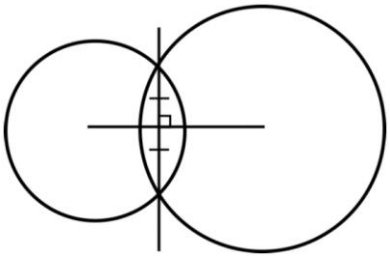
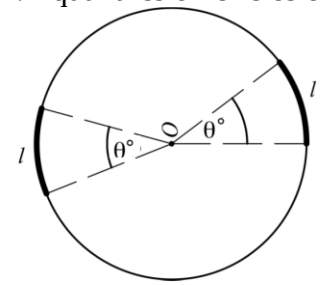
Geometry 2

Circle Geometry Theorems (Please take 10-15 minutes to revie the following concepts)

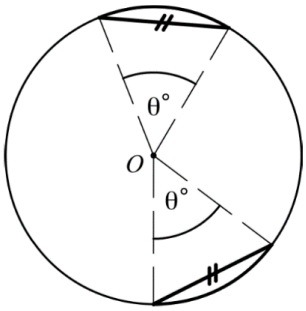
1. When two circles intersect, the line joining their centres bisects their common chord at right angles



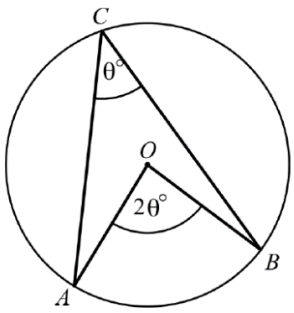
2. Equal arcs on circles of equal radii subtend equal angles at the centre, and conversely.



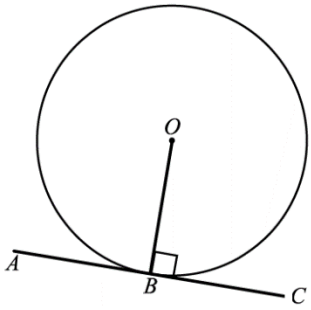
3. Equal angles at the centre stand on equal chords, and conversely.



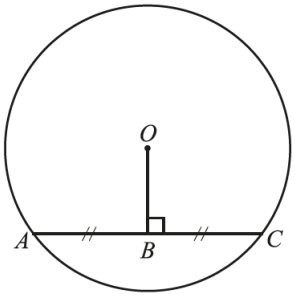
4. The angle at the centre is twice the angle at the circumference subtended by the same arc.



5. The tangent to a circle is perpendicular to the radius drawn to the point of contact and conversely.



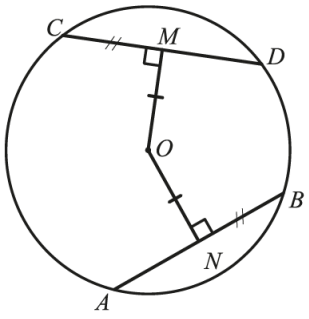
6. The perpendicular from the centre of a circle to a chord bisects the chord.



7. The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord (see above diagram).

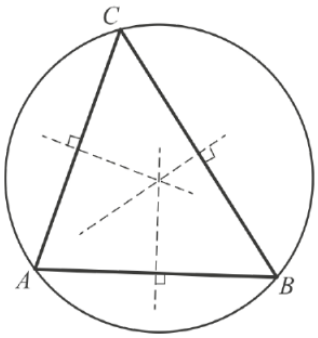
8. The perpendicular bisector of a chord passes through the centre of the circle (see above diagram).

9. Equal chords in equal circles are equidistant from the centres.

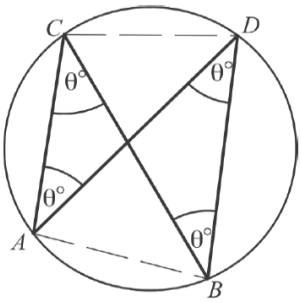


10. Chords in a circle which are equidistant from the centre are equal. (See above diagram)

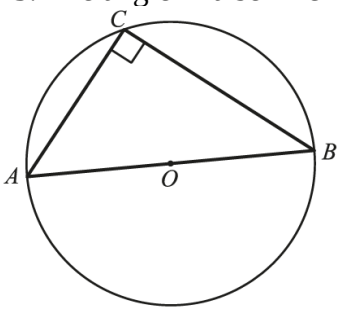
11. Any three non-collinear points lie on a unique circle, whose centre is the point of concurrency of the perpendicular bisectors of the intervals joining the points.



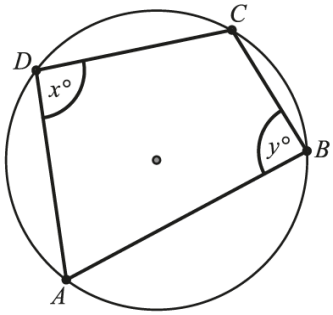
12. Angles in the same segment are equal.



13. The angle in a semi-circle is a right angle.

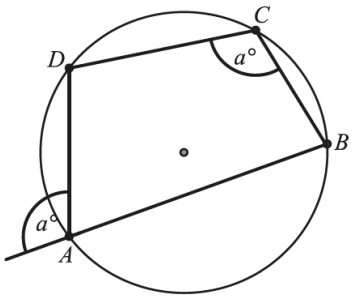


14. Opposite angles of a cyclic quadrilateral are supplementary.

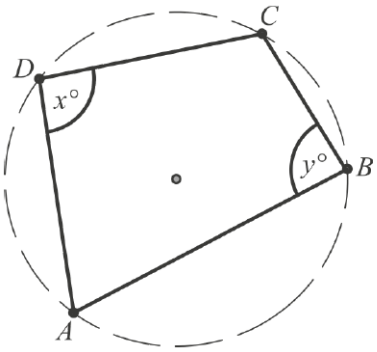


$x + y = 180$

15. The exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle.

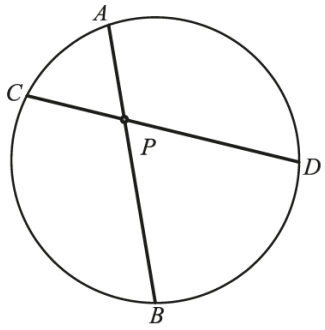


16. If the opposite angles in a quadrilateral are supplementary then the quadrilateral is cyclic.
Note: This theorem is also a test for four points to be concyclic.



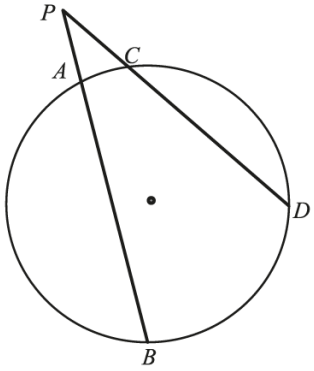
If $x + y = 180$ then $ABCD$ is a cyclic quadrilateral.

17. The products of the intercepts of two intersecting chords are equal.



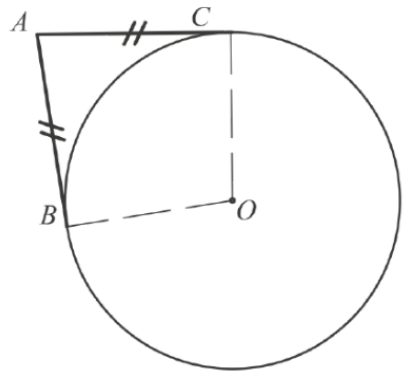
$AP \times BP = CP \times DP$

18. The products of the intercepts of two intersecting secants to a circle from an external point.

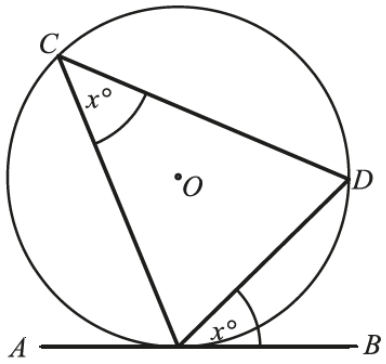


$AP \times BP = CP \times DP$

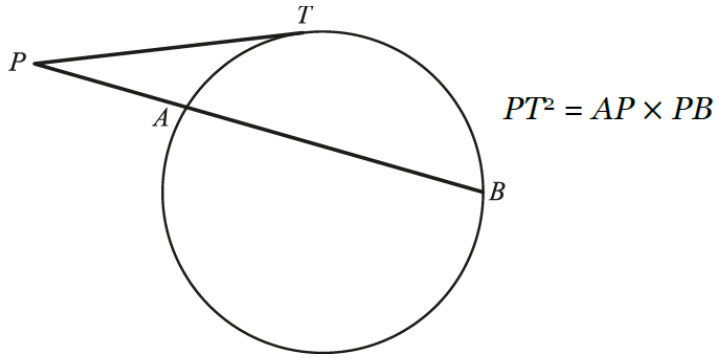
19. Tangents to a circle from an external point are equal.



20. The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

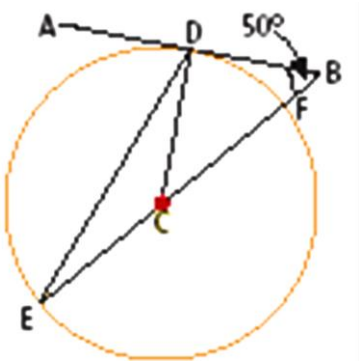


21. The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point ($PT^2 = AP \times PB$).



In-class questions

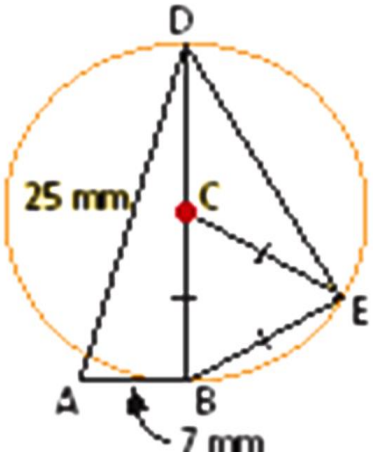
1. Determine Angle Measures in a Circle with a Tangent Line In the diagram shown, AB is tangent to the circle at point D, BE contains the diameter FE, and $\angle ABE = 50°$.



a) What is the measure of $\angle BDC$? Justify your answer.
b) What is the measure of central angle $\angle DCE$? Explain your reasoning.
c) What type of triangle is ΔCDE? Justify your answer.
d) What is the measure of $\angle DEC$? Explain your reasoning.

2. Use the Tangent Chord Relationship
In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle. AB = 7 mm, AD = 25 mm, and ΔBCE is an equilateral triangle.

a) What is the length of diameter BD? Justify your answer.
b) What is the length of chord BE? Explain your reasoning.
c) What is the measure of the inscribed angle $\angle BED$?
d) What is the length of chord DE? Justify your answer and express your answer to the nearest millimetre.

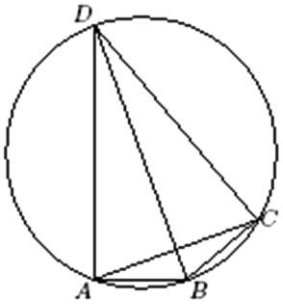


3. Three points A, B, C lie on the circumference of a circle K with centre O. The points C and O are on the same side of chord AB. Given $\angle OAB = 15^o$, find the size of $\angle ACB$.

4. In a quadrilateral ABCD, $\angle BAC = 30^o$, $\angle ADC = 80^o$, $\angle ADB = 50^o$, find the sizes of $\angle ACB$ and $\angle ABC$.

5. Two chords AC and BD of a circle with centre O, intersect at E. Show that $\angle AEB = \frac{1}{2}(\angle AOB + \angle COD)$.

6. In quadrilateral ABCD, $\angle ABC = 140^o$, $\angle CAD = 70^o$ and $AD = DC$, prove that BD bisects $\angle ABC$.



7. Chords AB and CD of a circle K are produced to meet at X. If O is the centre of K, prove that ∠AXC = $\frac{1}{2}$ (∠AOC - ∠BOD).