AP Calculus Final Review 5 – Differential Equations

7.4 Solving First-order Differential Equations; 7.5 Exponential Growth and Decay

1. What is the family of geometric figures represented by the general solution of the differential equation $y\,dy = x\,dx$?

2. What is the family of geometric curves represented by the general solution of the differential equation $\dfrac{dy}{dx} = y$?

3. Find a function that satisfies the equations $f(x)f'(x) = x$ and $f(0) = 1$.

4. What is the equation of the curve that passes through the point (1, 1) and whose slope at any point (x, y) is equal to $3y/x$?

5. If $\frac{dy}{dx} = \frac{k}{x}$, k is a constant, and if $y = 2$ when $x = 1$ and $y = 4$ when $x = e$, then, what is the value of y when $x = 2$?

6. If $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$ and $s = 1$ when $t = 0$, then, when $s = 3/2$, find the value of t.

7. If $(g'(x))^2 = g(x)$ for real x and $g(0) = 0$, $g(4) = 4$, find g(1).

8. Find the solution curve of $y' = y$ that passes through point (2, 3).

9. If radium decomposes at a rate proportional to the amount present, then find the amount R left after t years, if R_0 is the present amount and c is the negative constant of proportionality.

10. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40 g to 10 g in two hours, find the constant of proportionality.

11. According to Newton's law of cooling, the temperature of an object deceases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32ºC arrives at a mortuary where the temperature is kept at 10ºC. Determine the differential equation satisfied by the temperature T of the corpse t hours later.

12. If the corpse in Question 11 cools to 27ºC in one hour, determine a function for its temperature (in ºC) at time t.

13. The concentration of a medication injected into the bloodstream drops at a rate proportional to the existing concentration. If the factor of proportionality is 30% per hour, in how many hours will the concentration be one-tenth of the initial concentration?

Write a logistic growth equation and find the population after 55 years for a group of ducks with an initial population of P = 1,500, and a carrying capacity of K = 16,000. The duck population after 22 years is 2,000.

The carrying capacity of an environment is the maximum population size of a biological species that can be sustained in that specific environment, given the food, habitat, water, and other resources available. If letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

where the constant r defines the growth rate and K is the carrying capacity.