AP Calculus Homework 21

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the points.

a) $(-\sqrt{2}, 5\pi/4)$ b) $(1, 5\pi/2)$ c) $(2, -7\pi/6)$

2. The Cartesian coordinates of a point are given. Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leqslant \theta \leqslant 2\pi$.

a) $(2, -2)$ b) $(-1, \sqrt{3})$

3. Identify the curve by finding a Cartesian equation for the curve.

a) $r\cos\theta = 1$ b) $r = 3\sin\theta$

4. Find a polar equation for the curve represented by the given Cartesian equation.

a) $x^2 + y^2 = 9$ b) $x = -y^2$

5. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

a) $r = 2\sin\theta, \quad \theta = \pi/6$ b)* $r = 2 - \sin\theta, \quad \theta = \pi/3$

c) $r = \frac{1}{\theta}, \quad \theta = \pi$ d)* $r = \cos 2\theta, \quad \theta = \pi/4$

6. Find the points on the given curve where the tangent line is horizontal or vertical

a)** $r = e^{\theta}$ b) $r = 2 + \sin\theta$

7. Find the area of the region that is bounded by the given curve and lies in the specified sector.

a) $r = e^{\theta/2}, \quad \pi \leqslant \theta \leqslant 2\pi$ b) $r = \sin\theta, \quad \pi/3 \leqslant \theta \leqslant 2\pi/3$

8. Find the area that the curve $r = 3\cos\theta$ encloses.

9. Find the derivative of the vector function

a) $\mathbf{r}(t) = (t\sin t, t^2, t\cos 2t)$ b) $\mathbf{r}(t) = (\tan t, \sec t, 1/t^2)$

c) $\mathbf{r}(t) = \mathbf{i} - \mathbf{j} + e^{4t}\ \mathbf{k}$ d)* $\mathbf{r}(t) = \sin^{-1} t\ \mathbf{i} - \sqrt{1-t^2}\ \mathbf{j} + \mathbf{k}$

10. Evaluate the integral

a) $\displaystyle\int_0^1 (16t^3\ \mathbf{i} - 9t^2\ \mathbf{j} + 25t^4\ \mathbf{k})dt$ b) $\displaystyle\int_1^2 (t^2\ \mathbf{i} + t\sqrt{t-1}\ \mathbf{j} + t\sin \pi t\ \mathbf{k})dt$

11. The area of the region enclosed by the polar curve $r = 1 - \cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

12. The area of the region enclosed by the polar curve $r = \sin 2\theta$ for $0 \le \theta \le \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

13. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

14. For any time $t \ge 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is

(A) $\left(2t, \dfrac{2}{2t+3}\right)$ (B) $\left(2t, \dfrac{-4}{(2t+3)^2}\right)$ (C) $\left(2, \dfrac{4}{(2t+3)^2}\right)$

(D) $\left(2, \dfrac{2}{(2t+3)^2}\right)$ (E) $\left(2, \dfrac{-4}{(2t+3)^2}\right)$

15. If a particle moves in the xy-plane so that at time $t > 0$, its position vector is $(\ln(t^2+2t), 2t)$, then at time $t = 2$, its velocity vector is

(A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(\frac{-5}{16}, 4\right)$

16. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$

(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$