Unit: Derivatives (2)

Quotient Rule

If f and g are differentiable at x and $g(x) \neq 0$ then so is $\frac{f}{g}$ and:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[f(x)]^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Ex 1. For each function f(x) find the derivative f'(x).

1. f(x) = $\frac{x+1}{x-1}$	3. f(x) = $\frac{2\sqrt{x}}{\sqrt{x}+1}$

2. f(x) = $\frac{x^2}{2x+1}$	4. f(x) = $\frac{x^3-1}{x^3+1}$

Chain Rule(Leibniz Notation) $$\Delta x \underset{u=g(x)}{\rightarrow} \Delta u \underset{v=f(u)}{\rightarrow} \Delta v$$ and $$\frac{\Delta v}{\Delta x} = \frac{\Delta v}{\Delta u}\frac{\Delta u}{\Delta x} \quad \rightarrow \quad \frac{dv}{dx} = \frac{dv}{du}\frac{du}{dx}$$ Therefore: $$\frac{dv}{dx} = \frac{dv}{du}\frac{du}{dx}$$	**Chain Rule(**Lagrange Notation) $$v = f(u) = f(g(x)) = (f \circ g)(x)$$ $$\frac{dv}{dx} \rightarrow [f(g(x))]'$$ $$\frac{dv}{du} \rightarrow f'(u) = f'(g(x)$$ $$\frac{du}{dx} \rightarrow g'(x)$$ $$\frac{dv}{dx} = \frac{dv}{du}\frac{du}{dx} \quad \rightarrow \quad [f(g(x))]' = f'(g(x))g'(x)$$ If g is differentiable at x and f is differentiable at $f(x)$ then the composition $(f \circ g)(x) = f(g(x))$ is differentiable at x and $$(f \circ g)'(x) = [f(g(x))]' = f'(g(x))g'(x)$$ So, the derivative of $f(g(x))$ is the derivative of the *outside* function f evaluated of the *inside* function g times the derivative of the inside function g .

Ex 2. For each function f(x) find the derivative f'(x).

1. $f(x) = (3x - 2)^7$	2. $f(x) = (x^2 - 2x + 2)^{10}$
3. $f(x) = \left(\frac{x-2}{x-1}\right)^{10}$	4. $f(x) = \sqrt{x^2 + 1}$
5. $f(x) = \frac{-5}{(6x^2-7)^4}$	6. $f(x) = \sqrt{(3x^2 - 2)^3}$

Higher Order Derivatives - Notations

Let consider the function y = f (x).

The first derivative of f or " f prime" is: $f'(x) = y' = \frac{dy}{dx}$
The second derivative of f or " f double prime" is: $f''(x) = y'' = \frac{d^2y}{dx^2}$
The third derivative of f or " f triple prime" is: $f'''(x) = y''' = \frac{d^3y}{dx^3}$
The n-th derivative of f is: $f^{(n)}(x) = y^{(n)} = \frac{d^ny}{dx^n}$

Ex 3. Show that y = x^3+3x+1 satisfies $y'''+xy''-2y'= 0$.

Ex 4. Find $f^{(2017)}(x)$ if f is:

1. $f(x) = x^2 - x + 3$

2. $f(x) = x^{2017} - 3x^{2015} + 5x^2 + 10$

3. $f(x) = \frac{1}{x}$

Implicit differentiation

Ex 5. If we have $x^2 + y^2 = 25$, which represents a circle of radius five centered at the origin. Find the slope of the line tangent to the graph of this equation at the point (3, -4).

Ex 6. a. Find $\frac{dy}{dx}$ if $2x^5 + x^4y + y^5 = 36$

b. Find the slope of the tangent to the curve $2x^5 + x^4y + y^5 = 36$ at the point (1, 2)

Normal Line	**Ex 7**. Find the equation of the normal line to the curve $y = f(x) = x + \frac{2}{x}$ at P(1, 3).
If m_T is the slope of the tangent line, then slope of the normal line m_N is given by: $m_N = -\frac{1}{m_T}$