Lesson #6

Solving Rational Equations

- The root of the equation $\frac{ax+b}{cx+d}=0$ is the zero (x-intercept) of the function $f(x)=\frac{ax+b}{cx+d}$. Note that the zeroes of a rational function are the zeroes of the function in the numerator. Reciprocal functions do not have zeroes. All functions of the form $f(x)=\frac{1}{g(x)}$ have the x-axis as a horizontal asymptote. They do not intersect the x-axis.
- You can solve a rational equation algebraically by multiplying each term in the equation by the lowest common denominator and solving the resulting polynomial equation.
- When using graphing technology to solve a rational equation, you can either determine the zeroes of the corresponding rational function, or determine the intersection of two functions.
- When solving contextual problems, it is important to check for inadmissible solutions that are outside the domain determined by the context.

Example 1 Solve each equation algebraically.

a) $\frac{12}{x}+x=8$

b) $\frac{2x}{2x+3}-\frac{2x}{2x-3}=1$

Example 2 Solve each equation. Round your answers to two decimal places, if necessary.

a) $\frac{4}{x-2}=3$

b) $\frac{2}{x+1}+5=\frac{1}{x}$

Solving Rational Equations

- Factor and reduce where possible
- Multiply the equation by the least common multiple of the denominators
- Simplify and solve
- Make sure to state the restrictions
- Watch out for any extraneous roots

1. $\frac{2x}{3}+\frac{x-2}{5}=\frac{1}{6}$

2. $\frac{x+1}{2}-\frac{5x}{3}=-x$

3. $\frac{4}{x+2}+\frac{5}{x-2}=\frac{29}{x^2-4}$

4. $\frac{7}{x+2}-\frac{4}{x-3}=\frac{-14}{x^2-x-16}$

5. $\frac{x}{x-1} + \frac{3}{x} = \frac{5}{2x}$

6. $\frac{x}{x+1} = \frac{5}{2x-2} - \frac{1}{2}$

7. $\frac{x+3}{x+2} = 1 - \frac{x+1}{x+2}$

8. $\frac{4x}{x+1} + \frac{x+5}{x+1} = 3$

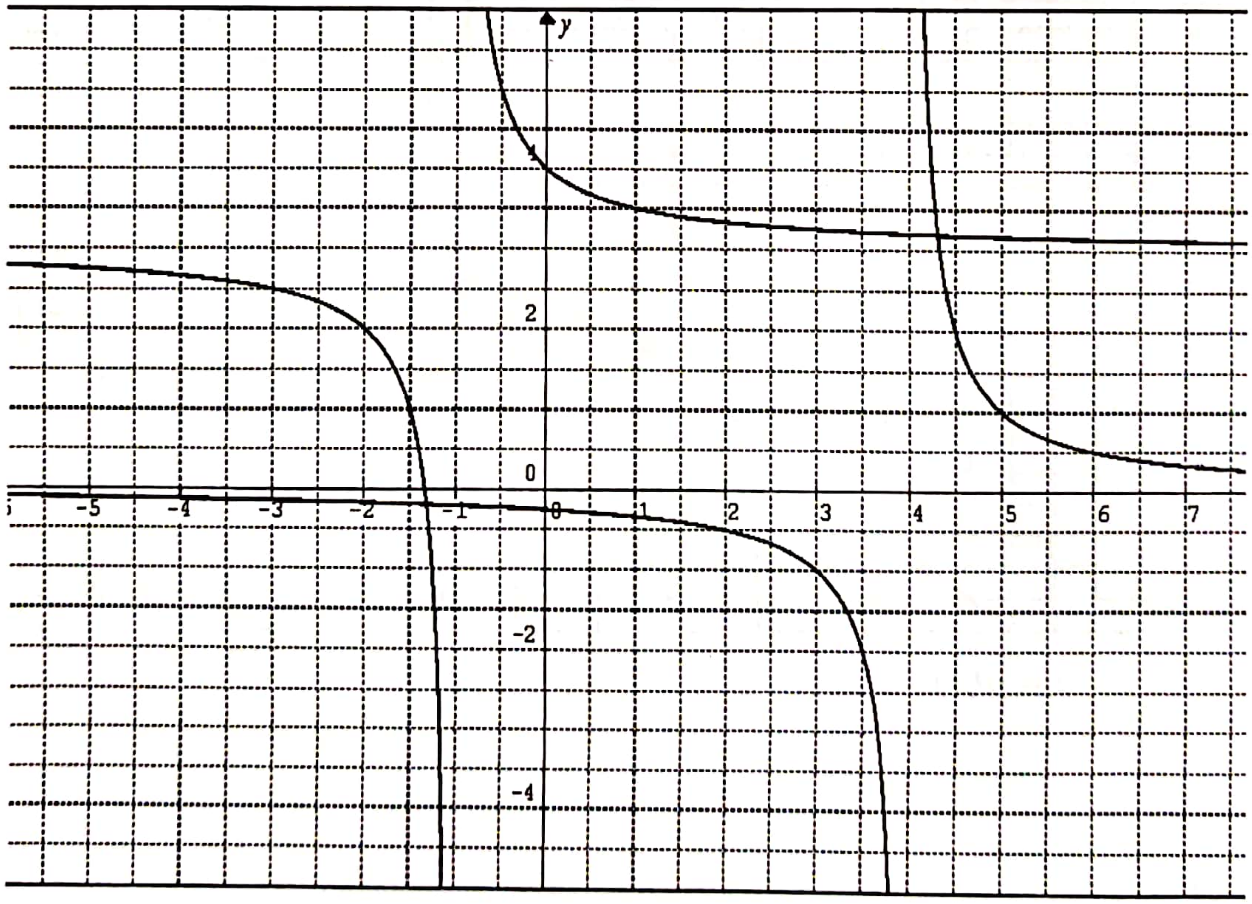
Solving Rational Functions by Graphing

- Look for the intersections between the graphs
- Look for regions where one graph is above or below the other graph

a) Determine the intervals where $f(x) > g(x)$

$$f(x) = \frac{1}{x+1} + 3$$
$$g(x) = \frac{1}{x-4}$$

$y = f(x)$



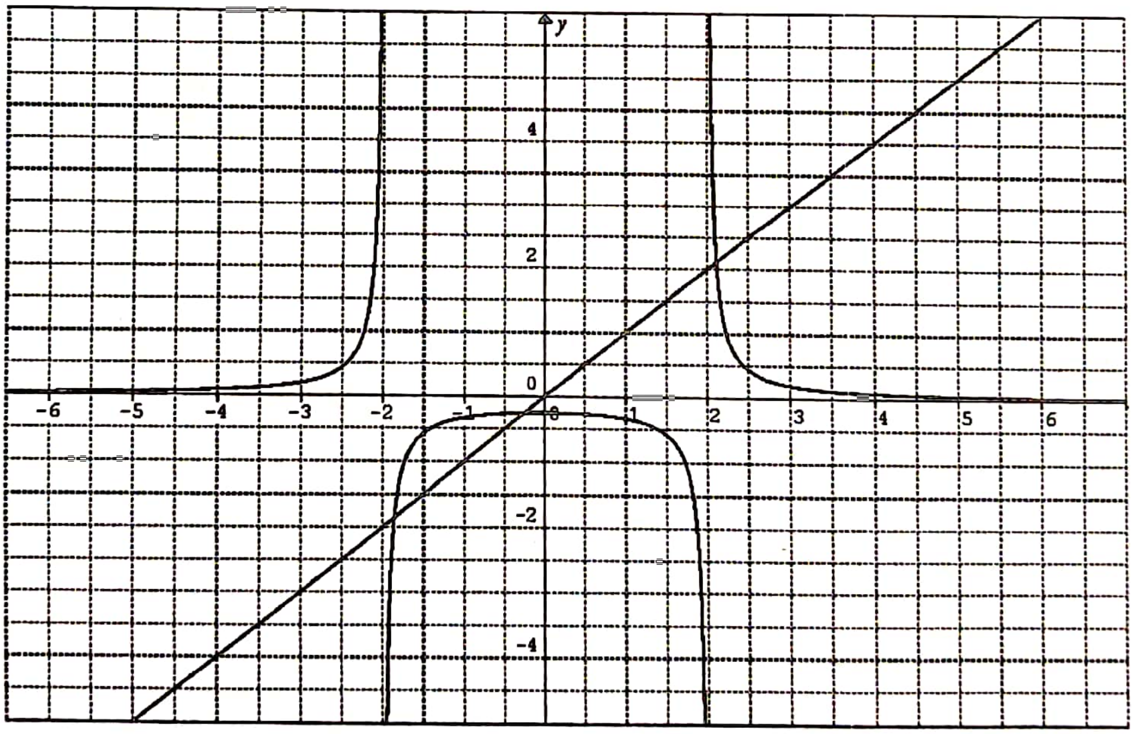
$y = g(x)$

b) Determine the intervals where $f(x) \leq g(x)$

$$f(x) = \frac{1}{x^2 - 4}$$

$$g(x) = x$$

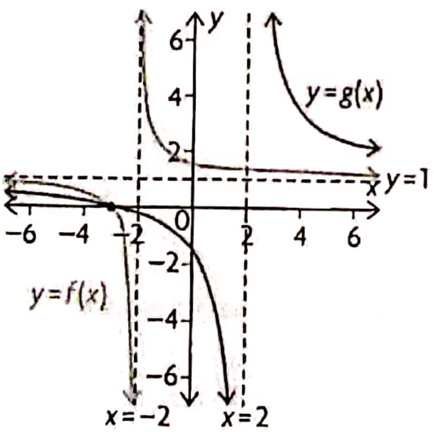
$y = f(x)$



$y = g(x)$

Solving Rational Inequalities

- To solve a rational inequality algebraically, rearrange the inequality so that one side is zero. Combine the expressions on the non-zero side using a common denominator. Make a table to examine the sign of each factor and the sign of the entire expression on the intervals created by the zeroes of the numerator and the denominator.
- You can solve an inequality using graphing technology by graphing the functions on each side of the inequality sign and then identifying all the intervals created by the vertical asymptotes and points of intersection. For x-values that satisfy $f(x) > g(x)$, identify the specific intervals where the graph of $f(x)$ is above the graph of $g(x)$. For x-values that satisfy $f(x) < g(x)$, identify the specific intervals where the graph of $f(x)$ is below the graph of $g(x)$. Consider the following graph:



In this graph, there are four intervals to consider: $(-\infty, -3)$, $(-3, -2)$, $(-2, 2)$ and $(2, \infty)$. In these intervals, $f(x) > g(x)$ when $x \in (-\infty, -3)$ or $(-2, 2)$, and $f(x) < g(x)$ when $x \in (-3, -2)$ or $(2, \infty)$.

- You can also solve an inequality using graphing technology by creating an equivalent inequality with zero on one side and then identifying the intervals created by the zeroes on the graph of the new function. Finding where the graph lies above the x-axis (where $f(x) > 0$) or below the x-axis (where $f(x) < 0$) defines the solutions to the inequality.

Example 1 Solve each inequality algebraically. Write the solution using interval notation.

a) $\dfrac{3x+1}{2x-4}<0$

b) $\dfrac{x+1}{x-2}\geq\dfrac{x+7}{x+1}$

Example 2 Solve each inequality. Write the solution using set notation.

a) $\dfrac{3}{2x+4}>-2$

b) $\dfrac{3}{x+2}\leq\dfrac{4}{x+3}$