AP Calculus Practice Test 1

1. What is the slope of the line tangent to the graph of $y = \dfrac{x^2 - 2}{x^2 + 1}$ when $x = 1$?

(A) $-\dfrac{3}{2}$ (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 1 (E) $\dfrac{3}{2}$

2. If $y^2 - 2x^2y = 8$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{4}{y - 2x}$ (B) $\dfrac{2xy}{y - x^2}$ (C) $\dfrac{4 + 2xy}{y - x^2}$ (D) $\dfrac{2xy}{y + x^2}$ (E) $\dfrac{2xy + x^2}{y}$

3. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

(A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

4. $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$ is

(A) ∞ (B) $e - 1$ (C) 1 (D) 0 (E) e^x

5. If $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$, for $t > 0$, then in terms of t, $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{1}{2}$ (B) 2 (C) $4t$ (D) $6t^2$ (E) $12t^2$

6. What are the equations of the horizontal asymptotes of the graph of $y = \dfrac{2x}{\sqrt{x^2 - 1}}$?

(A) $y = 0$ only

(B) $y = 1$ only

(C) $y = 2$ only

(D) $y = -2$ and $y = 2$ only

(E) $y = -1$ and $y = 1$ only

7. $\lim\limits_{h \to 0} \dfrac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

8. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

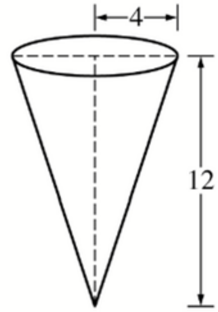
(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$

9. The first derivative of the function f is given by $f'(x) = \sin\left(x^2\right)$. At which of the following values of x does f have a local minimum?

(A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0



10. A container has the shape of an open right circular cone, as shown in the figure above. The container has a radius of 4 feet at the top, and its height is 12 feet. If water flows into the container at a constant rate of 6 cubic feet per minute, how fast is the water level rising when the height of the water is 5 feet? (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(A) 0.358 ft/min

(B) 0.688 ft/min

(C) 2.063 ft/min

(D) 8.727 ft/min

(E) 52.360 ft/min

1. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

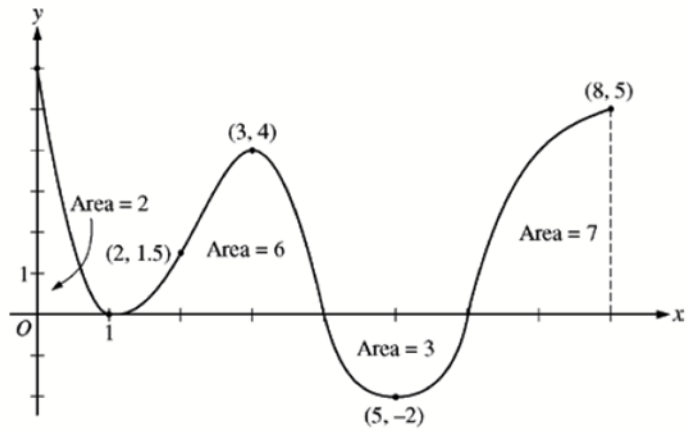
2. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

3. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.