Analytic Geometry

1. Linear Equation

1) Point-slope form

Given a point in the line (x_1, y_1), and the slope of the line, m, an equation of the line may be expressed as $y - y_1 = m(x - x_1)$

Example: Determine an equation of a line through point (3, 2) with slope m = 2.

Solution:
$(x_1, y_1) = (3, 2)$ and $m = 2$, so $y - 2 = 2(x - 3)$, this equation can be expressed in standard form:
$2x - y - 4 = 0$

2) Slope Y-intercept form

Given a slope and the y-intercept of the line, b, an equation of the line may be expressed in the form:
$y = mx + b$.

Example: Determine an equation of the line with m=3 and y-intercept 2.
Solution: $b = 2$ and $m = 3$, the $y = 3x + 2$

3) Two point solution

Given two points (x_1, y_1) and (x_2, y_2), then the equation of the line can be expressed as

$(y - y_1) = \frac{(y_1 - y_2)}{(x_1 - x_2)}(x - x_1)$ or $y - y_1 = m(x - x_1)$, here $m = \frac{(y_1 - y_2)}{(x_1 - x_2)}$

Example: given two points $P_1(2, 3)$ and $P_2(-1, 2)$, determine the equation of the line.

Solution: $m = \frac{(y_1 - y_2)}{(x_1 - x_2)} = \frac{(3-2)}{(2-(-1))} = \frac{1}{3}$, so $y - 3 = \frac{1}{3}(x - 2)$, this equation can be expressed in standard form: $x - 3y + 7 = 0$

2. Length of segment

The length of a line segment can be found by Pythagorean Theorem given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then the segment joining P_1 and P_2 may be expressed by following formula:

$L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Example: Find the length of the line segment joining points (3, 2) and (−1, 4)

Solution: $L=\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3 -(-1))^2 + (2 - 4)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = 4\sqrt{5}$

3. Midpoint of a line segment

We can calculate the coordinates of the midpoint of a line segment if the coordinates of the endpoints are given.

The coordinates of the midpoint M of the segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are:

$$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$$

► The relations between two lines with slope m_1 and m_2:

- if $m_1=m_2$, then two lines are parallel;
- if $m_1 \bullet m_2=-1$, then two lines are perpendicular;
- if $m_1 \neq m_2$, the two lines have one intersection.

In-class questions

1. Let points $A=(0,0,0)$, $B=(1,0,0)$, $C=(0,2,0)$, *and* $D=(0,0,3)$. Points *E*, *F*, *G*, and *H* are midpoints of line segments *BD*, *AB*, *AC*, and *DC* respectively. What is the area of *EFGH?*

2. Let points $A=(0,\ 0)$, $B=(1,\ 2)$, $C=(3,\ 3)$, *and* $D=(4,\ 0)$. Quadrilateral *ABCD* is cut into equal area pieces by a line passing through *A*. This line intersects *CD* at point (*p/q, r/s)*, where these fractions are in lowest terms. What is $p+q+r+s$ *?*

3. A dilation of the plane-that is, a size transofrmation with a positive scale factor-sends the circle of radius 2 centered at $A(2,\ 2)$ to the circle of radius 3 centered at $A'(5,\ 6)$. What distance does the origin $O(0,\ 0)$ more under this transformation?

4. What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

5. How many triangles with positive area have all their vertices at points (i, j) in the coordinate plane, where $i \, and \, j$ are integers between 1 and 5, inclusive?

(A) 2128 (B) 2148 (C) 2160 (D) 2200 (E) 2300

6. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = a^2 - a$ in the real xy-pane intersect at exactly 3 points?

7. points $A(6, 13)$ *and* $B(12, 11)$ lie on a circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at point on the x-axis. What is the area of ω?

8. A lattice point in an xy-coordinate system is any point where both *x* and yare integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \le 100$ for all *m* such that $1/2 < m < a$. What is the maximum possible value of *a?*