Lesson #13

Double Angle Formulas

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half Angle Formulas

$$\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\tan\theta = \pm\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

1. Use an appropriate double angle formula to rewrite each expression as a single trigonometric ratio.

a) $10 \, sin \, 5x \, cos \, 5x$

b) $1 - 2 \, sin^2 \left(\frac{\pi}{4}\right)$

c) $\frac{\sin^2(4x) - \cos^2(4x)}{\cos 3x \sin 3x}$

2. Derive a formula for

a) $cos \, 3\theta$ in terms for $cos \, \theta$

b) $sin \, 4\theta$ in terms for $sin \, \theta$ & $\cos\theta$

Prove each identity

3. $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

4. $\frac{\sin 2x}{2-2\cos^2 x} = 2\csc 2x - \tan x$

5. $\tan 2x = \frac{2}{\cot x - \tan x}$

6. $\frac{\sin 2\theta}{1-\cos 2\theta} = 2\csc 2\theta - \tan\theta$

7. $2\csc 2x = \sec x \csc x$

8. $2\cot 2x = \cot x - \tan x$

9. $\cot\left(\frac{x}{2}\right) = \frac{1+\cos x}{\sin x}$

10. $\frac{1-\sin 2x}{\cos 2x} = \frac{\cos 2x}{1+\sin 2x}$

Determine the exact value of each.

1. $sin(22.5^0)$

2. $tan\frac{5\pi}{12}$

3. $cos\left(\frac{7\pi}{12}\right)$

Math 12 – Double Angle Identities Worksheet

1. Write each expression in terms of a single trigonometric function.
   a) $2\sin 0.6\cos 0.6$ b) $2\sin 3\cos 3$ c) $2\sin2\cos2$
   d) $\cos^2 0.45-\sin^2 0.45$ e) $2\cos^2 5-1$ f) $1-2\sin^2 3$

2. Write each expression in terms of a single trigonometric function
   a) $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$ b) $\cos^2\frac{\pi}{10}-\sin^2\frac{\pi}{10}$ c) $2\cos^2 0.5-1$

3. If $\sin\theta=\frac{1}{3}$ and θ is in quadrant I, evaluate each expression.
   a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

4. A value of θ is defined. Evaluate the expressions $\sin 2\theta,\ \cos 2\theta,\ and\ \tan 2\theta$
   a) $\cos\theta=-\frac{1}{2}$, and θ is in Quadrant II
   b) $\sin\theta=-\frac{2}{3}$, and θ is in Quadrant III
   c) $\tan\theta=0.75$ and $\pi<\theta<\frac{3\pi}{2}$

5. Prove each identity:
   a) $1+\sin 2\theta=(\sin\theta+\cos\theta)^2$ b) $\sin 2\theta=2\cot\theta\sin^2\theta$
   c) $\cos 2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ d) $\sec^2\theta=\dfrac{2}{1+\cos 2\theta}$

6. Prove each identity:
   a) $\dfrac{1-\cos 2\theta}{2}=\sin^2\theta$ b) $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=-\sec 2\theta$
   c) $\dfrac{(\sin\theta+\cos\theta)^2}{\sin 2\theta}=\csc 2\theta+1$

Answers:

1a) $\sin 1.2$ b) $\sin 6$ c) $\sin 4$ d) $\cos 0.9$ e) $\cos 10$ f) $\cos 6$

2a) $\sin\frac{\pi}{3}$ b) $\cos\frac{\pi}{5}$ c) $\cos 1$

3a) $\frac{4\sqrt{2}}{9}$ b) $\frac{7}{9}$ c) $\frac{4\sqrt{2}}{7}$

4a) $-\frac{\sqrt{3}}{2},\ -\frac{1}{2},\ \sqrt{3}$ b) $\frac{4\sqrt{5}}{9},\ \frac{1}{9},\ 4\sqrt{5}$ c) $\frac{24}{25},\ \frac{7}{25},\ \frac{24}{7}$

Use a double-angle or half-angle identity to find the exact value of each expression.

1) $\cos\frac{7\pi}{8}$

2) $\sin\frac{7\pi}{8}$

3) $\sin 165°$

4) $\sin 112\frac{1}{2}°$

5) $\sin 15°$

6) $\cos\frac{23\pi}{12}$

7) $\sin 22\frac{1}{2}°$

8) $\sin -\frac{5\pi}{12}$

9) $\cos\frac{3\pi}{8}$

10) $\sin 75°$

Solving Trigonometric Equations (I)

Because of their periodic nature, trigonometric equations have an infinite number of solutions. When we use a trigonometric model, we usually want solutions within a specified interval.

- To find the exact solutions of a linear trigonometric equation, use special triangles, the CAST rule, the unit circle, and/or a sketch of the graph. In some cases, you may also need to use a trigonometric identity to help you solve a given equation.
- A scientific or graphing calculator can be used to find the approximate solutions of a linear trigonometric equation. The inverse trigonometric function of a positive ratio yields the related acute angle. Use this reference angle and the period of the corresponding function to determine all the solutions in the given interval.
- The same strategies can be used to solve linear trigonometric equations when the variable is measured in degrees or radians.

Example 1 Solve each equation for θ, where $0 \le \theta \le 360^\circ$.

a) $\sec\theta = -1$

b) $5\tan\theta + 7 = 0$

Example 2 Solve each equation for x, where $0 \le x \le 2\pi$.

a) $\sqrt{3}\sin x = 3\cos x$

b) $2 - 6\cot x = 5\cot x + 13$

Example 3

Solve for θ in the domain $[0, 2\pi]$.

a) $4\sin\theta\cos\theta - 1 = 0$

b) $\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3} = \frac{1}{\sqrt{2}}$

Solving Trigonometric Equations (II)

In some applications, the formula contains a square of a trigonometric ratio.

- Quadratic trigonometric equation can often be factored. You can then solve the resulting two linear trigonometric equations.
- In cases where the equation cannot be factored, use the quadratic formula and then solve the resulting linear trigonometric equations.

  Recall that the solutions to $ax^2+bx+c=0$ are determined by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- You may need to use a Pythagorean idenity, compound angle formula, or double angle formula to create a quadratic equation that contains only a single trigonometric function whose arguments all match.
- A quadratic equation may have multiple solutions in the interval $0\le x\le 2\pi$. Some of the solutions may be inadmissible, however, in the context of the problem.

Example 1 Solve each equation for x in the interval $0\le x\le 2\pi$.

a) $\csc^2 x-1=0$

b) $2\cos^2 x+5\cos x=3$

Example 2

Use the quadratic formula to solve each equation for θ, where $0 \le \theta \le 360^\circ$. Give answers to the nearest degree.

a) $\tan^2\theta - 5\tan\theta + 10 = 0$

b) $3\cot^2\theta - 1 = 4\cot\theta$

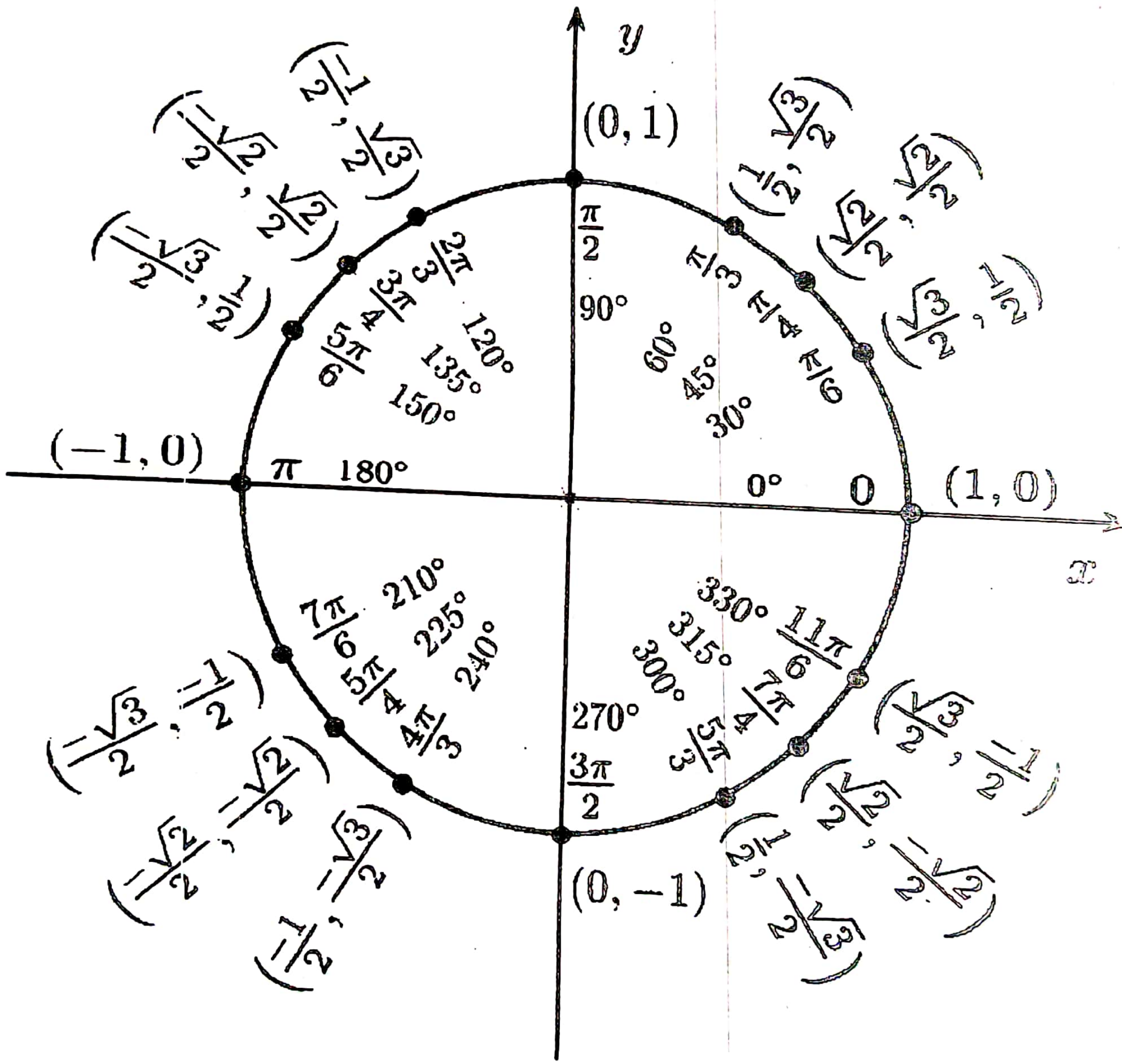
Example 3 For each equation, use a trigonometric identity to create a quadratic equation. Then solve the equation for x in the interval $[0, 2\pi]$. Give exact answers where possible. Round approximate answers to the nearest hundredth.

a) $5-5\cos x = 4\sin^2 x$

b) $2\cot^2 x + 3\csc x = 0$

c) $\sin x = 6\sin 2x$

d) $\cos 2x + \cos x + 1 = 0$



y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0 0°
$\frac{2\pi}{3}$ 120°
$\frac{3\pi}{4}$ 135°
$\frac{5\pi}{6}$ 150°
π 180°
$\frac{7\pi}{6}$ 210°
$\frac{5\pi}{4}$ 225°
$\frac{4\pi}{3}$ 240°
$\frac{3\pi}{2}$ 270°
$\frac{5\pi}{3}$ 300°
$\frac{7\pi}{4}$ 315°
$\frac{11\pi}{6}$ 330°
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
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$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
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$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$