Solving Polynomial Equations

- The solutions to a polynomial equation $f(x)=0$ are the zeroes of the corresponding polynomial function, $y=f(x)$.
- Many polynomial equations can be solved algebraically using a factoring strategy.
- If a given polynomial equation is not factorable, then graphing technology can be used to find the solution.
- When solving problems using polynomial models, it may be necessary to ignore the solutions that are outside the domain defined by the conditions of the problem.

Example 1 State the zeroes of the following functions.

a) $y=(3x^2-48)(4x^2-8x-5)$

b) $y=3x^4+81x$

Example 2 Solve the following equations algebraically by factoring.

a) $b^4-29b^2+100=0$

b) $9y^3-4y=8-18y^2$

c) $x^3-4x^2-7x+10=0$

Solving Polynomial Equations

- Factor where possible using the factor theorem.
- Find the roots or x-intercepts of the polynomial functions.

1. $x(x+2)(2x-5)=0$	2. $-3x(x^2-9)(2x^2+5x-3)=0$
3. $x^2+5x-6=0$	4. $x^3-3x^2-4x=0$
5. $x^3-3x-4x+12=0$	6. $6x^3-13x^2+x+2=0$

Solving Polynomial Inequalities

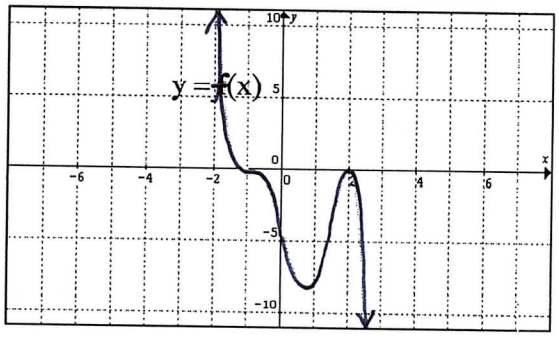
- A polynomial inequality can be solved algebraically by
  - using inverse operations to move all terms to one side of the inequality
  - factoring the polynomial to determine the zeroes of the corresponding polynomial equation
  - using a number line, a graph, or a factor table to determine the intervals on which the polynomial is positive or negative

- A polynomial inequality can be solved using graphing technology by
  - creating an equivalent inequality with zero on one side
  - identifying the intervals created by the zeroes of the graph of the new function
  - finding where the graph lies above the x-axis (where $f(x) > 0$) or below the x-axis (where $f(x) < 0$), as required

Example 1

Given the following polynomial function, state the intervals where

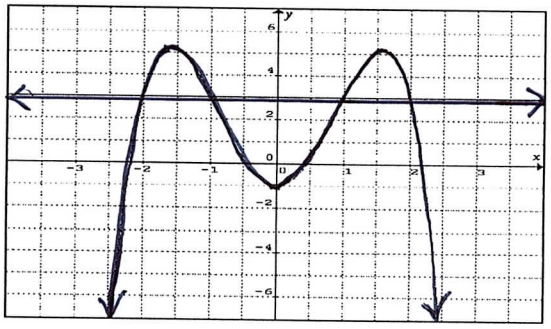
a) $f(x) \geq 0$ b) $f(x) < 0$



Example 2

For the following pair of functions, determine when

a) $f(x) > g(x)$ b) $f(x) \leq g(x)$

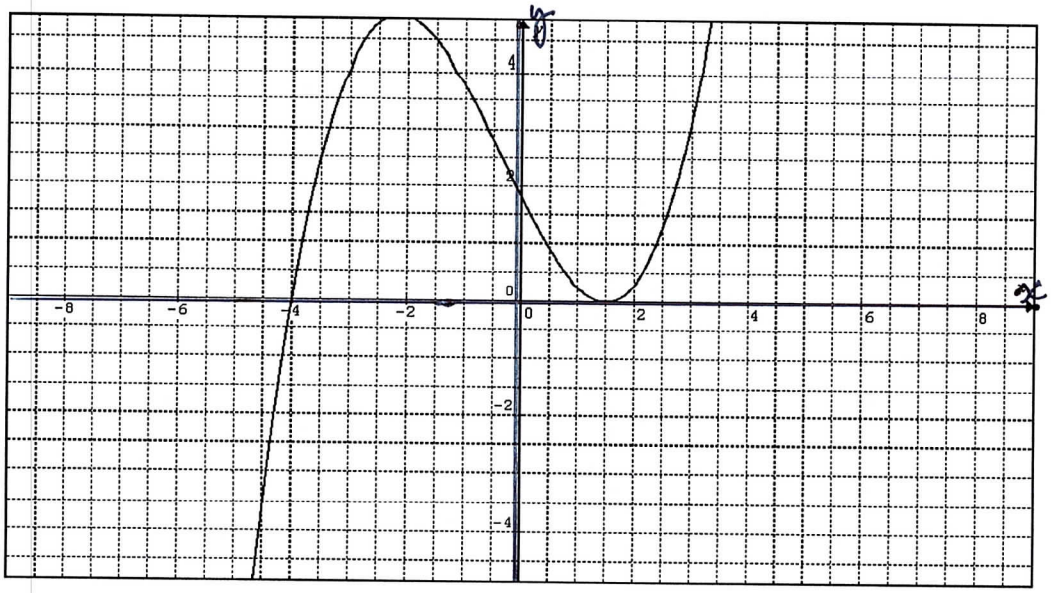


$y = g(x)$

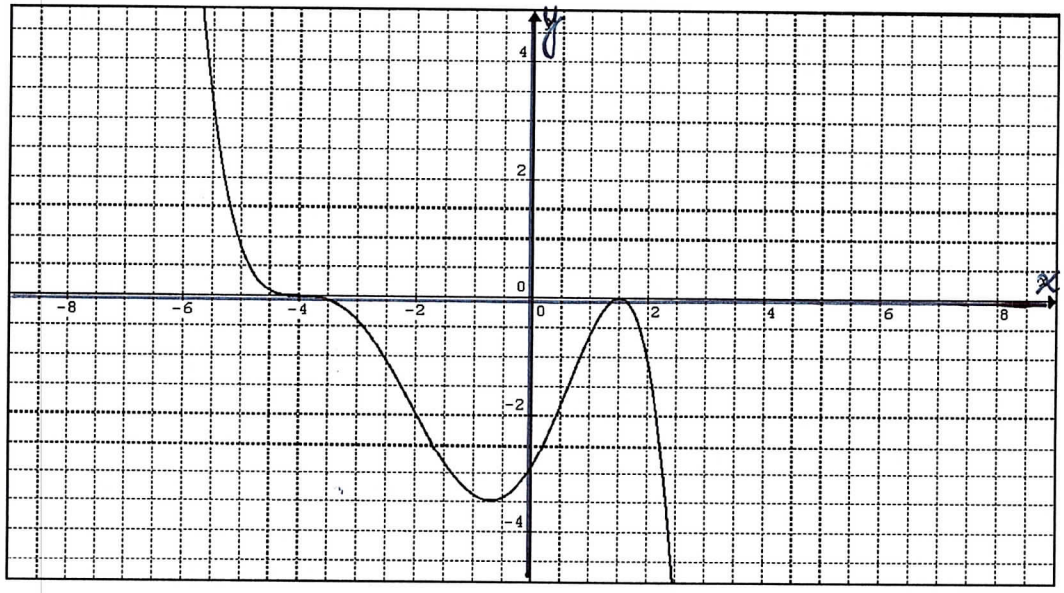
$y = f(x)$

Find the intervals where a) $f(x) \leq 0$ b) $f(x) > 0$

1)

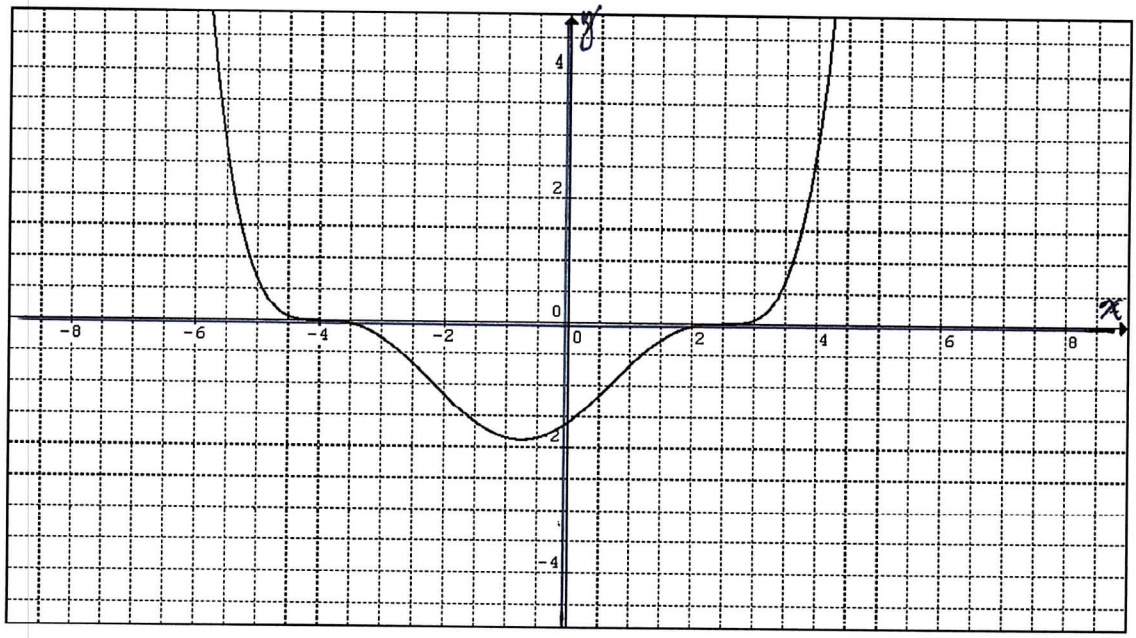


2)

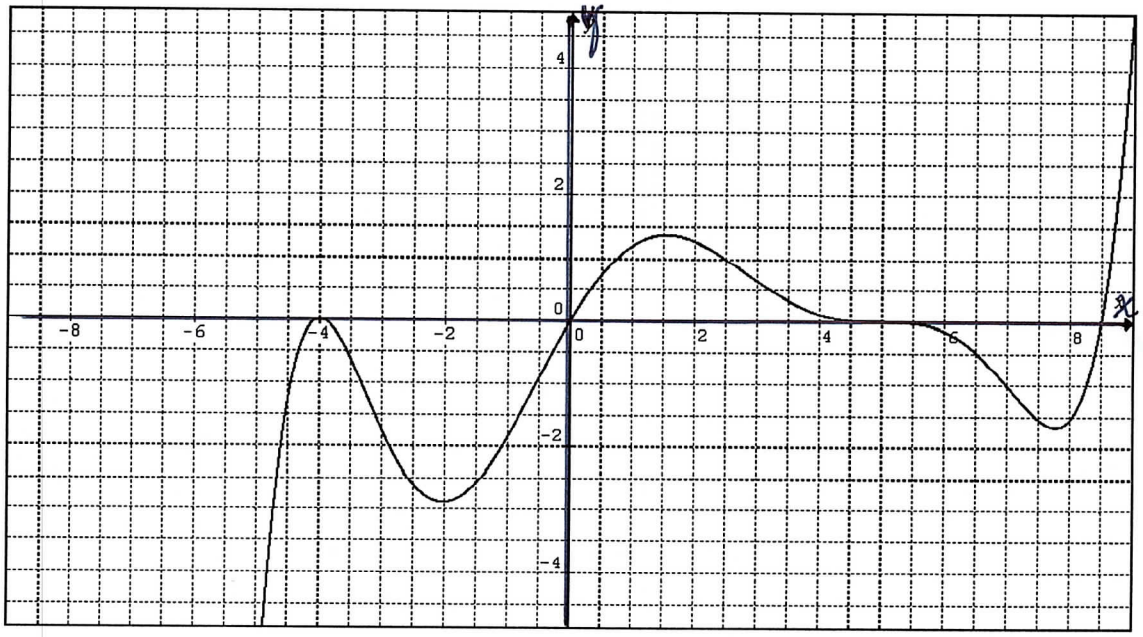


Find the intervals where a) $f(x) < 0$ b) $f(x) \geq 0$

3)



4)



Example 3 Solve the following inequalities using a number line strategy. Express your answers using interval notation.

a) $(x-2)(x+1)(x+5)<0$

b) $x^4 \geq 64x$

c) $5x^3+3x^2 \geq 12x-4$

Example 4 Solve the following inequalities using a factor table strategy. Express your answers using set notation.

a) $2x^3 + 50x > 20x^2$

b) $4x^3 + 3x^2 - 25x + 6 \leq 0$