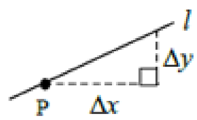
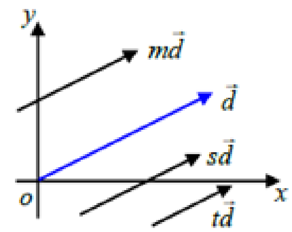
Unit: Equations of lines and planes

Vector and Parametric Equations of a line in Plane

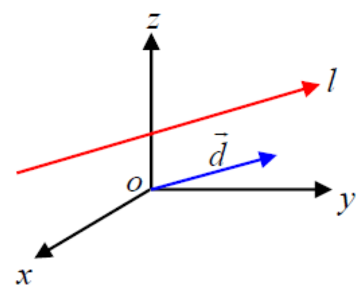
How can a line be formed?		
a) 2 points are given	b) 1 point + slope m	c) 1 point + its direction $\vec{d}$
		



<u>Direction vector</u> $\vec{d}$

- It is a non-zero vector issued from the origin.
- It represents the direction of a line.
- A line may have infinite number of direction vectors namely $t\vec{d}, s\vec{d}, m\vec{d}$, etc.
- Direction vectors can be written both ways: $\overrightarrow{AB}$ or $\overrightarrow{BA}$.

<u>Equation of a Line in 3-Space</u>



Given: $\vec{d}=(a,b,c)$ and Point : (x_0, y_0, z_0)

Vector equation: $(x,y,z)=(x_0,y_0,z_0)+t(a,b,c),\quad t\in R$

Parametric equation:

$$x = x_0 + ta$$
$$y = y_0 + tb$$
$$z = z_0 + tc$$

Symmetric equation: $\dfrac{x-x_0}{a}=\dfrac{y-y_0}{b}=\dfrac{z-z_0}{c}\quad a,b,c\neq 0$

Scalar equation: Not for 3-space.

> *Knowing one of these forms of the equation of a line enables you to find the other two, since all three forms depend on the same information about the line.*

Ex 1: Vector, Parametric and Symmetric Equation of lines

a) Find vector, parametric, and symmetric equations of line passing through the points *A*(2, -1, 5) and *B* (7, 4, 3).

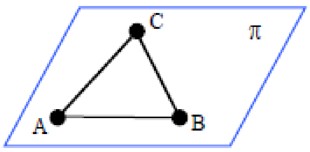
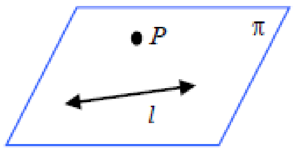
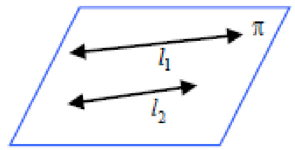
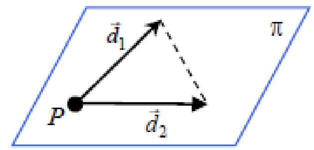
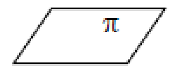
b) Does the point *Q*(1, 2, -3) lie on the line?

Ex 2: Symmetric Equation of line

Find the vector and symmetric equations of the line that passes through the point (-6, 4, 2) and is perpendicular to both of the lines

$l_1 : \frac{x}{-4} = \frac{y+10}{-6} = \frac{z+2}{3}$ and $l_2 : \frac{x-5}{3} = \frac{y-5}{2} = \frac{z+5}{4}$.

Equation of a Plane

How can a plane be formed?			
3 non-collinear points	**1 line and 1 point not on the line**	**2 parallel and non-coincident lines**	**1 point and 2 direction vectors (2 intersecting lines)**
			
To represent planes, parallelograms are used to represent a small part of the plane and are designated with the Greek letter π.			

Given: $\vec{u} = (u_x, u_y, u_z)$, $\vec{v} = (v_x, v_y, v_z)$ and point $P_0(x_0, y_0, z_0)$

Vector equation of the plane: $(x, y, z) = (x_0, y_0, z_0) + t(u_x, u_y, u_z) + s(v_x, v_y, v_z)$, $s, t \in R$

Parametric equations of a plane: $\begin{cases} x = x_0 + su_x + tv_x \\ y = y_0 + su_y + tv_y \\ z = z_0 + su_z + tv_z \end{cases}$; $s, t \in R$

Ex 3. Convert the vector equation to the parametric equations.

: $\vec{r} = (-1,0,2) + s(0,1,-1) + t(1,-2,0)$; $s,t \in R$

Ex 4. Convert the parametric equations to the vector equation.

$$\begin{cases} x = 1 + s - 2t \\ y = 3t \\ z = 4 - s \end{cases} \quad ; \quad s,t \in R$$

Ex 5. Find the vector equation of the plane π that passes through the points A(0,1,−1) , B(2,−1,0) and C(0,0,1) .

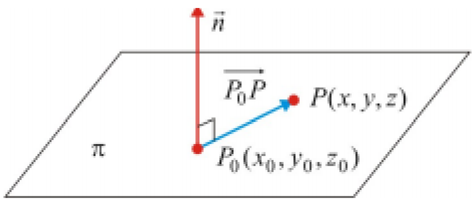
Ex 6. Find the vector and parametric equations of the plane π that contains the following parallel and distinct lines:

L_1 : $\vec{r}$ = (1, 2, 1) + s(0,-1,-2); s∈R

L_1 : $\vec{r}$ = (3, 4, 0) + t(0,1,2); t∈R

Cartesian/Scalar Equation of a Plane

Let's write the normal vector of a plane in the form: $\vec{n}$ = (A, B, C)



Then, the **normal equation** may be written as:

(x-x_0, y-y_0, z-z_0) · (A,B,C) = 0

Ax + By + Cz - Ax_0 - By_0 - Cz_0 = 0

or

Ax + By + Cz + D = 0 **Cartesian equation of a plane**.

Ex 7. Consider the plane π defined the Cartesian equation π : 2x – 3y + 6z +12 = 0 .

a) Find a normal vector to this plane.

b) Find two points on this plane

c) Find if the point P(1,2,3) is a point on this plane.

More practice, more fun 😊

1. Determine which of the following points lie on the line ℓ: (x, y, z) = (2, −3, 4) + *t*(1, 3, 2).

a. (3, 0, 6)
b. (−1, −12, −2)
c. (8, −8, 12)
d. (4.2, −5.6, 1.4)
e. (4.5, 4.5, 9)

2. Given the line ℓ: (x, y, z) = (8, 2, −3) + *t*(4, 1, −2)

a. Find the point on the line with an *x*-coordinate of 120.
b. Does the line have an *x*-intercept, a *y*-intercept, or a *z*-intercept? If so, find them.

3. For each of the following, find the vector equation of the line that:

a. is parallel to (6, 4, 1) and passes through the point (3, 0, −4)
b. passes through the points (2, −4, 3) and (−4, −8, 7)
c. is parallel to the *y*-axis and passes through the point (6, −2, −4)
d. has *x*-intercept 5 and *z*-intercept −10

4. If the points (4, 2, 7), (6, 19, −4), and (80, *b*, *c*) lie on the same straight line, find the values of *b* and *c*.

5. Determine the angle between each pair of lines:

a. l_1: (x, y, z) = (4, 5, −2) + *t*(3, −1, −1) and l_2: (x, y, z) = (4, 5, −2) + *s*(−2, −3, 2)

b. $l_1: \begin{cases} x = 20 + 3t \\ y = -10 + 2t \\ z = 4 \end{cases}$ and $l_2: \begin{cases} x = 20 + t \\ y = -10 + 5t \\ z = 4 \end{cases}$

6. Find a scalar equation for each of the following planes:

a. The plane with normal vector (5, 1, −1) and passing through (3, 0, 2)
b. The plane with vector equation $\vec{r}$ = (1, 0, 2) + s(1, 1, 1) + t(2, −1, 3)
c. $\begin{cases} x = 3 + 4s - t \\ y = s + 3t \\ z = -2 - s + 4t \end{cases}$
d. The plane that passes through the points (5, −2, 3), (−3, 1, 2), and (6, 0, 4).
e. The plane with a x-intercept of 12, a y-intercept of 3, and a z-intercept of −2.