

## Unit: Derivatives (2)

### Quotient Rule

If  $f$  and  $g$  are differentiable at  $x$  and  $g(x) \neq 0$  then so is  $\frac{f}{g}$  and:

$$\left( \frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}$$

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Ex 1.** For each function  $f(x)$  find the derivative  $f'(x)$ .

1.  $f(x) = \frac{x+1}{x-1}$

3.  $f(x) = \frac{2\sqrt{x}}{\sqrt{x}+1}$

2. $f(x) = \frac{x^2}{2x+1}$	4. $f(x) = \frac{x^3-1}{x^3+1}$
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<p><b>Chain Rule</b>(Leibniz Notation)</p> $\Delta x \xrightarrow{u=g(x)} \Delta u \xrightarrow{v=f(u)} \Delta v$ <p>and</p> $\frac{\Delta v}{\Delta x} = \frac{\Delta v}{\Delta u} \frac{\Delta u}{\Delta x} \rightarrow \frac{dv}{dx} = \frac{dv}{du} \frac{du}{dx}$ <p>Therefore:</p> $\frac{dv}{dx} = \frac{dv}{du} \frac{du}{dx}$	<p><b>Chain Rule</b>(Lagrange Notation)</p> $v = f(u) = f(g(x)) = (f \circ g)(x)$ $\frac{dv}{dx} \rightarrow [f(g(x))]'$ $\frac{dv}{du} \rightarrow f'(u) = f'(g(x))$ $\frac{du}{dx} \rightarrow g'(x)$ $\frac{dv}{dx} = \frac{dv}{du} \frac{du}{dx} \rightarrow [f(g(x))]' = f'(g(x))g'(x)$ <p>If <math>g</math> is differentiable at <math>x</math> and <math>f</math> is differentiable at <math>f(x)</math> then the composition <math>(f \circ g)(x) = f(g(x))</math> is differentiable at <math>x</math> and</p> $(f \circ g)'(x) = [f(g(x))]' = f'(g(x))g'(x)$ <p>So, the derivative of <math>f(g(x))</math> is the derivative of the <i>outside</i> function <math>f</math> evaluated of the <i>inside</i> function <math>g</math> times the derivative of the inside function <math>g</math>.</p>
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**Ex 2.** For each function  $f(x)$  find the derivative  $f'(x)$ .

1. $f(x) = (3x - 2)^7$	2. $f(x) = (x^2 - 2x + 2)^{10}$
3. $f(x) = \left(\frac{x-2}{x-1}\right)^{10}$	4. $f(x) = \sqrt{x^2 + 1}$
5. $f(x) = \frac{-5}{(6x^2 - 7)^4}$	6. $f(x) = \sqrt{(3x^2 - 2)^3}$

## Higher Order Derivatives - Notations

Let consider the function  $y = f(x)$ .

**The first derivative** of  $f$  or “ $f$  prime” is:

$$f'(x) = y' = \frac{dy}{dx}$$

**The second derivative** of  $f$  or “ $f$  double prime” is:

$$f''(x) = y'' = \frac{d^2y}{dx^2}$$

**The third derivative** of  $f$  or “ $f$  triple prime” is:

$$f'''(x) = y''' = \frac{d^3y}{dx^3}$$

**The  $n$ -th derivative** of  $f$  is:

$$f^{(n)}(x) = y^{(n)} = \frac{d^ny}{dx^n}$$

**Ex 3.** Show that  $y = x^3 + 3x + 1$  satisfies  $y''' + xy'' - 2y' = 0$ .

**Ex 4.** Find  $f^{(2017)}(x)$  if  $f$  is:

1.  $f(x) = x^2 - x + 3$

2.  $f(x) = x^{2017} - 3x^{2015} + 5x^2 + 10$

3.  $f(x) = \frac{1}{x}$

### Implicit differentiation

**Ex 5.** If we have  $x^2 + y^2 = 25$ , which represents a circle of radius five centered at the origin. Find the slope of the line tangent to the graph of this equation at the point (3, -4).

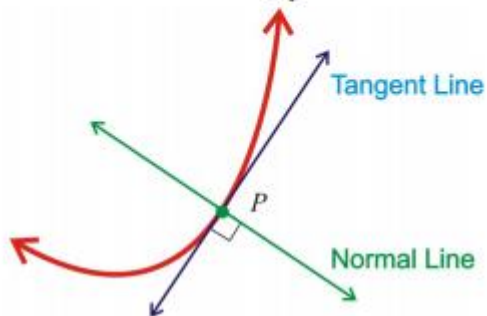
**Ex 6.** a. Find  $\frac{dy}{dx}$  if  $2x^5 + x^4y + y^5 = 36$

b. Find the slope of the tangent to the curve  $2x^5 + x^4y + y^5 = 36$  at the point  $(1, 2)$

### Normal Line

If  $m_T$  is the slope of the tangent line, then slope of the normal line  $m_N$  is given by:

$$m_N = -\frac{1}{m_T}$$



**Ex 7.** Find the equation of the normal line to the curve  $y = f(x) = x + \frac{2}{x}$  at  $P(1, 3)$ .