Unit: Derivatives (3)

Derivative of Exponential Function

Derivative of e^x	**Derivative of $e^{f(x)}$**
$(e^x)' = e^x$ $\frac{d}{dx}e^x = e^x$	$(e^{f(x)})' = e^{f(x)}f'(x)$ $\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$
Ex 1. Differentiate and simplify. a. x^2e^x	b. $e^{\sqrt{x^2+1}}$
Derivative of b^x , b > 0, b ≠ 1 $(b^x)' = (\ln b)b^x$ $\frac{d}{dx}b^x = (\ln b)b^x$ Proof: $(b^x)' = (e^{x\ln b})' = e^{x\ln b}(\ln b) = (\ln b)b^x$	**Derivative of $b^{f(x)}$** , b > 0, b ≠ 1 $(b^{f(x)})' = (\ln b)b^{f(x)}f'(x)$ $\frac{d}{dx}b^{f(x)} = (\ln b)b^{f(x)}f'(x)$

Ex 2. Differentiate. a. 3^x b. 2^{-x^3}	c. Determine f'(0) for f(t) = $2e^{3t}$-5t.

Derivative of Logarithmic Function

Derivative of $\ln x$	**Derivative of $\ln f(x)$**
$(\ln x)' = \frac{1}{x}$ $\frac{d}{dx}\ln x = \frac{1}{x}$ Proof: $y = \ln x \Rightarrow x = e^y \Rightarrow (x)' = (e^y)' \Rightarrow$ $1 = e^y y' \Rightarrow y' = \frac{1}{e^y} \Rightarrow y' = \frac{1}{x} \Rightarrow \therefore (\ln x)' = \frac{1}{x}$	$[\ln f(x)]' = \frac{f'(x)}{f(x)}$ $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$
Ex 3. Differentiate and simplify. a. $x^2 \ln x$	b. $\frac{\ln x}{x}$

c. $e^x \ln x$	d. If g(x)=$e^{2x-1}\ln(2x-1)$, evaluate g'(1).
Derivative of $\log_a x$ $(\log_b x)' = \frac{1}{(\ln b)x}$ $\frac{d}{dx}\log_b x = \frac{1}{(\ln b)x}$	**Derivative of $\log_a f(x)$** $[\log_b f(x)]' = \frac{f'(x)}{(\ln b)f(x)}$ $\frac{d}{dx}\log_b f(x) = \frac{f'(x)}{(\ln b)f(x)}$
Ex 4. Differentiate. a. $\log x$	b. f(x) = $x^2 + 2^x + \log_2 x$

Derivative of Trigonometric Functions

Derivative of $\sin x$	**Derivative of $\cos x$**
$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$
Ex 6. Differentiate. a. $y = x^3 \sin x$	d. $y = \tan x$

b. $f(x) = \cos\frac{\pi}{x}$

c. $f(x) = \sin^3 x + \sin 3x$

e. $y = x^{\sin x}$

Ex 7. Find $\frac{dy}{dx}$ at x=0 for $y = \frac{x\cos x}{1+e^x}$.

Ex 8. Find $y'(\frac{\pi}{2})$ for $y = x^{\sin x}$

Ex 9. An object is suspended from the end of a string. Its displacement from the equilibrium position is $s(t) = 8\sin(10\pi t)$. Calculate the velocity and acceleration of the object at any time t.