Chain Rule

More practice, more fun 😊

Differentiate each of the following

1. $f(x) = \ln x^{10}$
2. $f(x) = \cos(10x)$
3. $f(x) = \left(5x^4 + 12x + 3\right)^{100}$
4. $f(x) = \ln\left(\sqrt{x^3 - x - 10}\right)$
5. $f(x) = \sin\left(x^3 - 2x\right)$
6. $f(x) = \cos\left(\frac{\pi}{2}x\right)$
7. $f(x) = \sqrt{x^2 - \frac{1}{x^2}}$
8. $f(x) = \frac{1}{5x^2 - 6x + 19}$
9. $f(x) = \ln\left(\sin\left(1 - x^2\right)\right)$
10. $f(x) = \cos\left(\ln\left(1 - x^2\right)\right)$
11. $f(x) = \frac{1}{\cos^3 x}$
12. $f(x) = \sin^3 x + \cos^3 x$
13. $f(x) = (5 - x)^{100}$
14. $f(x) = (5 - x)^{17}$
15. $f(x) = \cos\left(x^4 + 3x^2 + 1\right)$
16. $f(x) = \sqrt{1 - x^2}$
17. $f(x) = \frac{1}{\sin x}$
18. $f(x) = \ln^3(2x - 1)$

19. Let g be a differentiable function with $g(2) = 4$ and $g'(2) = -3$. Compute the exact value of $f'(2)$ if f is defined as

a) $f(x) = 2g(x) - 3$

b) $f(x) = (g(x))^3$

c) $f(x) = \ln(g(x))$

d) $f(x) = \cos(g(x))$

e) $f(x) = \frac{1}{(g(x))^3}$

f) $f(x) = \frac{1}{g(x)}$

Answers

1.) $f'(x) = \dfrac{10}{x}$ 2.) $f'(x) = -10\sin(10x)$ 3.) $f'(x) = 100\left(12x + 5x^4 + 3\right)^{99}\left(20x^3 + 12\right)$

4.) $f'(x) = \dfrac{3x^2 - 1}{2\left(x^3 - x - 10\right)}$ 5.) $f'(x) = \left(3x^2 - 2\right)\cos\left(x^3 - 2x\right)$

6.) $f'(x) = -\dfrac{\pi}{2}\sin\left(\dfrac{\pi}{2}x\right)$ 7.) $f'(x) = \dfrac{x + \dfrac{1}{x^3}}{\sqrt{x^2 - \dfrac{1}{x^2}}}$ 8.) $f'(x) = -\dfrac{10x - 6}{\left(5x^2 - 6x + 19\right)^2}$

9.) $f(x) = -2x\dfrac{\cos\left(1 - x^2\right)}{\sin\left(1 - x^2\right)}$ 10.) $f'(x) = \dfrac{2x}{1 - x^2}\sin\left(\ln\left(1 - x^2\right)\right)$ 11.) $f'(x) = \dfrac{3}{\cos^4 x}\sin x$

12.) $f'(x) = 3\cos x\sin^2 x - 3\cos^2 x\sin x$ 13.) $f'(x) = -100\left(5 - x\right)^{99} = 100\left(x - 5\right)^{99}$

14.) $f'(x) = -17\left(5 - x\right)^{16} = -17\left(x - 5\right)^{16}$ 15.) $f'(x) = -\left(4x^3 + 6x\right)\sin\left(x^4 + 3x^2 + 1\right)$

16.) $f'(x) = \dfrac{-x}{\sqrt{1 - x^2}}$ 17.) $f'(x) = -\dfrac{\cos x}{\sin^2 x}$ 18.) $f'(x) = \dfrac{6\left(\ln^2\left(2x - 1\right)\right)}{2x - 1}$

19.) a) -6 b) -144 c) $-\dfrac{3}{4}$ d) $3\sin 4$ e) $\dfrac{9}{256}$ f) $\dfrac{3}{16}$