Algebra 2

1. **Linear equation**

1) What is linear equation?

A **linear equation** is any equation that can be written in the form: $ax+b=c$, where *a* and *b* are real numbers and *x* is a variable.

This form is sometimes called the **standard form** of a linear equation. Note that most linear equations will not start off in this form. Also, the variable may or may not be an *x* so don't get too locked into always seeing an *x* there.

To solve linear equations we will make heavy use of the following facts.

a) If $a=b$ then $a+c=b+c$, for any *c*. All this is saying is that we can add a number, *c*, to both sides of the equation and not change the equation.

b) If $a=b$ then $a-c=b-c$, for any *c*. As with the last property we can subtract a number, *c*, from both sides of an equation.

c) If $a=b$ then $ac=bc$, for any *c*. Like addition and subtraction we can multiply both sides of an equation by a number, *c*, without changing the equation.

d) If $a=b$ then $\frac{a}{c}=\frac{b}{c}$, for any non-zero *c*. We can divide both sides of an equation by a non-zero number, *c*, without changing the equation.

e) Solving an equation means find root(s) for the equation.

If *r* is a root of the equation, replacing *x* with *r* will satisfy the equation.
For example, $ax+b=c$, then $ax=b-c$ and $x=\frac{b-c}{a}$.

2) Solving Systems of Equations by the Substitution Method

We can solve a system of equations by solving one equation for one of the variables; then substitute this result into the second equation.

Here is an example of solving a system of equations by substitution.

Find the solution to the following system of equations.

$x = y + 3$
$y + 3x = 1$

Step 1: The first equation is solved for x. Substitute ($y + 3$) in for x in the second equation.

$y + 3x = 1$ becomes $y + 3(y + 3) = 1$

Step 2: Solve for y.

$y + 3y + 9 = 1$
$4y = -8$
$y = -2$

Step 3: Substitute -2 in for y in the first equation to solve for x.

$x = y + 3$ becomes
$x = -2 + 3$
$x = 1$

Step 4: Write the solution as an ordered pair.

The solution to the system of equations is (1, -2).

3) Solving Systems of Equations by the Elimination Method

Our goal is to eliminate one variable by adding or subtracting the equations together.

$5x + 4y = 24$
$3x = 2 + 2y$

Step 1. $5x + 4y = 24$
$3x – 2y = 2$

Step 2. $15x + 12y = 72$
$-15x + 10y = -10$

Step 3. $22y = 62$

Step 4. $y = {}^{31}/_{11}$

Step 5. $3x = 2 + 2({}^{31}/_{11})$

Step 6. $x = {}^{84}/_{33}$

2. Linear Inequalities

The best way to define interval notation is the following table. There are three columns to the table. Each row contains an inequality, a graph representing the inequality and finally the interval notation for the given inequality.

Inequality	Graph	Interval Notation
$a \le x \le b$		$[a,b]$
$a < x < b$		(a,b)
$a \le x < b$		$[a,b)$
$a < x \le b$		$(a,b]$
$x > a$		(a,∞)
$x \ge a$		$[a,\infty)$
$x < b$		$(-\infty,b)$
$x \le b$		$(-\infty,b]$

An equivalent set of facts for the remaining three inequalities.

1) If $a < b$ then $a + c < b + c$ and $a - c < b - c$ for any number *c*.

In other words, we can add or subtract a number to both sides of the inequality and we don't change the inequality itself.

2) If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

So, provided *c* is a positive number we can multiply or divide both sides of an inequality by the number without changing the inequality.

3) If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

In this case, unlike the previous fact, if *c* is negative we need to flip the direction of the inequality when we multiply or divide both sides by the inequality by *c*.

Example 1

When Betty substitutes $x=1$ into the expression $ax^3 - 2x + c$ its value is -5. When she substitutes $x=4$ the expression has value 52. Find one value of x that makes the expression equal to zero.

Solution
When $x=1$, we are told that $a(1)^3 - 2(1) + c = -5$, or $a + c = -3$ (1)
Similarly, when $x = 4$, $a(4)^3 - 2(4) + c = 52$, or $64a + c = 60$ (2)

Subtracting equation (1) from equation (2) gives $63a = 63$, or $a = 1$.
Substituting $a=1$ into equation (1) gives $c = -4$.
The original expression $ax^3 - 2x + c$ becomes $ax^3 - 2x - 4$.

By trial and error, using divisors of 4, when $x = 2$ we get
$2^3 - 2(2) - 4 = 0$.

Example 2

During a football game, Matt kicked the ball three times. His longest kick was 43 metres and the three kicks averaged 37 metres. If the other two kicks were the same length, find the distance, in metres, that each travelled

Solution 1
Matt's longest kick was 6 metres more than the average.

Thus, the other two kicks must be six metres less than the average when combined (that is, when we add up the difference between each of these kicks and the average, we get 6).

Since the other two kicks were the same length, then they each must have been 3 metres less than the average, or 34 metres each.

Solution 2
Since Matt's three kicks averaged 37 metres, then the sum of the lengths of the three kicks was $3 \times 37 = 111$ metres.

Let x be the length of each of the two kicks of unknown length.
Then $43 + 2x = 111$ or $x = 34$.

In-class questions

1. Given that $x + y = 1 \; and \; x^2 + y^2 = 4$, what is the value of $x^3 + y^3$?

2. If $a^2 + b^2 = c^2 + d^2 = 1 \; and \; ac + bd = 0$ then what is the maximum possible value of $ad - bc$?

3. If $ab = k$ and $\frac{1}{a^2} + \frac{1}{b^2} = m$, then what is the $(a-b)^2$ expressed in terms of m and k?

4. The cost of making a rectangular table is calculated by adding two variables. The first is proportional to the area of the table and the other to the square of the length of the longer side. A $2 \times 3 - metre$ table costs $50 to make and a $1.5 \times 4 - metre$ table costs $64 to make. What is the cost of making a 2.5metre square table, to the nearest cent?

5. A girl walks at 4 km/hr, a boy walks at 3 km/hr, and a dog runs at 6 km/hr. The girl and the boy are 2km apart on a straight road, and the dog is midway between them. The girl follows after the boy who walks away from her, and the dog runs back and forth between the two of them. If the dog starts by running after the boy, then what is the number of km between the girl and the dog after one hour?

6. Alistair, Conrad, Emma, and Salma compete in a three-sport race. They each swim 2 km, then bike 40 km, and finally run 10 km. Also, they each switch instantly from swimming to biking and from biking to running.

a) Emma has completed 113 of the total distance of the race. How many kilometers has she travelled?
b) Conrad began the race at 8.00 a.m. and completed the swimming portion in 30 minutes. Conrad biked 12 times as fast as he swam, and ran 3 times as fast as he swam. At what time did he finish the race?
c) Alistair and Salma also began the race at 8.00 a.m. Alistair finished the swimming portion in 36 minutes, and then biked at 28 km/h. Salma finished the swimming portion in 30 minutes, and then biked at 24 km/h. Alistair passed Salma during the bike portion. At what time did Alistair pass Salma?