First Name:____________________ Last Name: __________________ Student ID: ________

Trigonometric Functions (2)

Graphing Reciprocal Functions

$\frac{1}{\text{SinX}} =$ $\frac{1}{\text{CosX}} =$ $\frac{1}{\text{TanX}} =$

RECALL:

General steps for sketching **f(x) =** $\frac{\mathbf{1}}{\mathbf{g(x)}}$

1. Sketch the function y=g(x).

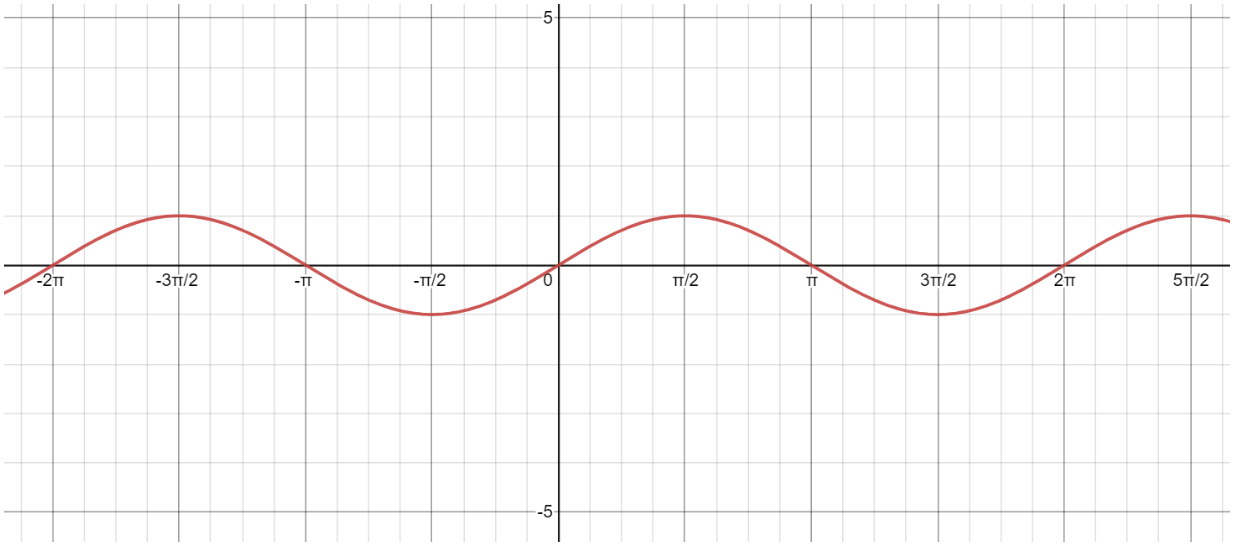
2. Identify the values of x where g(x)= 1 or g(x)= −1. At these points f(x)= g(x). That is, these points are on both f(x) and g(x). These points are called **fixed points** or **static points**.

3. Identify the x-intercepts of g(x). At these points, f(x) is undefined. There will be vertical asymptotes for these values of x.

4. If required, determine what happens to the reciprocal function as x approaches the vertical asymptotes from the left and from the right.

5. If required, determine the end behaviour of f(x).

1. Below is the graph $y = \sin x$. Recalling that $\csc x = \frac{1}{\sin x}$, sketch the graph of $y = \csc x$.



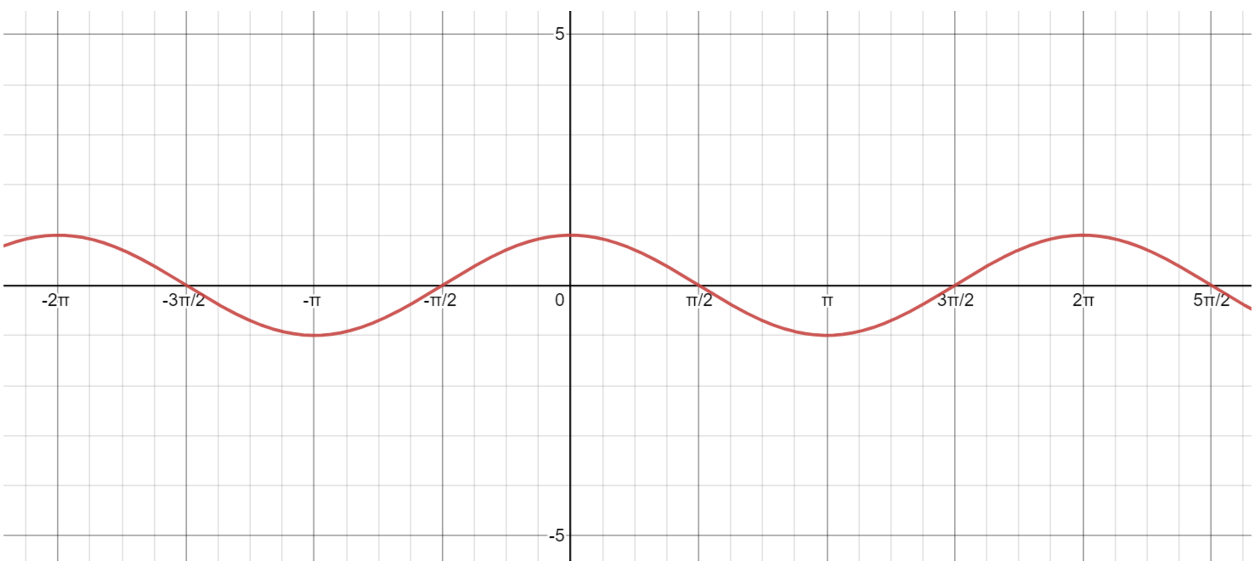
Domain:

Range:

Period:

Equations of Vertical Asymptotes:

2. Below is the graph $y = \cos x$. Recalling that $\sec x = \frac{1}{\cos x}$, sketch the graph of $y = \sec x$.



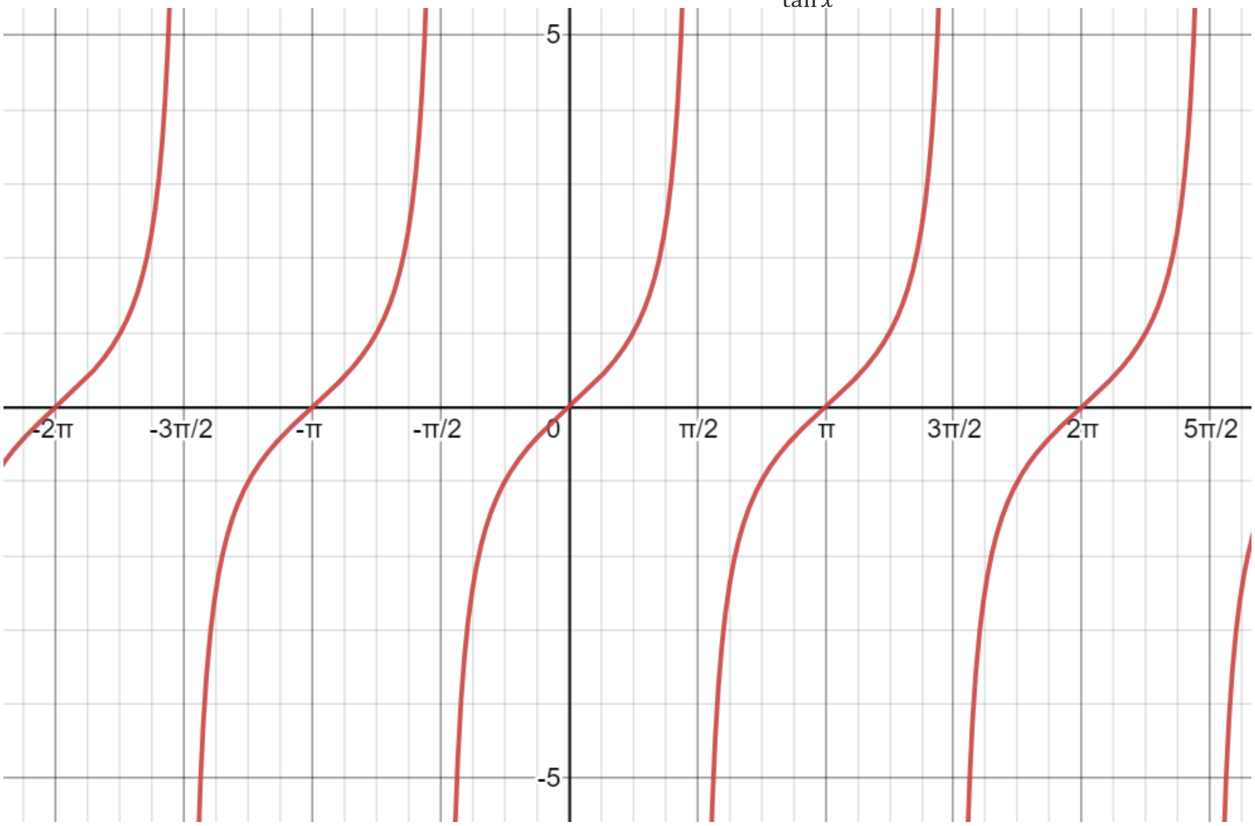
Domain:

Range:

Period:

Equations of Vertical Asymptotes:

3. Below is the graph $y = \tan x$. Recalling that $\cot x = \frac{1}{\tan x}$, sketch the graph of $y = \cot x$.



Domain: Range:

Period: Equations of Vertical Asymptotes:

Example: Sketch the graphs of the following transformations of

a) $y = \sec\left(x + \frac{\pi}{4}\right) - 1$

b) $y = 2\csc\left(x + \frac{\pi}{2}\right)$

NOTE:
The sine function, the cosine function and their transformations are referred to as **sinusoidal functions**. Their graphs have the property that they oscillate above and below a central horizontal axis.
For both $y = \sin(x)$ and $y = \cos(x)$, this central horizontal axis is $y = 0$ (Equation of Axis, **EOA**).

Graphing Trigonometric Functions Using Transformations

$$\boldsymbol{y = f(x) \rightarrow y = af[b(x - h)] + k}$$

Mapping Notation

$$\boldsymbol{(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)}$$

Example: For each of the following equations, state the transformations, period and amplitude of the function.

Equation	Transformations	Period	Amplitude
$y = 3\sin[2(x + \frac{\pi}{4})] - 3$			
$y = \frac{1}{3}\tan[\frac{1}{5}(x - \frac{\pi}{3})] + 10$			
$y = \frac{1}{10}\cos(-3x - \frac{\pi}{2}) - 7$			
$y = -4\sec(\frac{1}{2}x - \pi) + 6$			

Example: Write the equation of **y = cos(x)** if it has undergone the following transformations:

a) Up 6

Left 2π

Vertical Reflection

Vertical Compression of 8

Horizontal Stretch of 6

b) Down 3

Right π

Vertical Stretch of 6

Horizontal Compression of 5

Horizontal Reflection

Example : Graph one cycle of each of the following functions.

a) $y = 3sin[3\,(x - \frac{\pi}{2})] - 1$

b) $y = -\,cos[\frac{1}{2}(x + \frac{5\pi}{6})] + 3$

c) $y = 4sin\left[-\frac{\pi}{3}(x + 1)\right] - 2$

Example : The piston in an engine moves up and down along a crankshaft in the middle. The height of the piston over time is shown by the graph below.

<table>
<tr>
<td>
a) How long does it take for the piston to move up and down once?

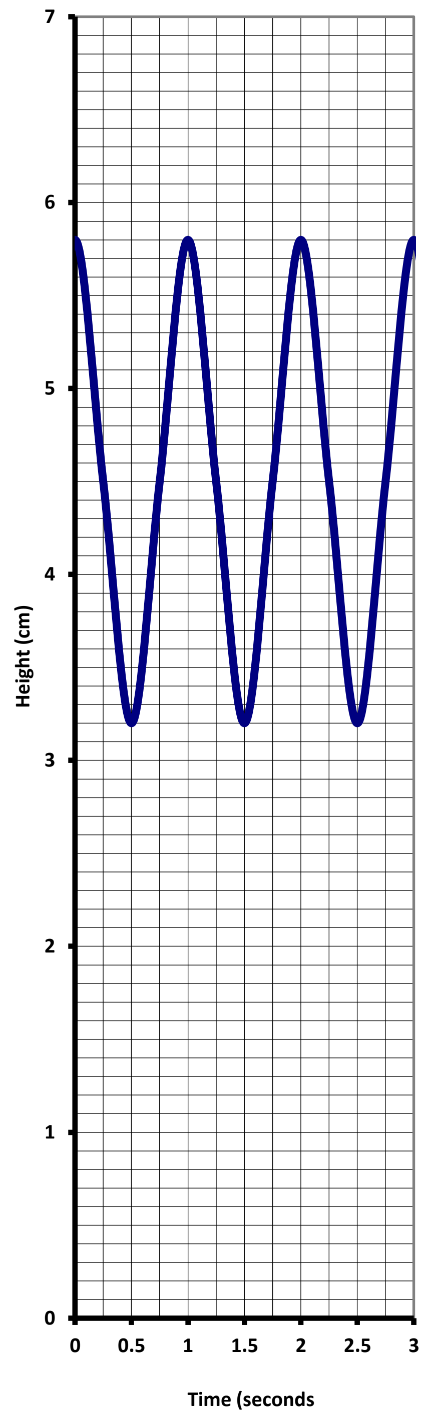
b) What is the maximum height that the piston reaches?

c) What is the lowest height that the piston reaches?

d) How high is the crankshaft?

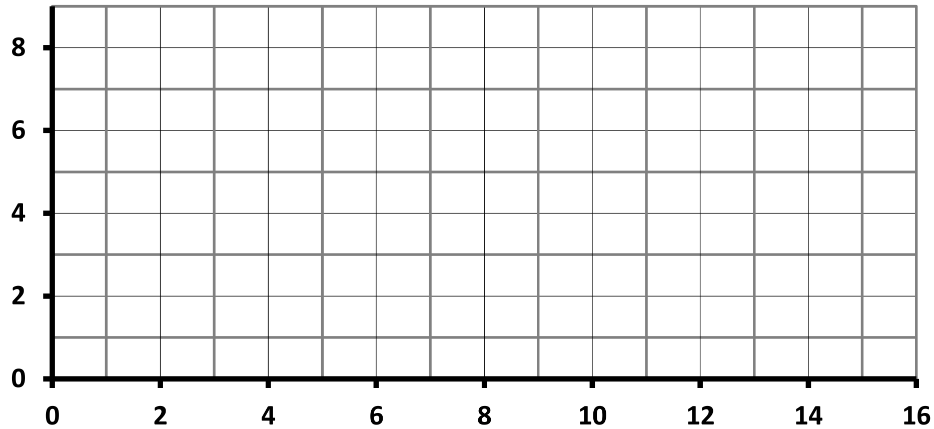
e) Determine the equation of the function.

f) What would you expect to happen to the graph if you revved the engine?

f) What would you expect to happen to the graph if the crankshaft was lower?
</td>
<td>


</td>
</tr>
</table>

8. The water level in ocean harbour is 5 m during low tide and 8 m during high tide. If takes 8 hours to complete one full tide cycle.



a) Sketch two tide cycles starting at high tide.

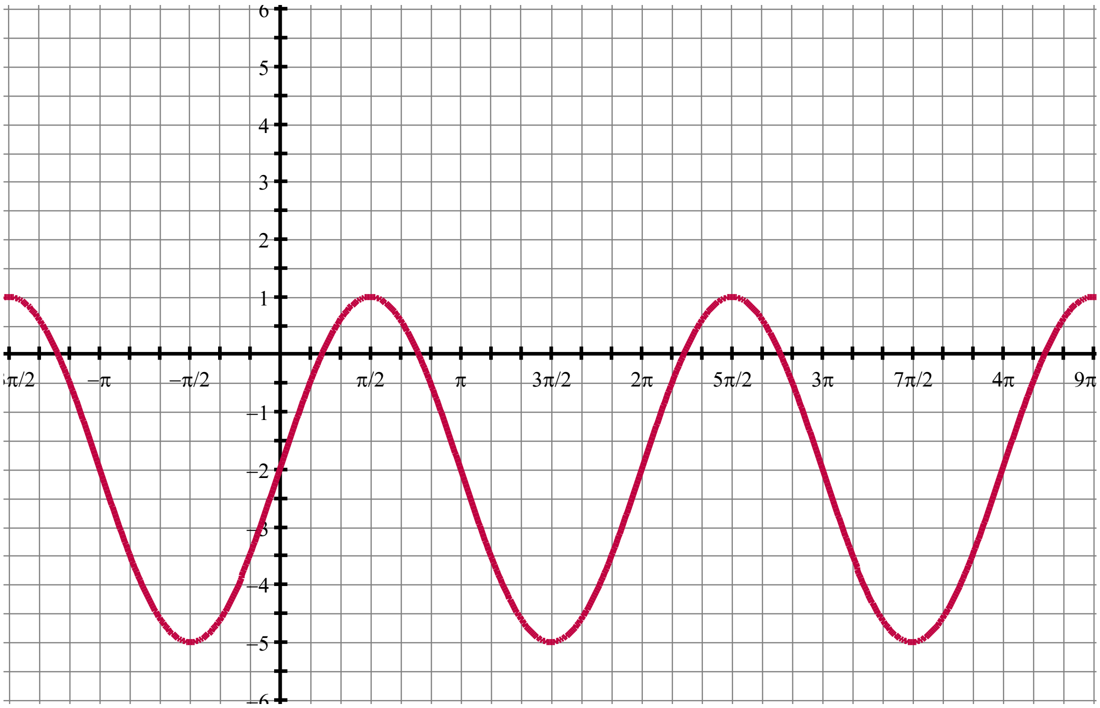
b) Determine an equation for the tide function.

c) If low tide occurs at 8:00 AM, at what time would you expect it to be high tide?

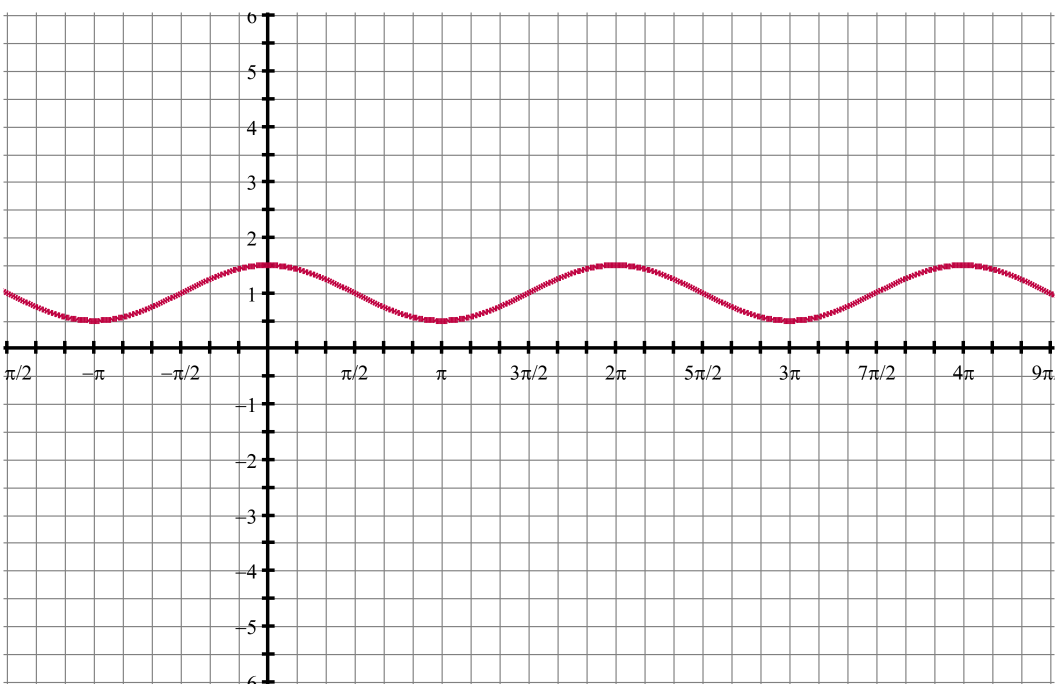
d) If low tide occurs at 8:00 AM, what would you expect the height of the water to be at 6:00 AM the next day?

Example: Find the equation of the following function as both a SINE function and a COSINE function.

a)

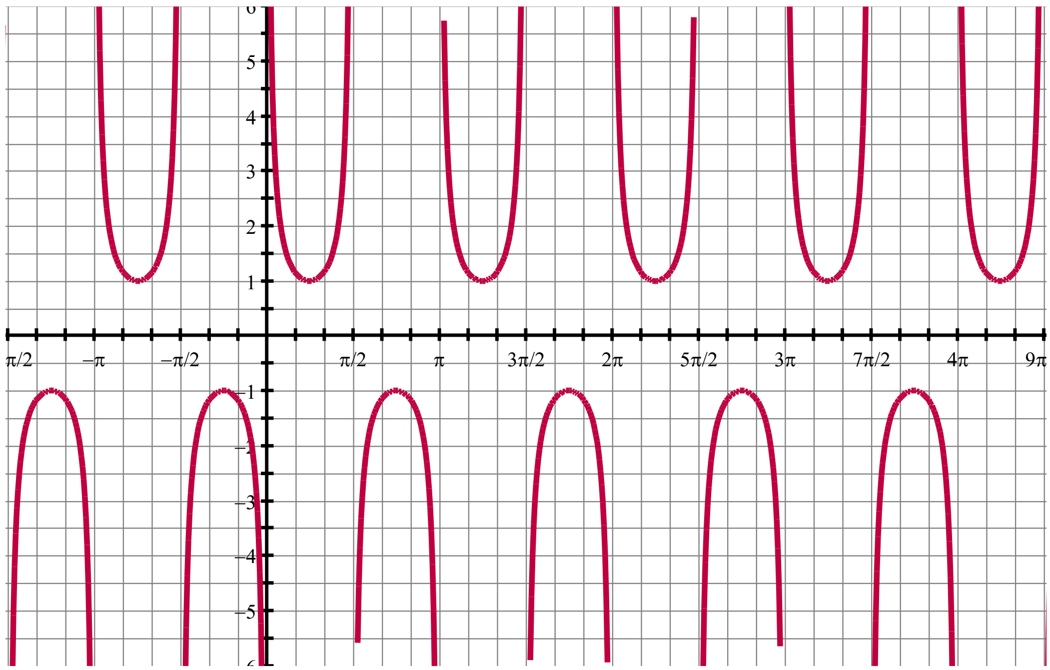


b)



Example: Determine the equation for each of the following graphs:

a)



b)