AP Calculus In-Class Eight – Antiderivatives and the Definite Integral

4.1 Antiderivatives; 4.2 Area

1. Find the most general antiderivatives of the functions.

(a) $f(x) = 4x^2 - 8x + 1$

(b) $f(x) = 4/x^7 - 7/x^4 + x$

2. Solve the differential equations subject to the given boundary conditions.

(a) $f'(x) = 9x^2 + x - 8, \quad f(-1) = 1$

(b) $f'''(x) = (\pi^3/2)\sin(\pi x/4), \quad f''(0) = 4 - 2\pi^2, \quad f'(0) = 0, \quad f(0) = 32$

3. Evaluate the integrals without using your calculator.

(a) $\int \left(3x^2 - 2x + 3\right) dx$

(b) $\int \left(x - \frac{1}{2x} \right)^2 dx$

(c) $\int \sqrt{4 - 2t}\ dt$

(d) $\int \frac{dx}{3(2x-1)^2}$

4. Evaluate the integrals without using your calculator.

(a) $\int \frac{dy}{\sqrt{4 - y^2}}$

(b) $\int \frac{ydy}{\sqrt{4 - y^2}}$

(c) $\int \frac{2x+1}{2x}\,dx$

(d) $\int \frac{(x-2)^3}{x^2}\,dx$

5. Evaluate the integrals without using your calculator.

(a) $\int \left(4x^{1/3} - 5x^{3/2} - x^{-1/2}\right) dx$

(b) $\int \left(\frac{x^3 - x - 1}{x^2}\right) dx$

(c) $\int \frac{dy}{\sqrt{y}\left(1-\sqrt{y}\right)}$

(d) $\int \frac{u\,du}{\sqrt{4-9u^2}}$

6. Evaluate the integrals without using your calculator.

(a) $\int x\cos x\ dx$

(b) $\int \frac{du}{\cos^2 3u}$

(c) $\int \frac{\cos x dx}{\sqrt{1+\sin x}}$

(d) $\int \frac{\cos(\theta-1)}{\sin^2(\theta-1)}\ d\theta$

7. Evaluate the integrals without using your calculator.

(a) $\int \frac{\sin 2t}{1-\cos 2t}\ dt$

(b) $\int \frac{e^x}{e^x-1}\ dx$

(c) $\int \frac{(x-1)dx}{x(x-2)}$

(d) $\int xe^{x^2} dx$

8. Evaluate the integrals without using your calculator.

(a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\int xe^{-x}dx$

(c) $\int \frac{e^{x}}{1+e^{2x}}\,dx$

(d) $\int \frac{\ln\sqrt{x}\;dx}{x}$

9. Evaluate the integrals without using your calculator.

(a) $\int \frac{dv}{v\ln v}$

(b) $\int \frac{y-1}{y+1}\,dy$

10. Evaluate the integrals without using your calculator.

(a) $\int \frac{2x+1}{4+x^2}\,dx$

(b) $\int \frac{1-x}{\sqrt{1-x^2}}\,dx$

(c) $\int \frac{e^{2x}}{1+e^x}\,dx$

(d) $\int \frac{\cos\theta}{1+\sin^2\theta}\,d\theta$

11. An object is dropped from a height of 300 m. Neglecting air resistance, find the distance it falls in *t* s. What is its velocity at the end of 3 s? When will it strike the ground?

12. If an automobile starts from rest, what constant acceleration will enable it travel 150 m in 10 s?

13. The volume V of as a balloon is changing with respect to time t at a rate given by $dV/dt = 3\sqrt{t} + t/4$ m^3/s. if, at t = 4s, the volume is 6 m^3, express V as a function of t.

14. If $f(x) = 8 - x^2/2$, find the Riemann sum R_p of $f(x)$ where P is the partition of [0, 6] into the six equal subintervals determined by $x_0 = 0$ and $x_6 = 6$, and w_i is the midpoint of the interval $[x_{i-1}, x_i]$.

15. Find the area under the graph of $f(x)$ from a to b using inscribed rectangles. Sketch the graph and typical rectangles, labeling the drawing.

    $f(x) = 3x^2 + 5; \quad a = 1, \ b = 4$

16. Find the area under the graph of $f(x)$ from a to b using circumscribed rectangles. Sketch the graph and typical rectangles, labeling the drawing.

    (a) $f(x) = x^3$; $a = -2$, $b = 6$