Section II: Free-Response Questions

This is the free-response section of the 2017 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION II: Free Response

2017

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4NBP4-S

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.

1. For $0 \le t \le 8$, a particle moving in the xy-plane has position vector $\langle x(t), y(t) \rangle = \langle \sin(2t), t^2 - t \rangle$, where $x(t)$ and $y(t)$ are measured in meters and t is measured in seconds.

   (a) Find the speed of the particle at time $t = 2$ seconds. Indicate units of measure.

   (b) At time $t = 4$ seconds, is the speed of the particle increasing or decreasing? Explain your answer.

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Continue question 1 on page 5.

(c) Find the total distance the particle travels over the time interval $0 \le t \le 5$ seconds.

(d) At time $t = 8$ seconds, the particle begins moving in a straight line. For $t \ge 8$, the particle travels with the same velocity vector that it had at time $t = 8$ seconds. Find the position of the particle at time $t = 10$ seconds.

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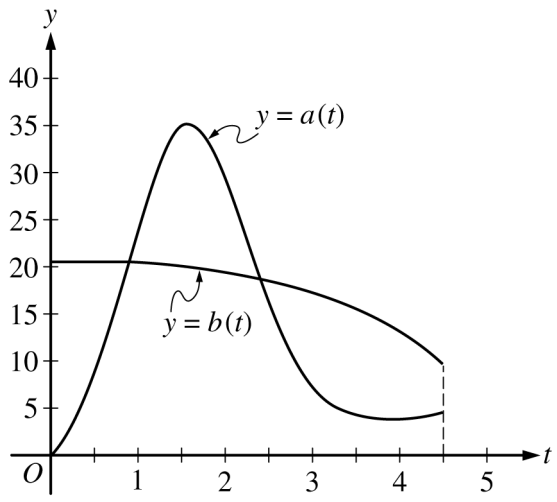

2. During the time interval $0 \le t \le 4.5$ hours, water flows into tank A at a rate of $a(t) = (2t - 5) + 5e^{2\sin t}$ liters per hour. During the same time interval, water flows into tank B at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank A at time $t = 4.5$?

(b) During the time interval $0 \le t \le k$ hours, water flows into tank B at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank A and the amount of water in tank B at time $t = k$?

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Continue question 2 on page 7.

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470. How much water is in tank B at time $t = 2.416$?

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank B is modeled by $w(t) = 21 - \dfrac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

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END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

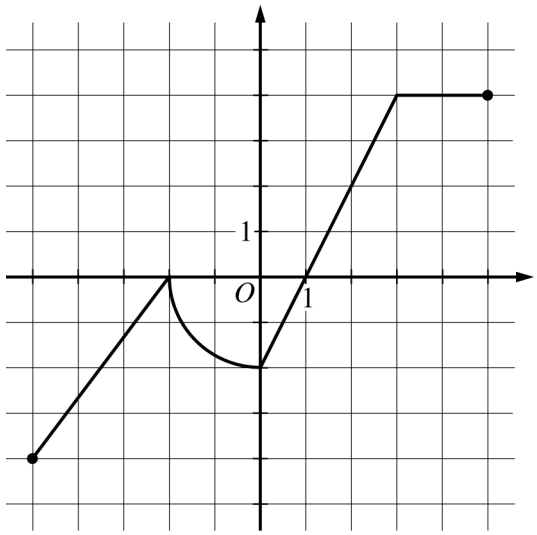
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist.

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Continue question 3 on page 15.

NO CALCULATOR ALLOWED

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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NO CALCULATOR ALLOWED

x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval $[0, 6]$. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

   (a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

   (b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

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Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(c) Evaluate $\int_2^4 f''(x)\,dx$.

(d) Find $\lim_{x \to 0} \dfrac{f(x) - 20e^x}{0.5f(x) - 10}$.

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NO CALCULATOR ALLOWED

5. Consider the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$.

(a) Show that $\frac{d^2y}{dx^2} = \frac{2y^3 - y^2 - 2xy}{x^2}$.

(b) Let $y = g(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$ with initial condition $g(4) = 2$. Does g have a relative minimum, a relative maximum, or neither at $x = 4$? Justify your answer.

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Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Let $y = h(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = -1 + \frac{y^2}{x}$ with initial condition $h(1) = 2$. Write the second-degree Taylor polynomial for h about $x = 1$.

(d) For the function h given in part (c), it is known that $\left|h'''(x)\right| \leq 60$ for all x in the interval $0.9 \leq x \leq 1.1$. Let A represent the approximation of $h(1.1)$ found by using the second-degree Taylor polynomial for h about $x = 1$ from part (c). Use the Lagrange error bound to show that A differs from $h(1.1)$ by at most 0.01.

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NO CALCULATOR ALLOWED

6. Let f be the function defined by $f(x) = \dfrac{1}{x^2 + 9}$.

(a) Evaluate the improper integral $\displaystyle\int_3^{\infty} f(x)\,dx$, or show that the integral diverges.

(b) Determine whether the series $\displaystyle\sum_{n=3}^{\infty} f(n)$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

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Continue question 6 on page 21.

(c) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\left(e^n \cdot f(n)\right)} = \sum_{n=1}^{\infty} \frac{(-1)^n\left(n^2+9\right)}{e^n}$ converges absolutely, converges conditionally, or diverges.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**