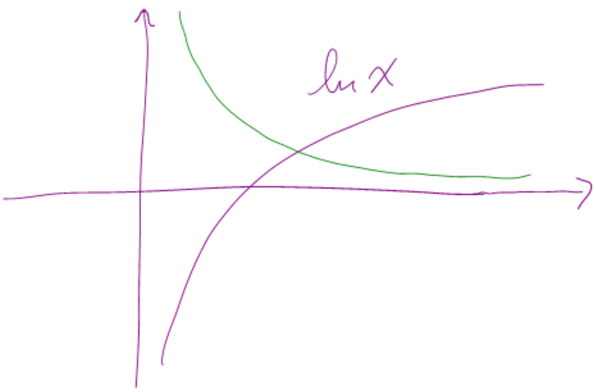
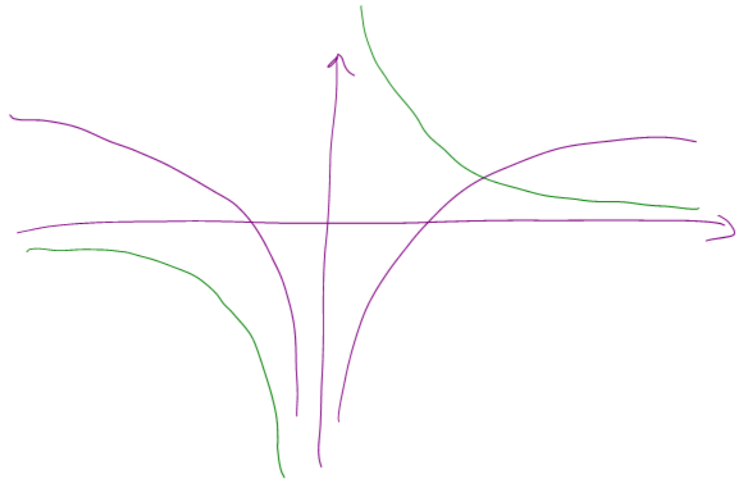
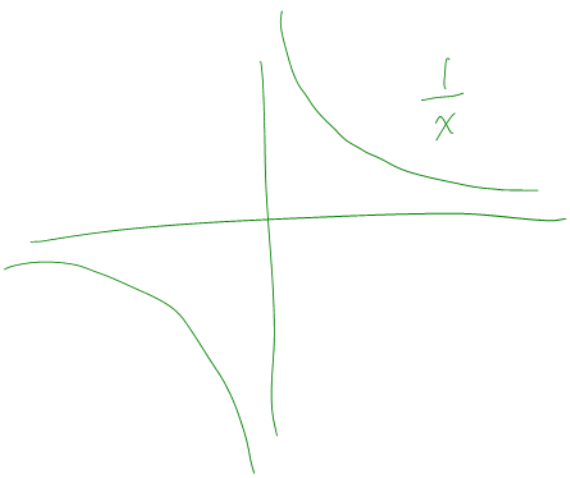
AP Calculus Class 10.

$\frac{d}{dx}\ln x = \frac{1}{x} \rightarrow x \in (0, \infty)$ vs. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$







Antiderivative.

fun^n $\rightarrow$ A.D.

$\frac{1}{x}$ $\qquad$ $\ln|x|$

$y = \ln|2 - x - 5x^2|$

$y' = \frac{1}{2 - x - 5x^2}(2 - x - 5x^2)'$

$= \frac{1}{2 - x - 5x^2}(-1 - 10x) = -\frac{1 + 10x}{2 - x - 5x^2}$

Homework 9.

3. $V_f = 120$ ft/s.

$a_g = -32\ ft/s^2$ $= 9.8\ m/s^2$

down is negative direction.

$s(t)$: displacement./distance

$v(t)$: velocity/speed.

$a(t)$: acceleration.

$s'(t) = v(t)$

$v'(t) = a(t)$.

The antiderivative of $a(t)$ is $v(t)$.

$\Rightarrow \quad v(t) = -32t + C.$

Since the stone was dropped, $v(0) = 0.$

We can solve for C.

$\Rightarrow \quad 0 = -32(0) + c \qquad \Rightarrow c = 0.$

$\Rightarrow \quad v(t) = -32t$

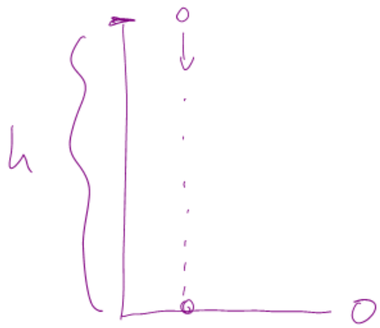
Next, find the time of impact.

$\Rightarrow \quad v(t) = -32t = -120.$

$\Rightarrow \quad t = \dfrac{-120}{-32} = 3.75\ s.$

The A.D. of $v(t)$ is the position fun^n $s(t)$.

$s(t) = -32\left(\frac{1}{2}t^2\right) + D = -16t^2 + D.$



$\Rightarrow \ s(3.75) = 0$

$\Rightarrow \ 0 = -16(3.75)^2 + D$

$\Rightarrow \ D = 225 \text{ ft}$

$s(t) = -16t^2 + 225$

2. d) $f''(x) = x^{-2} \qquad x > 0 \qquad f(1) = 0, \quad f(2) = 0$

A.D. $x^n = \dfrac{x^{n+1}}{n+1} + C$

$\Rightarrow f'(x) = \dfrac{x^{-2+1}}{-2+1} + C$

$= \dfrac{x^{-1}}{-1} + C = -x^{-1} + C$

$x^{-1} = \dfrac{1}{x}$

$\ln|x| + D + Cx + E$

A.D. $\dfrac{1}{x} = \ln|x| + D$

$\Rightarrow f(x) = -\ln|x| + Cx + D$

$\Rightarrow f(1) = -\ln 1 + C + D = 0$

$\Rightarrow C + D = 0 \qquad \Rightarrow C = -D$

$f(2) = -\ln 2 + 2(-D) + D = 0$

$\Rightarrow -\ln 2 - D = 0 \qquad \Rightarrow D = -\ln 2$

$\Rightarrow \quad c = \ln 2$

$f(x) = -\ln x + x\ln 2 - \ln 2$

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7. d) $\displaystyle\int_1^9 \frac{x-1}{\sqrt{x}}\,dx$

$$= \int_1^9 \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\,dx = \int_1^9 x^{\frac{1}{2}} - x^{-\frac{1}{2}}\,dx$$

$$= \int_1^9 x^{\frac{1}{2}}\,dx - \int_1^9 x^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^9 - \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^9 = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^9 - \left[2x^{\frac{1}{2}}\right]_1^9$$

$$= \left[\frac{2}{3}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)\right] - \left[2\left(9^{\frac{1}{2}} - 1^{\frac{1}{2}}\right)\right]$$

$$= \frac{2}{3}(26) - 4 = \frac{52}{3} - \frac{12}{3} = \frac{40}{3}$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right]_1^9 = \left[\left(\frac{2}{3}9^{\frac{3}{2}} - 2(9)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(1)^{\frac{3}{2}} - 2(1)^{\frac{1}{2}}\right)\right]$$

Indefinite Integrals.

$\int f(x)\,dx = F(x)$ $\qquad F'(x) = f(x)$

$\int x^2\,dx = \frac{x^3}{3} + C$ $\qquad \frac{d}{dx}\left(\frac{x^3}{3} + C\right) = x^2$

Note: Definite integrals are numbers.

Indefinite integrals are functions.

$(\sin x)' = \cos x$ $\qquad \left(\sin \frac{\pi}{2}\right)' = 1$

Example: $\int (10x^4 - 2\sec^2 x)\,dx.$

$$10\int x^4\,dx - 2\int \sec^2 x\,dx = 10\,\frac{x^5}{5} - 2\tan x + C.$$

$$= 2x^5 - 2\tan x + C.$$

Example: $\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx.$

$$= \left[2\frac{x^4}{4} - 6\,\frac{x^2}{2} + 3\tan^{-1} x\right]_0^2$$

$$= \left[\frac{1}{2}x^4 - 3x^2 + 3\tan^{-1} x\right]_0^2$$

$$= \left[\frac{1}{2}(2)^4 - 3(2)^2 + 3\tan^{-1} 2 \right] = -4 + 3\tan^{-1} 2.$$

Example: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\, dt.$

$$\int_1^9 2 + \sqrt{t} - t^{-2}\, dt = \int_1^9 2 + t^{\frac{1}{2}} - t^{-2}\, dt.$$

$$= \left[2t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{-1}}{-1} \right]_1^9 = 32\,\frac{4}{9}$$

$$\int_1^2 3x^2\, dx = [x^3 + C]_1^2$$

$$= (2^3 + c) - (1^3 + c) = 8 \cancel{+c} - 1 \cancel{-c} = 7$$

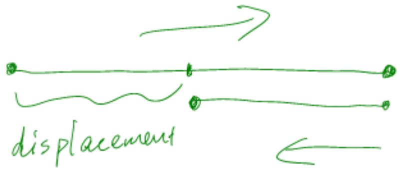
Velocity Problems.

If an object moves along a straight line with position fun^n $s(t)$, then it's velocity fun^n $v(t)$ is.

$$v(t) = s'(t)$$

$$\int_{t_1}^{t_2} v(t)\, dt = s(t_2) - s(t_1).$$

This integral is the net change of position, displacement.



$\int_{t_1}^{t_2} |v(t)|\,dt$ = the total distance travelled.

Example: A particle moves along a line

$v(t) = t^2 - t - 6 \quad (m/s).$

a) Find the displace of the particle $t \in [1, 4]$.

b) Find the distance travelled during this time

a) $s(4) - s(1) = \int_1^4 t^2 - t - 6\,dt.$

$$= \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big]_1^4 = -\frac{9}{2}$$

b) $\int_1^4 |t^2 - t - 6|\,dt$

$v(t) = t^2 - t - 6 = (t-3)(t+2)$

In this case, $t = 3$.

$v(t) \le 0$ on $[1, 3]$ $\qquad v(t) \ge 0$ $[3, 4]$.

$\Rightarrow$ The distance travelled is

$$\int_1^4 |v(t)|\, dt = \int_1^3 -v(t)\, dt + \int_3^4 v(t)\, dt.$$

$$= \int_1^3 -t^2+t+6\, dt + \int_3^4 t^2-t-6\, dt$$

$$= \left[-\frac{t^3}{3}+\frac{t^2}{2}+6t\right]_1^3 + \left[\frac{t^3}{3}-\frac{t^2}{2}-6t\right]_3^4$$

$$= \frac{61}{6}$$

The Substitution Rule.

$$\int 2x\sqrt{1+x^2}\, dx.$$

let $u = 1+x^2$

$\frac{du}{dx} = 2x$ differentials

$du = 2x\, dx$

$$\int \underbrace{\sqrt{1+x^2}}_{u}\ \underbrace{2x\, dx}_{du}$$

$$\int \sqrt{u}\, du = \int u^{\frac{1}{2}}\, du$$

$$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{3}(1+x^2)^{\frac{3}{2}} + C.$$

The Substitution Rule.

If $u = g(x)$ is a differentiable fun^n whose range is an interval I, and f is continuous on I, then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du.$$

Example: $\int x^3 \cos(x^4+2)\, dx$.

let $u = x^4 + 2$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\, dx$

$\frac{1}{4} du = x^3 dx$

$$\int \cos(x^4+2)\, x^3\, dx = \int \cos(u) \cdot \frac{1}{4}\, du$$

$$\frac{1}{4}\int \cos u\, du = \frac{1}{4} \sin u + C.$$

$$= \frac{1}{4} \sin(x^4+2) + C.$$

Example: $\int \sqrt{1+x^2} \cdot x^5\, dx$.

let $u = 1+x^2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$\Rightarrow x^2 = u-1$

$\Rightarrow x^4 = (u-1)^2$

$$\int \underbrace{\sqrt{1+x^2}}_{u} \cdot x^4 \cdot \underbrace{x\, dx}_{du}$$

$$\Rightarrow \int \sqrt{u}\, (u-1)^2\, \frac{1}{2}\, du$$

$$= \frac{1}{2} \int u^{\frac{1}{2}} (u^2 - 2u + 1)\, du$$

$= \frac{1}{2} \int u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}} \, du.$

Example: $\int_0^4 \sqrt{2x+1} \, dx$

Let $u = 2x+1$

$du = 2 \, dx$

$\frac{1}{2} du = dx$

$u(0) = 2(0)+1 = 1$

$u(4) = 2(4)+1 = 9$

$\frac{1}{2} \int_1^9 \sqrt{u} \, du = \frac{1}{2} \int_1^9 u^{\frac{1}{2}} \, du$

$= \frac{1}{2} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^9 = \frac{1}{2} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^9$

$= \frac{1}{2} \left[\frac{2}{3} 9^{\frac{3}{2}} - \frac{2}{3} 1^{\frac{3}{2}} \right]$

$= \frac{1}{2} \left[\frac{2}{3}(27) - \frac{2}{3} \right] = \frac{1}{2} \left[\frac{2}{3} \right] [26] = \frac{26}{3}$