

AP Calculus Homework 2

Please write your answer on a separate piece of paper and submit it on Classkick or write your answer directly on Classkick.

Please write all answers in exact forms. For example, write π instead of 3.14.

Questions with a * are optional. Questions with ** are optional and more challenging.

1. Find the limit

$$\text{a) } \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{x + 1}} \quad \text{b) } ** \lim_{x \rightarrow +\infty} \left(\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right)$$

2. Find an equation of the slant/oblique asymptote.

$$\text{a) } y = \frac{x^2 + 1}{x + 1} \quad \text{b) } * y = \frac{4x^3 - 2x^2 + 5}{2x^2 + x - 3}$$

3. Sketch the graph of a function that has a jump discontinuity at $x = 2$ and a removable discontinuity at $x = 4$, but is continuous elsewhere.

4. Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a .

$$f(x) = x^2 + \sqrt{7 - x}, \quad a = 4$$

5. Explain why the function is discontinuous at the given number a . Sketch the graph of the function.

$$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \quad a = 0$$

6. Use continuity to evaluate the limit.

$$\text{a) } \lim_{x \rightarrow 4} \frac{5 + \sqrt{x}}{\sqrt{5 + x}} \quad \text{b) } * \lim_{x \rightarrow 1} e^{x^2 - x}$$

7.* Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f .

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x - 3} & \text{if } x \geq 3 \end{cases}$$

8. Which of the following functions f has a removable discontinuity at a ? If the discontinuity is removable, find a function g that agrees with f for $x \neq a$ and is continuous at a .

a) $f(x) = \frac{x^4 - 1}{x - 1}, \quad a = 1$

b)* $f(x) = \frac{x^3 - x^2 - 2x}{x - 2}, \quad a = 2$

9. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$\sqrt[3]{x} = 1 - x, \quad (0, 1)$