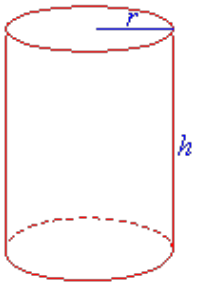
3D Geometry

1. Cylinder

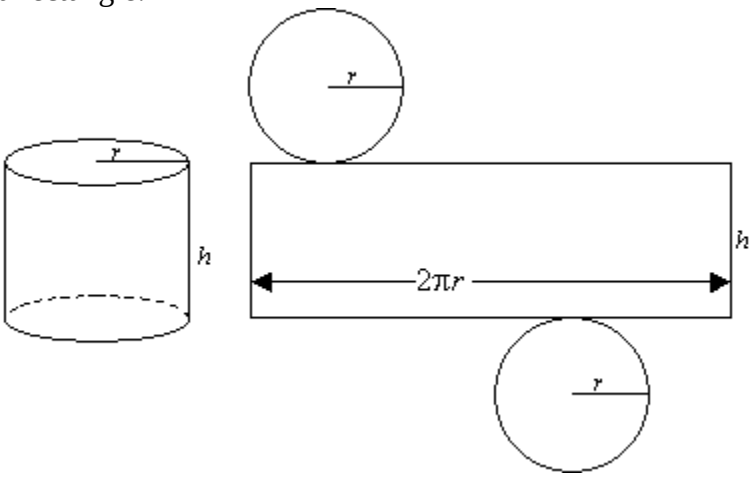
A cylinder is a solid with two congruent circles joined by a curved surface.



In the above figure, the radius of the circular base is r and the height is h. The volume of the cylinder is the area of the base × height.

$$\text{Volume of cylinder} = \pi r^2 h$$

The net of a solid **cylinder** consists of 2 circles and one rectangle. The curved surface opens up to form a rectangle.

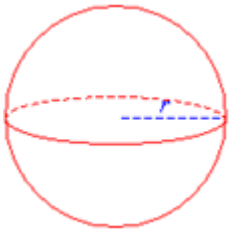


Surface area = 2 × area of circle + area of rectangle

Surface area of cylinder = $2\pi r^2 + 2\pi rh = 2\pi r\,(r + h)$

2. Sphere

A sphere is a solid with all its points the same distance from the center.



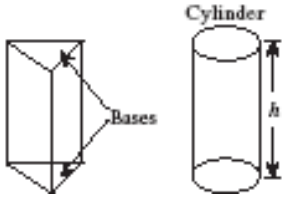
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$

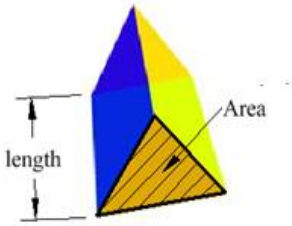
3. Cylinders and Prisms

A prism is a 3 dimensional figure, which consists of 2 parallel, congruent (identical) faces called bases and several lateral faces formed by connecting corresponding points on the lower and upper bases. Cubes and boxes are examples of regular prisms.

The height of the prism is the perpendicular distance between the bases. A cylinder is a prism whose bases are circular. In general the volume V of a prism is given by $V = Ah$ where A is the area of the bases and h is the height. Thus for a cylinder, the volume $V = \pi r^2 h$ and the surface area $S = 2\pi rh + 2\pi r^2$.



The Volume of a prism is simply the area of one end times the length of the prism

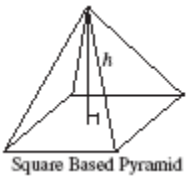


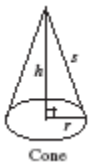
Volume = Area × Length

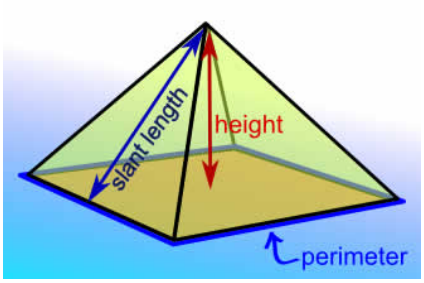
Example: What is the volume of a prism whose ends are 25 in^2 and which is 12 in long:
Answer: Volume = 25 in^2 × 12 in = 300 in^3

4. Cones and Pyramids

A pyramid is a solid figure with a polygonal base and a number of triangular lateral faces, all of which share a common point, called the vertex of the pyramid. The most common pyramids are the triangular pyramid or tetrahedron and the square or rectangular based pyramid. If the base is circular rather than polygonal we have a cone. For all these solid shapes the volume is $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height.



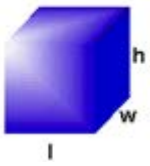
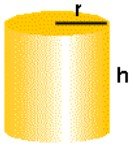
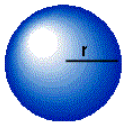
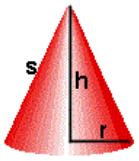
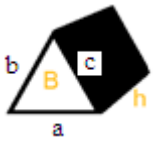
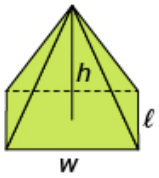
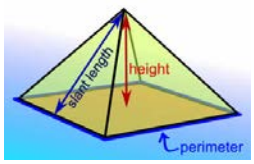




The Volume of a Pyramid
$^1/_3$ × [Base Area] × Height

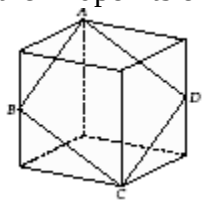
The Surface Area of a Pyramid
When all side faces are the same:
[Base Area] + $^1/_2$ × Perimeter × [Slant Length]
When side faces are different:
[Base Area] + [Lateral Area]

►Volume (V) and Surface Area (SA) Formulas

Name	Shapes	Formula
Rectangular Solid		$Volume = Length \cdot Width \cdot Height$ $V = lwh$ SA: Surface Area $SA = 2lh + 2hw + 2lw$
Cylinder		$Volume = \pi r^2 \cdot height$ $V = \pi r^2 h$ SA: Surface Area $SA = 2\pi rh + 2\pi r^2$
Sphere		$V = \frac{4}{3}\pi r^3$ SA: Surface Area $SA = 4\pi r^2 = \pi d^2$
Cone		$V = \frac{1}{3}\pi r^2 h$ SA: Surface Area $SA = s\pi r + \pi r^2$, $s = \sqrt{r^2 + h^2}$
Prism		$V = \frac{1}{2}Bh$ (B: Area) SA: Surface Area $SA = 2B + Ph$ $SA = 2B + (a + b + c) \cdot h$
Pyramid		$V = \frac{1}{3}Bh = \frac{1}{3}wlh$, where B is the area of the base.
Pyramid		SA: Surface Area When all side faces are the same: [Base Area] + $^1/_2$ × Perimeter × [Slant Length] When side faces are different: [Base Area] + [Lateral Area]

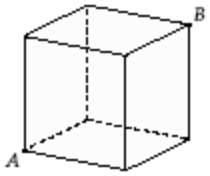
Questions in class

1. A cube with an edge length of 6 is cut by a plane to form a quadrilateral *ABCD*, where *B* and *D* are the midpoints of two edges of the cube. What is the area of the quadrilateral *ABCD?*



2. A company wants to construct a rectangular box that will hold exactly 150 cubes each of dimension $1 \times 1 \times 1$ centimetre. What is the minimum possible surface area of the box, measured in square centimetres?

3. The shortest path on the surface of a cube from vertex *A* to the furthest vertex *B* involves crossing a certain number of faces and edges of the cube. See the diagram.



(a) How many faces and how many edges must be crossed?
(b) How many such shortest paths are there from vertex *A* to vertex *B*?
(c) If we call the intersection of a shortest path with an edge a "corner", what is the figure formed by the set of all corners?

4. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.

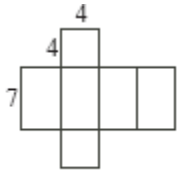


Figure 1

(b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point A, an ant walked to point B crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from A to B. Determine the length of AB in Figure 3.

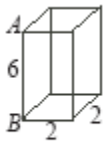


Figure 2

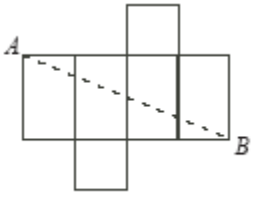


Figure 3

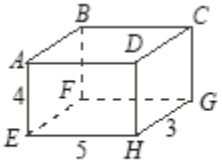
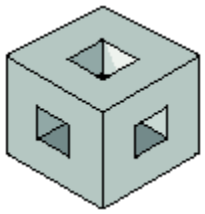


Figure 4

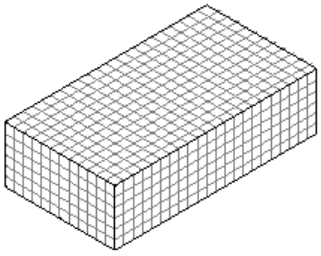
(c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner A. Determine, with justification, the shortest possible distance from A to G along the surface of the block.

5. A 3 by 3 by 3 cube has three holes, each with a 1 by 1 by 1 cross-section running from the center of each face to the center of the opposite face. What is the total surface area (in square units) of the resulting solid?



6. A solid rectangular block with a 5 cm by 5 cm square base has a height of x cm. If the *surface area* of the block is 120 cm^2, then x equals what?

7. A 6 cm × 12 cm × 22 cm rectangular block of wood is painted red and then cut into small cubes, each of which has a surface area of 6 cm^2. What is the number of small cubes that have red paint on exactly two faces?



8. A glass box 7 cm by 12 cm by 18 cm, closed on all six sides, is partly filled with coloured water. When the box is placed on one of its 7 by 12 sides, the water level is 15 cm above the table. When the box is placed on one of its 7 by 18 sides, what is the water level above the table, in cm?