AP Calculus Practice Test 3

1 A particle moves in the xy-plane with position given by $(x(t), y(t)) = \left(5 - 2t, t^2 - 3\right)$ at time t. In which direction is the particle moving as it passes through the point $(3, -2)$?

(A) Up and to the left

(B) Down and to the left

(C) Up and to the right

(D) Down and to the right

(E) Straight up

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2y - x$ with initial condition $f(1) = 2$. What is the approximation for $f(0)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) -1 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{27}{4}$

3. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{n+2}$

(B) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2}$

(C) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2n}$

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+2n}$

(E) $\sum_{n=1}^{\infty} \frac{3n^2}{n^4+2n}$

4. A population of wolves is modeled by the function P and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1 - \frac{P}{5000}\right)$, where t is the time in years and $P(0) = 1000$. Which of the following statements are true?

I. $\lim_{t \to \infty} P(t) = 5000$

II. $\frac{dP}{dt}$ is positive for $t > 0$.

III. $\frac{d^2P}{dt^2}$ is positive for $t > 0$.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

5. Which of the following integrals gives the area of the region that is bounded by the graphs of the polar equations $\theta = 0$, $\theta = \frac{\pi}{4}$, and $r = \frac{2}{\cos\theta + \sin\theta}$?

(A) $\int_0^{\pi/4} \frac{1}{\cos\theta + \sin\theta}\, d\theta$

(B) $\int_0^{\pi/4} \frac{2}{\cos\theta + \sin\theta}\, d\theta$

(C) $\int_0^{\pi/4} \frac{2}{(\cos\theta + \sin\theta)^2}\, d\theta$

(D) $\int_0^{\pi/4} \frac{4}{(\cos\theta + \sin\theta)^2}\, d\theta$

(E) $\int_0^{\pi/4} \frac{2(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)^4}\, d\theta$

6. The sum of the series $1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots + \frac{2^n}{n!} + \cdots$ is

(A) $\ln 2$ (B) e^2 (C) $\cos 2$ (D) $\sin 2$ (E) nonexistent

7. If $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$, for $t > 0$, then in terms of t, $\frac{d^2y}{dx^2} =$

(A) $\frac{1}{2}$ (B) 2 (C) $4t$ (D) $6t^2$ (E) $12t^2$

8. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

9. Let f be a function with second derivative $f''(x) = \sqrt{1 + 3x}$. The coefficient of x^3 in the Taylor series for f about $x = 0$ is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{3}{2}$

10. What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$?

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 4 (E) 6

11 $\int_1^{\infty} \frac{1}{x^p}\,dx$ and $\int_0^{1} \frac{1}{x^p}\,dx$ both diverge when $p =$

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) -1

12 Which of the following is the solution to the differential equation $\frac{dy}{dx} = -2xy$ with the initial condition $y(1) = 4$?

(A) $y = e^{x^2} + 4 - e$

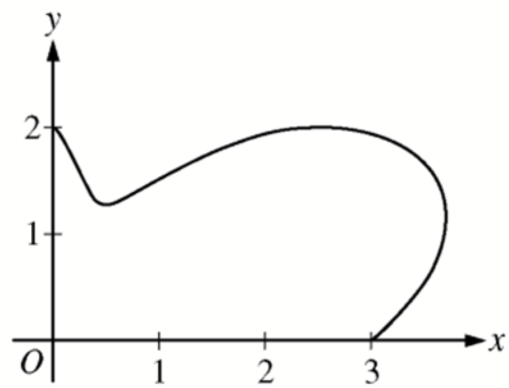
(B) $y = e^{-x^2} + 4 - \frac{1}{e}$

(C) $y = 4e^{x^2-1}$

(D) $y = 4e^{-x^2+1}$

(E) $y = e^{-x^2+16}$

Free-Response Questions



1. The figure above shows the graph of the polar equation $r = 2 + \sin(4\theta) + \cos(\theta)$ for $0 \le \theta \le \frac{\pi}{2}$. The derivative of r with respect to θ is given by $r'(\theta) = 4\cos(4\theta) - \sin(\theta)$.

   (a) Find the area of the region bounded by the graph of r and the lines $\theta = 0$ and $\theta = \frac{\pi}{2}$.

   (b) Find the area of the region in the first quadrant that is outside the graph of $r = 2 + \sin(4\theta) + \cos(\theta)$ but inside the graph of the circle of radius 2 centered at the origin.

   (c) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve $r = 2 + \sin(4\theta) + \cos(\theta)$ with greatest distance from the origin. Justify your answer.

2. The Maclaurin series for a function f is given by $\frac{x}{3} + \frac{x^2}{4} + \frac{x^3}{5} + \cdots + \frac{x^n}{n+2} + \cdots$.

(a) Use the ratio test to find the interval of convergence of the Maclaurin series for f.

(b) Let g be the function given by $g(x) = f(-2x)$. Find the first three terms and the general term of the Maclaurin series for g.

(c) The first two terms of the Maclaurin series for f are used to approximate $f(0.1)$. Given that $|f'''(x)| \le 2$ for $0 \le x \le 0.1$, use the Lagrange error bound to show that this approximation differs from $f(0.1)$ by at most $\frac{1}{3000}$.