AP Calculus Homework 10

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

For the following questions, please refer to the integral table posted on Classkick.

1. Find the general indefinite integral.

a) $\int \frac{x^3 - 2\sqrt{x}}{x} dx$ b) $\int (1 + \tan^2 \alpha) d\alpha$ c)* $\int \frac{\sin 2x}{\sin x} dx$

2. Evaluate the integral.

a) $\int_{-2}^{2} (3u + 1)^2 du$ b) $\int_{-2}^{-1} \left(4y^3 + \frac{2}{y^3}\right) dy$ c) $\int_{1}^{4} \sqrt{\frac{5}{x}} dx$

d) $\int_{\pi/4}^{\pi/3} \sec\theta \tan\theta d\theta$ e) $\int_{0}^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta} d\theta$

f)* $\int_{0}^{1/\sqrt{3}} \frac{t^2 - 1}{t^4 - 1} dt$ g)* $\int_{-1}^{2} (x - 2|x|) dx$

3. The velocity function (in meters per second) is given for a particle moving along a line. Find (a) the displacement and (b) the distance traveled by the particle during the given time interval.

$v(t) = t^2 - 2t - 8, \quad 1 \leqslant t \leqslant 6$

4.* The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find (a) the velocity at time t and (b) the distance traveled during the given time interval.

$a(t) = 2t + 3, \quad v(0) = -4, \quad 0 \leqslant t \leqslant 3$

5. Evaluate the indefinite integral.

a) $\int \frac{\cos\sqrt{t}}{\sqrt{t}} dt$ b) $\int \frac{z^2}{\sqrt[3]{1 + z^3}} dz$ c)* $\int e^{\tan x} \sec^2 x dx$ d) $\int \frac{\sin(\ln x)}{x} dx$

e)* $\int \frac{\sin 2x}{1 + \cos^2 x} dx$ f) $\int \frac{1}{\cos^2 t \sqrt{1 + \tan t}} dt$ g) $\int \frac{1 + x}{1 + x^2} dx$ h)* $\int \frac{x}{\sqrt[4]{x + 2}} dx$

6. Evaluate the definite integral.

a) $\int_0^1 x^2(1+2x^3)^5 dx$ b)* $\int_0^{\pi} \sec^2(t/4)dt$ c) $\int_1^2 \frac{e^{1/x}}{x^2}dx$

d) $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}dx$ e)* $\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$

7. If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.

8. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]dx =$

(A) $2\int_3^5 g(x)dx + 7$ (B) $2\int_3^5 g(x)dx + 14$ (C) $2\int_3^5 g(x)dx + 28$

(D) $\int_3^5 g(x)dx + 7$ (E) $\int_3^5 g(x)dx + 14$

9. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

10. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b (f(x) + g(x))dx = 4$

III. $\int_a^b (f(x)g(x))dx = -5$

(A) I only (B) II only (C) III only

(D) II and III only (E) I, II, and III

11. $\int (x^2+1)^2 dx =$

(A) $\frac{(x^2+1)^3}{3} + C$ (B) $\frac{(x^2+1)^3}{6x} + C$ (C) $\left(\frac{x^3}{3} + x\right)^2 + C$

(D) $\frac{2x(x^2+1)^3}{3} + C$ (E) $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

12. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

(A) $2\sqrt{x^3+1} + C$ (B) $\frac{3}{2}\sqrt{x^3+1} + C$ (C) $\sqrt{x^3+1} + C$

(D) $\ln\sqrt{x^3+1} + C$ (E) $\ln(2x^3+1) + C$