonstrate through our simulation experiments. In this article, we propose MMSN, which takes advantage of multifrequency availability while, at the same time, takes into consideration the restrictions of wireless sensor networks. Through extensive experiments, MMSN exhibits the prominent ability to utilize parallel transmissions among neighboring nodes. When multiple physical frequencies are available, it also achieves increased energy efficiency, demonstrating the ability to work against radio interference and the tolerance to a wide range of measured time synchronization errors.

Additional Key Words and Phrases: Wireless sensor networks, media access control, multi-channel, radio interference, time synchronization

1 INTRODUCTION

In addition, one may think that the estimator network can directly predict the system parameters given the input observation without taking the states. The reason to predict the states is that for some applications, it may be useful to know the state of the system. For example, for a self-driving car, we would like to know the speed of other vehicles by using observations from cameras. Knowing the speed of other vehicles (i.e., the state) allows the self-driving car to plan for a trajectory, which is important for the safe deployment of the system. In addition, we would like to increase the interpretability of the model. The inclusion of the state allows the system designer to ensure the representation learned by neural networks is informative.

2 BACKGROUND

3 ALPS ARCHITECTURE

The autoencoder with latent physics (ALPS) model (see Fig. 1) estimates states and physical parameters of the system using four main components. Firstly, encoder h estimates states \tilde{x}_s from an observation sequence o_s . Secondly, estimator f predicts physical parameters θ from the states \tilde{x}_s . Thirdly, using the initial state and parameters θ physics simulator generates a state trajectory \hat{x}_s consistent with the laws of physics. Lastly, A decoder g reconstructs observations \hat{o}_s from the state trajectory \hat{x}_s . The autoencoder is trained to minimize the observation reconstruction loss $\sum_s \|o_s - \hat{o}_s\|_2^2$. In addition, the encoder h is trained to minimize the sum of squared state errors $\sum_s \|\tilde{x}_s - \hat{x}_s\|_2^2$.

The encoder network. Depending on the type of observations – pixel images or direct measurements – a convolution or feedforward network is used to represent an observation o as a vector

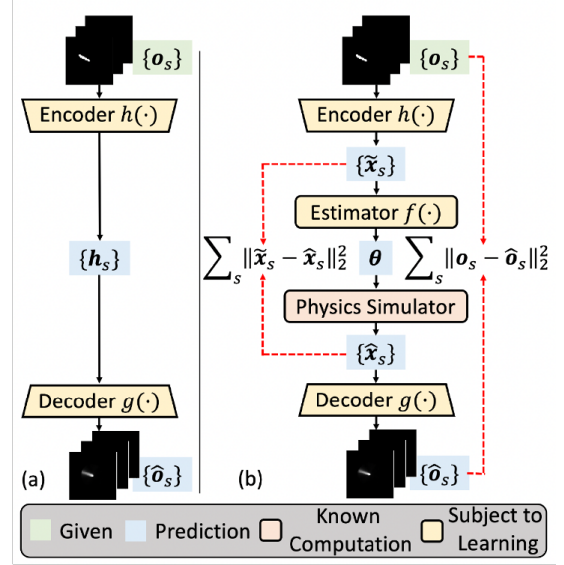


Fig. 1. ALPS Architecture

embedding z' . To understand the dynamics within the data, the local and global context of the dynamics are extracted by aggregating the vector embedding z' using a self-attention network [Vaswani et al. 2017].

Firstly, to incorporate positional information, a positional encoding is added to the embeddings which are then stacked over τ steps to form a matrix. Secondly, to calculate the attention softmax is taken over the τ steps. Thirdly, to allow the model to jointly attend to information from different representation subspaces at different positions the multi-head attention mechanism is applied (1,2).

$$\text{Multihead} = \text{Concat}(\text{head}_1, \dots, \text{head}_h) \cdot W^O \quad (1)$$

$$\text{head}_i = \text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right)V, \quad (2)$$

where Q, K, V represent the query, key, and value matrices of dimensionality d respectively and W^O are learnable matrices. Finally, using Gaussian and Mises distributions as posterior for translational and rotational coordinates accordingly, a feedforward network produces the parameters of the distribution for each state in the sequence.

The parameter estimator network. It was shown that multilayer perceptrons (MLPs) are affected by spectral bias [Rahaman et al. 2019] i.e. lower frequencies are learned first. Therefore, for systems that involve periodical or vibrational behavior, such as pendulum and oscillator, the input is passed through a Fourier transform on each component j of state trajectory (3) in order to capture high frequency content in the data [Tancik et al. 2020]. For other systems, which do not have periodic behavior the transformation is not needed. For each Fourier feature mapping a residual network [He

et al. 2016] is used to get a representation. The parameter estimator MLP network predicts physical parameters θ by taking a concatenation of the magnitudes of Fourier features $|\tilde{X}_\omega(j)|$ from states \tilde{x}_s .

$$\tilde{X}!(j) := \sum_{k=t-\tau+1}^t \tilde{x}_k(j) \left(\cos\left(\frac{2\pi}{\tau}\omega k\right) - i \cdot \sin\left(\frac{2\pi}{\tau}\omega k\right) \right) \quad (3)$$

The physics simulator. The core of the physics simulator is a neural ordinary differential equation (ODE). Neural ODE [Chen et al. 2018] is a type of neural network (NN) models, where instead of specifying a discrete sequence of hidden layers, the derivative of the hidden state is parameterized using a NN. Given the initial state \tilde{x}_s predicted by the encoder and physical parameters θ predicted by the estimator, the ODE solver rollouts state trajectories \hat{x}_s (4).

$$\begin{aligned} & \{\hat{x}_s\}_{s=t-\tau+1}^t : \hat{x}_{t-\tau+1}, \dots, \hat{x}_{t-1}, \hat{x}_t \\ & = \text{ODESolver}(\tilde{x}_{t-\tau+1}, \dot{x} = A(\theta)x + B(\theta)u, \tau, \Delta), \end{aligned} \quad (4)$$

where Δ is a sampling start interval.

The decoder network. Depending on the type of input observations — images or sensor measurements — the decoder network is either a deconvolutional or feedforward network, that generates a reconstructed observation \hat{o}_s from each of the states \hat{x}_s , simulated by the ODE solver.

The loss function. The loss function consists of three losses (5). Firstly, the variational autoencoder (VAE) loss is used to train the encoder h , the estimator f and the decoder g . Secondly, the observation reconstruction loss used to minimize the error between input o_s and reconstructed \hat{o}_s observations, which improves the image reconstruction quality. Thirdly, the state reconstruction loss encourages states, defined by encoder \tilde{x}_s to follow the ones, generated by physics simulator \hat{x}_s and constrains the encoder from predicting arbitrary sequences.

$$\begin{aligned} L = & \sum_{s=t-\tau+1}^t D_{KL}(Q(\tilde{x}_s|o_s)||P(\tilde{x}_s)) \\ & + \sum_{s=t-\tau+1}^t \|o_s - \hat{o}_s\|_2^2 + \sum_{s=t-\tau+1}^t \|\tilde{x}_s - \hat{x}_s\|_2^2 \end{aligned} \quad (5)$$

4 PERFORMANCE EVALUATION

5 ALPS LIMITATIONS

A Impact of ALPS (and assumptions)

6 FUTURE WORK

We think the future extension of ALPS to differentiable rendering engines is a valuable future research direction.

We see that the network mostly uses the observation in the beginning and the end of the attention mask to infer states. This suggests that the network learns to predict an average velocity. In addition, we see that there are vertical attention patterns, which happens to be at the peak of the cosine wave. We suspect that the network uses this information as an anchor to capture the periodic behavior of the pendulum. We leave this as a future work to understand the network.

7 CONCLUSIONS

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