Probability (2)

1. Probability

Probability is defined as the measure of the *"likelihood"* of an event, or *outcome*.
Probability is all about the *certainty*, the *uncertainty*, and the *prediction* of something happening.

Definition of Odds
The odds of an event occurring is the ratio of the number of ways the event can occur (successes) to the number of ways the event cannot occur (failures).

Odds = successes / failures

2. Probability of an event

The probability of an event is the chance that it will occur, expressed as a ratio of a specific event to all possible events.

$$Probability = \frac{number\ of\ actual\ events}{number\ of\ possible\ events}$$

Or....

$$Probability = \frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes}$$

- The probability of an event that is *certain* to occur is 1.
- The probability of an event that is *impossible* to occur is 0.
- The probability of an event that is *possible* is >0 and <1

Example
In this experiment we will simply roll a single die one time, record the result, and roll the die again.

What is the probability of rolling a "3"?
Since a die has 6 faces, but only one of them a "3"

$$P(3) = \frac{1}{6}$$

Now, what is the probability of NOT rolling a "3"?
Since there are 5 other possible outcomes

$$P(not\ 3) = \frac{5}{6}$$

Do you see that if you add the probability of rolling the "3" to the probability of NOT rolling the "3" that the sum is 1?

$$\frac{1}{6} + \frac{5}{6} = 1!$$

That means that if you know the probability of something happening, to determine the probability of it NOT happening, simply subtract the favorable probability from 1.

If the probability of it raining today is 2/5, the probability of it NOT raining is 1 - 2/5 = 3/5!

3. Probability of two or more Independent Events

1) Compound event
A compound event consists of two or more events.

There are a couple of things to note about this experiment. Choosing two pairs of socks from the same drawer is a compound event. Since the first pair was replaced, choosing a red pair on the first try has no effect on the probability of choosing a red pair on the second try. Therefore, these events are independent.

2) Independent event
Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

Some other examples of independent events are:

- Landing on heads after tossing a coin AND rolling a 5 on a single 6-sided die.
- Choosing a marble from a jar AND landing on heads after tossing a coin.
- Choosing a 3 from a deck of cards, replacing it, AND then choosing an ace as the second card.
- Rolling a 4 on a single 6-sided die, AND then rolling a 1 on a second roll of the die

To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities.

This multiplication rule is defined symbolically below. Note that multiplication is represented by AND.

Multiplication Rule:
When two events, A and B, are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

(Note: Another multiplication rule will be introduced in the next lesson.) Now we can apply this rule to find the probability for Experiment 1.

Example

A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and an eight?

Solution:

$$P(Event) = \frac{number\ of\ \ favorable\ outcomes}{number\ of\ \ total\ possible\ outcomes}$$

$$P(Jack) = \frac{4}{52}, \quad P(8) = \frac{4}{52}$$

$$P(Jack\ \ and\ \ 8) = P(Jack) \bullet P(8) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

3. Probability With / Without Replacement

Suppose you have 2 bags with the exact same sets of marbles inside. Let's say there are 4 red, 5 blue and 9 yellow marbles in each bag.

From the first bag, you reach in and make a selection. You record the color and then drop the marble back into the bag. You then repeat the experiment a second time.

From the second bag you do exactly the same thing EXCEPT, after you select the first marble and record it's color, you do NOT put the marble back into the bag, You then select a second marble, just like the other experiment.

The first experiment involves a process called
"with replacement". You put the object back into the bag so that the number of marbles to choose from is the same for both draws.

The second experiment involves a process called
"without replacement". You do not put the object back in the bag so that the number of marbles is one less than for the first draw.

An Important Note
Sometimes a problem will not specifically state whether it is a problem "with or without replacement". In these cases it is very important to ask yourself this question:

"Is this problem with replacement?"
or
"Is this problem without replacement?"

Let common sense and a little intuition guide you through these types of problems.

A player is dealt 2 cards from a standard deck of 52 cards.
What is the probability of getting a pair of aces?

Think about it....obviously this is an experiment "without replacement" because the player was given 2 cards.

To calculate the probability of a pair of aces you use the rules for compound events:

$P(\text{ace on first card}) = \frac{4}{52}$ (remember, there are 4 aces in the deck)

$P(\text{ace on the second card}) = \frac{3}{51}$! (the first card drawn was an ace!)

So the probability of getting 2 aces is: $P(ace,\ ace) = \left(\frac{4}{52}\right) \times \left(\frac{3}{51}\right) = \frac{12}{2652} = \frac{1}{221}$!

Example
A jar contains 2 red and 5 green marbles. A marble is drawn, it's color noted, and put back in the jar. This process is repeated a total of 4 times. What is the probability that you selected 4 green marbles?

Since you put the marble back in the jar after each selection this is an experiment "with replacement".

So, the probability for each draw will be exactly the same:

$P(green) = \frac{5}{7}$

Therefore:

$P(green,\ green,\ green,\ green) = \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{625}{2401}$

4. Empirical vs. Theoretical Probability

In this lesson we will examine two different types of probability, *empirical and theoretical* probability.

Although both of these methods for determining the probability of an event involve the concept of "chance", the solutions are obtained in two very different ways.

1) Empirical Probability

When you perform an experiment, or sample a number of people or objects to determine the probability of an event, you are determining the "empirical probability" of an event.

Example 1
Let's say that a manufacturer tested 1000 radios, at random, and found 15 of them to be defective.

We can easily determine that the empirical probability that a radio is defective would be:

$$P(defective\ radio)\ = \frac{15}{1000} = \frac{3}{200} = 0.015 = 1.5\%$$

Now the manufacturer can use this result to *predict* that in the production of 7500 radios, 1.5% of them will *probably* be defective. Or, (0.015) × (7500) = 112.5 defective radios.

Example 2
Let's say that Sheila tossed a coin 200 times and recorded whether it was a "head", or a "tail" after each toss.

After the experiment, Sheila found that she had recorded 122 "heads" and 78 "tails."

From this experiment she could predict that the *empirical probability* of getting a "head" is:

$$P(Head) = \frac{122}{200} = \frac{61}{100}$$

As a decimal it would be .61, and as a percent 61%.

Because in determining empirical probability you actually perform an experiment, it is sometimes called: "experimental probability."

2) Theoretical Probability

An empirical approach is required whenever we want to determine the probability of an event by actually performing the experiment. However, common sense tells us that there is also a simple, mathematical, way to determine the probability of an event in another way.

Example 1
Jack is playing a game in which each player must roll a die. To win the game Jack must roll a number greater than 4. What is the probability that Jack will win on his next turn?

Common sense tells us that:
(a) The die has an equal chance of landing in 6 ways.
(b) There are only 2 ways for Jack to win the game.
(c) Therefore the probability of Jack winning is:

$$P(Jack\ wins) = \frac{2}{6} = \frac{1}{3}$$

As a decimal it would be .333..., as a percent, 33 1/3%

Example 2
In this game a standard 52 card deck of playing cards is used. In order to win you must pick a "face card."

What is the probability that you will win the game on the next draw?

Again, common sense tells us:

(a) Each card in the deck has an equal chance of being drawn.
(b) There are 12 face cards (winning cards) in the deck.

Therefore the probability of winning on the next draw is:

$$P(\,face\ card\,) = \frac{12}{52} = \frac{3}{13}$$

As a decimal it would be about .23, and as a percent, 23%

An *empirical* example

During the last 15 basketball games, Sam has made 64 and missed 32 foul shots. What is the empirical (or experimental) probability that Sam will make his next foul shot?

Using the statistical information in the problem we can determine that the empirical probability of Sam making his next foul shot would be:

$$P(making\ the\ next\ shot) = \frac{64\ (made)}{96\ (total)} = \frac{2}{3}$$

as a decimal it would be .666..., as a percent, 66 2/3%

A *theoretical* example

In a box there are 4 red marbles, 3 blue marbles and 5 yellow marbles. If one marble is chosen at random from the box, what is the probability of choosing:
(a) a red marble
(b) a blue or yellow marble

Remember...don't forget your "common sense!"

$$P(red) = \frac{4}{12} = \frac{1}{3}$$

(decimal): 0.333.., (percent): 33 1/3%

$$P(blue\ or\ yellow) = \frac{8}{12} = \frac{2}{3}$$

(decimal): 0.666.., (percent): 66 2/3%

Questions in class

1. How many different arrangements are there of the letters in the word BANANA?

2. How many odd four digit numbers have four different digits?

3. Two cards are dealt from a standard deck of 52 cards. Find the probability that both cards will have the same color.

4. How many different ’words’ can be made by rearranging the letters in the word POTPOURRI? (The ‘words’ do not have to make sense)

5. If you have three cents, two nickels and one dime, how many different amounts of money can you made using one or more of these coins?

6. A card is selected at random from a deck of 52 cards. What are the odds in favor of selecting a heart? What is the probability of selecting an ace?

7. Find the number of ways to sit a delegation of 6 people for round-table talks. Note: It only matters who sits next to whom.

8. A city council is composed of 6 men and 3 women. Four members are to be chosen as delegates. In how many ways can exactly 2 men and 2 women be chosen?

9. A four digit number is formed by randomly assigning a "2" or a "6" to each digit. What is the probability that the number is divisible by 11?