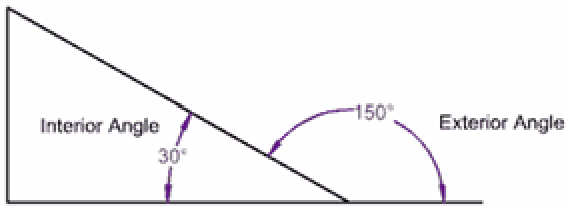
Geometry 1

1. Interior and Exterior Angle

An Interior Angle is an angle inside a shape.

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



Note: If you add up the Interior Angle and Exterior Angle you get a straight line, 180°.

1) Interior Angles sum of Polygons

The sum of the measures of the interior angles of a polygon with n sides is (n-2)180.

For examples:

- Triangle or ('3 - gon')
    - sum of interior angles: (3-2) 180 = 180°

- Quadrilateral which has four sides (' 4 - gon')
    - sum of interior angles: (4-2)180 = 360°

- Hexagon which has six sides ('6 - gon')
    - sum of interior angles: (6-2)180 = 720°

An interior angle of a regular polygon with n sides is $\frac{(n-2)\times 180}{n}$.

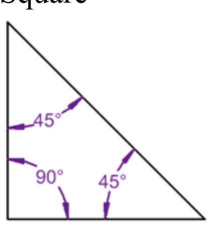
Example:

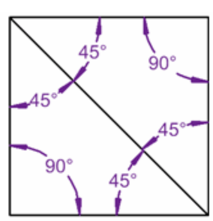
To find the measure of an interior angle of a regular octagon, which has 8 sides, apply the formula above as follows:
((8-2) × 180) /8 = 135°

There are Two Triangles in a Square

The internal angles in this triangle add up to 180°

(90°+45°+45°=180°)





... and for this square they add up to **360°**

... because the square can be made from two triangles!

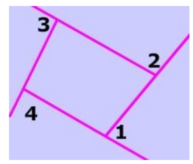
2) Exterior Angles sum of Polygons

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

For example:
A Polygon is any flat shape with straight sides.

The Exterior Angles of a Polygon add up to 360°.
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$



2. Pythagorean Triples

A "Pythagorean Triple" is a set of positive integers, **a**, **b** and **c** that fits the rule:

$$a^2 + b^2 = c^2$$

1) Endless

The set of Pythagorean Triples is endless.

It is easy to prove this with the help of the first Pythagorean Triple, (3, 4, and 5):
Let n be any integer greater than 1, then 3n, 4n and 5n would also be a set of Pythagorean Triple.

This is true because: $(3n)^2 + (4n)^2 = (5n)^2$

Examples:

n	**(3n, 4n, 5n)**
2	(6,8,10)
3	(9,12,15)
...	... etc ...

So, you can make infinite triples just using the (3,4,5) triple.

2) Euclid's Proof that there are Infinitely Many Pythagorean Triples

However, Euclid used a different reasoning to prove the set of Pythagorean Triples is unending.
The proof was based on the fact that the difference of the squares of any two consecutive numbers is always an odd number.

Examples:

$2^2 - 1^2 = 4 - 1 = \mathbf{3}$ (an odd number),
$15^2 - 14^2 = 225\text{-}196 = \mathbf{29}$ (an odd number)

3) Properties

It can be observed that a Pythagorean Triple always consists of:

- all even numbers, or
- two odd numbers and an even number.

A Pythagorean Triple can never be made up of all odd numbers or two even numbers and one odd number. This is true because:

- The square of an odd number is an odd number and the square of an even number is an even number.
- The sum of two even numbers is an even number and the sum of an odd number and an even number is in odd number.

Therefore, if one of a and b is odd and the other is even, c would have to be odd. Similarly, if both a and b are even, c would be an even number too!

4) Constructing Pythagorean Triples

It is easy to construct sets of Pythagorean Triples.

When **m** and **n** are any two positive integers ($m < n$):
$a = n^2 - m^2$
$b = 2nm$
$c = n^2 + m^2$
Then, a, b, and c form a Pythagorean Triple.

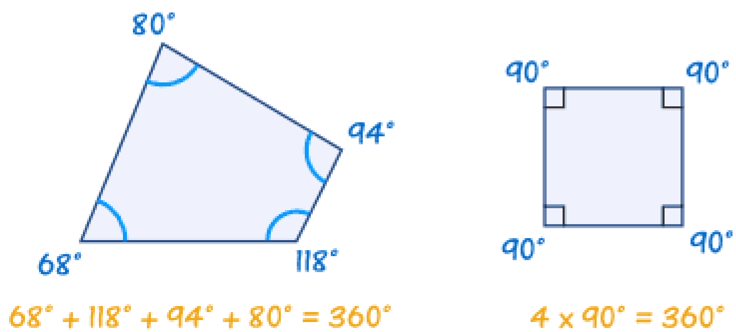
Example:
$m=1$ and $n=2$
$a = 2^2 - 1^2 = 4 - 1 = \mathbf{3}$
$b = 2 \times 2 \times 1 = \mathbf{4}$
$c = 2^2 + 1^2 = \mathbf{5}$
Thus, we obtain the first Pythagorean Triple (3,4,5).

Similarly, when m=2 and n=3 we get the next Pythagorean Triple (5,12,13).

3. Four sides (or edges)

1) Properties

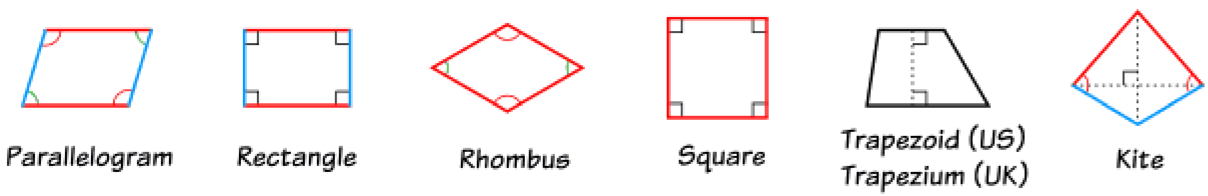
Four vertices (or corners). The interior angles add up to **360 degrees**:



Try drawing a quadrilateral, and measure the angles. They should add to **360°**

2) Types of Quadrilaterals

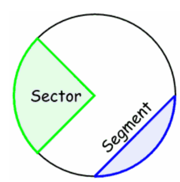
There are special types of quadrilateral:



Some types are also included in the definition of other types! For example a **square**, **rhombus** and **rectangle** are also ***parallelograms***.

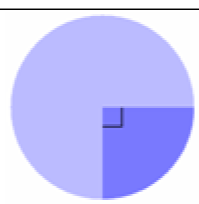
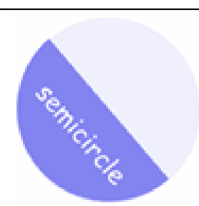
4. Circle Sector and Segment

Slices
There are two main "slices" of a circle:
The "pizza" slice is called a **Sector**.
And the slice made by a chord is called a **Segment**.



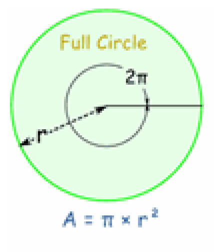
1) Common Sectors

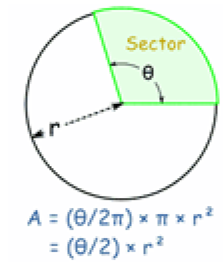
The Quadrant and Semicircle are two special types of Sector:

Quarter of a circle is called a **Quadrant**	Half a circle is called a **Semicircle.**
	

2) Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
Note: I am using radians for the angles.





This is the reasoning:

- A circle has an angle of 2π and an Area of: πr^2
- So a Sector with an angle of θ (instead of 2π) must have an area of: $(\theta/2\pi) \times \pi r^2$
- Which can be simplified to: $(\theta/2) \times r^2$

Area of Sector = $\frac{1}{2} \times \theta \times r^2$ ***(when θ is in radians)***

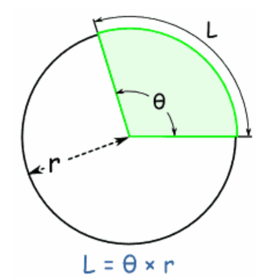
Area of Sector = $\frac{1}{2} \times (\theta \times \pi/180) \times r^2$ ***(when θ is in degrees)***

3) Arc Length of Sector or Segment

By the same reasoning, the arc length (of a Sector or Segment) is:

Arc Length "L" = $\theta \times r$ ***(when θ is in radians)***

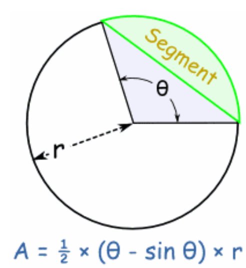
Arc Length "L" = $(\theta \times \pi/180) \times r$ ***(when θ is in degrees)***



4) Area of Segment

The Area of a Segment is the area of a sector minus the triangular piece (shown in light blue here).

There is a lengthy derivation, but the result is a slight modification of the Sector formula:



Area of Segment = $\frac{1}{2} \times (\theta - \sin\theta) \times r^2$ ***(when θ is in radians)***

Area of Segment = $\frac{1}{2} \times ((\theta \times \pi/180) - \sin\theta) \times r^2$ ***(when θ is in egrees)***

Please remember following formulas

1. Area of a triangle $= \frac{1}{2}$ (base) (height).

2. Area of a rectangle = (length) (breadth).

3. Perimeter of a rectangle = 2 (length) +2 (breadth).

4. Area of a square = (side)^2.

5. Perimeter of a square = 4 (Side).

6. Volume of a cube = x^3 where (x) is the length of the side.

7. Surface area of a cube = $6x^2$.

8. Area of a trapezium = $\frac{1}{2}$ (sum of parallel side) × (distance between the parallel sides).

9. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ where (a) is the length of a side.

10. Parallelogram Area = base × height

11. Area of a circle of radius (r) is πr^2.

12. Perimeter (or circumference) of a circle of radius (r) is $2\pi r$.

13. Volume of a sphere of radius (r) is $\frac{4}{3}\pi r^3$.

14. Surface area of a sphere of radius (r) is $4\pi r^2$.

15. Volume of right circular cone is $\frac{1}{3}\pi r^2 h$, where (r) is the radius of the base and (h) is the height of the cone.

16. Curved surface area of a cone is πrl where (l) is the slant height of the cone.

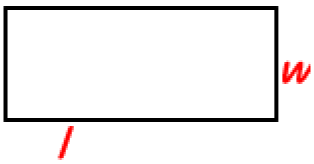
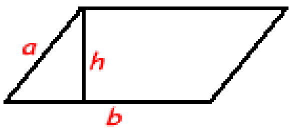
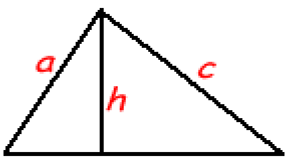
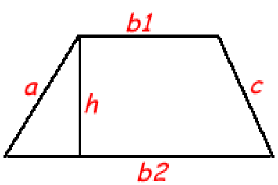
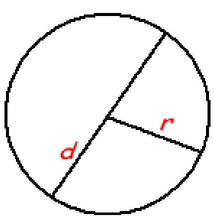
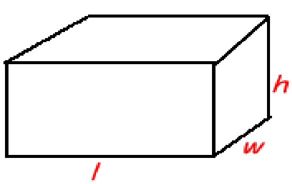
17. Total surface area of a cone is $\pi rl + \pi r^2$.

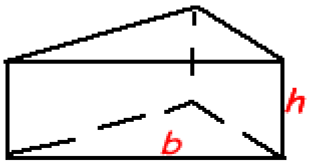
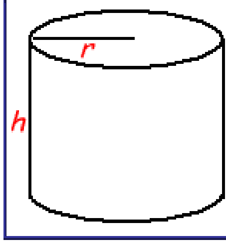
18. Volume of a right circular cylinder is $\pi r^2 h$.

19. Curved surface area of a right circular cylinder is $2\pi rh$.

20. Total surface area of a right circular cylinder is $2\pi rh + 2\pi r^2$.

►Geometric formula

Shapes	**Formula**
	Rectangle: Area = Length X Width A = lw Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w
	Parallelogram Area = Base X Height a = bh
	Triangle Area = 1/2 of the base X the height a = 1/2 bh Perimeter = a + b + c (add the length of the three sides)
	Trapezoid area $A = \left(\frac{b_1 + b_2}{2}\right)h$ Perimeter = a + b1 + b2 + c
	Circle Try the Online tool. The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14) More about circles. d = 2r c = d = 2π r A = πr^2 □(π=3.14)
	Rectangular Solid Volume = Length X Width X Height V = lwh Surface = 2lw + 2lh + 2wh

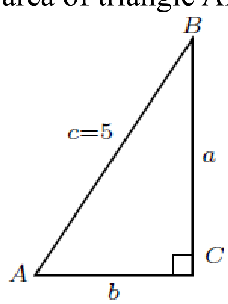
	Prisms Volume = Base X Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	Cylinder Volume = πr^2 x height V = πr^2 h Surface = 2π radius x height S = $2\pi rh + 2\pi r^2$

These Geometric formulae come from TDSB website

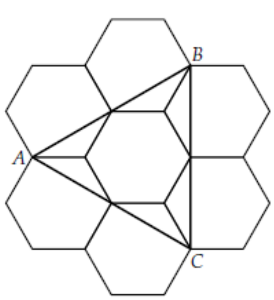
In-class questions

1. A square whose side length is *r* has a square inside it whose area is one-half the area of the larger square. There is a uniform border between the two squares. What is the width of the border?

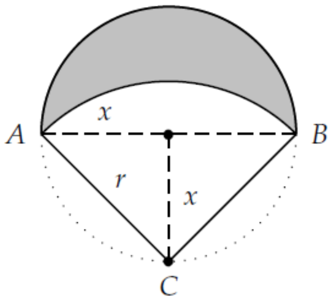
2. In the diagram ΔABC is a right triangle with the right angle at C, $a+b=\sqrt{45}$, *and* $c=5$. What is the area of triangle ABC?



3. In the diagram, six regular hexagons surround a regular hexagon of side length 1. What is the area of triangle *ABC?*



4. The shaded crescent shown is bounded by a semi-circle and the arc of a second circle whose centre is on the first circle. If the second circle has radius r, then what is the area of the crescent?



5. The longer base of a trapezoid has length 15 and the line segment joining the midpoints of the two diagonals has length 3. What is the length of the shorter base of the trapezoid?

6. In Figure 1, ΔABC has points D and E on AB and AC, respectively, so that DE is parallel to BC. In this case, ΔABC and ΔADE are similar triangles because their corresponding angles are equal. These triangles thus have the property that

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}.$$

In Figure 2, WXY Z is a trapezoid with WX parallel to ZY. Also, points M and N and are on WZ and XY, respectively, with MN parallel to WX and ZY.

a) Suppose that, in Figure 1, $DE = 6,\ BC = 10,\ EC = 3,\ and\ AE = x.$ Determine the value of x.

b) Suppose that, in Figure 2, $\frac{WX}{ZY} = \frac{3}{4}$, and $\frac{WM}{MZ} = \frac{XN}{MZ} = \frac{2}{3}$. Determine the value of $\frac{WX}{MN}$

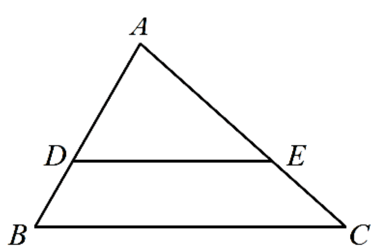


Figure 1

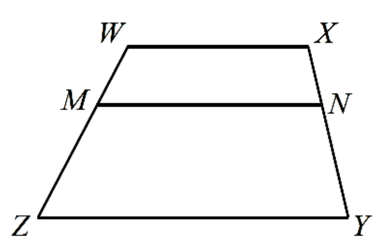


Figure 2