AP Calculus Homework Four – Differentiation
2.3 Implicit Differentiation; 2.4 Estimating a Derivative; 2.5 Derivative of the Inverse of a Function

1. Find $\dfrac{dy}{dx}$

(a) $x + \cos(x + y) = 0$

(b) $\sin x - \cos y - 2 = 0$

(c) $3x^2 - 2xy + 5y^2 = 1$

(d) $\sin(xy) = x$

(e) $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$

(f) $\begin{cases} x = 1 - e^{-t} \\ y = t + e^{-t} \end{cases}$

(g) $\begin{cases} x = \dfrac{1}{1-t} \\ y = 1 - \ln(1-t) \end{cases}$

2. Find $\dfrac{d^2 y}{dx^2}$

(a) $x^2 + y^2 = 25$

(b) $\begin{cases} x = t^2 - 1 \\ y = t^4 - 2t^3 \end{cases}$

(c) $\begin{cases} x = \cos t \\ y = \cos 2t \end{cases}$

3. In the following eight sub questions, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	−4
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

(a) If $A = f + 2g$, find $A'(3)$

(b) If $B = f \times g$, find $B'(2)$

(c) If $D = \dfrac{1}{g}$, find $D'(1)$

(d) If $H(x) = \sqrt{f(x)}$, find $H'(3)$

(e) If $K(x) = \left(\dfrac{f}{g}\right)(x)$, find $K'(0)$

(f) If $M(x) = f(g(x))$, find $M'(1)$

(g) If $P(x) = f(x^3)$, find $P'(1)$

(h) If $S(x) = f^{-1}(x)$, find $S'(3)$

4. From the values of f shown in the table below, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

5. Using the values shown in the table in Q7, estimate $\left(f^{-1}\right)'(4)$.

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6. The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \leq 4$ is a one-to-one function; therefore its inverse is also a function. Call that inverse *g*. Find $g'(3)$.

7. At how many points on the interval [a, e] does the function graphed satisfy the Mean Value Theorem?