Algebra and Equations 1

1. Equations and Formulas

1) What is an Equation?

An equation says that two things are equal. It will have an equals sign "=" like this:

$$x + 2 = 6$$

That equations says: what is on the left (x + 2) is equal to what is on the right (6)
So an equation is like a statement "*this* equals *that*"

2) Solve an Equation

Solving Equations Procedure

The procedure for solving equations in general is:

- Step 1 Remove parentheses.
- Step 2 Combine like terms on each side of the equation.
- Step 3 Use the addition and/or subtraction principle to get the variables on one side and the constant terms on the other side.
- Step 4 Use the division principle to solve for x.
- Step 5 Check the equation.

Example 1: Solve 3(2x + 6) = 30.

$$3(2x + 6) = 30$$
$$6x + 18 = 30 \quad \text{Remove parentheses}$$
$$6x + 18 - 18 = 30 - 18 \quad \text{Subtract 18}$$
$$6x = 12$$
$$\frac{\cancel{6}x}{\cancel{6}} = \frac{12}{6} \quad \text{Divide by 6}$$
$$x = 2$$

Example 2: Solve 5(2x – 7) – 3x = 5x + 9.

$$5(2x - 7) - 3x = 5x + 9$$
$$10x - 35 - 3x = 5x + 9 \quad \text{Remove parentheses}$$
$$7x - 35 = 5x + 9 \quad \text{Combine like terms}$$
$$7x - 35 - 5x = 5x - 5x + 9 \quad \text{Get variables on one side and the}$$
$$2x - 35 = 9 \quad \text{constants on the other side}$$
$$2x - 35 + 35 = 9 + 35$$
$$2x = 44$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{44}{2} \quad \text{Divide by 2}$$
$$x = 22$$

Example 3: Solve $\sqrt{(2x-5)} - \sqrt{(x-1)} = 1$

Isolate one of the square roots: $\sqrt{(2x-5)} = 1 + \sqrt{(x-1)}$

Square both sides: $2x - 5 = (1 + \sqrt{(x-1)})^2$

We have removed one square root.

Expand right hand side: $2x - 5 = 1 + 2\sqrt{(x-1)} + (x-1)$

Simplify: $2x - 5 = 2\sqrt{(x-1)} + x$

Simplify more: $x - 5 = 2\sqrt{(x-1)}$

Now do the "square root" thing again:

Isolate the square root: $\sqrt{(x-1)} = (x-5)/2$

Square both sides: $x - 1 = ((x-5)/2)^2$

We have now successfully removed both square roots.

Let us continue on with the solution.

Expand right hand side: $x - 1 = (x^2 - 10x + 25)/4$

It is a Quadratic Equation! So let us put it in standard form.

Multiply by 4 to remove division: $4x - 4 = x^2 - 10x + 25$

Bring all to left: $4x - 4 - x^2 + 10x - 25 = 0$

Combine like terms: $-x^2 + 14x - 29 = 0$

Swap all signs: $x^2 - 14x + 29 = 0$

Using the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (a=1, b=-14, c=29), I found the solutions to be:

2.53 and 11.47 (to 2 decimal places)

Let us check the solutions:

2.53: $\sqrt{(2 \cdot 2.53-5)} - \sqrt{(2.53-1)} \approx -1$ Oops! Should be plus 1!

11.47: $\sqrt{(2 \cdot 11.47-5)} - \sqrt{(11.47-1)} \approx 1$ Yes that one works.

There is really only one solution: 11.47 (to 2 decimal places)

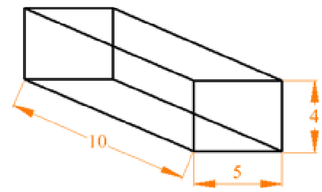
3) What is a Formula?
A formula is a special type of equation that shows the relationship between different variables. (A variable is a symbol like x or V that stands in for a number we don't know yet).

Example: The formula for finding the volume of a box is **V = wdh**

V stands for volume, **w** for width, **d** for depth and **h** for height.

If w = 5, d = 10 and h = 4, then **V = 5 × 10 × 4 = 200**

A formula will have **more than one variable**.



These are all equations, but only some are formulas:

x = 2**y** - 7	Formula (relating **x** and **y**)
$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$	Formula (relating **a**, **b** and **c**)
x/2 + 7 = 0	Not a Formula (just an equation)

4) Without the Equals
Sometimes a formula is written without the "=":

Example: The formula for the volume of a box is: **lwh**
But in a way the "=" is still there, because you could write **V = lwh** if you wanted to.

5) Subject of a Formula
The "subject" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

Example: in the formula
$\mathbf{s = ut + ½ at^2}$
"s" is the subject of the formula

6) Changing the Subject
One of the very powerful things that Algebra can do is to "rearrange" a formula so that another variable is the subject.

Rearrange the volume of a box formula (**V = lwh**) so that the width is the subject:

Start with:	**V = lwh**
divide both sides by d:	**V / l = wh**
divide both sides by h:	**V / lh = w**
swap sides:	**w = V / lh**

So now if you have a box with a depth of 2m, a height of 2m and a volume of $12m^3$, you can calculate its width:

w = V / lh
w = 12m^3 / (2m×2m) = 12/4 = 3m

2. Inequalities

Inequality tells you about the relative size of two values.

Mathematics is not always about "equals"! Sometimes you only know that something is bigger or smaller.

Example: Alex and Billy have a race, and Billy wins!

What do we know?
We don't know how fast they ran, but we do know that Billy was faster than Alex:
Billy was faster than Alex

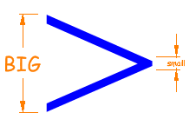
We can write that down like this: $b > a$ (Where "b" means how fast Billy was, "a" means how fast Alex was, and ">" means "greater than")
We call things like that **inequalities** (because they are not "equal")

Greater or Less Than

The two most common inequalities are:

Symbol	Words	Example Use
>	greater than	$5 > 2$
<	less than	$7 < 9$

They are easy to remember: the "small" end always points to the smaller number, like this:



Greater Than Symbol: BIG > small

Example: Alex plays in the under age of 15 soccer team. How old is Alex?

We don't know exactly how old Alex is, because it doesn't say "equals"
But we do know "less than 15", so we can write: Age < 15!

You can also have inequalities like:

Symbol	Words	Example Use
≥	greater than or equal to	$x \geq 1$
≤	less than or equal to	$y \leq 3$

Example: you must be 13 or older to watch a movie.

The "inequality" is between your age and the age of 13.
Your age must be "greater than or equal to 13", which would be written: Age $\geq$ 13

3. Solve Inequalities

Sometimes we need to solve Inequalities like these:

Symbol	Words	Example
$>$	greater than	$x + 3 > 2$
$<$	less than	$7x < 28$
$\geq$	greater than or equal to	$5 \geq x - 1$
$\leq$	less than or equal to	$2y + 1 \leq 7$

Our aim is to have x (or whatever the variable is) on its own on the left of the inequality sign:

Something like: $x < 5$

or: $y \geq 11$

We call that "solved".

How to Solve? Solving inequalities is very like solving equations, you do most of the same things, but you must also pay attention to the direction of the inequality.

These are things you can do **without** affecting the direction of the inequality:

- Add (or subtract) a number from both sides
- Multiply (or divide) both sides by a positive number
- Simplify a side

Example: $3x < 7 + 3$

$3x < 10$

$x < 10/3$

But these things **will change** the direction of the inequality ("<" becomes ">" for example):

- Multiply (or divide) both sides by a negative number
- Swapping left and right hand sides

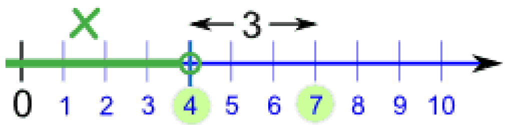
Example: $12 > 2x + 4$

When you swap the left and right hand sides, you must also change the direction of the inequality:

$2x + 4 < 12$

$2x < 8$

$x < 4$



In other words, x can be any value less than 4. When graphing the solution, we put a hole on 4 since the solution doesn't include 4. If it does, we put a dot.

Example: Solve: $-2y < -8$

Let us divide both sides by -2 ... and reverse the inequality!

$-2y < -8$

$-2y/-2 > -8/-2$

$y > 4$

Example: Solve: $bx < 3b$

It seems easy just to divide both sides by b, which would give us: $x < 3$

... but wait ... if b is negative we need to reverse the inequality like this: $x > 3$

But we don't know if b is positive or negative, so we have to put it into two cases.

If $b > 0$, $x < 3$

If $b < 0$, $x > 3$

Example: Solve: $(x-3)/2 < -5$

$(x-3) < -10$

$x < -7$

Two Inequalities At Once

Example: Solve: $-2 < (6 - 2x)/3 < 4$

$-6 < 6 - 2x < 12$

$-12 < -2x < 6$

$6 > x > -3$

But to be neat it is better to have the smaller number on the left, larger on the right. So let us swap them over (and make sure the inequalities point correctly):

$-3 < x < 6$

Questions in class

1. The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

2. Suppose A, B, and C are three numbers for which 1001C− 2002A = 4004 and 1001B + 3003A = 5005. What is the average of the three numbers A, B, and C?

3. The arithmetic mean of the nine numbers in the set {9,99,999,9999,… ,999999999} is a 9−digit number M, all of whose digits are distinct. What digit does the number M not contain?

4. It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?

5. Let d and e denote the solutions of $2x^2 + 3x - 5 = 0$. What is the value of (d−1)(e−1)?

6. The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd counting numbers. What is the smallest of the even integers?

7. In the expression $c \cdot a^b - d$. The values of a, b. c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

8. On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

9. A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans. How many rows does it contain?

10. The equations $2x + 7 = 3$ and $bx - 10 = -2$ have the same solution x. What is the value of b?