

AP Calculus Homework 1

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write $/\pi$ instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Sketch the graph of an example of a function f that satisfies all of the given conditions

a) $\lim_{x \rightarrow 0^-} f(x) = 1$, $\lim_{x \rightarrow 0^+} f(x) = -1$, $\lim_{x \rightarrow 2^-} f(x) = 0$, $\lim_{x \rightarrow 2^+} f(x) = 1$,

$f(2) = 1$, $f(0) = \text{undefined}$

b) $\lim_{x \rightarrow 1} f(x) = 3$, $\lim_{x \rightarrow 4^-} f(x) = 3$, $\lim_{x \rightarrow 4^+} f(x) = 3$, $f(1) = 1$, $f(4) = -1$

2. Given that

$$\lim_{x \rightarrow 2} f(x) = 4, \lim_{x \rightarrow 2} g(x) = -2, \lim_{x \rightarrow 2} h(x) = 0$$

find the following limits

a) $\lim_{x \rightarrow 2} [f(x) + 5g(x)]$ b) $\lim_{x \rightarrow 2} \sqrt{f(x)}$ c) $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$

3. Evaluate the limit, if it exists.

a) $\lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2}$ b) $\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$ c) $\lim_{x \rightarrow -4} \frac{1/4 + 1/x}{4 + x}$

d) $\lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$ e)* $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$

4. Prove the following limits

a) $\lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = 0$ b)* $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\sin \pi/x} = 0$

5. Find the limit, if it exists. If the limit does not exist, explain why.

a) $\lim_{x \rightarrow -6} \frac{2x + 12}{|x + 6|}$ b) $\lim_{x \rightarrow 0.5^-} \frac{2x - 1}{|2x^3 - x^2|}$

6. Find the limit

$$\begin{array}{lll} \text{a)* } \lim_{x \rightarrow 0} \frac{\sin 3x}{x} & \text{b) } \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos^2 x} & \text{c) } \lim_{x \rightarrow 0} \frac{4x}{\tan x} \\ \text{d) } \lim_{x \rightarrow 0} 3 \frac{x}{\sin x} & \text{e)* } \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x} & \text{f)* } \lim_{x \rightarrow 0} \frac{x^2 \sin x}{1 - \cos^2 x} & \text{g) } \lim_{x \rightarrow 0} \frac{\sin^2 7x}{\sin^2 11x} \end{array}$$

7. Determine the infinite limit.

$$\text{a) } \lim_{x \rightarrow 1} \frac{2 - x}{(x - 1)^2} \quad \text{b) } \lim_{x \rightarrow 3^+} \ln(x^2 - 9) \quad \text{c) } \lim_{x \rightarrow 2\pi^-} x \csc x$$

8. Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

9. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x \rightarrow -2} f(x) = \infty, \quad \lim_{x \rightarrow -\infty} f(x) = 3, \quad \lim_{x \rightarrow \infty} f(x) = -3$$

10. Find the limit

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} & \text{b)* } \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x}) & \text{c) } \lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2} \\ \text{d)* } \lim_{x \rightarrow \infty} \frac{1 - e^x}{1 + 2e^x} & \text{e) } \lim_{x \rightarrow \infty} (e^{-2x} \cos x) & \end{array}$$

11*. Find the horizontal and vertical asymptotes of the curve

$$y = \frac{1 + x^4}{x^2 - x^4}$$