Algebra and Equations 2

1. One Unknown

A linear equation in one unknown can always be stated into the standard form $\boldsymbol{ax = b}$ where x is an unknown and a and b are constants. If a is not equal to zero, this equation has a unique solution $\boldsymbol{x = b/a}$

2. Two Unknowns

A linear equation in two unknown, x and y, can be put into the form $\boldsymbol{ax + by = c}$ where x and y are two unknowns and a, b, c are real numbers. Also, we assume that a and b are not zero.

3. Solution of Linear Equation

1) Algebraically

A solution of the equation consists of a pair of number, $u = (k_1, k_2)$, which satisfies the equation $\boldsymbol{ax + by = c}$.

Mathematically speaking, a solution consists of $u = (k_1, k_2)$ such that $ak_1 + bk_2 = c$.

Solution of the equation can be found by assigning arbitrary values to x and solving for y OR assigning arbitrary values to y and solving for x.

2) Geometrically

Geometrically, any solution $u = (k_1, k_2)$ of the linear equation $ax + by = c$ determine a point in the Cartesian plane. Since a and b are not zero, the solution u correspond precisely to the points on a straight line.

4. Graphing Linear Equations

a) Making a table of values

Graphing linear equations is pretty simple, but only if you work neatly. If you're messy, you'll often make extra work for yourself, and you'll frequently get the wrong answer.

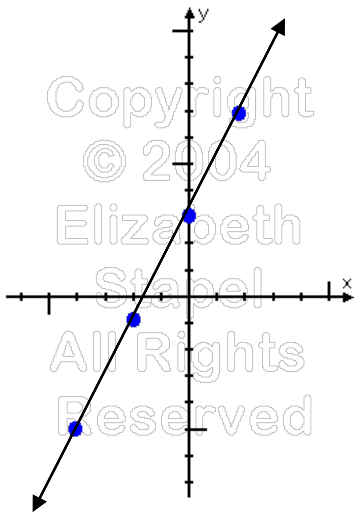
Example: Graph $y = 2x + 3$

Pick some values for x. Then your y-values will come from evaluating the equation at the x-values you've chosen.

Once you've picked x-values, you have to compute the corresponding y-values:

x	y = 2x + 3
-4	2(-4) + 3 = -8 + 3 = -5
-2	2(-2) + 3 = -4 + 3 = -1
0	2(0) + 3 = 0 + 3 = 3
2	2(2) + 3 = 4 + 3 = 7

Now we have 4 points and we can plot them. Then join them and put arrows on both sides.



b) x- and y-Intercepts

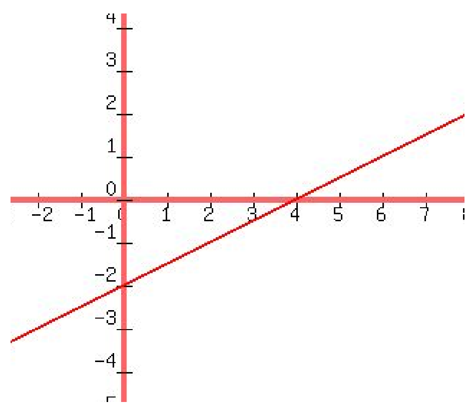
The **x-intercepts** are where the graph crosses the x-axis, and the **y-intercepts** are where the graph crosses the y-axis. Algebraically, an x-intercept is a point on the graph where y is zero, and a y-intercept is a point on the graph where x is zero.

Example: $2x - 4y = 8$

The y intercept is when x = 0, so make x = 0 and solve for y.
$-4y = 8$
$y = 8/-4$
$y = -2$

The x intercept is when y = 0, so make y = 0 and solve for x.
$2x = 8$
$x = 4$

Now, we have 2 points on your line: 1 point is (0, -2), the other point is (4, 0), and we can graph.



5. Systems of Two Equations in Two variables

Given the linear system
ax + by = c
dx + ey = f

• A solution is an ordered pair that will satisfy each equation
• The solution set is the set of all ordered pairs that satisfy both equations. (x_0, y_0).

1) Method of Substitution

- Solve one of the equations for either x or y.
- Substitute that result into the other equation to obtain an equation in a single variable (either x or y).
- Solve the equation for that variable.
- Substitute this value into any convenient equation to obtain the value of the remaining variable.

Example: Solve the following system

$$3x - y = 7$$
$$2x + 3y = 1$$

Solution
In this case it looks like it will be really easy to solve the first equation for *y* so let's do that.

$$3x - 7 = y$$

Now, substitute this into the second equation.

$$2x + 3(3x - 7) = 1$$

This is an equation in *x* that we can solve so let's do that.

$$\begin{aligned} 2x + 9x - 21 &= 1 \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

So, there is the *x* portion of the solution.

Finally, do NOT forget to go back and find the *y* portion of the solution. This is one of the more common mistakes students make in solving systems.

To so this we can either plug the *x* value into one of the original equations and solve for *y* or we can just plug it into our substitution that we found in the first step. That will be easier so let's do that.

$$y = 3x - 7 = 3(2) - 7 = -1$$

So, the solution is $x = 2$ and $y = -1$ as we noted above.

2) Method of elimination

In this method we multiply one or both of the equations by appropriate numbers (*i.e.* multiply every term in the equation by the number) so that one of the variables will have the same coefficient with opposite signs.

Then next step is to add the two equations together. Because one of the variables had the same coefficient with opposite signs it will be eliminated when we add the two equations.

The result will be a single equation that we can solve for one of the variables. Once this is done substitute this answer back into one of the original equations.

Example: Solve the following system of equations

$$5x+4y=1$$
$$3x-6y=2$$

Solution

This is the system in the previous set of examples that made us work with fractions. Working it here will show the differences between the two methods and it will also show that either method can be used to get the solution to a system.

So, we need to multiply one or both equations by constants so that one of the variables has the same coefficient with opposite signs. So, since the *y* terms already have opposite signs let's work with these terms.

It looks like if we multiply the first equation by 3 and the second equation by 2 the *y* terms will have coefficients of 12 and -12 which is what we need for this method.

Here is the work for this step.

$$\begin{array}{lcr} 5x+4y=1 & \xrightarrow{\times 3} & 15x+12y=3 \\ 3x-6y=2 & \xrightarrow{\times 2} & \underline{6x-12y=4} \\ & & 21x \qquad\;\; =7 \end{array}$$

So, as the description of the method promised we have an equation that can be solved for

x. Doing this gives, $x=\dfrac{1}{3}$ which is exactly what we found in the previous example.

To find *y* we need to substitute the value of *x* into either of the original equations and solve for *y*.

$$3\left(\frac{1}{3}\right)-6y=2 \qquad 1-6y=2 \qquad -6y=1 \qquad y=-\frac{1}{6}$$

6. Rational Exponent

You already know of one relationship between exponents and radicals: the appropriate radical will "undo" an exponent, and the right power will "undo" a root. For example:

$$\sqrt[3]{(2)^3} = 2$$
$$\sqrt[4]{(3)^4} = 3$$

But there is another relationship (which, by the way, can make computations like those above much simpler): For the square (or "second") root, we can write it as the one-half power, like this:

$$\sqrt{2} = 2^{1/2} \qquad \text{or} \qquad \sqrt{4} = 4^{1/2} = 2$$

The cube (or "third") root is the one-third power:

$$\sqrt[3]{8} = 8^{1/3} = 2$$

The fourth root is the one-fourth power:

$$\sqrt[4]{81} = 81^{1/4} = 3$$

The fifth root is the one-fifth power; and so on.

Looking at the first examples, we can re-write them like this:

$$\sqrt[3]{(2)^3} = \left(2^3\right)^{1/3} = \left(2^{3/1}\right)^{1/3} = 2^{\frac{3}{1}\cdot\frac{1}{3}} = 2^1 = 2$$
$$\sqrt[4]{(3)^4} = \left(3^4\right)^{1/4} = \left(3^{4/1}\right)^{1/4} = 3^{\frac{4}{1}\cdot\frac{1}{4}} = 3^1 = 3$$

Fractional exponents allow greater flexibility (you'll see this a lot in calculus), are often easier to write than the equivalent radical format, and permit you to do calculations that you couldn't before. For instance:

$$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{\frac{1}{10}\cdot\frac{5}{1}} = 25^{1/2} = \sqrt{25} = 5$$

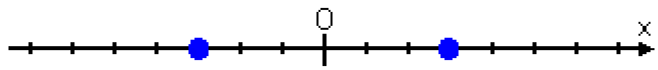
Whenever you see a fractional exponent, remember that the top number is the power, and the lower number is the root (if you're converting back to the radical format). For instance:

$$7^{2/3} = \sqrt[3]{7^2} = \left(\sqrt[3]{7}\right)^2$$

By the way, some decimal powers can be written as fractional exponents, too. If you are given something like "$3^{5.5}$", recall that $5.5 = {}^{11}/_{2}$, so: $3^{5.5} = 3^{11/2}$

7. Absolute Value

The absolute value of *x*, denoted "| *x* |" (and which is read as "the absolute value of *x*"), is the distance of *x* from zero. This is why absolute value is never negative; absolute value only asks "how far?" not "in which direction?" This means not only that | 3 | = 3, because 3 is three units to the right of zero, but also that | –3 | = 3, because –3 is three units to the left of zero.



Warning: The absolute-value notation is *bars*, not parentheses or brackets. Use the proper notation; the other notations do *not* mean the same thing.

► Questions in class

1. If $\frac{x-y}{x+y}=\frac{5}{2}$, then $\frac{x}{y}$ is equal to what?

2. The number $2^n + 2^n + 2^n + 2^n$ can be written as

a. 2^{4n} b. 16^n c. $24(4n^2-2)$ d. 2^{2n+2} e. $16^{\frac{n+2}{4}}$

3. If *a* is any real number, for what real value(s) of *b* does the equation $|x + a| = b$ have NO solutions for *x*?

4. How many integers are there such that $7x + 2 \le 23$ and $3x - 5 \ge 1$?

5. A square has an area of R^2. An equilateral triangle has a perimeter of E. If *r* is the perimeter of the square and *e* is a side of the equilateral triangle, then, in terms of R and E, $e + r$ = ?

6. Given the line $4x - 3y = 12$, what is the distance between it and the origin?

7. Given $x + 3y = a$ and $2x + 5y = b$, solve for *x* in terms of a and b.

8. In December 2001, the first snow occurred in New York with a depth of 12.1 cm. At the same time, the snow in Toronto reached a depth of 18.6 cm. It continued to snow at a constant rate of 2.6 cm per hour in New York and y cm per hour in Toronto over the next 13 hours. After 13 hours, the depth of snow in two cities are the same. What is the value of y?

9. Let s and w represent positive integers and x, y satisfy $\frac{x}{s}+\frac{y}{w}=1$ and $\frac{s}{x}+\frac{w}{y}=4$, Find $x + y$ in terms of s and w.

10. The average of four positive integers less than 100 is 94. What is the minimum possible value for one of the integers?