Unit: Relationships between points, lines and planes (1)

The intersection of 2 lines (2 and 3-D)

To check if $\ell_1 \& \ell_2$ represent the same line or intersection of $\ell_1 // \ell_2$ in 3-space	
1) Check that $\vec{d_1} = k\vec{d_2} \Rightarrow \ell_1 // \ell_2$ 2) Substitute the point from ℓ_1 into symmetric equation of ℓ_2.	
(i) If $t \neq t \neq t$ $\Rightarrow \ell_1 // \ell_2$ and distinct; $\Rightarrow \ell_1 \& \ell_2$ are not the same line; l_1 $\Rightarrow \ell_1 \& \ell_2$ have no intersection. l_2	(ii) If $t = t = t$ $\Rightarrow \ell_1 // \ell_2$ and coincident; l_1 $\Rightarrow \ell_1 \& \ell_2$ represent the same line; l_2 $\Rightarrow \ell_1 \& \ell_2$ have infinite number of intersections.

Ex 1: Intersection of lines in Space

a) Prove, in each case, $l_1 \parallel l_2$:

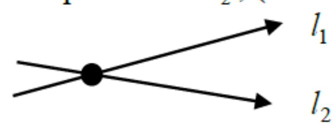
b) Find the intersection of l_1 and l_2, if any.

i) $l_1: \vec{r} = (1, 0, 3) + s(3, -6, 3)$
$l_2: \vec{r} = (2, -2, 5) + t(2, -4, 2)$

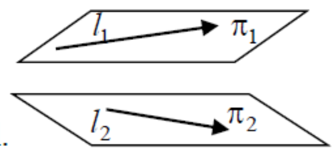
ii) $l_1: \vec{r} = (1, -1, 1) + s(6, 2, 0)$
$l_2: \vec{r} = (-5, -3, 1) + t(-9, -3, 0)$

Test for the Intersection of $l_1 \nparallel l_2$ in 3-Space

1) Check that $\vec{d}_1 \neq k\vec{d}_2 \Rightarrow l_1 \nparallel l_2$
2) Substitute parametric equations of ℓ_1 into symmetric equation of ℓ_2, (or vice versa)
3) 2 equations are formed.
4) Solve for t.
   (i) If $t = t \Rightarrow$ there is a point of intersection.



   (ii) If $t \neq t \Rightarrow$ no intersection (ℓ_1 & ℓ_2 are skew lines).
   This occurs when the lines are on different dimensional planes. They are not considered parallel.



Ex 2: Intersection of lines in Space

a) Prove, in each case, $l_1 \nparallel l_2$
b) Find the intersection of l_1 and l_2, if any.

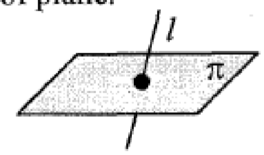
i) $l_1 : (x, y, z) = (-1, 1, 0) + t(3, 4, -2)$
$l_2 : x = -1 + 2t$
$y = 3t$
$z = -7 + t$

ii) $l_1 : \vec{r} = (2, 1, 0) + t(1, -1, 1)$
$l_2 : \vec{r} = (3, 0, -1) + t(2, 3, -1)$

The intersection of Lines with Plane

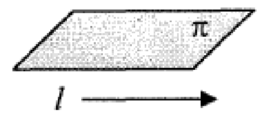
Test for the intersection of a line and a plane

1) Substitute the parametric equations of the line into scalar equation of plane.
2) Solve for t.
   i) If $t = 5 \Rightarrow$ a point of intersection.



   ii) If $0t = 0 \Rightarrow$ line lies on plane $\Rightarrow$ infinite number of intersection

   iii) If $0t \neq 0 \Rightarrow$ line // plane and distinct $\Rightarrow$ no intersection



Ex 3: Intersection of lines with plane

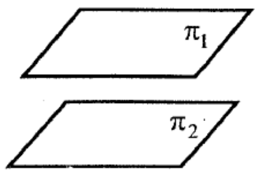
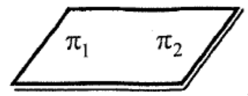
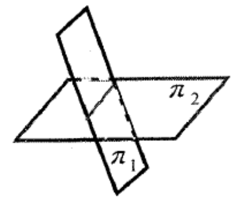
Find the intersection of l and π in each case:

a) $l: (x, y, z) = (1, -6, -5) + t\,(2, 3, 2)$ & $\pi: 4x - 2y + z - 19 = 0$

b) $l: x = 2t,\ y = 1 - t,\ z = -4 + t$
$\pi: x + 4y + 2z - 4 = 0$

c) $l: (x, y, z) = (-4, 0, 0) + t\,(3, 0, 1)$
$\pi: x - 2y - 3z + 4 = 0$

THE INTERSECTION OF TWO PLANES (3 CASES)

$\pi_1 \parallel \pi_2$ & distinct $\Rightarrow$ (no intersection)	$\pi_1 \parallel \pi_2$ & coincident $\Rightarrow$ (infinite intersections)	$\pi_1 \not\parallel \pi_2$ $\Rightarrow$ (a line of intersection) Let $z = t$, solve x and y in terms of t.
		

Ex 4: Intersection of 2 planes

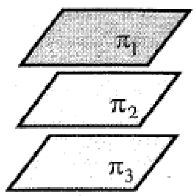
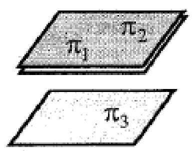
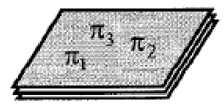
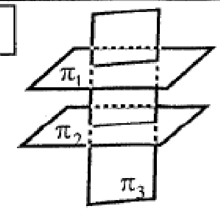
Find the intersection of π_1 and π_2, if any

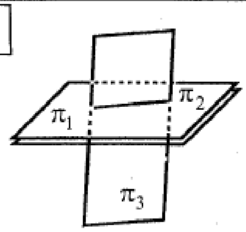
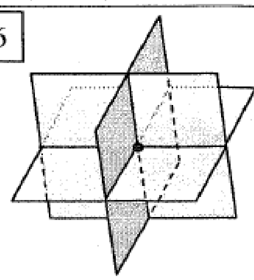
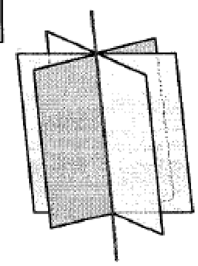
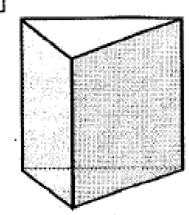
a) π_1: $2x + y - 3z - 6 = 0$
π_2: $4x + 2y - 6z + 5 = 0$

b) π_1: $2x + y - 3z - 6 = 0$
π_2: $-6x - 3y + 9z + 18 = 0$

c) π_1: $2x - 2y + 5z = -10$
π_2: $2x + y - 4z = -7$

THE INTERSECTION OF THREE PLANES (8 CASES)

$\pi_1 // \pi_2 // \pi_3$ (All are distinct) (No Intersection)	$\pi_1 // \pi_2$ (Coincident) and $// \pi_3$ (Distinct) No Intersection	$\pi_1 // \pi_2 // \pi_3$ (All are Coincident) (Infinite Intersections)	$\pi_1 // \pi_2$ (Distinct) $\nparallel \pi_3$ (No Intersection)
1 	2 	3 	4 

$\pi_1 // \pi_2$ (Coincident) $\nparallel \pi_3$ (Line of Intersection)	$\pi_1 \nparallel \pi_2 \nparallel \pi_3$ (Point of Intersection) $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) \neq 0$	$\pi_1 \nparallel \pi_2 \nparallel \pi_3$ (Line of Intersection) $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) = 0$	$\pi_1 \nparallel \pi_2 \nparallel \pi_3$ (No Intersection) $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) = 0$
5 	6 	7 	8 

Ex 5: Intersection of 3 planes

Describe geometrically the intersection of π_1, π_2, and π_3 in each case:

a) π_1: $2x + y - 2z + 6 = 0$
π_2: $4x + 2y - 4z - 5 = 0$
π_3: $6x + 3y - 6z + 11 = 0$

b) π_1: $2x + y - 2z + 6 = 0$
π_2: $4x + 2y - 4z + 12 = 0$
π_3: $6x + 3y - 6z + 11 = 0$

c) π_1: $2x + y - 2z + 6 = 0$
π_2: $4x + 2y - 4z + 12 = 0$
π_3: $6x + 3y - 6z + 18 = 0$

d) π_1: $2x + y - 2z + 6 = 0$
π_2: $4x + 2y - 4z - 5 = 0$
π_3: $x + y - z - 2 = 0$

e) π_1: $2x + y - 2z + 6 = 0$
π_2: $4x + 2y - 4z + 12 = 0$
π_3: $x + y - z - 2 = 0$