Analytic Geometry

1. Linear Equation

1) Point-slope form

Given a point in the line (x_1, y_1), and the slope of the line, m, an equation of the line may be expressed as $y - y_1 = m(x - x_1)$

Example: Determine an equation of a line through point (3, 2) with slope m = 2.

Solution:
$(x_1, y_1) = (3, 2)$ and $m = 2$, so $y - 2 = 2(x - 3)$, this equation can be expressed in standard form:
$2x - y - 4 = 0$

2) Slope Y-intercept form

Given a slope and the y-intercept of the line, b, an equation of the line may be expressed in the form: $y = mx + b$.

Example: Determine an equation of the line with m=3 and y-intercept 2.
Solution: $b = 2$ and $m = 3$, the $y = 3x + 2$

3) Two point solution

Given two points (x_1, y_1) and (x_2, y_2), then the equation of the line can be expressed as

$(y - y_1) = \dfrac{(y_1 - y_2)}{(x_1 - x_2)}(x - x_1)$ or $y - y_1 = m(x - x_1)$, here $m = \dfrac{(y_1 - y_2)}{(x_1 - x_2)}$

Example: given two points $P_1\,(2, 3)$ and $P_2\,(-1, 2)$, determine the equation of the line.

Solution: $m = \dfrac{(y_1 - y_2)}{(x_1 - x_2)} = \dfrac{(3-2)}{(2-(-1))} = \dfrac{1}{3}$, so $y - 3 = \dfrac{1}{3}(x - 2)$, this equation can be expressed in standard form: $x - 3y + 7 = 0$

2. Length of segment

The length of a line segment can be found by Pythagorean Theorem given two points $P_1\,(x_1, y_1)$ and $P_2(x_2, y_2)$, then the segment joining P_1 and P_2 may be expressed by following formula:

$L=\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Example: Find the length of the line segment joining points (3, 2) and (−1, 4)

Solution: $L=\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3 -(-1))^2 + (2 - 4)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = 4\sqrt{5}$

3. Midpoint of a line segment

We can calculate the coordinates of the midpoint of a line segment if the coordinates of the endpoints are given.

The coordinates of the midpoint M of the segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

► The relations between two lines with slope m_1 and m_2:

- if $m_1=m_2$, then two lines are parallel;
- if $m_1 \cdot m_2=-1$, then two lines are perpendicular;
- if $m_1 \neq m_2$, the two lines have one intersection.

In-class questions

1. The vertices of the quadrilateral ABCD in counter-clockwise order are $A(0,\ 0),\ B(k,\ 0),$ $C(k+m,\ n),\ and\ D(m,\ n)$, where $k>0,\ m>0,\ n>0$. What is the area of the quadrilateral ABCD?

2. A square in the coordinate plane has vertices whose y-coordinates are 1, 3, 6, and 8. What is the area of the square?

3. Two perpendicular lines L_1 and L_2 intersect at the point $Q(p,\ 2p)$ in the first quadrant. If $S(p-6,\ p)$ *is on* L_1 *and* $T(p+6,\ -p)$ *is on* L_2, which of the following is true?

   a) Q may be any point on the line $y = 2x$
   b) there is no such point Q
   c) there is exactly one possible position for the point Q
   d) there are exactly two possible positions for the point Q
   e) the number of possible positions for the point Q is greater than two, but finite

4. A bug following the line $4x+3y=60$ wants to move to the line $4x+3y=120$. What is the shortest distance that she can travel to get from one line to the other?

5. The line L_1 has equation $y=-\frac{4}{3}x$ and passes through the origin, O. The line L_2 has equation $y=-\frac{1}{2}x+5$ and crosses the x-axis at P. Lines L_1 *and* L_2 intersect at Q.

   a) What are the coordinates of points P and Q? (No justification is required.)
   b) Find the area of ΔOPQ.
   c) Point R is on the positive x-axis so that the area of ΔOQR is three times the area of ΔOPQ. Determine the coordinates of R.