Unit: Relationships between points, lines and planes (2)

Distance from a Point to a Line (2D & 3D)

Distance from a point to a line or distance between l_1 // l_2 (in 2-Space)

The distance from a point $Q(x_1, y_1)$ to a line: $Ax + By + C = 0$ is given by

$$D = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Ex 1: Distance from a point to a line in plane

Find the distance from the point $Q(4, -3)$ to the line : $5x - 2y + 7 = 0$.

Ex 2: Distance between two parallel lines in plane

Show $l_1 : 2x - y + 6 = 0$ and $l_2 : 4x - 2y - 5 = 0$ are parallel and then find the distance between l_2 and l_2.

Ex 3: Distance from a point to a line or distance between 2 parallel lines in 3-space

a) Prove that the distance from a point Q in **space** to a line through a point P with direction vector $\vec{d}$ is equal to $\dfrac{\left|\vec{d}\times\overrightarrow{PQ}\right|}{\left|\vec{d}\right|}$.

b) Find the distance from the point Q(1, -2, -3) to the line $\vec{r} = (3, 1, 0) + t(1, 1, 2)$.

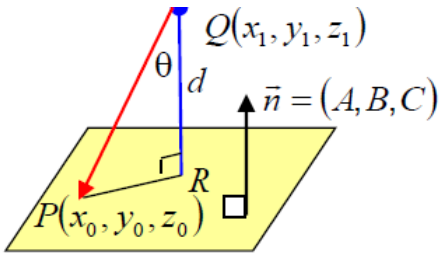
c) Find the distance between the parallel lines l_1: $\vec{r} = (-2, 2, 1) + t(7, 3, -4)$ & l_2: $\vec{r} = (2, -1, -2) + u(7, 3, -4)$.

Distance from a Point to a Plane

Distance from a point to a plane OR distance between 2 parallel planes (3-Space)

Distance from a point $Q(x_1, y_1, z_1)$ to a plane $Ax + By + Cz + D = 0$ is:

$$d = \frac{\left|Ax_1 + By_1 + Cz_1 + D\right|}{\sqrt{A^2 + B^2 + C^2}}$$



Ex 4: Distance from a point to a Plane

Find the distance from the point $Q(4, -3, 2)$ to the plane π: $5x - 2y + 4z - 6 = 0$.

Ex 5: Distance between two parallel Planes

a) prove that $\pi_1 \,//\, \pi_2$,

π_1: $2x - y + 3z - 6 = 0$

π_2: $4x - 2y + 6z + 9 = 0$

b) Find the distance between π_1 and π_2

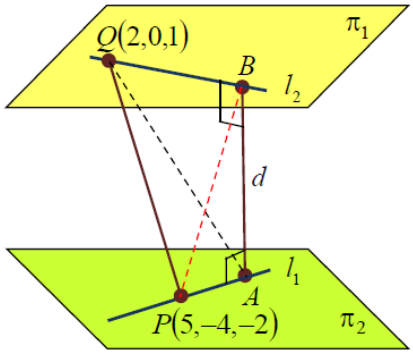
Ex 6: Distance between two skew lines

Determine the distance between the skew lines

$l_1 : (x, y, z) = (5,-4,-2) + s(1,2,3)$

$l_2 : (x, y, z) = (2,0,1) + t(2,-1,1)$

Recall: Skew lines are lines in two different dimensional planes and therefore the lines never intersect.



Ex 7. Find the distance from the point $Q(4,-1,1)$ to the line

$$l: \begin{cases} x = 1 + 2t \\ y = 3 - t \\ z = -1 + t \end{cases}, t \in R$$