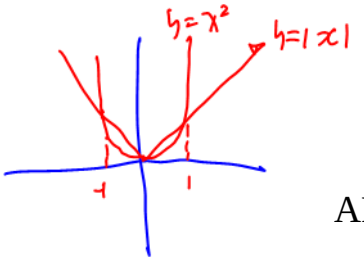

AP Calculus In-Class Two – Limit and Continuity

1.4 Other Basic Limits; 1.5 Asymptotes

1. If c is a nonnegative real number and $0 \le f(x) \le c$ for every x. Prove that $\lim_{x\to 0} x^2 f(x) = 0$.

Proof : Let $h(x) = x^2 f(x)$. consider $x \in [-\frac{1}{2}, \frac{1}{2}]$,

2. Find limits.

(a) $\lim_{x\to 0} \dfrac{\sin x}{\sqrt[3]{x}}$

(b) $\lim_{x\to 0} \dfrac{x + \tan x}{\sin x}$

(c) $\lim_{t\to 0} (1-t)^{1/t}$

3. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statement is (are) true?

I. $\lim_{x\to -3} f(x) = -1$

II. f is continuous everywhere except at $x = -3$.

III. f has a removable discontinuity at $x = -3$.

(A) None of them (B) I only (C) III only
(D) I and III only (E) All of them

4. Find a value of c that makes $h(x)$ is continuous at $x = 0$.

$$h(x) = \begin{cases} \dfrac{1-\cos 3x}{x^2}, & \text{if } x \neq 0 \\ c, & \text{if } x = 0 \end{cases}$$

5. Find all asymptotes of the graph of $y = \dfrac{2x^2 + 2x + 3}{4x^2 - 4x}$.

6. Find all asymptotes for the graph of $g(x) = \arctan x$.

7. Find all vertical and horizontal asymptotes for the graph of $y = \dfrac{\ln x}{1 - \ln x}$.

8. Show that equation $|x| = \cos x$ has at least one positive root.