Notice: The notes are the same as before, but the questions in class and homework are different with before.

Number Theory

1. Commutative, Associative, and Distributive Laws

1) Commutative Laws
The "Commutative Laws" say you can swap numbers over and still get the same answer.

when you add: $a + b = b + a$
or when you multiply: $a \times b = b \times a$

2) Associative Laws
The "Associative Laws" say that it doesn't matter how you group the numbers (i.e. which you calculate first).

when you add: $(a + b) + c = a + (b + c)$
or when you multiply: $(a \times b) \times c = a \times (b \times c)$

Examples:

This:	$(2 + 4) + 5 = 6 + 5 = 11$
Has the same answer as this:	$2 + (4 + 5) = 2 + 9 = 11$
This:	$(3 \times 4) \times 5 = 12 \times 5 = 60$
Has the same answer as this:	$3 \times (4 \times 5) = 3 \times 20 = 60$

Sometimes it is easier to add or multiply in a different order:

What is $19 + 36 + 4$?
$19 + 36 + 4 = 19 + (36 + 4) = 19 + 40 = 59$

Or to rearrange a little: What is $2 \times 16 \times 5$?
$2 \times 16 \times 5 = (2 \times 5) \times 16 = 10 \times 16 = 160$

3) Distributive Law
The "Distributive Law" is the BEST one of all, but needs careful attention.

This is what it lets you do:
3 lots of $(2+4)$ is the same as 3 lots of 2 plus 3 lots of 4
So, the $3\times$ can be "distributed" across the $2+4$, into 3×2 and 3×4

Try the calculations yourself:
$3 \times (2 + 4) = 3 \times 6 = 18$
$3\times2 + 3\times4 = 6 + 12 = 18$

You get the same answer when you: multiply a number by a group of numbers added together, or do each multiply separately then add them

Like this: $a \times (b + c) = a \times b + a \times c$

Sometimes it is easier to break up a difficult multiplication:

Example: What is 6×204?
$6 \times 204 = 6 \times 200 + 6 \times 4 = 1{,}200 + 24 = 1{,}224$
Or to combine:

Example: What is $16 \times 6 + 16 \times 4$?
$16 \times 6 + 16 \times 4 = 16 \times (6+4) = 16 \times 10 = 160$
You can use it in subtraction too:

Example: $26\times3 - 24\times3$
$26 \times 3 – 24 \times 3 = (26 - 24) \times 3 = 2 \times 3 = 6$
You could use it for a long list of additions, too:

Example: $6 \times 7 + 2\times7 + 3\times7 + 5\times7 + 4\times7$
$6\times7 + 2\times7 + 3\times7 + 5\times7 + 4\times7 = (6+2+3+5+4) \times 7 = 20 \times 7 = 140$

The Associative Law does not work for subtraction:
Example:
$(9 – 4) – 3 = 5 – 3 = 2$, but
$9 – (4 – 3) = 9 – 1 = 8$

The Distributive Law does not work for division:
Example:
$24 / (4 + 8) = 24 / 12 = 2$, but
$24 / 4 + 24 / 8 = 6 + 3 = 9$

Summary

Commutative Laws:	$a + b = b + a$ $a \times b = b \times a$
Associative Laws:	$(a + b) + c = a + (b + c)$

$$(a \times b) \times c \;=\; a \times (b \times c)$$

Distributive Law:

$$a \times (b + c) \;=\; a \times b \;+\; a \times c$$

2. Exponents

Rule 1: $(x^{m})(x^{n}) = x^{(m+n)}$

Example 1: Simplify $(x^3)(x^4)$

$(x^3)(x^4) = (xxx)(xxxx) = xxxxxxx = x^7$ or $(x^3)(x^4) = x^{3+4} = x^7$

Example 2: Simplify $(x^2)^4$

$(x^2)^4 = (x^2)(x^2)(x^2)(x^2)$
$= (xx)(xx)(xx)(xx) = xxxxxxxx = x^8$

or, $(x^2)^4 = x^{(2\times4)} = x^8$

Rule 2: $(x^{m})^{n} = x^{mn}$

Example 1: $(xy^2)^3 = (xy^2)(xy^2)(xy^2) = (xxx)(y^2y^2y^2) = (xxx)(yyyyyy) = x^3y^6 = (x)^3(y^2)^3$.

Another example would be:

$$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$$

Example 2: $(x - 2)^2 = (x - 2)(x - 2) = xx - 2x - 2x + 4 = x^2 - 4x + 4$

Rule 3: Anything to the power zero is just "1"
$x^0 = 1$

Example 1: $(x + y)^0 = 1$

Example 2: Simplify $[(3x^5y^7z^3)^5(\text{-}5xyz)^2]^0$

$[(3x^5y^7z^3)^5(\text{-}5xyz)^2]^0 = 1$

2. Fractions in Algebra

You can add, subtract, multiply and divide fractions in algebra in the same way that you do in simple arithmetic.

Adding Fractions

To add fractions there is a simple rule:

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

Example: $\frac{x+4}{3}+\frac{x-3}{4}=\frac{(x+4)(4)+(3)(x-3)}{3\times 4}=\frac{4x+16+3x-9}{12}=\frac{7x+7}{12}$

Subtracting Fractions

Subtracting fractions is very similar to adding, except that the + is now -

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$

Example: $\frac{x+2}{x}+\frac{x}{x-2}=\frac{(x+2)(x-2)-(x)(x)}{x(x-2)}=\frac{x^2-2^2-x^2}{x^2-2x}=\frac{-4}{x^2-2x}$

Multiplying Fractions

Multiplying fractions is the easiest one of all, just multiply the tops together, and the bottoms together:

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

Example: $\frac{3x}{x-2}\times\frac{x}{3}=\frac{(3x)(x)}{3(x-2)}=\frac{x^2}{x-2}$

Dividing Fractions

To divide fractions, first "flip" the fraction you want to divide by, then use the same method as for multiplying:

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

Example: $\frac{3y^2}{x+1}\div\frac{y}{2}=\frac{3y^2}{x+1}\times\frac{2}{y}=\frac{(3y^2)(2)}{(x+1)(y)}=\frac{6y^2}{(x+1)(y)}=\frac{6y}{x+1}$

3. Complex Fractions

A complex fraction is a fraction where the numerator, denominator, or both contain a fraction.

Example 1: $\frac{3}{1/2}$ is a complex fraction. The numerator is 3 and the denominator is 1/2.

Example 2: $\frac{3/7}{9}$ is a complex fraction. The numerator is 3/7 and the denominator is 9.

Example 3: $\frac{3/4}{9/10}$ is a complex fraction. The numerator is 3/4 and the denominator is 9/10.

Rule: To manipulate complex fractions, just convert them to simple fractions.

Example: Convert $\frac{3}{1/2}$ to a simple fraction and reduce.

Answer: $\frac{3}{1/2} = 3 \div \frac{1}{2} = \frac{3}{1} \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = 6$

4. Remember these Identities

1) $(x+a)(x+b) = x^2 + (a+b)x + ab$

2) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

3) $(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$

4) $a^2 - b^2 = (a-b)(a+b)$

5) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

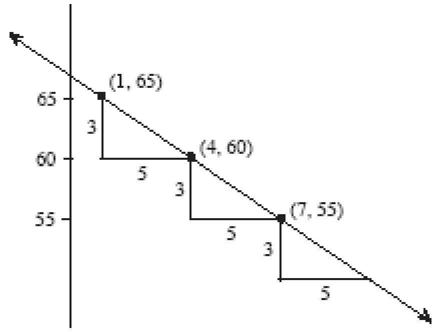
6) $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$ (n is a positive integer)

7) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

8) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

► Questions in class

1. In how many ways is it possible to spend exactly $200 on pumpkins if large pumpkins cost $5 and small $3?



2. Show that the equation $155x - 465y = 35$ has no integer solutions.

3. Find the largest positive integer that leaves the same remainder when divided into each of 889, 961, 1009, and 1189.

4 a) Peter notices that the teenage students who are members of the math club have ages whose product is 611520. How many members does the club have?

b) Is it true that whatever number replaces 611520 that this problem has a unique solution?

c) Suppose the club included 12 year olds and the product was 163762560, how many members does the club now have?

5. a) A simple calculation will reveal that 72 has 12 factors including 1 and 72. How many natural numbers less than 100 have 12 such factors?

b) Find the only natural number less than 1 million with 77 factors.

6 (a) How many of the positive integers from 1 to 100, inclusive, do not contain the digit 7? Explain how you got your answer.

(b) How many of the positive integers from 1 to 2000, inclusive, do not contain the digit 7? Explain how you got your answer.

(c) Determine the sum of all of the positive integers from 1 to 2006, inclusive, that do not contain the digit 7. Explain how you got your answer.