Counting and Patterns

1. Skip Counting

"Skip Counting" is counting by a number that is not 1.

1) Learn to Skip Count by 2

Learning to skip count by 2 means you can count things faster!
Try this example, who will be the winner?

Example: You Skip Count by 2 like this:
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

Learning to "Skip Count" helps you: count many things quickly learn your multiplication tables

2) Skip Counting by 10s

Skip Counting by 10s is like normal counting, except there is an extra "0":
10, 20, 30, 40, 50, 60, 70, 80, 90, 100, ...

3) Skip Counting by 5s

Skip Counting by 5s has a nice pattern:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

4) Skip Counting by 3s and 4s

Skip Counting by 3s is:
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

Skip Counting by 4s is:
4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

2. Patterns

A **pattern** is a group of numbers, shapes, or objects that follow a rule while repeating or changing.

To extend a pattern you can use a table or a pattern rule that relates the term number to the pattern rule.

A **term number** is the number that tells the position of an item in a pattern.

For example, the pattern 2, 4, 6, 8, 10, … can be shown in a table like this:

Term number	Number in pattern
1	2
2	4
3	6
4	8
5	10

A pattern rule to get any number in the pattern is multiply 2 by the term number.
10th term = $2 \times 10 = 20$

Patterns can be represented using a spreadsheet. A **spreadsheet** is a computer program that has columns of data that are related. Each number in a spreadsheet has its own cell.

To represent a pattern, enter information for the first term. Use one or more operations to get the rest of the terms in the pattern.

For example, the spreadsheet below shows a pattern. The first term is $2.

The formula to get the second term is B3 = 2*B2, the third term is B4 = 2*B3, and so on.

	A	**B**
1	**Term number**	**Cost**
2	1	$2
3	2	$4
4	3	$8
5	4	$16

1) General Number Patterns

These general number patterns encourage the children to look carefully at the differences between the numbers in the sequence, and use that information to work out what comes next, or to fill in the gaps.

The worksheet here can be printed and photocopied, and the answers are listed below:

1) A very simple pattern, involving repeated addition of one (one times table)
1, 2, 3, 4, 5, <u>6</u>, <u>7</u>, ...

2) Another simple pattern, involving repeated addition of two (two times table)
2 4 6 <u>8</u> 10 <u>12</u> <u>14</u>

3) Repeated addition of five (five times table) ... 5 <u>10</u> 15 20 25 <u>30</u> <u>35</u>

4) Doubling each time ... <u>1</u> 2 4 8 <u>16</u> 32 <u>64</u>

5) Adding one more each time ... 1 2 4 7 11 16 22

i.e. 1 + ***1*** = 2, 2 + ***2*** = 4, 4 + ***3*** = 7, 7 + ***4*** = 11, 11 + ***5*** = 16, 16 + ***6*** = 22

6) Halving each time ... 1600 800 400 200 100 50 25

7) Adding three each time ... 8 11 14 17 20 23 26

8) Halving each time ... 8 4 2 1 1/2 1/4 1/8

9) Adding 22 each time ... 12 34 56 78 100 122 144

10) Repeated pattern ... 0 15 30 0 15 30 0

2) The Sums of the Rows in Pascal's Triangle

The sum of the numbers in any row is equal to 2 to the n^{th} power or 2^n, when n is the number of the row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 200 252 200 120 45 10 1

For example:

$2^0 = 1$
$2^1 = 1+1 = 2$
$2^2 = 1+2+1 = 4$
$2^3 = 1+3+3+1 = 8$
$2^4 = 1+4+6+4+1 = 16$

3) Prime Numbers

If the first element in a row is a prime number (remember, the 0th element of every row is 1), all the numbers in that row (excluding the 1's) are divisible by it.

For example, in row 7 (1 7 21 35 35 21 7 1) 7, 21, and 35 are all divisible by 7.

3. Figure pattern

1) Polygonal Numbers

Polygonal Numbers are really just the number of vertexes in a figure formed by a certain polygon.

The first number in any group of Polygonal Numbers is always 1, or a point. The second number is equal to the number of vertexes of the polygon.

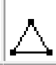
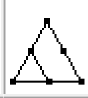
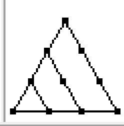
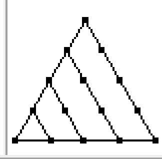
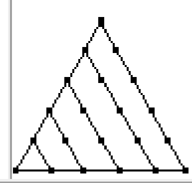
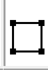
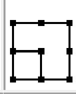
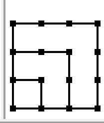
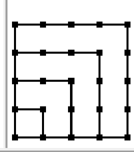
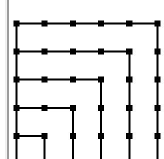
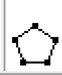
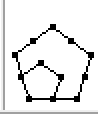
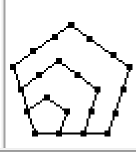
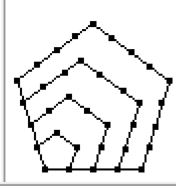
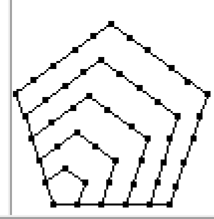
For example, the second Pentagonal Number is 5, since pentagons have 5 vertexes (and sides).

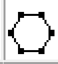
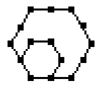
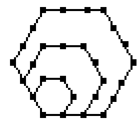
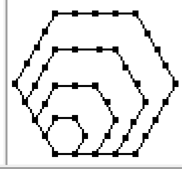
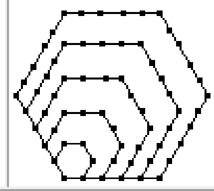
The third Polygonal Number is made by extending two of the sides of the polygon from the second Polygonal Number, completing the larger polygon, and placing vertexes *and other points where necessary*.

The third Polygonal Number is found by adding all the vertexes *and points* in the resulting figure. (Look at the table below for a clearer explanation).

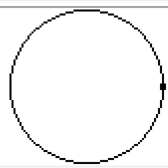
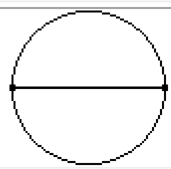
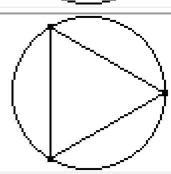
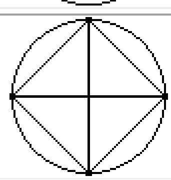
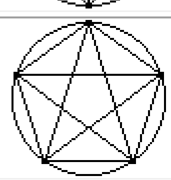
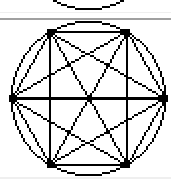
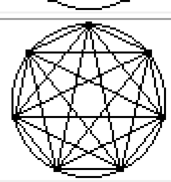
A formula that will generate the n^{th} x-gonal number (for example: the 2^{nd} 3-gonal, or triangular number) is:

$$\frac{n^2-n}{2}\times(x-2)+n$$

Type	**1^{st}**	**2^{nd}**	**3^{rd}**	**4^{th}**	**5^{th}**	**6^{th}**
Triangular						
Value	1	3	6	10	15	21
Square						
Value	1	4	9	16	25	36
Pentagonal						

Value	1	5	12	22	35	51
Hexagonal						
Value	1	6	15	28	45	66

2) Points on a Circle

Image	Points	Segments	Triangles	Quadrilaterals	Pentagons	Hexagons	Heptagons
	1						
	2	1					
	3	3	1				
	4	6	4	1			
	5	10	10	5	1		
	6	15	20	15	6	1	
	7	21	35	35	21	7	1

As you may have noticed, the numbers in the chart above are actually the tip of the right-angled form of Pascal's Triangle, except the preceding 1's in each row are missing. The circular figures are formed

4. Counting Principles

To determine the number of ways that one action can be performed after another, use the **Fundamental Counting Principle**.

1) **Fundamental Counting Principle (FCP)**

If one action can be performed in n ways, then another in m ways, then both actions can be performed, in order, in n x m ways.

Example: In how many ways can a Canadian postal code be made? A postal code has the format A9A 9A9, where A is any letter and 9 is any number.

Think of this problem as selecting a letter, then selecting a number, then a letter, and so on.
There are 26 possible letters, and 10 possible numbers, for each position.
According to the FCP, the total number of postal codes is
26 x 10 x 26 x 10 x 26 x 10 = 17 576 000

Example: A cafeteria offers lunch specials consisting of one item from each category.

Entree	**Beverage**	**Dessert**
Hamburger	Soft Drink	Ice Cream
Sandwich	Milk	Fruit Cup
Wrap	Juice	
Pasta		
Chicken Salad		

Determine the number of possible lunch specials.

Using the FCP, there are 5 x 3 x 2 = 30 lunch specials.

2) **Rule of Sum (RoS)**

If one action can be performed in n ways, and another in m ways, and both actions cannot be performed together, then either action can be performed in n + m ways.

Example: Determine the number of ways of drawing a red face card or a spade from a standard deck of 52 cards.

There are 6 red face cards (J Heart, J Diamond, Q Heart, Q Diamond, K Heart, K Diamond) and 13 spades. Thus, according to the RoS, there are 6 + 13 = 19 ways of drawing a red face card or a spade.

3) Factorial

Question: How many different ways can 8 girls be arranged in line.

1^{st} girl could be anyone of 8 of them, there are 8 choices.
If the 1^{st} girl is decided, then the 2^{nd} girl has 7 choices, and so on.
Therefore by FCP we have 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 40320 ways.

To work more easily with calculations like 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1, we can use factorial notation (**!**).

Example:
(a) 5! .....″five factorial″
    5! = 5 x 4 x 3 x 2 x 1 = 120
(b) 1! = 1
(c) 0! = 1

For a natural number "n" ,
n! = n x (n - 1) x (n - 2) x ....x 3 x 2 x 1
and is read "n factorial"

4) Permutation

A permutation is an ordered selection (or arrangement) of elements from a given set.
A permutation of ″n″ distinct objects taken ″r″ at a time is an arrangement of ″r″ of the ″n″ object in a definite order. The total number of such permutations (arrangements) is denoted by:

$$nPr = P(n,r) = \frac{n!}{(n-r)!}$$

Example: Eight runners are in the 100m final at the track meet. How many different ways could the g old, silver and bronze medal be awarded after the race?

Way 1: There are 8 choices of runner can be awarded with gold, 7 for silver, and 6 for bronze.
8 x 7 x 6 = 336 ways

Way 2: P(8, 3) = 336 ways

5) Combination

A combination of "n" distinct objects, taken "r" at a time is a selection of "r" of the "n" objects witho ut regard for order.

The total # of such combinations is:

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Example: For the school dance a clean up crew of 3 people is needed.
If there are 8 members of students' council available to help, how many different clean up crew groups could be formed?

Order doesn’t matter → Combination → ${}_8C_3$ = 56 groups

Questions in class

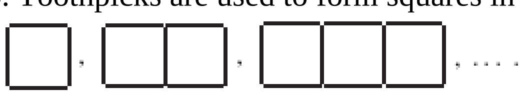
1. The digits 1, 2, 3, 4 can be arranged to form different four-digit numbers. If these numbers are then listed from the smallest to largest, in what position is 3142?

2. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2’s, three 3’s, four 4’s, five 5’s, six 1’s, seven 2’s, and so on. The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ... What is the 100th digit in the sequence?

3. How many integers between 100 and 1000 are multiples of 7?

4. At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?

5. Toothpicks are used to form squares in the pattern shown:



Four toothpicks are used to form one square, seven to form two squares, and so on. If this pattern continues, how many toothpicks will be used to form 10 squares in a row?

6. A woman with a basket of eggs finds that, if she removes the eggs from the basket 2, 3, 4, 5, or 6 at a time, there is always one egg left. However, if she removes the eggs 7 at a time, there are no eggs left. If the basket holds up to 500 eggs, how many eggs does the woman have?

7. Mr. McMath has a penny, nickel, dime, quarter and a half-dollar. How many different sums of money can he form by choosing three of the coins?

8. A whole number is called *decreasing* if each digit of the number is less than the digit to its left. For example, 8540 is a decreasing four-digit number. How many decreasing numbers are there between 100 and 500?

9. The increasing list of five different integers {3, 4, 5, 8, 9} has a sum of 29. How many increasing lists of five different single-digit positive integers have a sum of 33?

10. At a certain party, the first time the doorbell rang 1 guest arrived. On each succeeding ring, two more guests arrived than the previous ring. After 20 rings, what was the number of guests at the party?