AP® Calculus BC Exam

SECTION I: Multiple Choice **2014**

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

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Form I
Form Code 4JBP6-S

68

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2014 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\displaystyle\int \frac{x^3 + 5}{x^2}\, dx =$

(A) $1 - \dfrac{10}{x^3} + C$

(B) $\dfrac{3x}{4} + \dfrac{15}{x^2} + C$

(C) $\dfrac{x^2}{2} - \dfrac{5}{x} + C$

(D) $\dfrac{x^2}{2} - \dfrac{5}{3x^3} + C$

(E) $-\dfrac{x^3}{4} - 5 + C$

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2. What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$ (E) 4

GO ON TO THE NEXT PAGE.

3. $\lim\limits_{x \to 0} \dfrac{x^2}{1 - \cos x}$ is

(A) -2 (B) 0 (C) 1 (D) 2 (E) nonexistent

Apply l'Hospital's rule.

$$\lim_{x \to 0} \frac{x^2}{1-\cos x} \overset{H}{=} \lim_{x \to 0} \frac{2x}{\sin x} \overset{H}{=} \lim_{x \to 0} \frac{2}{\cos x} = 2$$

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4. $\displaystyle\int \frac{1}{x^2 - 7x + 10}\, dx =$

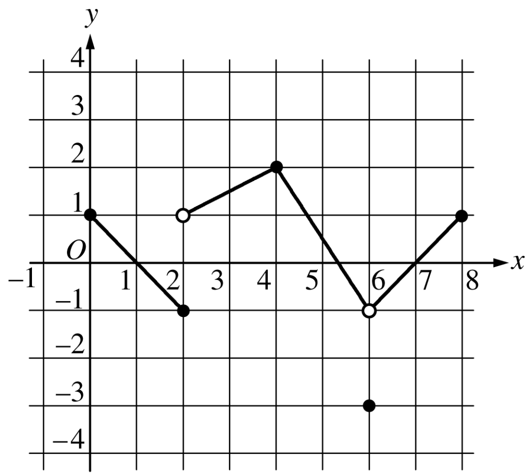
(A) $\ln\left|(x-2)(x-5)\right| + C$

(B) $\frac{1}{3}\ln\left|(x-2)(x-5)\right| + C$

(C) $\frac{1}{3}\ln\left|\dfrac{2x-7}{(x-2)(x-5)}\right| + C$

(D) $\frac{1}{3}\ln\left|\dfrac{x-2}{x-5}\right| + C$

(E) $\frac{1}{3}\ln\left|\dfrac{x-5}{x-2}\right| + C$



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim\limits_{x \to 2^-} f(x) = f(2)$

II. $\lim\limits_{x \to 6^-} f(x) = \lim\limits_{x \to 6^+} f(x)$

III. $\lim\limits_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

6. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

7. Let f be the function defined by $f(x) = \begin{cases} x^2 + 2 & \text{for } x \le 3, \\ 6x + k & \text{for } x > 3. \end{cases}$

If f is continuous at $x = 3,$ what is the value of k ?

(A) -7 (B) 2 (C) 3 (D) 7 (E) There is no such value of k.

---

8. $\int_0^1 x\sqrt{1 + 8x^2}\, dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

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10. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e+2}$ (D) $\frac{e}{e+2}$ (E) The series diverges.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{e^n \cdot e^1} = \sum_{n=1}^{\infty} \frac{1}{e}\left(\frac{-2}{e}\right)^n = \sum_{n=1}^{\infty} \frac{1}{e}\left(\frac{-2}{e}\right)\left(\frac{-2}{e}\right)^{n-1}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad a = \frac{1}{e}\left(\frac{-2}{e}\right) \qquad r = \left(\frac{-2}{e}\right)$$

$$= \frac{\frac{1}{e}\left(-\frac{2}{e}\right)}{1-\left(-\frac{2}{e}\right)} = \frac{-\frac{2}{e^2}}{1+\frac{2}{e}} = -\frac{2}{e^2+2e}$$

$$f(x) = \begin{cases} 2x + 5 & \text{for } x < -1 \\ -x^2 + 6 & \text{for } x \geq -1 \end{cases}$$

11. If f is the function defined above, then $f'(-1)$ is

(A) -2 (B) 2 (C) 3 (D) 5 (E) nonexistent

For $x < -1$, $\lim_{x \to -1^-} 2x+5 = 3$

For $x \geq -1$, $\lim_{x \to 1^+} -x^2+6 = 5$

$\Rightarrow \lim_{x \to 1} f(x)$ DNE

$\Rightarrow f'(-1)$ DNE.

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12. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

GO ON TO THE NEXT PAGE.

13. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

(A) $\frac{9}{2}\text{ m}^2$ (B) $\frac{81}{4}\text{ m}^2$ (C) 27 m^2 (D) 40 m^2 (E) $\frac{81}{2}\text{ m}^2$

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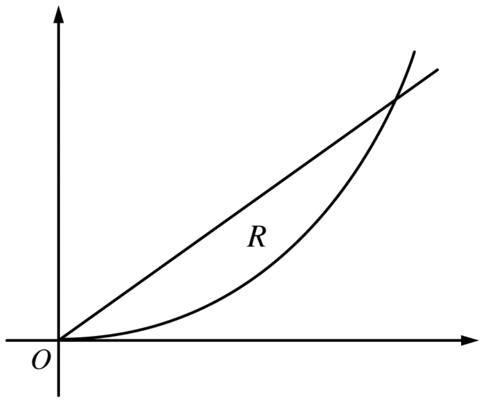
14. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

(A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 (E) 144

15. Suppose $\ln x - \ln y = y - 4$, where y is a differentiable function of x and $y = 4$ when $x = 4$. What is the value of $\frac{dy}{dx}$ when $x = 4$?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{17}{5}$



16. Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k ?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

$A = \int_a^b \frac{1}{2} [r(\theta)]^2 \, d\theta$

$r = \theta \rightarrow \theta = k.$

$0 \leq \theta \leq k$

$r(\theta) = \theta$

$\Rightarrow A = \int_0^k \frac{1}{2} \theta^2 \, d\theta = \frac{1}{2} \int_0^k \theta^2 \, d\theta$

$= \frac{1}{2} \left[\frac{1}{3}\theta^3\right]_0^k = \frac{1}{6} k^3 = \frac{k^3}{6}$

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17. Which of the following is the Maclaurin series for e^{3x} ?

(A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(B) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$

(C) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$

(D) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$

(E) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$

18. $\displaystyle\int_1^{\infty} \frac{x^2}{\left(x^3+2\right)^2}\,dx$ is

let $u = x^3 + 2 \Rightarrow du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx \qquad u(1) = 3 \qquad u(\infty) = \infty$

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 1 (E) divergent

$$\Rightarrow \int_3^{\infty} \frac{1}{3}\frac{1}{u^2}\,du = \frac{1}{3}\lim_{t\to\infty}\int_3^{t}\frac{1}{u^2}\,du = \frac{1}{3}\lim_{t\to\infty}\left[-\frac{1}{u}\right]_3^{\infty}$$

$$= \frac{1}{3}\lim_{t\to\infty}\left[-\frac{1}{t}+\frac{1}{3}\right] = \frac{1}{3}\left[0+\frac{1}{3}\right] = \frac{1}{9}$$

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19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

Point of inflection occurs when y'' changes sign.

$y' = 15x^4 + 40x^3$

$y'' = 60x^3 + 120x^2$

$\Rightarrow x^2(60x + 120) = 0 \quad \Rightarrow x = -2$

20. If $f'(x) = \dfrac{(x-2)^3\left(x^2-4\right)}{16}$ and $g(x) = f\left(x^2-1\right)$, what is $g'(2)$?

(A) 2 (B) $\frac{5}{4}$ (C) $\frac{5}{8}$ (D) $\frac{5}{16}$ (E) 0

$$g'(x) = f'(x^2-1)(x^2-1)^2 = \frac{(x^2-1-2)^3(x^2-1)^2-4}{16}(2x)$$

$$= \frac{(1)^3(3)^2-4}{16}(4) = \frac{5}{4}$$

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x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	-1

21. The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6 (B) 34 (C) $2\sqrt{3}+2\sqrt{2}+2$ (D) $2\sqrt{5}+2\sqrt{2}+2$ (E) $2\sqrt{5}+4\sqrt{2}+2$

$$L = \int_1^7 \sqrt{1+(f'(x))^2}\,dx \approx \sum_{i=1}^{3} \sqrt{1+(f'(x_i))^2}\,\Delta x_i$$

$$\approx \frac{7-1}{3}\left(\sqrt{1+2^2}+\sqrt{1+1^2}+\sqrt{1+0^2}\right)$$

$$= 2\left(\sqrt{5}+\sqrt{2}+1\right)$$

GO ON TO THE NEXT PAGE.

22. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$?

(A) $1 < x < 5$

(B) $1 \le x < 5$

(C) $1 \le x \le 5$

(D) $2 < x < 4$

(E) $2 \le x \le 4$

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23. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2} - 2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6 - x^2}$

(E) $y = \frac{6 - x^2}{2}$

$\frac{dy}{dx} = xy^2$

$\Rightarrow \frac{1}{y^2}\, dy = x\, dx \Rightarrow \int \frac{1}{y^2}\, dy = \int x\, dx$

$\Rightarrow \frac{y^{-1}}{-1} = \frac{x^2}{2} + C \Rightarrow -\frac{1}{y} = \frac{x^2}{2} + C$

$\Rightarrow y = -\frac{1}{\frac{x^2}{2} + C} \Rightarrow y = -\frac{2}{x^2 + 2C}$

$\Rightarrow 1 = -\frac{2}{4 + 2C} \Rightarrow 4 + 2C = -2 \Rightarrow C = -3$

$\Rightarrow y = -\frac{2}{x^2 - 6} = \frac{2}{6 - x^2}$

24. Which of the following series converge?

I. $1+(-1)+1+\cdots+(-1)^{n-1}+\cdots$ $\times$

II. $1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}+\cdots$ $\times$ $\longrightarrow$ Harmonic series with odd terms.

III. $1+\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^{n-1}}+\cdots$

$\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=1}^{\infty} 1\left(\frac{1}{3}\right)^{n-1}$ Geometric Series with $r<1$.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

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25. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$\frac{dr}{d\theta} = -\sin\theta$$

$$= \frac{(-\sin\theta)\sin\theta + (\cos\theta)\cos\theta}{(-\sin\theta)\cos\theta - (\cos\theta)(\sin\theta)}$$

$$= \frac{-\sin^2\theta + \cos^2\theta}{-2\sin\theta\cos\theta}$$

GO ON TO THE NEXT PAGE.

26. For $x > 0$, $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt =$

(A) $\frac{1}{2\sqrt{x}\,(1+x)}$ (B) $\frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)}$ (C) $\frac{1}{1+x}$ (D) $\frac{\sqrt{x}}{1+x}$ (E) $\frac{1}{1+\sqrt{x}}$

$$(\tan^{-1}x)' = \frac{1}{1+x^2}$$

$$\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt = [\tan^{-1}t]_1^{\sqrt{x}}$$

$$= [\tan^{-1}\sqrt{x} - \tan^{-1}1] = \tan^{-1}\sqrt{x} - \frac{\pi}{4}$$

$$\frac{d}{dx}\left(\tan^{-1}\sqrt{x} - \frac{\pi}{4}\right) = \frac{1}{1+\sqrt{x}^2}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{z}\,(1+x)}$$

$\boxed{A}$

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27. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

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28. The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $\left(h^{-1}\right)'(2)$?

(A) $\frac{1}{83}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1 (E) 8

$f^{-1} \neq \frac{1}{f}$

Reflection property of inverse funⁿ.

$f(1) = 2 \Rightarrow f^{-1}(2) = 1.$

$h(h^{-1}(x)) = x$

$h'(h^{-1}(x)) = h'(h^{-1}(x)) \cdot (h^{-1})' x = 1$

$\Rightarrow (h^{-1})'(x) = \dfrac{1}{h'(h^{-1}(x))}$

$\Rightarrow (h^{-1})'(2) = \dfrac{1}{h'(h^{-1}(2))} = \dfrac{1}{h'(1)}$

$h'(x) = 5x^4 + 3$

$\Rightarrow h'(1) = 8 \Rightarrow (h^{-1})'(2) = \dfrac{1}{8}$

GO ON TO THE NEXT PAGE.

CALCULUS BC

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

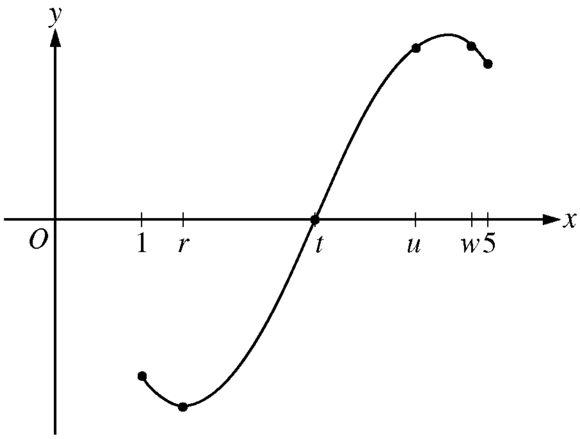
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

76. The figure above shows the graph of the differentiable function f for $1 \le x \le 5$. Which of the following could be the x-coordinate of a point at which the line tangent to the graph of f is parallel to the secant line through the points $(1, f(1))$ and $(5, f(5))$?

(A) r (B) t (C) u (D) w (E) There is no such point.

GO ON TO THE NEXT PAGE.

77. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025(5000 - y)$

(E) $\frac{dy}{dt} = 0.025y(5000 - y)$

78. What is the area of the region enclosed by the graphs of $y = \dfrac{1}{1 + x^2}$ and $y = x^2 - \dfrac{1}{3}$?

(A) 0.786 (B) 0.791 (C) 1.582 (D) 1.837 (E) 1.862

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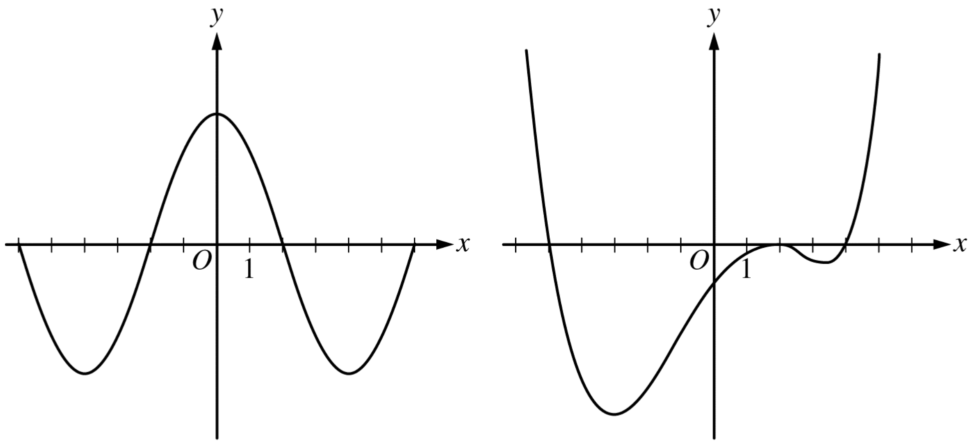
79. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115



Graph of f Graph of g

80. The graphs of two differentiable functions f and g are shown above. Given $p(x) = f(x)g(x)$, which of the following statements about $p'(-2)$ is true?

(A) $p'(-2) < 0$

(B) $p'(-2) = 0$

(C) $p'(-2) > 0$

(D) $p'(-2)$ is undefined.

(E) There is not enough information given to conclude anything about $p'(-2)$.

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81. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

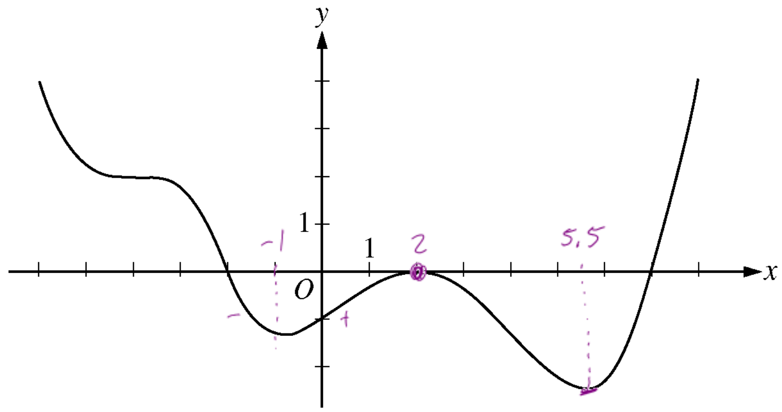
(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628

$\frac{dP}{dt} = kP \Rightarrow \frac{dP}{dt} = 2000e^{0.23t}$ $P(t) = ?$

$\Rightarrow \int dP = 2000 \int e^{0.23t}\, dt \Rightarrow P(t) = 2000 \cdot \frac{1}{0.23} \cdot e^{0.23t} + C.$

$\Rightarrow P(0) = 1500 = \frac{2000}{0.23} e^{0.23(0)} + C \Rightarrow C = -7195.$

$\Rightarrow P(3) = 10141$



Graph of f'

82. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

$x = -1, 2, 5.5$

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

83. The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

(A) 2.35

(B) 3.65

(C) 4.35

(D) 4.70

(E) 4.80

84. A sphere is expanding in such a way that the area of any circular cross section through the sphere's center is increasing at a constant rate of $2\,\text{cm}^2/\text{sec}$. At the instant when the radius of the sphere is 4 centimeters, what is the rate of change of the sphere's volume? (The volume V of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $8\,\text{cm}^3/\text{sec}$

(B) $16\,\text{cm}^3/\text{sec}$

(C) $8\pi\,\text{cm}^3/\text{sec}$

(D) $64\pi\,\text{cm}^3/\text{sec}$

(E) $128\pi\,\text{cm}^3/\text{sec}$

Let A be the area of the circular cross section.

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2$

Since $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 2r\,(\underbrace{2\pi r \frac{dr}{dt}}_{2})$

when $r = 4$,

$\frac{dV}{dt} = (2)(4)(2) = 16$

GO ON TO THE NEXT PAGE.

85. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane are given by the parametric equations $x'(t) = \frac{1}{t+1}$ and $y'(t) = ke^{kt}$, where k is a positive constant. The line $y = 4x + 3$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k ?

(A) 0.072 (B) 0.433 (C) 0.495 (D) 0.803 (E) 0.828

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

86. The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

(A) 37 (B) 49 (C) 58 (D) 65 (E) 92

GO ON TO THE NEXT PAGE.

87. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

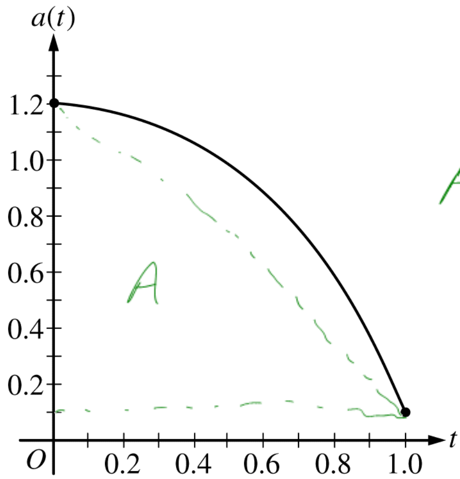
(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.



$A = \frac{1}{2}(1.1) = 0.55 > \frac{1}{2}$

A is change in velocity.

88. A particle moves along the x-axis so that its acceleration $a(t)$ is given by the graph above for all values of t where $0 \le t \le 1$. At time $t = 0$, the velocity of the particle is $-\frac{1}{2}$. Which of the following statements must be true?

(A) The particle passes through $x = 0$ for some t between $t = 0$ and $t = 1$.

(B) The velocity of the particle is 0 for some t between $t = 0$ and $t = 1$.

(C) The velocity of the particle is negative for all values of t between $t = 0$ and $t = 1$.

(D) The velocity of the particle is positive for all values of t between $t = 0$ and $t = 1$.

(E) The velocity of the particle is less than $-\frac{1}{2}$ for all values of t between $t = 0$ and $t = 1$.

89. The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2}\, dt$. What is the average rate of change of f over the interval $[0, 3]$?

(A) 1.324 (B) 1.497 (C) 1.696 (D) 2.266 (E) 2.694

$$\text{Avg} = \frac{f(3) - f(0)}{3 - 0} = \frac{\int_1^3 \sqrt{t^3+2}\,dt - \int_1^0 \sqrt{t^3+2}\,dt}{3} = \frac{1}{3}\int_0^3 \sqrt{t^3+2}\,dt$$

$\approx 2.6939.$

---

90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

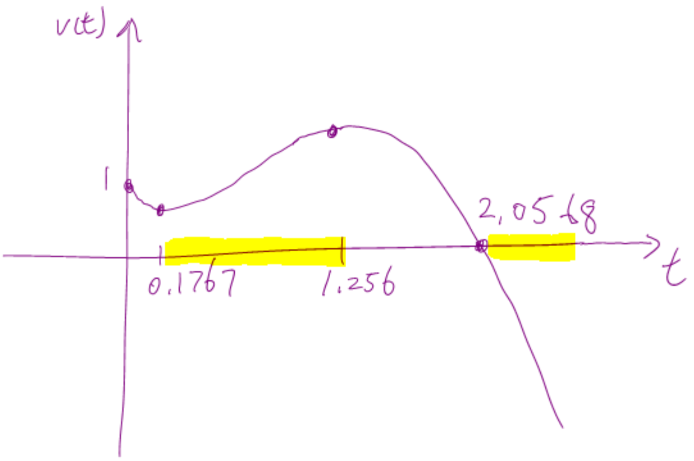
(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

Use a graphing calculator to graph $v(t)$.



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91. Line ℓ is tangent to the graph of $y = \cos x$ at the point $(k, \cos k)$, where $0 < k < \pi$. For what value of k does line ℓ pass through the origin?

(A) 0.860

(B) 1.571

(C) 2.356

(D) 2.798

(E) There is no such value of k.

---

x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

92. The table above gives selected values of twice-differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?

(A) 63 (B) 69 (C) 78 (D) 84 (E) 103

$\int_2^4 f(x)g''(x)dx = [f(x)g'(x)]_2^4 - \int_2^4 f'(x)g'(x)\,dx$

$f = f(x) \quad g' = g''(x)$

$f' = f'(x) \quad g = g'(x)$

$= [f(x)g'(x)]_2^4 - \int_2^4 3g'(x)\,dx = [f(4)g'(4) - f(2)g'(2)] - 3[g(x)]_2^4$

$= [13\cdot7 - 7\cdot1] - 3[9-2] = [91-7] - [21]$

$= [84-21] = 63$

GO ON TO THE NEXT PAGE.

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

Section II: Free-Response Questions

This is the free-response section of the 2014 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2014

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

   First letter of your first name

2. Date of birth

   Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark “No” with no effect on my score or its reporting.

   No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4JBP6-S

66

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

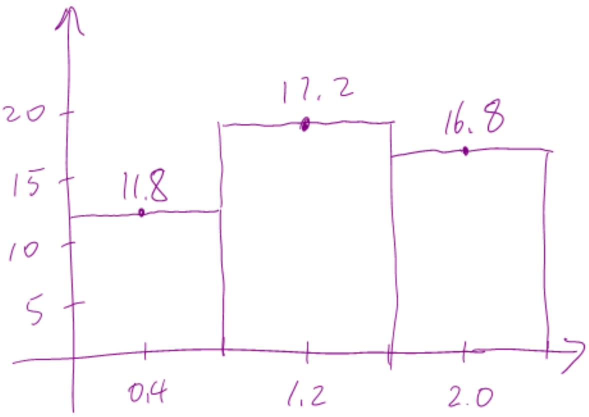
1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

$$a = \frac{v}{t} = \frac{v_f - v_i}{t_f - t_i} = \frac{v(1.6) - v(1.2)}{1.6 - 1.2} = \frac{16.3 - 17.2}{0.4}$$

$$\approx -2.25 \text{ miles}/h^2$$

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.



$$\int_0^{2.4} v(t)dt$$

$$= (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8)$$

$$= 36.64 \text{ miles}$$

$s(t) = \int_0^{2.4} v(t)\,dt$ is the total distance travelled in miles from $t = 0$ to 2.4 hours

Continue problem 1 on page 5.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

$$g_{avg} = \frac{1}{2.4-0}\int_0^{2.4} \frac{24t - 5\sin(6t)}{t+0.7}\,dt.$$

$$\Rightarrow g(t)_{avg} \approx 14.064 \text{ miles.}$$

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

$v(t) = g(1.3) = 18.0963 > 0$

$a(t) = g'(1.3) = 3.7612 > 0$

$\Rightarrow$ the speed is increasing.

Since both the $v(t)$ and $a(t)$ are positive, the speed is increasing.

speed	$v(t)$	$a(t)$
Inc	+	+
Inc	—	—
Dec	+	—
Dec	—	+

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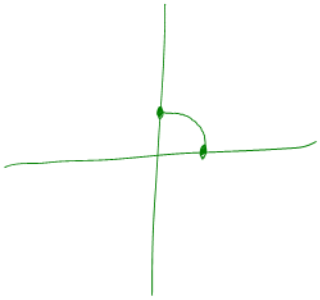

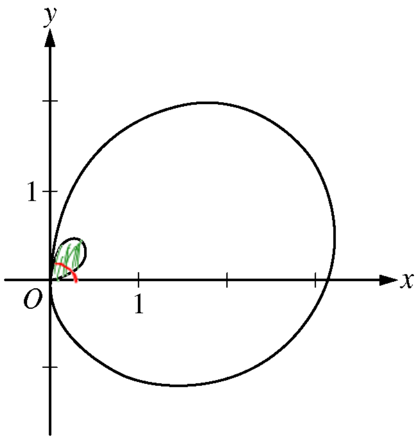

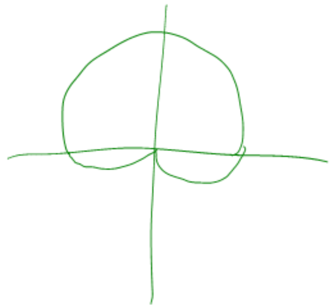

2. Consider the polar curve defined by the function $r(\theta) = \theta\cos\theta,$ where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.

(a) Find the area of the region enclosed by the inner loop of the curve.

$$A = \frac{1}{2}\int_a^b [r(\theta)]^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\theta\cos\theta)^2 \, d\theta = 0.127$$

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Continue problem 2 on page 7.

(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

$r(\theta) = \theta \cos\theta$

$\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta = 0.$

$\Rightarrow \theta = 0.860$ or $\theta = 3.426.$

$\Rightarrow r(0.860) = 0.561$

$\boxed{r(3.426) = -3.288}$

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

$\frac{dy}{dx} = \frac{2}{2-\pi}$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{1}{2}}{\frac{dx}{d\theta}} = \frac{2}{2-\pi}$

$\Rightarrow \frac{dx}{d\theta} = \frac{\frac{1}{2}}{\frac{2}{2-\pi}} = \frac{2-\pi}{4}$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

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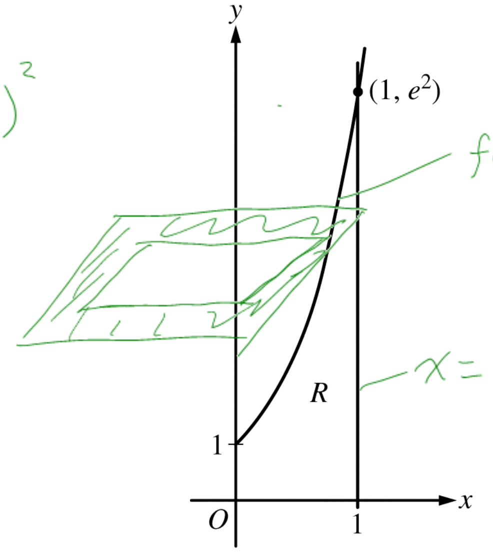
NO CALCULATOR ALLOWED

$A_c = \pi(R - r)^2$

$A_s = (S - s)^2$



$x = \frac{1}{2}\ln y$

$f(x) = e^{2x}$

$(x=1) - (x = \frac{1}{2}\ln y)$

$x = 1$

3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$.

$f(x) = e^{2x}$ $\quad$ $f'(x) = 2e^{2x}$

$f'(1) = 2e^2$

$y - y_0 = m(x - x_0)$ $\quad$ $(x_0, y_0) = (1, e^2)$

$\Rightarrow y - e^2 = 2e^2(x-1)$

$\Rightarrow y = 2e^2(x-1) + e^2$

Continue problem 3 on page 15.

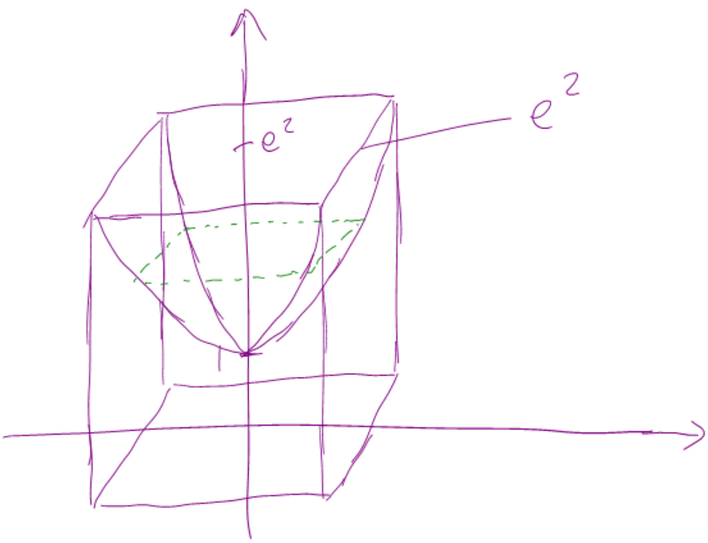
(b) Find the area of R.

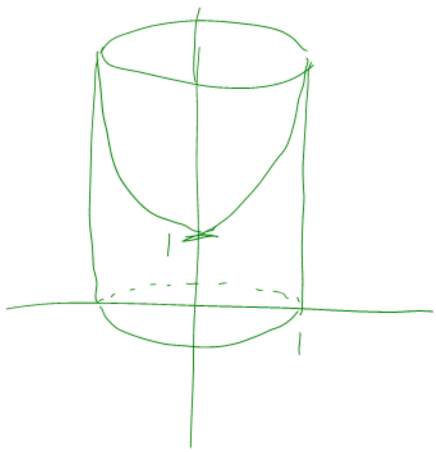
$$A = \int_0^1 f(x)\,dx$$

$$= \int_0^1 e^{2x}\,dx = \left[\tfrac{1}{2}e^{2x}\right]_0^1$$

$$= \frac{1}{2}(e^2 - 1)$$

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

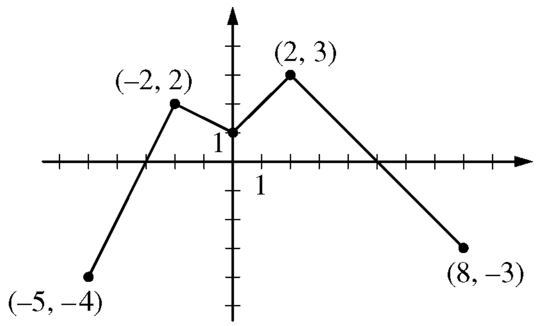




$$y = e^{2x} \Rightarrow \ln y = 2x$$

$$\Rightarrow x = \frac{1}{2}\ln y$$

$$V = 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$$



Graph of f

4. The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

$$g(0) = 2\cdot 0 + \int_{-2}^{0} f(t)\,dt$$

$$= 0 + 3 = 3$$

$$g(-5) = 2\cdot(-5) + \int_{-2}^{-5} f(t)\,dt$$

$$= -10 - \int_{-5}^{-2} f(t)\,dt = -10 - (-4 + 1) = -7$$

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

$$g'(x) = 2 + f(x)$$

$$g''(x) = f'(x)$$

$$g''(4) = f'(4) = -1$$

$$g''(-2) = f'(-2) \Rightarrow \text{DNE.}$$ (Derivate at a corner).

Continue problem 4 on page 17.

NO CALCULATOR ALLOWED

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

When a funⁿ is concave down, its avg rate of change is decreasing.

$\Rightarrow$ check $g'(x) = 2 + f(x)$ for decreasing intervals.

Observe that $f(x)$ is decreasing on $(-2, 0)$ and $(2, 8]$

$\Rightarrow$ g is concave down on $(-2, 0)$ and $(2, 8)$.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

$h(x) = g(x^3+1)$

$h'(x) = g'(x^3+1)(3x^2)$ (chain rule).

$h'(1) = g'(2)\,3$

$= (2 + f(2)) \cdot 3$

$= (2+3) \cdot 3 = 15$

5. A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \le t \le 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \le t \le 4$, find $x(t)$.

$x(t) = -6 + \int_1^t v(u)\,du$

$= -6 + \int_1^t 3u^2 - 12\,du$

$= -6 + [u^3 - 12u]_1^t$

$= t^3 - 12t + 5.$

(b) Find the total distance traveled by the train during the time interval $0 \le t \le 4$.

Distance $= \left|\int_a^b v(t)\,dt\right|$.

Find the time when the train changed direction.

$v(t) = 3t^2 - 12t = 3(t^2 - 4) = 0.$

$\Rightarrow$ ~~$t = -2$~~ or $t = 2$. (time can't be negative).

Distance $= \left|\int_0^2 v(t)\,dt\right| + \left|\int_2^4 v(t)\,dt\right|.$

$= |x(2) - x(0)| + |x(4) - x(2)|.$

$= 16 + 32 = 48.$

Continue problem 5 on page 19.

(c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \le t \le 4$.

Parametric equ'n's.

$x(t) = t^3 - 12t + 5$

$y(t) = 2 + 12\sin(\frac{t}{4})$.

Total distance = arc length.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

$\frac{dx}{dt} = 3t^2 - 12$ $\qquad$ $\frac{dy}{dt} = 3\cos(\frac{t}{4})$

$$\Rightarrow L = \int_0^4 \sqrt{(3t^2-12)^2 + (3\cos(\frac{t}{4}))^2}\, dt.$$

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6. Let $a_n = \frac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \frac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive. Use the integral test to show that $\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3 \ln 3} - \frac{1}{4 \ln 4} + \frac{1}{5 \ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\frac{1}{3}$.

Continue problem 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

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GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**