AP Calculus Practice Test 2

1. $\int x^2\left(x^3+5\right)^6\,dx =$

(A) $\frac{1}{3}\left(x^3+5\right)^6+C$

(B) $\frac{1}{3}x^3\left(\frac{1}{4}x^4+5x\right)^6+C$

(C) $\frac{1}{7}\left(x^3+5\right)^7+C$

(D) $\frac{3}{7}x^2\left(x^3+5\right)^7+C$

(E) $\frac{1}{21}\left(x^3+5\right)^7+C$

2. Which of the following gives the length of the curve $y=\sqrt{x}$ over the closed interval $[1,4]$?

(A) $\int_1^4 \sqrt{1+\frac{1}{2\sqrt{x}}}\,dx$

(B) $\int_1^4 \sqrt{1+\frac{1}{2x}}\,dx$

(C) $\int_1^4 \sqrt{1-\frac{1}{4x}}\,dx$

(D) $\int_1^4 \sqrt{1+\frac{1}{4x}}\,dx$

(E) $\int_1^4 \sqrt{1+\frac{1}{4}x^2}\,dx$

3. $$\int \frac{6}{x^2 + 10x + 16}\,dx =$$

(A) $-\ln\left|(x+8)(x+2)\right| + C$

(B) $\ln\left|\dfrac{x+2}{x+8}\right| + C$

(C) $\ln\left|\dfrac{x+8}{x+2}\right| + C$

(D) $6\ln\left|(x+8)(x+2)\right| + C$

(E) $6\ln\left|\dfrac{2x+10}{(x+2)(x+8)}\right| + C$

4. $$\int \left(2^t + e^\pi\right) dt =$$

(A) $\dfrac{2^{t+1}}{t+1} + \dfrac{e^{\pi+1}}{\pi+1} + C$

(B) $\dfrac{2^t}{\ln 2} + e^\pi t + C$

(C) $\dfrac{2^t}{\ln 2} + e^\pi + C$

(D) $2^t \ln 2 + \dfrac{e^{\pi+1}}{\pi+1} + C$

(E) $2^t \ln 2 + e^\pi t + C$

5. $\int_1^\infty \frac{1}{x^p}\,dx$ and $\int_0^1 \frac{1}{x^p}\,dx$ both diverge when $p =$

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) -1

6. If $F(x) = \int_4^{x^2} \sqrt{t}\,dt$ for all real numbers $x > 0$, then $F'(x) =$

(A) $-\frac{1}{2x}$ (B) $\sqrt{x}$ (C) x (D) $2x^2$ (E) $\frac{2x^3 - 16}{3}$

7. Let g be the function defined by $g(x) = \int_{-1}^{x} \frac{t^3 - t^2 - 6t}{\sqrt{t^2 + 7}}\,dt$. On which of the following intervals is g decreasing?

(A) $x \le -2$ and $0 \le x \le 3$

(B) $x \le -2$ and $x \ge 3$

(C) $-2 \le x \le 0$ and $x \ge 3$

(D) $-2 \le x \le 3$

(E) $x \le -1$

The following questions may require the use of a graphing calculator.

8. Let g be a function such that $g(-1) = 0$ and $g(2) = 5$. Which of the following conditions guarantees that there is an x, $-1 < x < 2$, for which $g(x) = 3$?

(A) g is defined for all x in $(-1, 2)$.

(B) g is continuous for all x in $[-1, 2]$.

(C) g is increasing on $[-1, 2]$.

(D) There exists an x in $(-1, 2)$ such that $g'(x) = 5$.

(E) $\int_{-1}^{2} g(x)\,dx = 3$

9. If $f(x) = (x + 2)\sin\left(\sqrt{x + 2}\right)$, what is the average value of f on the closed interval $[0, 6]$?

(A) 2.220 (B) 3.348 (C) 4.757 (D) 20.090 (E) 28.541

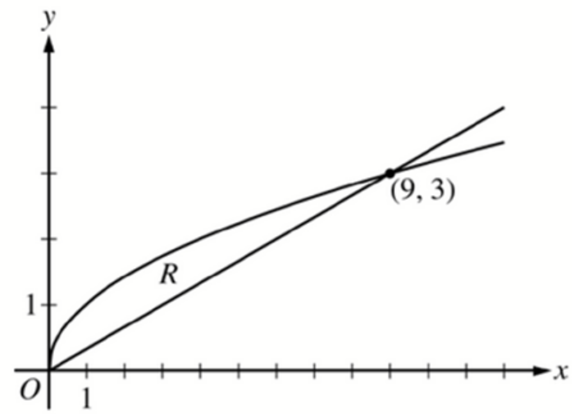
x	$f(x)$	$f'(x)$	$f''(x)$
0	1	–2	5
1	2	6	–1

10. Let f be a twice-differentiable function with selected values of f and its derivatives shown in the table above. What is the value of $\int_{0}^{1} x f''(x)\,dx$?

(A) 6 (B) 5 (C) 3 (D) $-\frac{1}{2}$ (E) -1

Free Response Questions

1. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.



(a) Find the area of region R.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

2. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.