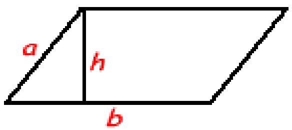
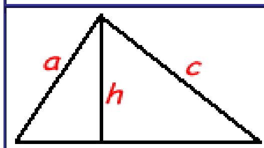
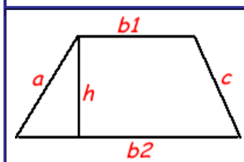
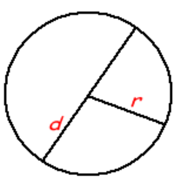
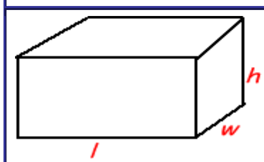
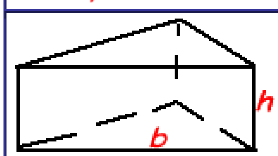
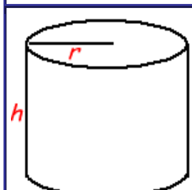
Geometry

1. Geometric formula

Shapes	**Formula**
	Rectangle: Area = Length × Width A = lw Perimeter = 2 × Lengths + 2 × Widths P = 2l + 2w
	Parallelogram Area = Base X Height a = bh
	Triangle Area = 1/2 of the base × the height a = 1/2 bh Perimeter = a + b + c (add the length of the three sides)
	Trapezoid area $A = \left(\frac{b_1 + b_2}{2}\right)h$ Perimeter = a + b1 + b2 + c
	The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (Pi = 3.14) More about circles. d = 2r c = d = 2π r A = πr^2 □(π=3.14)
	Rectangular Solid Volume = Length × Width × Height V = lwh Surface = 2lw + 2lh + 2wh
	Prisms Volume = Base × Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	Cylinder Volume = $\pi r^2 \times$ height V = πr^2 h Surface = 2π × radius × height S = $2\pi rh + 2\pi r^2$

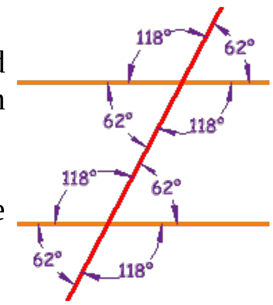
These Geometric formulae come from TDSB website

2. Parallel Lines and Pairs of Angles

1) Pairs of Angles

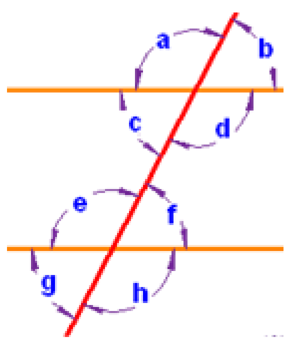
When parallel lines get crossed by another line (which is called a Transversal), you can see that many angles are the same, as in this example:

These angles can be made into pairs of angles which have special names.



2) Testing for Parallel Lines

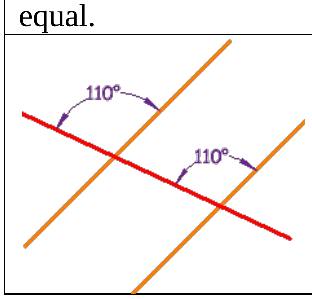
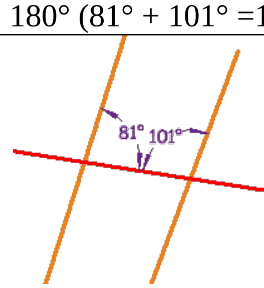
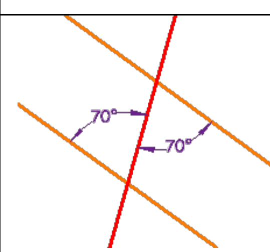
Some of those special pairs of angles can be used to test if lines really are parallel:



If Any Pair Of ...	*Example:*
Corresponding Angles are equal, or	$a = e$
Alternate Interior Angles are equal, or	$c = f$
Alternate Exterior Angles are equal, or	$b = g$
Consecutive Interior Angles add up to 180°	$d + f = 180°$

... then the lines are Parallel

Examples

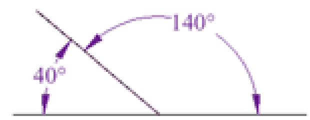
These lines are parallel, because a pair of Corresponding Angles are equal.	These lines are not parallel, because a pair of Consecutive Interior Angles do not add up to 180° (81° + 101° =182°)	These lines are parallel, because a pair of Alternate Interior Angles are equal
		

3) Supplementary Angles

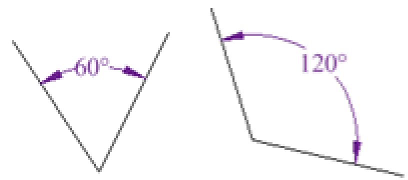
Two Angles are Supplementary if they add up to 180 degrees.

These two angles (140° and 40°) are Supplementary Angles, because they add up to 180°.

Notice that together they make a straight angle.



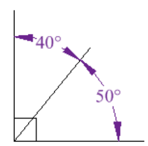
But the angles don't have to be together.
These two are supplementary because 60° + 120° = 180°



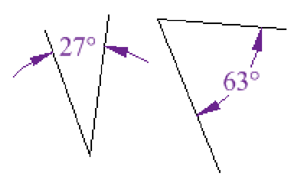
4) Complementary Angles

Two Angles are Complementary if they **add up to 90 degrees** (a Right Angle).

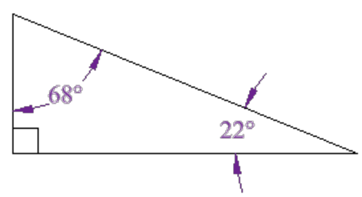
These two angles (40° and 50°) are Complementary Angles, because they add up to 90°.
Notice that together they make a right angle.



But the angles don't have to be together.
These two are complementary because
27° + 63° = 90°



Right Angled Triangle
In a right angled triangle, the two acute angles are complementary, because in a triangle the three angles add to 180°, and 90° have been taken by the right angle.



5) Complementary vs Supplementary

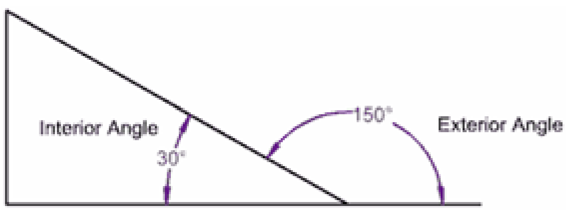
A related idea is Complementary Angles, they add up to 90°.

How can you remember which is which? Easy! Think:

- "C" of Complementary stands for "Corner" ∟ (a Right Angle), and
- "S" of Supplementary stands for "Straight" (180 degrees is a straight line)

3. Interior and Exterior Angle

An Interior Angle is an angle inside a shape.
The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



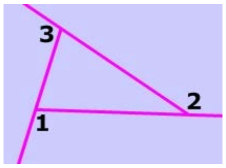
Note: If you add up the Interior Angle and Exterior Angle you get a straight line, 180°.

1) Exterior Angles of Polygons

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.
For example:

A Polygon is any flat shape with straight sides.

The Exterior Angles of a Polygon add up to 360°.
$\angle 1 + \angle 2 + \angle 3 = 360^0$



2) Interior Angles sum of Polygons

An interior angle of a regular polygon with n sides is $\dfrac{(n-2)\times 180}{n}$.

Example:

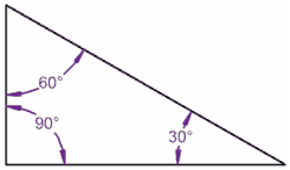
To find the measure of an interior angle of a regular octagon, which has 8 sides, apply the formula above as follows: ((8-2) × 180) /8 = 135°

The sum of the measures of the interior angles of a polygon with *n* sides is (n-2)180.

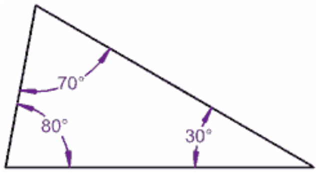
3) Triangles

The Interior Angles of a Triangle add up to 180°

90° + 60° + 30° = 180°

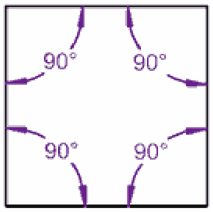


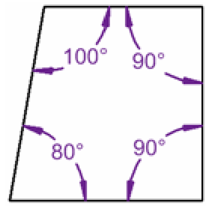
80° + 70° + 30° = 180°



4) Quadrilaterals (Squares, etc)

(A Quadrilateral is any shape with 4 sides)





90° + 90° + 90° + 90° = 360°

A Square adds up to 360°

80° + 100° + 90° + 90° = 360°

Let's tilt a line by 10° ... still adds up to 360°!

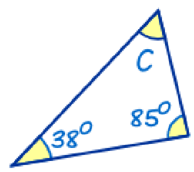
The Interior Angles of a Quadrilateral add up to 360°

4. Triangles

In a triangle, the three angles always add to 180°: **A + B + C = 180°.**

Example: Find the Missing Angle "C"

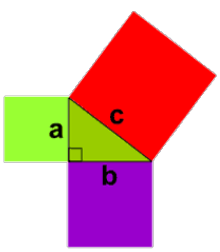
Start With: A + B + C = 180°
Fill in what we know: 38° + 85° + C = 180°
Rearrange: C = 180° - 38° - 85°
Calculate: C = 57°

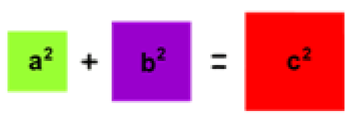


5. Pythagorean Triples

A "Pythagorean Triple" is a set of positive integers, **a**, **b** and **c** that fits the rule: $a^2 + b^2 = c^2$

Triangles
And when you make a triangle with sides **a**, **b** and **c** it will be a right angled triangle:

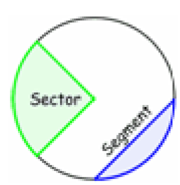




Note:
c is the **longest side** of the triangle, called the "hypotenuse"
a and **b** are the other two sides

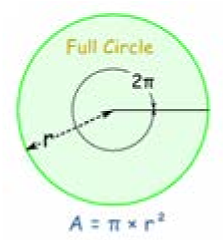
6. Circle Sector and Segment

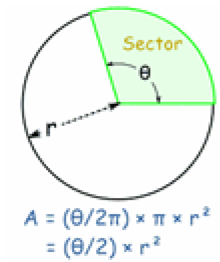
Slices
There are two main "slices" of a circle:
The "pizza" slice is called a **Sector**.
And the slice made by a chord is called a **Segment**.



Area of a Sector

You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
Note: I am using radians for the angles.





This is the reasoning:

- A circle has an angle of 2π and an Area of: πr^2
- So a Sector with an angle of θ (instead of 2π) must have an area of: $(\theta/2\pi) \times \pi r^2$
- Which can be simplified to: $(\theta/2) \times r^2$

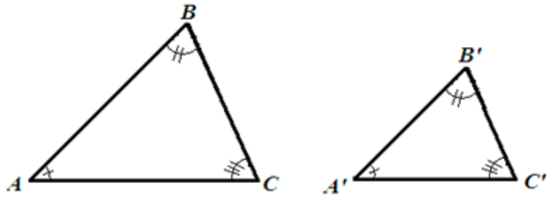
Area of Sector = $\frac{1}{2} \times \theta \times r^2$ *(when θ is in radians)*
Area of Sector = $\frac{1}{2} \times (\theta \times \pi/180) \times r^2$ *(when θ is in degrees)*

7. Similar triangle

1) Definition

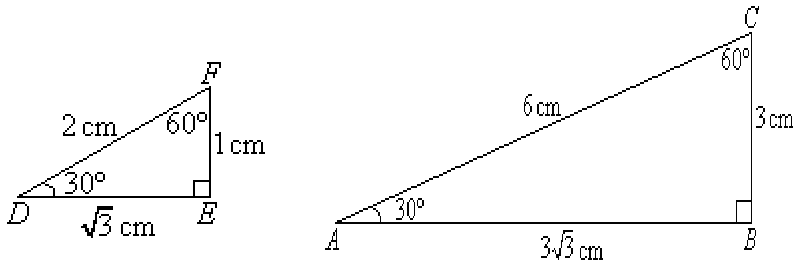
Two triangles ABC and A'B'C' are similar if the three angles of the first triangle are congruent to the corresponding three angles of the second triangle and the lengths of their corresponding sides are proportional as follows.

$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$ or AB: A'B'= BC: B'C' = CA: C'A'



Equiangular triangles have the same shape but may have different sizes. So, equiangular triangles are also called similar triangles.

For example, triangle *DEF* is similar to triangle *ABC* as their three angles are equal (equal angles are marked in the same way in diagrams).



The length of each side in triangle *DEF* is multiplied by the same number, 3, to give the sides of triangle *ABC*.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = 3$$

Generally speaking, if two triangles are similar, then the corresponding sides are in the same ratio.

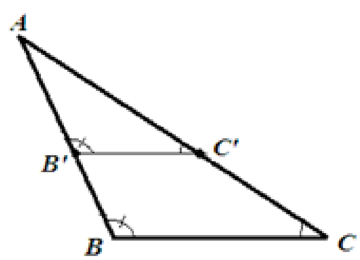
2) Angle-Angle (AA) Similarity

Theorem: If two angles in a triangle are congruent to the two corresponding angles in a second triangle, then the two triangles are similar. This is because sum of three angles of a triangle equals 180^o which assures the third pair of corresponding angles must be equal.

Example 1: Let ABC be a triangle and B'C' a segment parallel to BC. Prove triangles ABC is similar triangle A'BC'.

Solution

Since B'C' is parallel to BC, angles AB'C' and ABC are congruent (alternate angles).
Also angles AC'B' and ACB are congruent (alternate angles).
Since the two triangles have two corresponding congruent angles, they are similar by Angle-Angle.

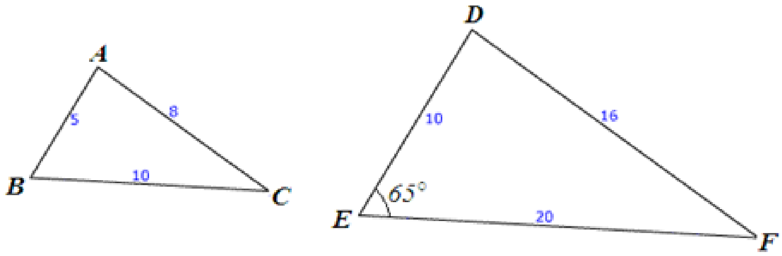


3) Side-Side-Side (SSS) Similarity

Theorem: If the three sides of a triangle are proportional to the corresponding sides of a second triangle, then the triangles are similar.

Or, the lengths of the corresponding sides are proportional and therefore the two triangles are similar.

Example 2: Prove the two triangles shown below are similar triangles and determine the angle of B.

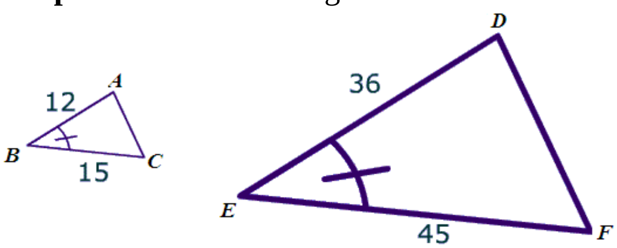


Solution
Since AB : DE =5:10= 1: 2
and AC: DF =8:16=1:2
and BC: EF=10:20=1:2
Therefore triangle ABC and triangle DEF are similar triangles.
Since corresponding angles are congruent for similar triangles, and angle E in triangle DEF is 65^{o}, then angle B = angle E =65^{o}

4) Side-Angle-Side (SAS) Similarity

Theorem
If an angle of a triangle is congruent to the corresponding angle of a second triangle, and the lengths of the two sides including the angle in one triangle are proportional to the lengths of the corresponding two sides in the second triangle, then the two triangles are similar.

Example 3: Prove the triangles shown below are similar. If DF equals 27, determine the length of AC.



Solution
Since angle B equals angle E,
and BC : EF = 15 : 45 =1:3
and BA : ED=12: 36=1:3,

The two triangles have two sides whose lengths are proportional and a congruent angle included between the two sides. Therefore the two triangles are similar(SAS). We may calculate the ratios of the lengths of the corresponding sides.

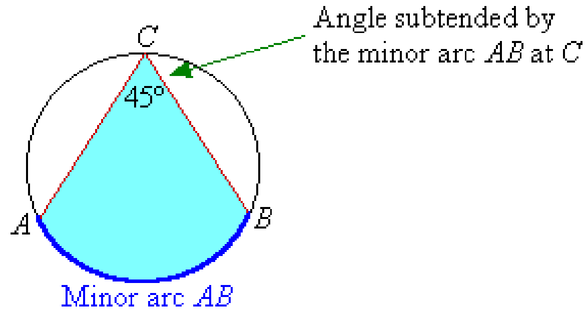
Then AB : DE = AC : DF, substitute the given value in written proportion, we have 12 : 36 =AC : 27, we get AC =9

8. Angle at the Circumference

If the end points of an arc are joined to a third point on the circumference of a circle, then an angle is formed. It is called the **inscribed angle**.

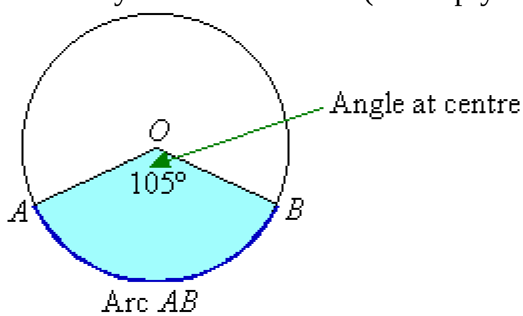
For example, the minor arc *AB* subtends an angle of 45º at *C*. The angle *ACB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at *C*.

The angle *ACB* is an angle at the circumference standing on the arc *AB*.



9. Angle at the Centre

If the end points of an arc are joined to the centre of a circle, then an angle is formed.
For example, the minor arc *AB* subtends an angle of 105º at *O*. The angle *AOB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at the centre *O*.

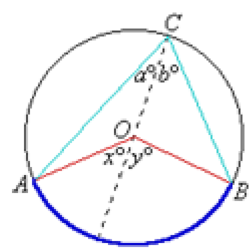


The angle *AOB* is an angle at the centre *O* standing on the arc *AB*, it is also called the **central angle**.

10. Angle at Centre Theorem

Theorem

Use the information given in the diagram to prove that the angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc.



Given: $\angle AOB$ and $\angle ACB$ stand on the same arc; and O is the centre of the circle.

To prove: $\angle AOB = 2\angle ACB$

Proof:

From ΔOAC, $x = a + a$ {Exterior angle of a triangle}

$\therefore x = 2a$ ... (1)

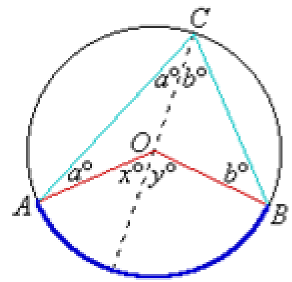
From ΔOBC, $y = b + b$ {Exterior angle of a triangle}

$\therefore y = 2b$ ... (2)

Adding (1) and (2) gives:

$x + y = 2a + 2b = 2(a + b)$ $\therefore \angle AOB = 2\angle ACB$

As required.



In general: The angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc. This is called the **Angle at Centre Theorem**.

We also call this the **basic property**, as the other angle properties of a circle can be derived from it.

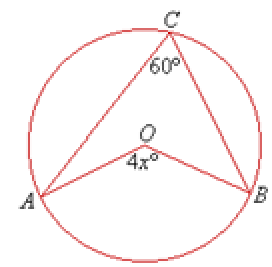
Example 1: Find the value of unknown in the following circle centred at *O*.

Solution:

$4x = 2 \times 60$ {Angle at Centre Theorem}

$4x = 120$

$\frac{4x}{4} = \frac{120}{4}$ $x = 30$



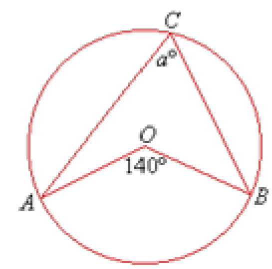
Example 2: Find the value of the unknown in the following circle centred at *O*.

Solution:

$2a = 140$ {Angle at Centre Theorem}

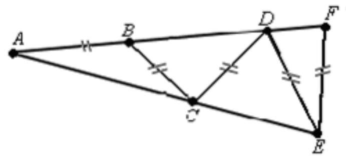
$\frac{2a}{2} = \frac{140}{2}$

$a = 70$



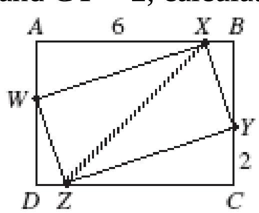
Questions in class

1. Given the isosceles triangle AEF (where AE = AF) with a path of 5 congruent segments A – B – C – D – E – F, what is the degree measure of angle A?



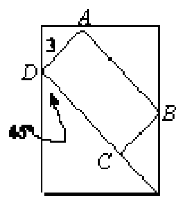
2. The floor of a room is a 10 m by 10 m square and the room is 4 m high. A spider is in one of the corners of the floor and sees a fly across the room at the diagonally opposite corner on the ceiling. If the fly does not move, what is the shortest distance (in meters) the spider must travel to catch the fly?

3. A rectangle ABCD has a second rectangle XYZW inscribed in it as shown. If XZ = $5\sqrt{2}$, AX = 6 and CY = 2, calculate the area of ABCD.

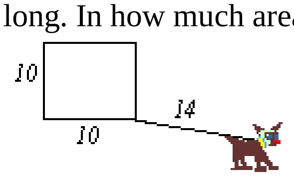


4. A square of perimeter 20 is inscribed in a square of perimeter 28 (inscribed means the vertices of the smaller square are on the sides of the larger square). What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

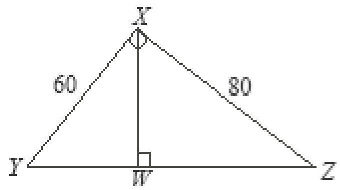
5. In the figure, *ABCD* is a rectangle and *B* is a midpoint of the larger rectangle. What is the area of *ABCD*?



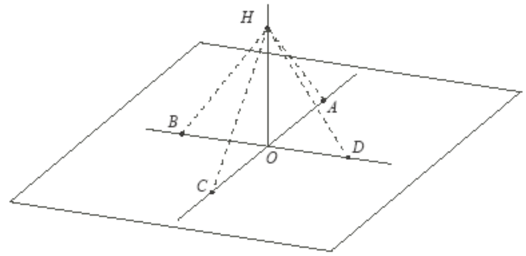
6. A dog is tethered to the corner of a ten-foot-by-ten-foot square pen by a leash that is fourteen feet long. In how much area outside the pen can the dog roam?



7. (a) In the diagram, ΔXY Z is right-angled at X, with Y X = 60 and XZ = 80. W is the point on Y Z so that WX is perpendicular to Y Z. Determine the length of WZ. Explain how you obtained your answer.



(a)



(b)

(b) Five points A, B, C, D, and O lie on a flat field. A is directly north of O, B is directly west of O, C is directly south of O, and D is directly east of O. The distance between C and D is 140 m. A hot-air balloon is positioned in the air at H directly above O. The balloon is held in place by four ropes HA, HB, HC, and HD. Rope HC has length 150 m and rope HD has length 130 m. Determine how high the balloon is above the field (that is, determine the length of OH). Explain how you obtained your answer.

(c) To reduce the total length of rope used, rope HC and rope HD are to be replaced by a single rope HP where P is a point on the straight line between C and D. (The balloon remains at the same position H above O as in part (b).) Determine the greatest length of rope that can be saved. Explain how you obtained your answer.

8. In the diagram, K, O and M are the centres of the three semi-circles. Also, OC = 32 and CB = 36.

(a) What is the length of AC?
(b) What is the area of the semi-circle with centre K?
(c) What is the area of the shaded region?
(d) Line l is drawn to touch the smaller semi-circles at points S and E so that KS and ME are both perpendicular to l. Determine the area of quadrilateral KSEM.