AP Calculus Homework One – Limit and Continuity
1.1 Definitions of Limits; 1.2 Continuity; 1.3 Limits Properties

1. Show that limits do not exist.

(a) $\lim\limits_{x \to -2} \dfrac{x+2}{|x+2|}$

(b) $\lim\limits_{x \to 0} \sin \dfrac{1}{x}$

(c) $\lim\limits_{x \to 0} \sqrt{3 + \arctan \dfrac{1}{x}}$

2. Find limits.

(a) $\lim\limits_{x \to 0} \dfrac{x^2}{2x-1}$

(b) $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$

(c) $\lim\limits_{x \to -1} \dfrac{2 + 2/x}{x^2 - 4x - 5}$

(d) $\lim\limits_{h \to 0} \dfrac{5(h-1)^2 + (h-1) - 4}{h}$

(e) Explain, using examples, when substitution can not be used to solve a limit.

3. Discuss the continuity and sketch the graph of $f(x) = \begin{cases} \dfrac{x^2 + x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$.

4. If $[x]$ is the greatest integer not greater than x, then $\lim\limits_{x \to \frac{1}{2}}[x]$ is

    (A) 1/2　　　　(B) 1　　　(C) nonexistent　　(D) 0　　　(E) none of these

5. Find a value of k such that $f(x)$ is continuous at $x = 0$.
    $$f(x) = \begin{cases} \dfrac{x^2 - x}{2x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

6. The function $s(x)$ is defined as follows. Find a value of k such that $s(x)$ is continuous for all x.
    $$s(x) = \begin{cases} 4x - 11, & \text{if } x < 3 \\ kx^2, & \text{if } x \geq 3 \end{cases}$$

7. Discuss the continuity of the graph of $y = \dfrac{x^2 - 9}{3x - 9}$, indicating type of discontinuity if there is one.