Implicit Differentiation

More practice, more fun 😊

1. Find the slope of the tangent line drawn to the graph of $x^4 - y^4 = 2x^2y + 23$ to the point $(2, -1)$.
2. Find an equation for the tangent line drawn to the graph of $x^3 + y^3 - 5y^2 = 6x^2 + 13x - 42$ at the point $(-3, 5)$.
3. Find an equation for all tangent lines drawn to the graph of $2x^2 + y^2 = 5y - x$ at $x = -2$.
4. Find an equation of all tangent lines drawn to the curve $x^2 - xy + y^2 = 16$ at $x = 0$.
5. Use implicit differentiation to compute y' in terms of x and y.

a) $2x^2 + 4xy = 10$

b) $x^4 + y^4 = 20y$

c) $x^3 + y^3 = 2xy$

d) $x^3 + y^3 = x^2 + y^2$

e) $\ln x - 2 + y^2 = y^5$

f) $x^2 + y^2 = \dfrac{1}{y}$

g) $\sin x + \cos y = -2y^3$

h) $x^4y - xy^4 = y$

i) $x^3 + y^3 = (x - y)^5$

j) $y^3 + y = \sqrt{x^2 - y^2}$

k) $2^{x+y} = xy^3$

l) $y + xy = \sqrt{xy - 2}$

m) $\ln y = \sin(xy) - 1$

n) $\left(\sin^3 x + \sin^3 y\right)^2 = x + y$

Answers

1. 10
2. $-2(x + 3) = y - 5$
3. $y = -7x - 12$ and $y = 7x + 17$
4. $y = \frac{1}{2}x + 4$ and $y = \frac{1}{2}x - 4$
5. a) $y' = -\dfrac{x + y}{x}$ b) $y' = -\dfrac{x^3}{y^3 - 5}$ c) $y' = \dfrac{3x^2 - 2y}{2x - 3y^2}$ d) $y' = \dfrac{-3x^2 + 2x}{3y^2 - 2y}$

e) $y' = -\dfrac{1}{x\left(2y - 5y^4\right)}$ f) $y' = -\dfrac{2xy^2}{2y^3 + 1}$ g) $y' = \dfrac{\cos x}{\sin y - 6y^2}$ h) $y' = \dfrac{y^4 - 4x^3y}{x^4 - 4xy^3 - 1}$

i) $y' = \dfrac{-3x^2 + 5(x - y)^4}{3y^2 + 5(x - y)^4}$ j) $y' = \dfrac{x}{y + \left(y + y^3\right)\left(3y^2 + 1\right)}$ k) $y' = \dfrac{y^3 - (\ln 2)\, 2^{x+y}}{-3xy^2 + (\ln 2)\, 2^{x+y}}$

l) $y' = \dfrac{y - 2y\sqrt{xy - 2}}{2\sqrt{xy - 2} - x + 2x\sqrt{xy - 2}}$ m) $y' = \dfrac{y^2 \cos xy}{-xy \cos xy + 1}$

n) $y' = \dfrac{-6\left(\cos x \sin^2 x\right)\left(\sin^3 x + \sin^3 y\right) + 1}{6\left(\cos y \sin^2 y\right)\left(\sin^3 x + \sin^3 y\right) - 1}$