AP Calculus Homework 18

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Show that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

2.* Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial-value problem

$$y' + (\tan x)y = \cos^2 x \qquad y(0) = -1$$

on the interval $-\pi/2 < x < \pi/2$.

3.* Which of the following functions are the solutions of the differential equation $y'' + y = \sin x$?

a) $y = \sin x$ b) $y = \cos x$ c) $y = \frac{1}{2}x \sin x$ d) $y = -\frac{1}{2}x \cos x$

4. Solve the differential equation.

a) $\dfrac{dy}{dx} = \dfrac{\sqrt{x}}{e^y}$ b) $(x^2 + 1)y' = xy$ c) $\dfrac{du}{dr} = \dfrac{1 + \sqrt{r}}{1 + \sqrt{u}}$

d)** $\dfrac{dy}{d\theta} = \dfrac{e^y \sin^2 \theta}{y \sec \theta}$ e)* $\dfrac{du}{dt} = 2 + 2u + t + tu$

5. Find the solution of the differential equation that satisfies the given initial condition.

a) $\dfrac{dy}{dx} = \dfrac{y \cos x}{1 + y^2}, \qquad y(0) = 1$

b) $x \cos x = (2y + e^{3y})y', \qquad y(0) = 0$

c)** $xy' + y = y^2, \qquad y(1) = -1$

6. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at (x, y) is xy.

7.** Find the function f such that $f'(x) = f(x)(1 - f(x))$ and $f(0) = \frac{1}{2}$.