AP Calculus Homework Three – Differentiation
2.1 Definition of Derivative; 2.2 Differentiation Rules

1. Find $\dfrac{dy}{dx}$

(a) $y = x^5 \tan x$

(b) $y = \sqrt{3 - 2x}$

(c) $y = \dfrac{2}{(5x+1)^3}$

(d) $y = 3x^{2/3} - 4x^{1/2} - 2$

(e) $y = \dfrac{x^2}{\cos x}$

(f) $y = \ln \dfrac{e^x}{e^x - 1}$

(g) $y = \tan^{-1} \dfrac{x}{2}$

(h) $y = \ln(\sec x + \tan x)$

(i) $y = \sin\left(\dfrac{1}{x}\right)$

(j) $y = e^{-x} \cos 2x$

(k) $y = \sec^2(x)$

(l) $y = \sin^{-1} x - \sqrt{1 - x^2}$

2. Find limits.

(a) $\lim\limits_{h \to 0} \dfrac{\sqrt[3]{8+h}-2}{h}$

(b) $\lim\limits_{h \to 0} \dfrac{\ln(e+h)-1}{h}$

(c) $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x}$

(d) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 4x}$

(e) $\lim\limits_{x \to 0} \dfrac{\tan \pi x}{x}$

(f) $\lim\limits_{x \to \infty} x^2 \sin \dfrac{1}{x}$

3. At how many points on the interval $[-5,\ 5]$ is a tangent to the curve of $y = x + \cos x$ parallel to the secant line that passes the two endpoints of the curve?

4. If f is differentiable and difference quotients overestimate the slope of f at $x = a$ for all $h > 0$, which must be true?
   (A) $f'(a) > 0$ (B) $f'(a) < 0$ (C) $f''(a) > 0$ (E) $f''(a) < 0$ (D) none of these

5. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, find $(f \circ g)'(3)$.

6. If $f(x) = \dfrac{x}{(x-1)^2}$, find the set of x's for which $f'(x)$ exists.

7. If $y = \sqrt{x^2 + 1}$, find the derivative of y^2 with respect to x^2.

8. Find the value of $f'(0)$ obtained using the symmetric difference quotient with $f(x) = |x|$ and $h = 0.001$. (the formula of symmetric difference quotient is $\dfrac{1}{2}\left[\dfrac{f(a+h)-f(a)}{h} + \dfrac{f(a)-f(a-h)}{h} \right]$)