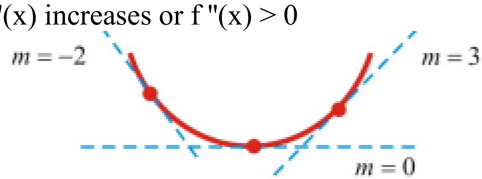
Unit: Derivatives and their applications (2)

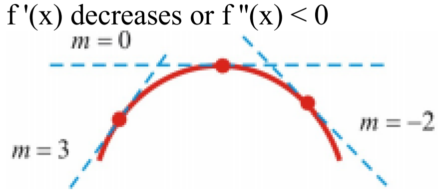
Concavity and Points of Inflection

Concavity

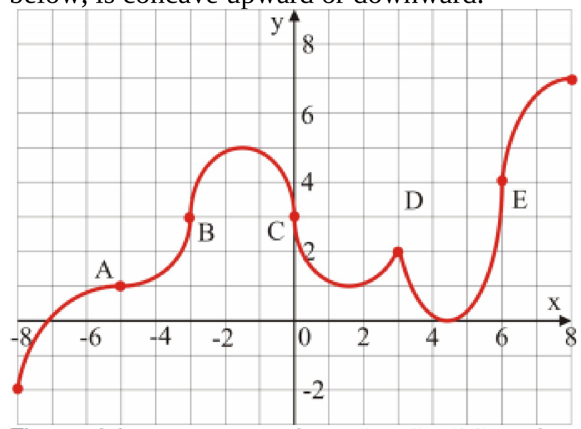
The graph of a function has a concavity upward if:
- Graph lies above all its tangents
- Tangents rotate counter-clockwise
- Slope of tangent lines increases
- f '(x) increases or f "(x) > 0



The graph of a function has a concavity downward if:
- Graph lies below all its tangents
- Tangents rotate clockwise
- Slope of tangent lines decreases
- f '(x) decreases or f "(x) < 0

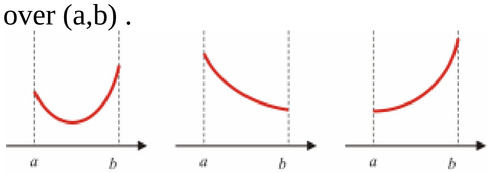


Ex 1. Find the intervals on which the graph, given below, is concave upward or downward.

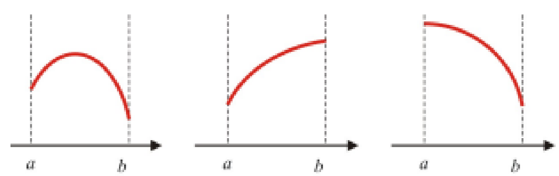


Test for Concavity
Let f be a function twice differentiable (f "(x) exists) over (a,b) .

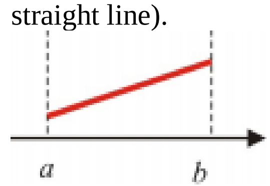
1. If f "(x) > 0 for all x∈(a,b) , then the graph of f is concave upward (has a concavity upward) over (a,b) .



2. If f "(x) < 0 for all x∈(a,b) , then the graph of f is concave downward (has a concavity downward) over (a,b) .

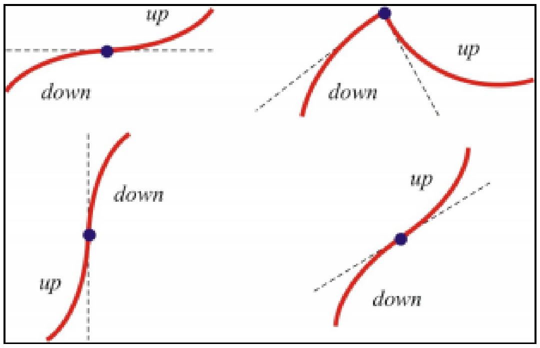


3. If f "(x) = 0 for all x∈(a,b) , then the graph of f has no concavity over (a,b) (f '(x) = const ; the graph is a straight line).



Point of Inflection

A point P(i, f (i)) on the graph of y = f (x) is called ***point of inflection*** if the concavity of the graph changes at P (from concave upward to concave downward or from concave downward to concave upward).



Ex 2. Use the function given at Ex. 1 to identify the points of inflection.

Ex 3. Find the intervals of concavity and the inflection points for

a. f(x) = $x^4 - 2x^3$.

b. g(x) = x^2

c. f(x) = $(x^2 - 1)^3$

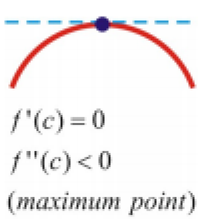
d. f(x) = $\begin{cases} 2x + 4, & x \le -1 \\ 3 - x^2, & x > -1 \end{cases}$

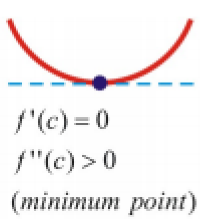
Second Derivative Test

Let f be a twice differentiable function over an open interval containing the critical number c and f '(c) = 0 ((c, f (c)) is a stationary point).

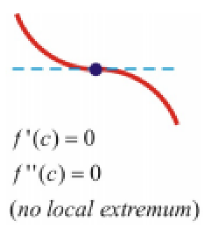
1. If f "(c) > 0 then f has a local minimum at x = c .

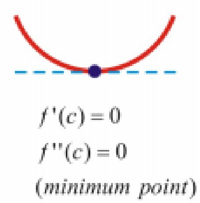
2. If f "(c) < 0 then f has a local maximum at x = c .





3. If f "(c) = 0 then the function may have a local minimum, maximum, or neither (inconclusive case). Use the first derivative test to conclude.





Ex 4. Use the second derivative test to find the local extrema. If the second derivative test is not conclusive (fails), then use the first derivative test to conclude. a. $f(x) = (x-2)^3$	b. $g(x) = x^4$
Algorithm for Solving Optimization Problems **1.** Read and understand the problem's text. **2.** Draw a diagram (if necessary). **3.** Assign variables to the quantities involved and state restrictions according to the situation. **4.** Write relations between these variables. **5.** Identify the variable that is minimized or maximized. This is the dependant variable. **7.** Find extrema (maximum or minimum) for the dependant variable (using global extrema algorithm, first derivative test or the second derivative test). **8.** Check if extrema satisfy the conditions of the application. **9.** Find the value of other variables at extrema (if necessary). **10.** Write the conclusion statement.	**Ex. 5**. Find two positive numbers with a product equal to 200 such that the sum of one number and twice the other number is as small as possible. What is the minimum value of the sum?

Ex. 6. If 2700cm^2 of material is available to make a box with a square base and an open top, find the dimensions (length, width, and height) of the box that give the largest volume of the box. What is the maximum volume of the box?