Unit: Curves sketching

Algorithm for Curve Sketching

1. Domain a. denominator $\neq 0$ (rational functions) b. radicand ≥ 0 (even roots) c. logarithmic argument > 0 (logarithmic functions)	**2. Intercepts** a. f (x) = 0 (x-int or zeroes) b. numerator = 0 (for rational functions) c. y − int = f (0) (if exists)
3. Symmetry a.f (−x) = f (x) (even functions are symmetric about the y-axis) b. f (−x) = − f (x) (odd functions are symmetric about the origin) c. f (x + T) = f (x) (periodic functions have cycles)	**4. Asymptotes** a. compute $\lim_{x \to \pm\infty} f(x)$ (horizontal asymptote) b. compute $\lim_{x \to a^-} f(x)$, $\lim_{x \to a^+} f(x)$ (vertical asymptote where a is a zero of the denominator but not of the numerator) c. compute long division (to find the oblique asymptotes for rational functions)
5. First Derivative a. compute f '(x) b. find critical points (f '(x) = 0 or f '(x) DNE) c. create the sign chart for f '(x) d. find intervals of increase/decrease e. find the local extrema (using first derivative test) and global extrema (if function is defined on a closed interval)	**6. Second Derivative** a. compute f ' '(x) find points where f ' '(x) = 0 or f ' '(x) DNE b. create the sign chart for f "(x) c. find points of inflection d. find intervals of concavity upward/downward e. check the local extrema using the second derivative test (if necessary)
7. Curve Sketching a. use broken lines to draw the asymptotes b. plot x- and y- intercepts, extrema, and inflection points c. draw the curve near the asymptotes d. sketch the curve	

Ex 1. Sketch the graph for the following functions:

a) y = $x^3 - 6x^2 + 9x + 1$

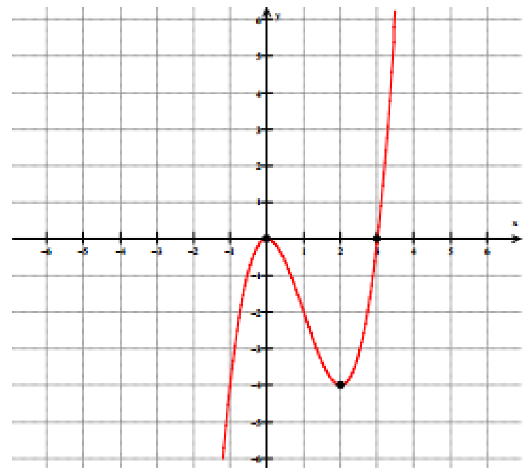
b) y = $\frac{4x}{x^2 + 1}$

Link between a function and its derivative

Consider a double differentiable function $y = f(x)$ ($f'(x)$ and $f''(x)$ exist). Then:

1. $f'(x)$ is the slope of the tangent at $P(x, f(x))$.
2. If $f'(x) = 0$, then $P(x, f(x))$ is a local extrema and tangent is horizontal.
3. If $f'(x) > 0$, then the function $y = f(x)$ is increasing.
4. If $f'(x) < 0$, then the function $y = f(x)$ is decreasing.
5. If $f''(x) = 0$, then $f'(x)$ has a local extrema and $y = f(x)$ has an inflection point.
6. If $f''(x) > 0$, then $f'(x)$ is increasing and $y = f(x)$ is concave upward.
7. If $f''(x) < 0$, then $f'(x)$ is decreasing and $y = f(x)$ is concave downward.

Ex 2. In the next figure is given the graph of the derivative f '(x) of a function f (x).



a) Find intervals where the function f (x) is increasing or decreasing.

b) Find intervals where the graph of f (x) is concave upward or downward.