AP Calculus Homework 25

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

1. Find the power series representation for the function and determine the interval of convergence.

a) $f(x) = \dfrac{1}{1+x}$ b) $f(x) = \dfrac{2}{3-x}$ c) $f(x) = \dfrac{1}{x+10}$

d) $f(x) = \dfrac{x}{9+x^2}$ e) $f(x) = \dfrac{x}{2x^2+1}$ f) $f(x) = \dfrac{1+x}{1-x}$

2. Differentiate to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

What is the radius of convergence?

3. Find the Maclaurin series for $f(x)$ using the definition of Maclaurin series. Also find the associated radius of convergence.

a) $f(x) = (1-x)^{-2}$ b) $f(x) = \ln(1+x)$ c) $f(x) = \sin \pi x$

d) $f(x) = e^{5x}$ e) $f(x) = xe^x$

4. Find the Taylor series for $f(x)$ centered at the given value of a.

a) $f(x) = x^4 - 3x^2 + 1, \quad a = 1$

b) $f(x) = x - x^3, \quad a = -2$

b) $f(x) = 1/x, \quad a = -3$

5. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

(A) $\dfrac{1}{6}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{9}{2}$

6. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

(A) $-\dfrac{1}{6}$ (B) 0 (C) $\dfrac{1}{120}$ (D) $\dfrac{1}{6}$ (E) 1

7. Let f be a function given by $f(x) = \ln(3 - x)$. The third degree Taylor polynomial for f about $x = 2$ is

(A) $-(x-2) + \dfrac{(x-2)^2}{2} - \dfrac{(x-2)^3}{3}$

(B) $-(x-2) - \dfrac{(x-2)^2}{2} - \dfrac{(x-2)^3}{3}$

(C) $(x-2) + (x-2)^2 + (x-2)^3$

(D) $(x-2) + \dfrac{(x-2)^2}{2} + \dfrac{(x-2)^3}{3}$

(E) $(x-2) - \dfrac{(x-2)^2}{2} + \dfrac{(x-2)^3}{3}$

8. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

(A) $1 - \dfrac{1}{2} + \dfrac{1}{24}$

(B) $1 - \dfrac{1}{2} + \dfrac{1}{4}$

(C) $1 - \dfrac{1}{3} + \dfrac{1}{5}$

(D) $1 - \dfrac{1}{4} + \dfrac{1}{8}$

(E) $1 - \dfrac{1}{6} + \dfrac{1}{120}$

9. If $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\displaystyle\sum_{n=0}^{\infty} a_n$ (D) $\displaystyle\sum_{n=1}^{\infty} n a_n$ (E) $\displaystyle\sum_{n=1}^{\infty} n {a_n}^{n-1}$