AP Calculus Homework 5

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Differentiate the following functions.

a) $y = \ln|2 - x - 5x^2|$ b)* $H(z) = \ln\sqrt{\dfrac{a^2 - z^2}{a^2 + z^2}}$ c) $y = [\ln(1 + e^x)]^2$

d) $y = 2x \log_{10}\sqrt{x}$ e)* $y = \log_2(e^{-x}\cos \pi x)$

2. Find y' and y'' if $y = \dfrac{\ln x}{x^2}$.

3. Using logarithmic differentiation to find the derivative of the function.

a) $y = \sqrt[4]{\dfrac{x^2 + 1}{x^2 - 1}}$ b) $y = x^{\sin x}$ c)** $y = (\tan x)^{1/x}$ d)* $y = (\ln x)^{\cos x}$

4. Find y' if $x^y = y^x$.

5.* Find $\dfrac{d^9}{dx^9}(x^8 \ln x)$

6. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

7.* At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

8.** A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with water at the rate of 0.2 m^3/min, how fast is the water level rising when the water is 30 cm deep?

9. A kite 100 ft above the ground moves horizontally at a speed 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

10. A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of $\pi/6$ rad/min. How fast is the plane traveling at that time?

11. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

A) $\frac{1}{\pi}$ B) $\frac{1}{2}$ C) $\frac{2}{\pi}$ D) 1 E) 2

12. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

A) A is always increasing

B) A is always decreasing

C) A is decreasing only when $b < h$

D) A is decreasing only when $b > h$

E) A remains constant

13. The radius of a circle is decreasing at a constant rate of 0.1 cm per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square cm per second?

A) $-(0.2)\pi C$ B) $-(0.1)C$ C) $-\frac{(0.1)C}{2\pi}$ D) $(0.1)^2 C$ E) $(0.1)^2\pi C$