AP Calculus Homework 3

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

a) $\lim_{x\to 5} \dfrac{2^x - 32}{x - 5}$ b) $\lim_{x\to \pi/4} \dfrac{\tan x - 1}{x - \pi/4}$

2. A particle moves along a straight line with equation of motion $s = f(t)$, where s is measured in meters and t in seconds. Find the velocity and the speed when $t = 5$.

a) $f(t) = 100 + 50t - 4.9t^2$ b) $f(t) = t^{-1} - t$

3. Differentiate the function. (Choose any five problems)

a) $f(x) = x^3 - 4x + 6$ b) $h(x) = (x - 2)(2x + 3)$ c) $R(t) = 5t^{-3/5}$

d) $G(x) = \sqrt{x} - 2e^x$ e) $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}}$ f) $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

g) $y = \dfrac{x^2 - 2\sqrt{x}}{x}$ h) $u = \sqrt[5]{t} + 4\sqrt{t^5}$ i) $v = \left(\sqrt{x} + \dfrac{1}{\sqrt[3]{x}}\right)^2$

j) $z = \dfrac{A}{y^{10}} + Be^y$ k) $y = e^{x+1} + 1$

4. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

5.** Find the value of c such that the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$.

6. Differentiate (Choose any five problems)

a) $f(x) = (x^3 + 2x)e^x$ b) $\dfrac{e^x}{1 + x}$ c) $f(t) = \dfrac{2t}{4 + t^2}$

d) $V(x) = (2x^3 + 3)(x^4 - 2x)$ e) $R(t) = (t + e^t)(3 - \sqrt{t})$ f) $y = \dfrac{x + 1}{x^3 + x - 2}$

g) $y = \dfrac{t^2 + 2}{t^4 - 3t^2 + 1}$ h) $y = (r^2 - 2r)e^r$ i) $z = w^{3/2}(w + ce^w)$

j) $f(t) = \dfrac{2t}{2 + \sqrt{t}}$

7. Suppose that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$. Find $h'(2)$ if

a) $h(x) = 5f(x) - 4g(x)$ b) $h(x) = f(x)g(x)$

c)* $h(x) = \dfrac{f(x)}{g(x)}$ d)* $h(x) = \dfrac{g(x)}{1 + f(x)}$

8. If f is a differentiable function, find an expression for the derivative of each of the following functions.

a) $y = x^2 f(x)$ b)* $y = \dfrac{x^2}{f(x)}$ c)* $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

9.* Find equations of the tangent lines to the curve

$$y = \frac{x - 1}{x + 1}$$

that are parallel to the line $x - 2y = 2$.

10. If $y = \dfrac{3}{4 + x^2}$, then $\dfrac{dy}{dx} =$

A) $\dfrac{-6x}{(4 + x^2)^2}$ B) $\dfrac{3x}{(4 + x^2)^2}$ C) $\dfrac{6x}{(4 + x^2)^2}$ D) $\dfrac{-3}{(4 + x^2)^2}$ E) $\dfrac{3}{2x}$

11. An equation of the line tangent to the graph of $y = \dfrac{2x + 3}{3x - 2}$ at the point $(1, 5)$ is

A) $13x - y = 8$ B) $13x + y = 18$ C) $x - 13y = 64$

D) $x + 13y = 66$ E) $-2x + 3y = 13$

12. If $f(x) = 2x^2 + 1$, then $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x^2}$ is

A) 0 B) 1 C) 2 D) 4 E) nonexistent

13. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

A) 3 B) 1 C) -1 D) -3 E) -5