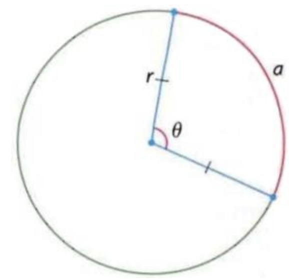
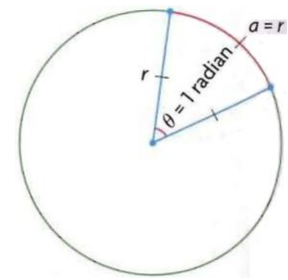
Radian Measure

- The radian is an alternative way to represent the size of an angle.
  The arc length, *a*, of a circle is proportional to its radius, *r*,
  and the central angle that it subtends, θ, by the formula $\theta = \dfrac{a}{r}$.



- One radian is defined as the angle subtended by an arc that is
  the same length as the radius, $\theta = \dfrac{a}{r} = \dfrac{r}{r} = 1$. 1 radian is about 57.3°.



- Using radians enables you to express the size of an angle as a real number without any units, often in terms of π. It is related to degree measure by the following conversion factor: $\pi \quad radians = 180^{\circ}$.
  - To convert from degree measure to radians, multiply by $\dfrac{\pi}{180^{\circ}}$.
  - To convert from radians to degrees, multiply by $\dfrac{180^{\circ}}{\pi}$.

Example 1 Determine the exact radian measure of each angle.

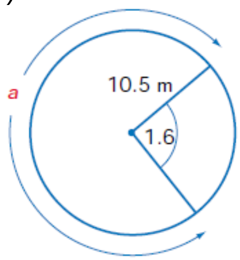
a) 75° b) 300° c) 540°

Example 2 Determine the exact degree measure of each angle.

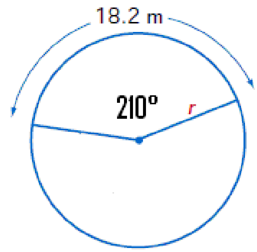
a) $\dfrac{\pi}{4}$ b) $\dfrac{13\pi}{15}$ c) $\dfrac{11\pi}{2}$

Example 3 Find the indicated quantity in each diagram. Round lengths to the nearest tenth of a metre.

a)



b)

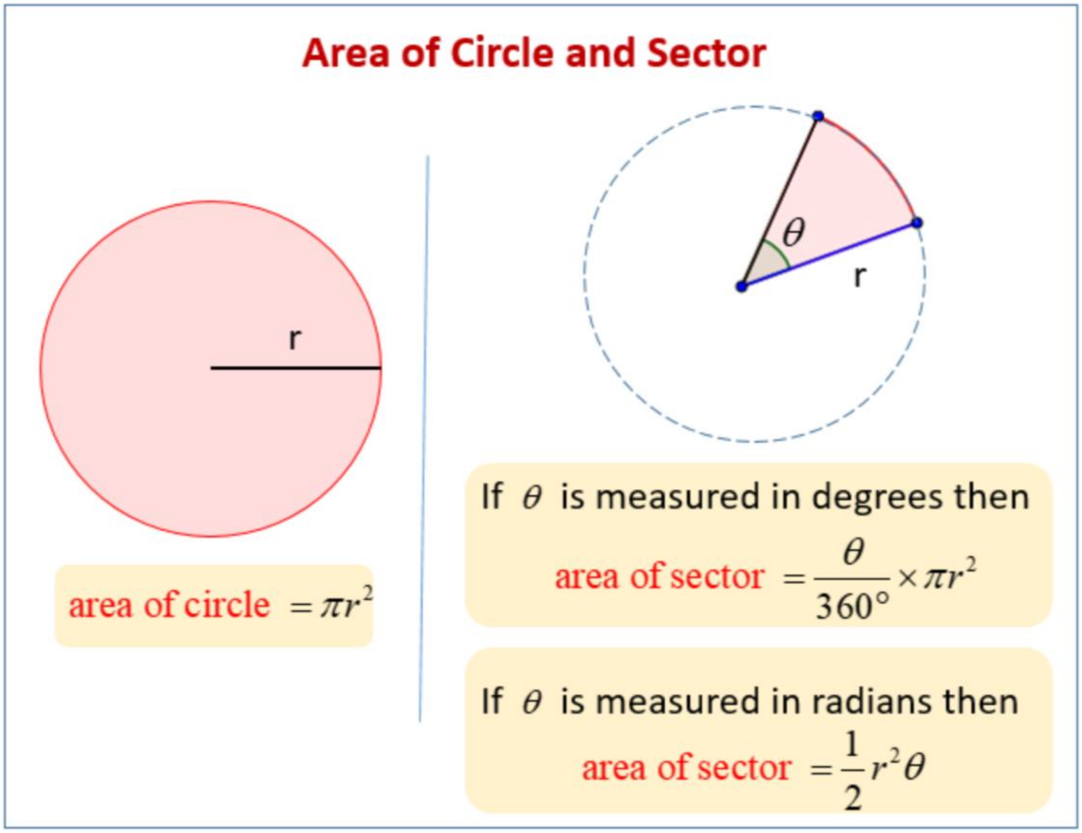


Example 4 David made a swing for his niece Sarah using ropes 2.4 m long, so that Sarah swings through an arc of length 1.2 m. Determine the angle through which Sarah swings, in both radians and degrees.

Example 5 A Ferris wheel with a radius of 32 m makes two revolutions every minute.

a) Find the exact angular velocity, ω, in radians per second.

b) If the ride lasts 3 min, how far does a rider travel, to the nearest metre?



Given a sector with radius r = 3 and a corresponding arc length of 5π, find the area of the sector.

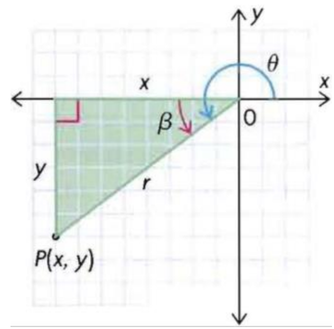
Radian Measure and Angles on the Cartesian Plane

- The trigonometric ratios for any principal angle, θ, in standard position can be determined by finding the related acute angle, β, using coordinates of any point that lies on the terminal arm of the angle.

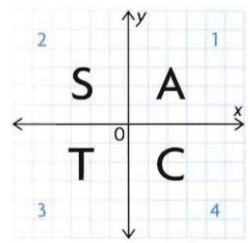
  From the Pythagorean theorem, $r^2 = x^2 + y^2$, if $r > 0$.

  $$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

  $$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

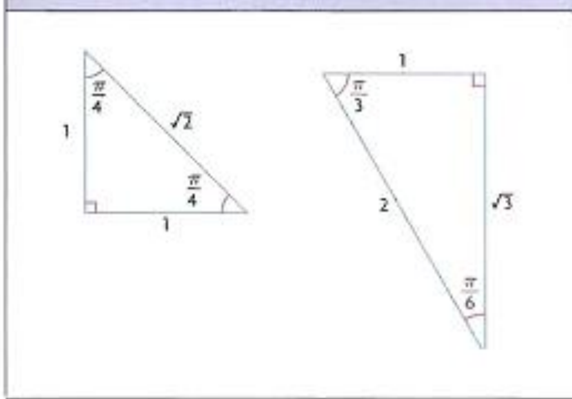
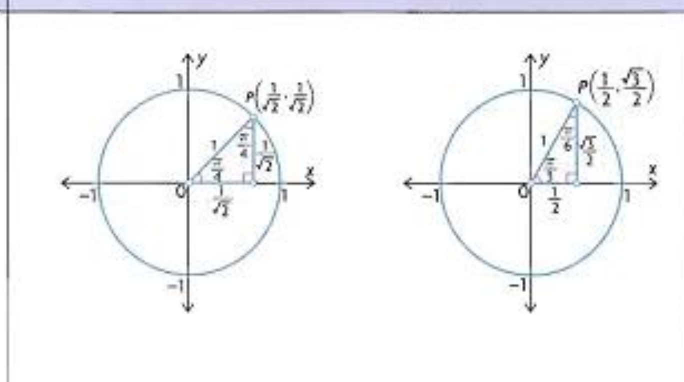


- The CAST rule is an easy way to remember which primary trigonometric r[...] quadrant. Since *r* is always positive, the sign of each primary ratio depends on the signs of the coordinates of the point.
  - ➢ In quadrant 1, All (A) ratios are positive because both x and y are positive.
  - ➢ In quadrant 2, only Sine (S) is positive, since x is negative and y is positive.
  - ➢ In quadrant 3, only Tangent (T) is positive because both x and y are negative.
  - ➢ In quadrant 4, only Cosine (C) is positive, since x is positive and y is negative.



- The angles in the special triangles can be expressed in radians, as well as in degrees. The radian measures can be used to determine the exact values of the trigonometric ratios for multiples of these angles between 0 and 2π.

- The strategies that are used to determine the values of the trigonometric ratios when an angle is expressed in degrees on the Cartesian plane can also be used when the angle is expressed in radians.

The Special Triangles	The Special Triangles on the Cartesian Plane Using a Circle of Radius 1
	

Example 1

Each of the following points lies on the terminal arm of an angle in standard position.

a) Sketch each angle.

b) Determine the value of r.

c) Determine the six trigonometric ratios for the angle.

d) Calculate the radian value of θ, to the nearest hundredth, where $0 \leq \theta \leq 2\pi$.

i) $(8, 15)$

ii) $(-2, -7)$

Example 2 Determine the exact value of each trigonometric ratio.

a) $\tan\frac{\pi}{2}$

b) $\csc\frac{3\pi}{2}$

c) $\cot\frac{5\pi}{4}$

d) $\cos\frac{11\pi}{6}$

Example 3 If $0 \le \theta \le 2\pi$, find the possible measures of θ

a) $\sin\theta = 0$

b) $\cot\theta = \frac{1}{\sqrt{3}}$

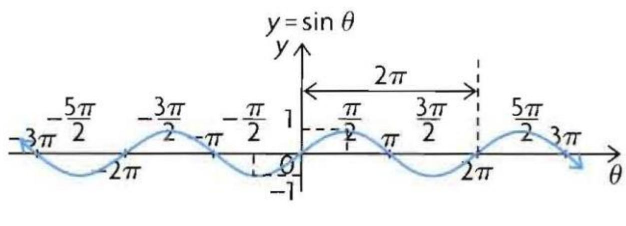
c) $\cos\theta = -\frac{1}{\sqrt{2}}$

Exploring Graphs of the Primary Trigonometric Functions

The graphs of the primary trigonometric functions can be summarized as follows:

Key points when
$0 \leq \theta \leq 2\pi$

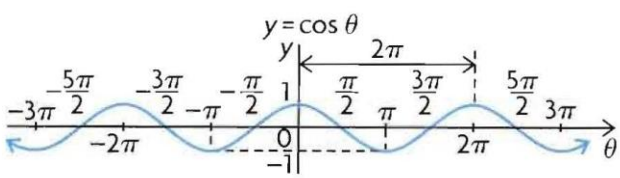
θ	$y = \sin(\theta)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



Period $= 2\pi$
Axis: $y = 0$
Amplitude $= 1$
Maximum value $= 1$
Minimum value $= -1$
$D = \{\theta \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

Key points when
$0 \leq \theta \leq 2\pi$

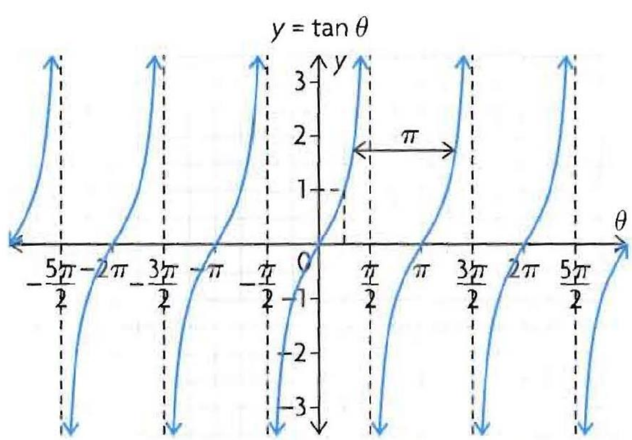
θ	$y = \cos(\theta)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



Period $= 2\pi$
Axis: $y = 0$
Amplitude $= 1$
Maximum value $= 1$
Minimum value $= -1$
$D = \{\theta \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

Key points:

- y-intercept $= 0$
- θ-intercepts $= 0, \pm\pi, \pm 2\pi, \ldots$



Period $= \pi$
Axis: $y = 0$
Amplitude: undefined
No maximum or minimum values
Vertical asymptotes:
$\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

$$D = \left\{\theta \in \mathbf{R} \,\middle|\, \theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\right\}$$

$R = \{y \in \mathbf{R}\}$