AP Calculus Homework 20

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Eliminate the parameter to find a Cartesian equation of the curve

a) $x = t^2 - 2$, $y = 5 - 2t$, $-3 \leqslant x \leqslant 4$

b) $x = \sqrt{t}$, $y = 1 - t$

c) $x = 4\cos\theta$, $y = 5\sin\theta$, $-\pi/2 \leqslant \theta \leqslant \pi/2$

d) $x = e^{2t}$, $y = t + 1$

2. Find an equation of the tangent to the curve at the given point.

a) $x = 1 + \ln t,\;\; y = t^2 + 2;\quad (1, 3)$

b) $x = 6\sin t,\;\; y = t^2 + t;\quad (0, 0)$

3. Find the area enclosed by the x-axis and the curve $x = 1 + e^t$, $y = t - t^2$.

4. Set up an integral that represents the length of the curve. Then use a calculator or online integrator to find the length correct to four decimal places.

a) $x = t - t^2,\;\; y = \frac{4}{3}t^{3/2},\;\; 1 \leqslant t \leqslant 2$

b) $x = t + \cos t,\;\; y = t - \sin t,\;\; 0 \leqslant t \leqslant 2\pi$

5. Find the exact length of the curve.

a) $x = 1 + 3t^2,\;\; y = 4 + 2t^3,\;\; 0 \leqslant t \leqslant 1$

b) $x = \dfrac{t}{1+t},\;\; y = \ln(1 + t),\;\; 0 \leqslant t \leqslant 2$

6. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x - 1$

(D) $y = 4x - 5$ (E) $y = 8x + 13$

7. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only (B) 1 only (C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1 (E) No value

8. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \leq t \leq 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

9. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \leq t \leq 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1}dt$ (B) $\int_0^1 \sqrt{t^2 + t}dt$ (C) $\int_0^1 \sqrt{t^4 + t^2}dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}dt$ (E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}dt$