

## AP Calculus In-Class One – Limit and Continuity

1.1 Definitions of Limits; 1.2 Continuity; 1.3 Limits Properties

1. Show that limits do not exist.

$$(a) \lim_{x \rightarrow 0} \frac{|x|}{x} \quad , \quad \text{fix} = \frac{|x|}{x}$$

$$(b) \lim_{x \rightarrow 3} \frac{|x-3|}{x-3}$$

$$(c) \lim_{x \rightarrow 0} \frac{1}{2 + 10^{\frac{1}{x}}}$$

2. Find limits.

$$(a) \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 4}$$

$$(b) \lim_{x \rightarrow 3} \frac{3-x}{x^2 - 2x - 3}$$

$$(c) \lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{2x^2 - 7x - 4}$$

$$(d) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{2x^2 - 17x - 9}$$

$$(e) \lim_{x \rightarrow a} (2 - e)$$

3. Use one-sided limits to discuss the continuity, indicating the type of discontinuity, and sketch

the graph of the function:

$$f(x) = \begin{cases} 1 - x, & \text{if } -1 \leq x < 0 \\ 2x^2 - 2, & \text{if } 0 \leq x \leq 1 \\ -x + 2, & \text{if } 1 < x < 2 \\ 1, & \text{if } x = 2 \\ 2x - 4, & \text{if } 2 < x < 3 \end{cases}$$

4. If  $\lim_{x \rightarrow a} f(x) = L \neq 0$  and  $\lim_{x \rightarrow a} g(x) = 0$ . Prove that  $\lim_{x \rightarrow a} [f(x)/g(x)]$  does not exist.

5. Find limits.

(a)  $\lim_{x \rightarrow \infty} \frac{4 - x^2}{x^2 - 1}$

(b)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x)$

(c)  $\lim_{x \rightarrow 0} \frac{\sqrt{1 - 2x + x^2} - (1 + x)}{x}$

(d)  $\lim_{t \rightarrow 0} \frac{(\sqrt{1+t^2} + t)^n - (\sqrt{1+t^2} - t)^n}{t}$

6. Discuss the continuity and sketch the graph of  $g(x) = \begin{cases} \frac{|x+1|}{x+1}, & \text{if } x \neq -1 \\ 2, & \text{if } x = -1 \end{cases}$ .