Dividing Polynomials

- Polynomials can be divided in much the same way that numbers are divided.
- A polynomial can be divided by a polynomial of the same degree or less.
- Synthetic division is a shorter form of polynomial division. It can only be used when the divisor is linear (that is, $(x-k)$ or $(ax-k)$).
- When using polynomial or synthetic division,
  - terms should be arranged in descending order of degree, in both the divisor and the dividend, to make the division easier to perform
  - zero must be used as the coefficient of any missing powers of the variable in both the divisor and the dividend
- If the remainder of polynomial or synthetic division is zero, both the divisor and the quotient are factors of the dividend.

Example 1 Calculate each of the following using long division.

a) $(3x^2+10x-2+9x^3)\div(3x+2)$

b) $(6m^4-13m^2+m+4)\div(2m^2-3)$

Example 2

Calculate each of the following using synthetic division.

a) $(2x^4 - 3x^2 + 1) \div (x + 1)$

b) $(4q^3 - 10q^2 + 6q - 18) \div (2q - 5)$

Example 3

$x - 5$ is a factor of the function $x^3 - 7x^2 + 11x - 5$. Determine the other factors. Then determine the zeroes, and sketch a graph of the polynomial.

Divide using Long Division. Express your answer in the forms of $P(x) = Q(x)D(x) + R(x)$ and $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$.

1. $(x^2 + 3x + 7) \div (x + 1)$	2. $(9x^2 + 6x - 11) \div (x + 2)$
3. $(2x^3 + 5x^2 - 6x + 15) \div (2x - 3)$	4. $(2x^4 - 3x^2 + x - 1) \div (x^2 - 2x + 1)$

Divide using Synthetic Division. Express your answer in the forms of $P(x) = Q(x)D(x) + R(x)$ and $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$.

1. $(x^2 + 7x + 9) \div (x + 1)$	2. $(9x^2 + 6x - 11) \div (3x - 2)$
3. $(2x^3 + 5x^2 - 6x + 15) \div (2x - 3)$	4. $(3x^4 + 4x^3 - 15x^2 + 6x - 10) \div (3x - 5)$

The Remainder Theorem

- If the polynomial $f(x)$ is divided by $(x-b)$, then the remainder is $f(b)$.
- If the polynomial $f(x)$ is divided by $(ax-b)$, then the remainder is $f\left(\frac{b}{a}\right)$.

Find the remainder of the following using the Remainder Theorem

1. $(x^2+2x+5)\div(x-1)$	2. $(3x^3-4x^2+x+3)\div(x+3)$
3. $(3x^3-5x^2-2x-1)\div(3x+2)$	4. $(x^4+6x^3+2x^2+9x+12)\div(2x-5)$

Find the value of the unknown.

1. When $(2x^4-kx^3+kx+2)$ is divided by $(x+2)$, the remainder is 10. Find k.	2. When $(2x^3-3x^2+kx-1)$ is divided by $(2x-1)$, the remainder is 1. Find k.

Find the values of the unknowns.

3. When $f(x) = ax^3 + bx^2 - x + 3$ is divided by $(x+1)$, the remainder is 3. When $f(x)$ is divided by $(x+2)$, the remainder is –7. What are the values of a and b?

Factoring Polynomials

- The remainder theorem: When a polynomial, $f(x)$, is divided by $x-a$, the remainder is equal to $f(a)$.
- The factor theorem: $x-a$ is a factor of $f(x)$ if and only if $f(a)=0$.
- To factor a polynomial, $f(x)$, of degree 3 or greater,
  - use the factor theorem to determine a factor of $f(x)$
  - divide $f(x)$ by $x-a$
  - factor the quotient, if possible
- If a polynomial, $f(x)$, has a degree greater than 3, it may be necessary to use the factor theorem more than once.
- Not all polynomial functions are factorable.

Example 1 Use the remainder theorem to determine the remainder for each division.

a) $(3m^2+7m+1)\div(m+3)$

b) $(8x^3+12x^2-4x+5)\div(2x+3)$

Example 2 Show that the binomial $y+2$ is a factor of the polynomial $y^4+4y^3-9y^2-16y+20$.

Example 3

Factor fully.

a) $4n^3 - 8n^2 + n + 3$

b) $2x^3 - x^2 - 2x + 1$

c) $m^4 - 20m^2 + 64$

d) $y^4 + 4y^3 - 7y^2 - 34y - 24$

Example 4

For what value of b will the polynomial $P(x) = -2x^3 + bx^2 - 5x + 2$ have the same remainder when it is divided by $x - 2$ and by $x + 1$?

Example 5

When the polynomial $4x^3 + vx^2 + wx + 11$ is divided by $x + 2$, the remainder is -7. When the polynomial is divided by $x - 1$, the remainder is 14. What are the values of v and w?

The Factor Theorem

- $(x-b)$ is a factor of the polynomial $f(x)$ if and only if $f(b)=0$.
- $(ax-b)$ is a factor of the polynomial $f(x)$ if and only if $f\left(\frac{b}{a}\right)=0$.

1. Is $(x+2)$ a factor of $f(x)=x^2+8x+6$?	2. Is $(2x-1)$ a factor of $f(x)=4x^3-6x^2+8x-3$?
3. Factor completely $f(x)=12x^3+8x^2-3x-2$	4. Factor completely $f(x)=x^4+3x^3-7x^2-27x-18$

Factoring a Sum or Difference of Cubes

Sum of Cubes

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

Difference of Cubes

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

Example 1

Factor each expression.

a) $v^3 + 1000$

b) $343c^3 - 729d^3$

Example 2

Completely factor each of the following expressions.

a) $500m^5n - 256m^2n^4$

b) $512x^9 + y^9$

Example 3

Factor each expression.

a) $\dfrac{27}{64}a^3 + \dfrac{216}{1331}b^3$

b) $40e^3 - 5(e+f)^3$