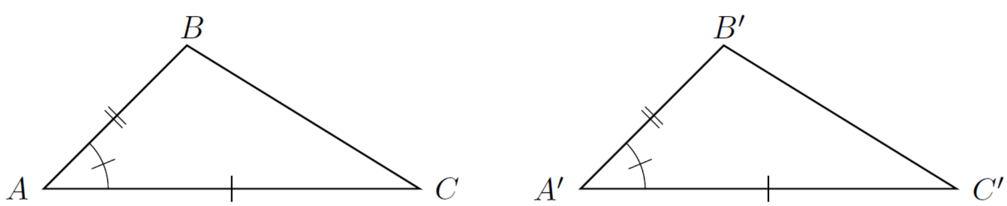
Geometry 1

Basic concepts

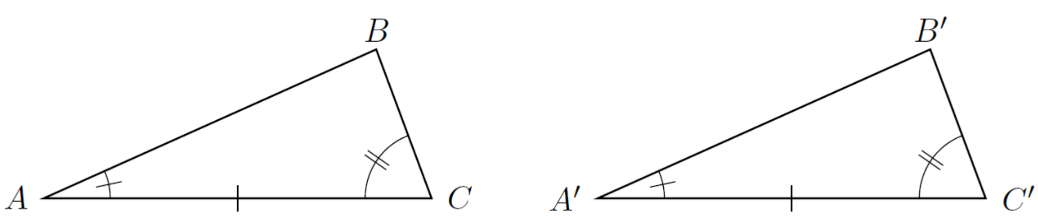
1. Ways to prove triangles are congruent

Side-Side-Side (SSS)
If all three sides of a triangle are congruent to all three corresponding sides of another triangle, the two triangles are congruent.

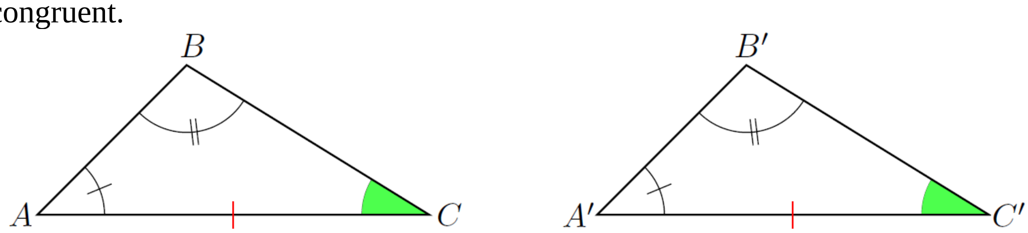
Side-Angle-Side (SAS):
If two sides of a triangle are congruent to two corresponding sides of another triangle, and the angle formed by the two sides is also congruent, then the two triangles are congruent.



Angle-Side-Angle (ASA):
If two angles of a triangle are congruent to two corresponding angles of another triangle, and the side between the two angles is also congruent, then the two triangles are congruent.

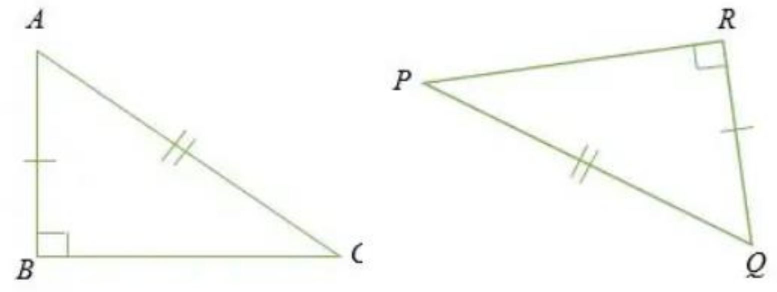


Angle-Angle-Side (AAS):
If two angles of a triangle are congruent to two angles of another triangle, and a corresponding side (not necessarily between the two angles) is also congruent, then the two triangles are congruent.



Hypotenuse-Leg (HL), right-triangle only:

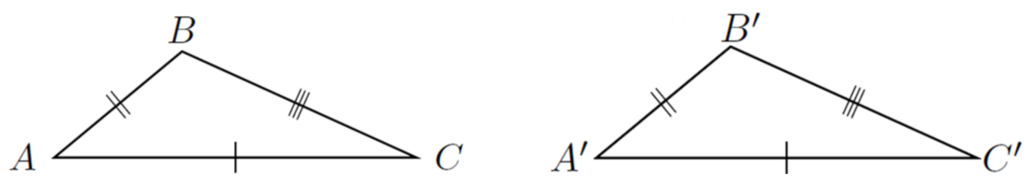
If the hypotenuse of two right-triangles is congruent and one of their legs is congruent, then the two right-triangles are congruent.



2. Ways to prove triangles are similar

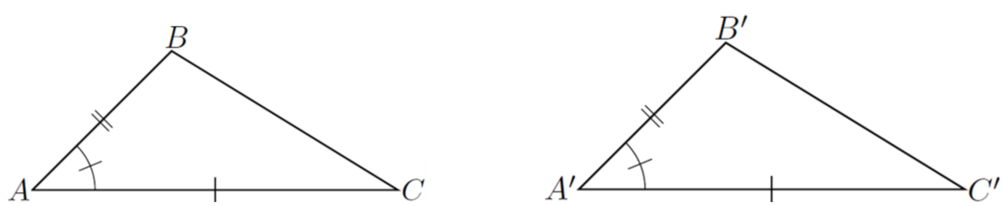
Side-Side-Side (SSS)

If all three sides of a triangle are proportional to all three corresponding sides of another triangle (it is called similarity ratio), the two triangles are similar.



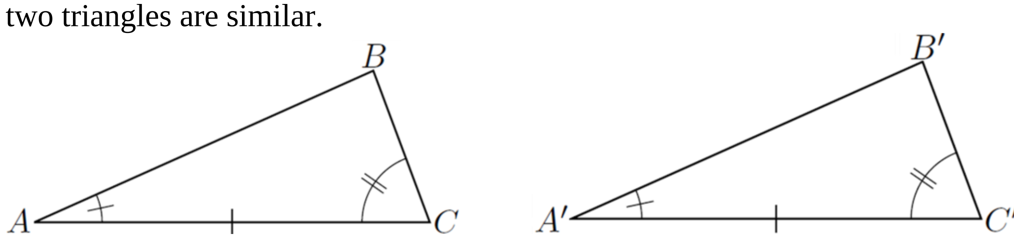
Side-Angle-Side (SAS):

If two sides of a triangle are proportional to two corresponding sides of another triangle, and the angle formed by the two sides is also congruent, then the two triangles are similar.



Angle-Angle (AA):

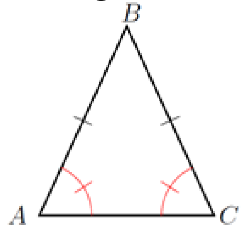
If two angles of a triangle are congruent to two corresponding angles of another triangle, then the two triangles are similar.



3. Triangle Theorems

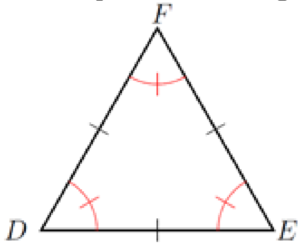
Isosceles Triangle

In an isosceles triangle, the base angles (the angles on the opposite sides of the congruent sides) are congruent.



Equilateral Triangle

In an equilateral triangle, all three angles are congruent.

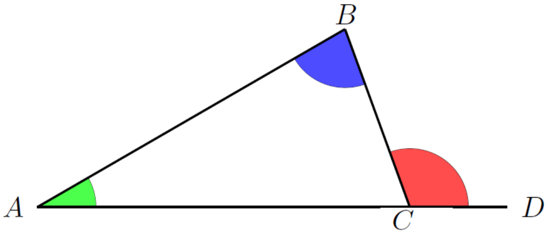


Sum of Interior Angles of a Triangle

The sum of the three interior angles of any triangle is always equal to 180^0 (two right angles)

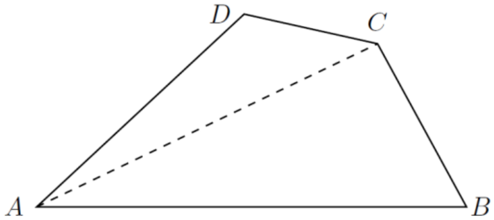
If two angles of one triangle is congruent to two angles of another triangle, then the third angle must also be congruent

The sum of any two interior angles of a triangle is equal to the opposite exterior angle. In the picture below, $m\angle A + m\angle B = m\angle BCD$.



Sum of Interior Angles of a Quadrilateral:

The sum of the interior angles of a quadrilateral is 360^0.

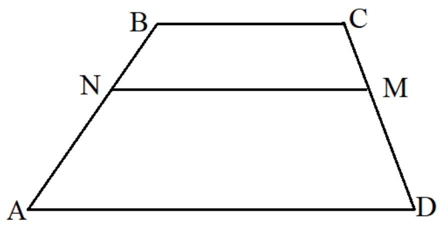


In picture above, notice that AC divides quadrilateral ABCD into two triangles, ΔACD and ΔACB. The sum of the interior angles of each triangle is 180◦, so the sum of the interior angles of a quadrilateral is 360^0.

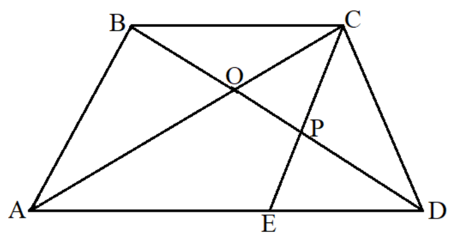
In general, for a polygon with n sides, the sum of its interior angles is equal to $(n-2)180^0$.

In-class questions

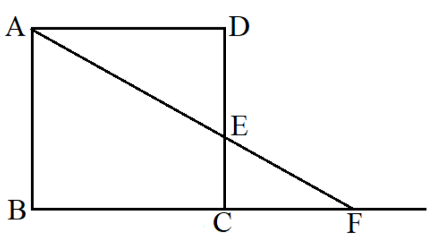
1. Let the lengths of bases AD and BC of trapezoid ABCD be a and b (a > b). Find the length of segment MN whose endpoints divide AB and CD in the ratio of $AM : MB = DN : NC = p : q$.



2. Point E on base AD of trapezoid ABCD is such that $AE = BC$. Segments CA and CE intersect diagonal BD at O and P, respectively. Prove that if $BO = PD$, then $AD^2 = BC^2 + AD \cdot BC$.

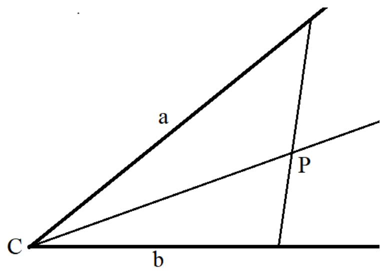


3. A straight line passing through vertex A of square ABCD intersects side CD at E and line BC at F. Prove that $\frac{1}{AE^2} + \frac{1}{AF^2} = \frac{1}{AB^2}$

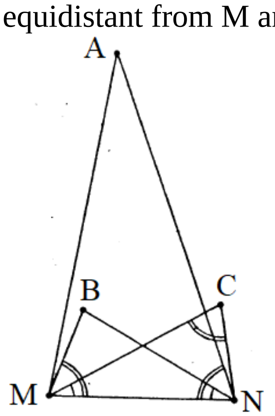


4. Through an arbitrary point P on side AC of △ABC straight lines are drawn parallelly to the triangle's medians AK and CL. The lines intersect BC and AB at E and F, respectively. Prove that AK and CL divide EF into three equal parts.

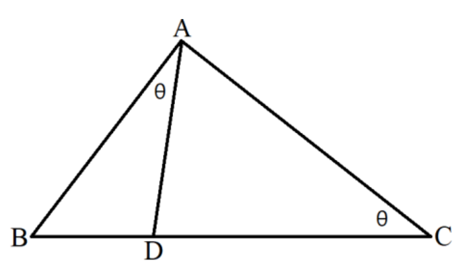
5. Point P lies on the bisector of an angle with vertex C. A line passing through P intercepts segments of lengths a and b on the angle's legs. Prove that the value of $\frac{1}{a}+\frac{1}{b}$ does not depend on the choice of the line.



6. Equally oriented similar triangles AMN, NBM and MNC are constructed on segment MN. Prove that △ABC is similar to all these triangles and the center of its circumscribed circle is equidistant from M and N.



7. Line segment BE divides △ABC into two similar triangles, their similarity ratio being equal to √3. Find the angles of △ABC.



Extra questions

8. In right triangle ABC with right angle $\angle C$, points D and E divide leg BC of into three equal parts. Prove that if $BC = 3AC$, then $\angle AEC + \angle ADC + \angle ABC = 90°$.