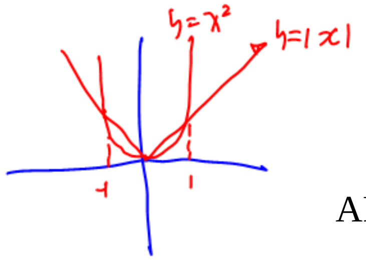

AP Calculus In-Class Two – Limit and Continuity

1.4 Other Basic Limits; 1.5 Asymptotes

1. If c is a nonnegative real number and $0 \le f(x) \le c$ for every x. Prove that $\lim_{x\to 0} x^2 f(x) = 0$.

Proof: Let $h(x) = x^2 f(x)$. consider $x \in [-\frac{1}{2}, \frac{1}{2}]$, $u(x) = 0$, $v(x) = cx^2$, then $u(x) \le h(x) \le v(x)$, $\lim_{x\to 0} u(x) = \lim_{x\to 0}(0) = 0$, $\lim_{x\to 0} v(x) = \lim_{x\to 0} cx^2 = c(0)^2 = 0$

Hence by the squeeze theorem. $\lim_{x\to 0} h(x) = \lim_{x\to 0} x^2 f(x) = 0$.

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{x}{\sin x} = 1$

2. Find limits.

(a) $\lim_{x\to 0} \frac{\sin x}{\sqrt[3]{x}}$ $= \lim_{x\to 0} \frac{x^{2/3} \sin x}{x^{2/3} x^{1/3}} = \lim_{x\to 0} x^{2/3} \lim_{x\to 0} \frac{\sin x}{x} = 0^{2/3}(1) = 0$

(b) $\lim_{x\to 0} \frac{x + \tan x}{\sin x}$ $= \lim_{x\to 0} \frac{x}{\sin x} + \lim_{x\to 0} \frac{\tan x}{\sin x} = 1 + \lim_{x\to 0} \frac{\frac{\cancel{\sin x}}{\cos x}}{\cancel{\sin x}} = 1 + \frac{\frac{1}{1}}{1} = 2$

let $-t = x \Rightarrow t = -x$.

(c) $\lim_{t\to 0} (1 - t)^{1/t}$

$= \lim_{x\to 0} (1+x)^{\frac{1}{-x}} = [\lim_{x\to 0} (1+x)^{\frac{1}{x}}]^{-1} = e^{-1} = \frac{1}{e}$

3. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statement is (are) true?

I. $\lim_{x\to -3} f(x) = -1$

II. f is continuous everywhere except at $x = -3$.

III. f has a removable discontinuity at $x = -3$.

(A) None of them (B) I only (C) III only

(D) I and III only (E) All of them

4. Find a value of c that makes $h(x)$ is continuous at $x = 0$.

$$h(x) = \begin{cases} \dfrac{1-\cos 3x}{x^2}, & \text{if } x \neq 0 \\ c, & \text{if } x = 0 \end{cases}$$

5. Find all asymptotes of the graph of $y = \dfrac{2x^2 + 2x + 3}{4x^2 - 4x}$.

6. Find all asymptotes for the graph of $g(x) = \arctan x$.

7. Find all vertical and horizontal asymptotes for the graph of $y = \dfrac{\ln x}{1 - \ln x}$.

8. Show that equation $|x| = \cos x$ has at least one positive root.