AP Calculus Homework 7

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

For questions 1 - 4,

a) Find the vertical and horizontal asymptotes.

b) Find the intervals of increase or decrease.

c)* Find the local maximum and minimum values.

d)* Find the intervals of concavity and the inflection points.

1. $f(x) = \sqrt{x^2+1} - x$

2. $f(x) = x\tan x, \quad -\pi/2 < x < \pi/2$

3. $f(x) = \ln(1 - \ln x)$

4. $f(x) = e^{-1/(x+1)}$

5. Suppose the derivative of a function f is $f'(x) = (x+1)^2(x-3)^5(x-6)^4$. On what interval is f increasing?

6. For what values of the numbers a and b does the function

$$f(x) = axe^{bx^2}$$

have the maximum value $f(2) = 1$?

7.** Show that the curve $y = (1+x)/(1+x^2)$ has three points of inflection and they all lie on one straight line.

8. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital Rule doesn't apply, explain why.

a) $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$ b)* $\lim_{x\to\infty} \dfrac{e^x}{x^3}$ c) $\lim_{x\to 0} \dfrac{\sin^{-1} x}{x}$

d)* $\lim_{x\to 1} \dfrac{x^a - ax + a - 1}{(x-1)^2}$ e) $\lim_{x\to 0} \cot 2x \sin 6x$ f) $\lim_{x\to\infty} (\sqrt{x^2+x} - x)$

g) $\lim_{x\to 0^+} x^{x^2}$ h)* $\lim_{x\to\infty} \left(1 + \dfrac{3}{x} + \dfrac{5}{x^2}\right)^x$ i) $\lim_{x\to 0^+} (4x+1)^{\cot x}$

9. What happens if you try to use l'Hospital's Rule to evaluate

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

Evaluate the limit using another method.

10. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

11. A box with a square base and open top must have a volume of 32 000 cm^2. Find the dimensions of the box that minimize the amount of material used.

12.* Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

13. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius r.

14.* The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area.

15. A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.

16.** At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?