

# AP Calculus Homework 4

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write  $\pi$  instead of 3.14.

Questions with a \* are optional. Questions with \*\* are optional and more challenging.

1. Differentiate the following functions.

a)  $y = e^u(\cos u + cu)$       b)  $y = \frac{\sin x}{x^2}$       c)\*  $f(x) = xe^x \csc x$

d)\*\*  $y = \frac{(1+x^2)\tan^{-1}x - x}{2}$

2. Find an equation of the tangent line to the curve  $y = \sec x - 2\cos x$  at the point  $(\pi/3, 1)$ .

3. Find the derivative of the function (Choose any five problems)

a)  $g(t) = \frac{1}{(t^4 + 1)^3}$       b)  $y = xe^{-kx}$       c)  $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

d)  $y = e^{-5x} \cos 3x$       e)  $y = \sin(\tan 2x)$       f)  $y = 2^{3x^2}$       g)  $y = [x + (x + \sin^2 x)^3]^4$

4. Find an equation of the tangent line to the curve  $y = (1+2x)^{10}$  at the point  $(0, 1)$ .

5.\* If  $F(x) = f(xf(xf(x)))$ , where  $f(1) = 2$ ,  $f(2) = 3$ ,  $f'(1) = 4$ ,  $f'(2) = 5$ , and  $f'(3) = 6$ , find  $F'(1)$ .

6.\*\* Find the derivative of the following function

$$f(x) = \frac{1}{x - \frac{2}{x + \sin x}}$$

7. Find  $\frac{dy}{dx}$  by implicit differentiation.

a)  $y^5 + x^2y^3 = 1 + ye^{x^2}$       b)  $y \sin(x^2) = x \sin(y^2)$       c)  $\sqrt{xy} = 1 + x^2y$

8.\* Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$  at  $(0, 1/2)$ .

9. Find the derivative of the function. Simplify where possible.

a)  $g(x) = \sqrt{x^2 - 1} \sec^{-1} x$       b)  $h(t) = \cot^{-1}(t) + \cot^{-1}(1/t)$

c)\*  $F(\theta) = \arcsin \sqrt{\sin \theta}$

10. Find the second derivative of the function  $y = xe^{cx}$

11. If  $g$  is a twice differentiable function and  $f(x) = xg(x^2)$ , find  $f''$  in terms of  $g$ ,  $g'$ , and  $g''$ .

12. If  $f(x) = x\sqrt{2x-3}$ , then  $f'(x) =$

A)  $\frac{3x-3}{\sqrt{2x-3}}$       B)  $\frac{x}{\sqrt{2x-3}}$       C)  $\frac{1}{\sqrt{2x-3}}$       D)  $\frac{-x+3}{\sqrt{2x-3}}$       E)  $\frac{5x-6}{2\sqrt{2x-3}}$

13.  $\frac{d}{dx} \left( xe^{\ln x^2} \right) =$

A)  $1+2x$       B)  $x+x^2$       C)  $3x^2$       D)  $x^3$       E)  $x^2+x^3$

14. The slope of the tangent line to the curve  $y^2 + (xy+1)^3 = 0$  at  $(2, -1)$  is

A)  $-\frac{3}{2}$       B)  $-\frac{3}{4}$       C)  $0$       D)  $\frac{3}{4}$       E)  $\frac{3}{2}$

15. If  $\frac{dy}{dx} = \sqrt{1-y^2}$ , then  $\frac{d^2y}{dx^2} =$

A)  $-2y$       B)  $-y$       C)  $\frac{-y}{\sqrt{1-y^2}}$       D)  $y$       E)  $\frac{1}{2}$