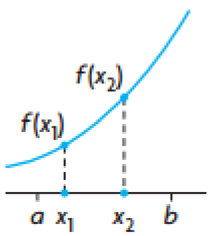
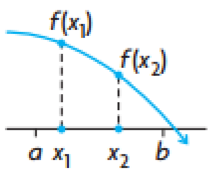
Unit: Derivatives and their applications (1)

Increasing and Decreasing Functions

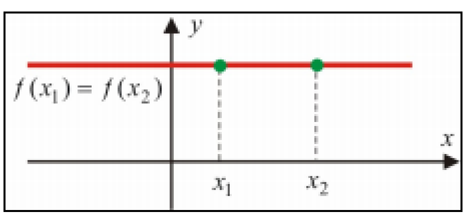
A function f is **increasing** over the interval (a,b) if f (x_1) < f(x_2) whenever $x_1 < x_2$ in the interval (a,b) .



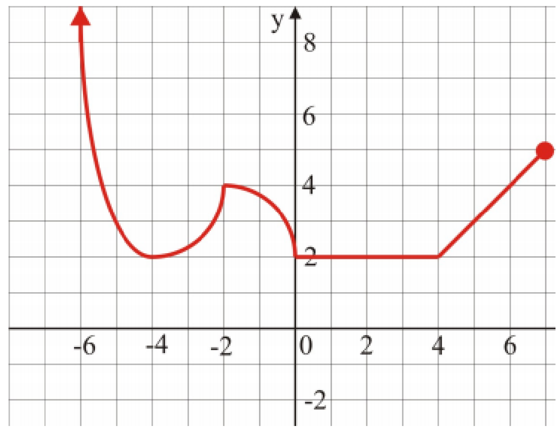
A function f is **decreasing** over the interval (a,b) if f $(x_1) > $ f(x_2) whenever $x_1 < x_2$ in the interval (a,b) .



A function f is ***constant*** over the interval (a,b) if f (x_1) = f(x_2) whenever $x_1 < x_2$ in the interval (a, b).



Ex 1. Find the intervals where the function y = f (x) is increasing, decreasing, or is constant.



Test for Intervals of Increase or Decrease

Let y = f (x) be a differentiable function over (a,b) Then:

1. If **f '(x)** > **0** for all x∈ (a, b) then **f is increasing** over (a, b).

2. If **f '(x)** < **0** for all x∈ (a, b) then **f is decreasing** over (a, b).

3. If **f '(x) = 0** for all x∈(a, b) then **f is constant** over (a, b)

Ex 2. Find the intervals of increase or decrease for

a. f(x) = $2x^3 + 3x^2 - 12x$

b. $g(x) = x^2$

c. $h(x) = x^3$

Critical Points. Local Maxima and Minim

Critical Points (Critical Number)

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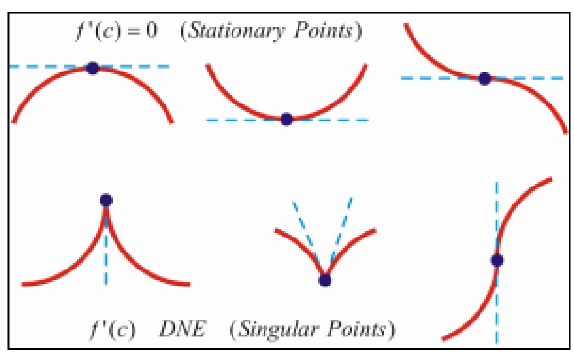
<td>A critical number c is a number in the domain of f where either f'(c) = 0 or f'(c) does not exist. The point (c, f (c)) is called a critical point.

If f '(c) = 0 , the critical point is called stationary point. If f '(c) does not exist, the critical point is called point of non-differentiability.</td>
<td>
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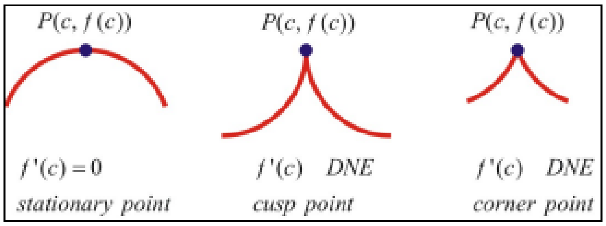
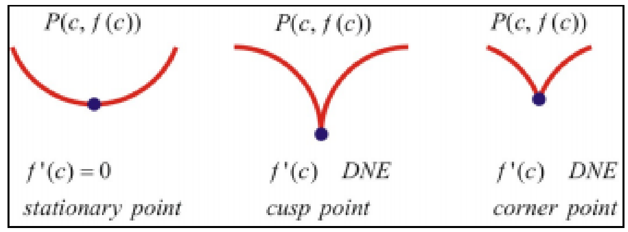
Local Maximum

A function has a local (relative) maximum at x = c if f (x) ≤ f (c) when x is sufficiently close to c(on both sides of c). f(c) is called local (relative) maximum value and (c, f (c)) is called local (relative) maximum point.

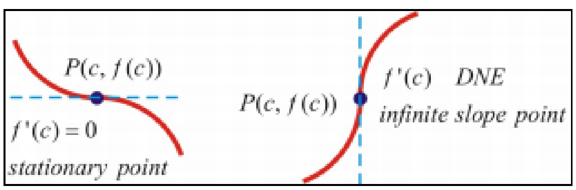


Local Minimum

A function has a local (relative) minimum at f x = c if f (x) ≥ f (c) when x is sufficiently close to con both sides of c). f(c) is called local (relative) minimum value and (c, f (c)) is called local (relative) minimum point.



Note: The following points are neither local minimum or maximum points.



Global Maximum

A function f has a global (absolute) maximum at x = c if f (x) ≤ f (c) for all x∈ D_f .

f (c) is called the global (absolute) maximum value.

(c, f (c)) is called the global (absolute) maximum point

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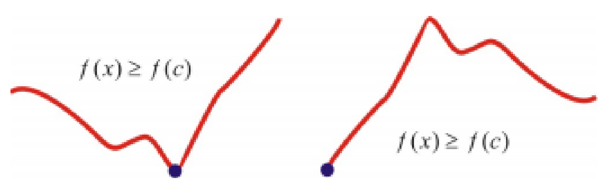
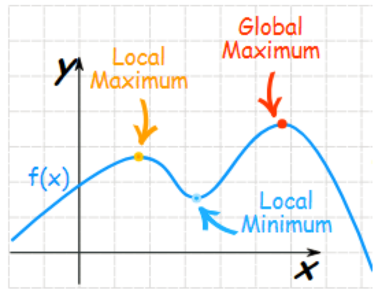
Global Minimum

A function f has a global (absolute) minimum at x = c if f (x) ≥ f (c) for all x∈ D_f.

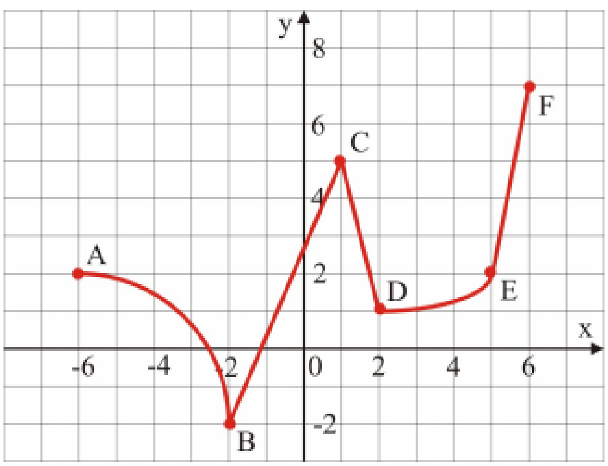
f (c) is called the global (absolute) minimum value.

(c, f (c)) is called the global (absolute) minimum point.



Extremum and Extrema	
An extremum is either a minimum or a maximum (value, point, local or global). Extrema is the plural of extremum.	

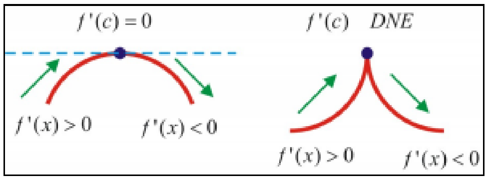
Ex 3. Find extrema for the function represented in the figure below by its graph.



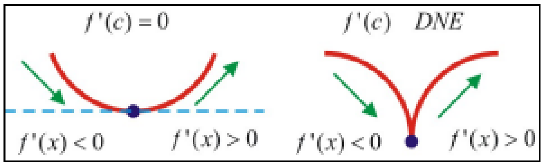
Local (Relative) Extrema - First Derivative Test

Let c be a critical point of a continuous function f.

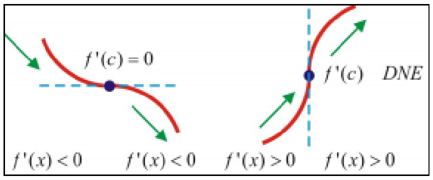
a. If *f'* changes from positive to negative at c, then *f* has a local maximum at c.



b. If *f'* changes from negative to positive at c, then *f* has a local minimum at c.



c. If *f'* does not change sign at c, then has no maximum or minimum at c.



Ex 4. Given $f(x) = x^3 - 6x^2$.

a. Find the critical numbers

b. Find the intervals of increase and decrease

c. Find the local maximum and local minimum points.

Ex 5. Find the local extrema for

$y = \text{f}(x) = \left(\frac{1+x}{1-x}\right)^2$.

Global (Absolute) Extrema Algorithm

Global (Absolute) Extrema Algorithm	**Ex 6.** Find the global extrema for
To find the global (absolute) extrema for a *continuous* function f over a closed interval $[a,b]$: 1) identify the *critical* numbers over (a,b) 2) find the *values* of the function $f(c)$ at each critical number c in (a,b) 3) find the *values* $f(a)$ and $f(b)$ 4) from the values obtained at part 2) and 3): - the *largest* represents the *global (absolute) maximum* value - the *least* represents the *global (absolute) minimum* value Note. c is a critical number if either $f'(c)=0$ or $f'(c)$ *DNE*	f(x) = -2x + 3, for $x \in [-1,2]$

Ex 7. Find a function of the form $f(x) = ax^4 + bx^2 + cx + d$ with a local maximum at $(0, -6)$ and a local minimum at $(1, -8)$.

Ex 8. For each case, use the first derivative sign to find the intervals of increase or decrease.

a. $f(x) = x^3(x-1)^4$

b. $f(x) = \begin{cases} \frac{x}{2} + 2, & x < 1 \\ x^3, & x \geq 1 \end{cases}$