AP Calculus Homework 6

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Sketch the graph of a function f that is continuous on [1,5] and has the given properties:

a) Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4.

b) f has no local maximum or minimum, but 2 and 4 are critical numbers.

2. Find the critical numbers of the functions (Choose any 5 problems).

a) $f(x) = x^3 + 3x^2 - 24x$ b) $g(t) = |3t - 4|$ c) $g(y) = \dfrac{y-1}{y^2 - y + 1}$

d) $g(x) = \sqrt{1 - x^2}$ e) $g(x) = x^{1/3} - x^{-2/3}$ f) $g(\theta) = 4\theta - \tan\theta$

g) $f(x) = x^2 e^{-3x}$ h) $f(x) = x^{-2} \ln x$

3. Find the absolute maximum and absolute minimum values of f on the given interval (Choose any 4 problems).

a) $f(x) = 3x^2 - 12x + 5, \quad [0, 3]$ b) $f(x) = (x^2 - 1)^3, \quad [-1, 2]$

c) $f(x) = \dfrac{x}{x^2 + 1}, \quad [0, 2]$ d) $f(t) = \sqrt[3]{t}(8 - t), \quad [0, 8]$

e) $f(t) = 2\cos t + \sin 2t, \quad [0, \pi/2]$ f) $f(x) = xe^{-x^2/8}, \quad [-1, 4]$

g) $f(x) = \ln(x^2 + x + 1), \quad [-1, 1]$

4. Prove that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum.

5. Verify that the function $f(x) = e^{-2x}, \quad [0, 3]$ satisfies the hypotheses of the Mean Value Theorem on the given internal. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

6. If $f(1) = 10$ and $f'(x) \geqslant 2$ for $1 \leqslant x \leqslant 4$, how small can $f(4)$ possibly be?

7.* Show that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.

8. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

A) $(-\infty, 2)$ B) $(-\infty, \infty)$ C) $(-2, 4)$ D) $(-2, \infty)$ E) $(2, \infty)$

9. What are all the values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

A) There are no such values of x.

B) $x < -1$ and $x > 3$

C) $-3 < x < 1$

D) $-1 < x < 3$

E) All values of x.

10. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

A) -0.46 B) 0.20 C) 0.91 D) 0.95 E) 3.73