First Name:____________________ Last Name: __________________ Student ID: ________

Test 2

Show your work!

1. Consider the following vectors: $\vec{a}$= 2$\vec{i}$ + 4$\vec{j}$ - 6$\vec{k}$, $\vec{b}$= -3$\vec{i}$ + $\vec{k}$, and $\vec{c}$ = 3$\vec{j}$ - 2$\vec{k}$

a. Find the magnitude of $\vec{a}$

b. Find vector $\vec{d}$= -2$\vec{a}$ + 3$\vec{b}$ - 4$\vec{c}$ written as an algebraic vector in components form

c. Find a unit vector parallel to vector $\vec{a}$

d. Find $\vec{c} \times \vec{b}$

e. Find $\vec{a} \cdot \vec{b}$

2. Given $|\vec{a}|$ = 8, $|\vec{b}|$ = 12, and $|\vec{a} + \vec{b}|$ = 16, find the angle θ = $\angle(\vec{a}, \vec{b})$ and $\vec{a} \cdot \vec{b}$. (The angle between the vectors $\vec{a}$ and $\vec{b}$ when positioned tail to tail)

3. Find if the points P(0, 1, 2), Q(-1, 2, -3), and R(-3, 4, -13) are collinear or not.

4. Find the angle between the vectors $\vec{u}$ = (1, -2, 4) and $\vec{v}$ = (-2, 0, 3).(when positioned tail to tail)

5. Consider the parallelogram ABCD where A(0, 1, 2), B(1, -2, 3), and D(2, 1, 0).

a. Find the angle ∠A.

b. Find the area of triangle ΔABD.

c. Find the coordinate of vertex C.

6. Given the plane **π**: $x - 2y + 3z - 12 = 0$ and the point $P(1, -2, 1)$

a. Find the distance between the point and the plane.

b. Find the equation of the line passing through P and perpendicular to **π**.

7. Given the equation of a line L: $\vec{r} = (1, 0, 1) + s(-1, 1, 1)$

a. Find the equation of the plane that passes through $O(0, 0, 0)$ and contains the line L.

b. Find the equation of the plane that passes through $O(0, 0, 0)$ and is perpendicular to the line L.

c. Find the distance from $O(0, 0, 0)$ to the line L.

8. Find the distance between the parallel planes

$x + y - z + 1 = 0$ and $-3x - 3y + 3z - 4 = 0$.

9. Find the vector equation of the plane passing through A(-1, 0, 1), B(0, 1, 2), and C(1, -1, 0).

10. Find the image of A(-1, 3, 5) if it is reflected in the plane whose equation is

$2x + y + 3z - 2 = 0$.

11. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ prove that $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{1}{2}(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$

12. Given three no-coplanar vectors $\vec{a}$ = (2, 1, 0), $\vec{b}$ = (1, 0, -1), and $\vec{c}$ = (-1, -1, 0).

Find the volume of the tetrahedron defined the three vectors.

Bonus questions

13. Given three-dimensional vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, define

$\vec{u} = (\vec{b}\cdot\vec{c})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}$,
$\vec{v} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$,
$\vec{w} = (\vec{b}\cdot\vec{a})\vec{c} - (\vec{b}\cdot\vec{c})\vec{a}$.

Prove that if $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a triangle then $\vec{u}$, $\vec{v}$ and $\vec{w}$ also form a triangle similar with the first one.

14. a. Prove that the scalar equation of the plane that cuts the axes at *A* (a,0,0), *B* (0,b,0), and *C* (0,0,c) is given by $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

b. Calculate the value of the solid bounded by the planes with equations $3x+2y+5z=120$, $6x+4y+3z=60$, $x=0$, $y=0$, and $z=0$.