First Name:____________________ Last Name: __________________ Student ID: ________

Trigonometric Functions (1)

Measuring Angles

There are more ways of measuring angles: ___________________ and _____________________.

An angle θ, when measured in radian, is the _____________ of the arc length subtended by θ as a central angle of a circle to the radius of the circle.

$360^o =$	$90^o =$	$45^o =$	$120^o =$
$180^o =$	$60^o =$	$30^o =$	$15^o =$

Convert each degree measure into radians and each radian measure into degrees.

1) $-\frac{4\pi}{3}$

2) $55°$

3) $135°$

4) $\frac{52\pi}{9}$

5) $\frac{23\pi}{18}$

6) $785°$

7) $-30°$

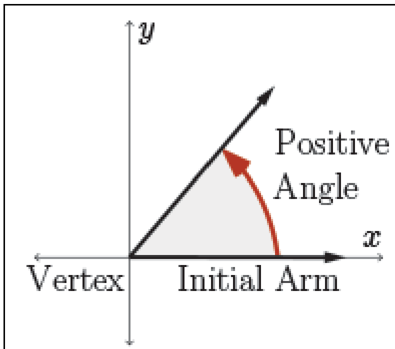
8) $-125°$

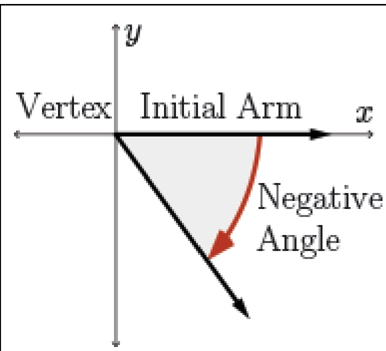
Arc Length Formula

Area of a Sector Formula

Example: Determine the arc length of a sector with an area of $100\pi\ cm^2$ and radius of 40 cm.

Angles in Standard Position



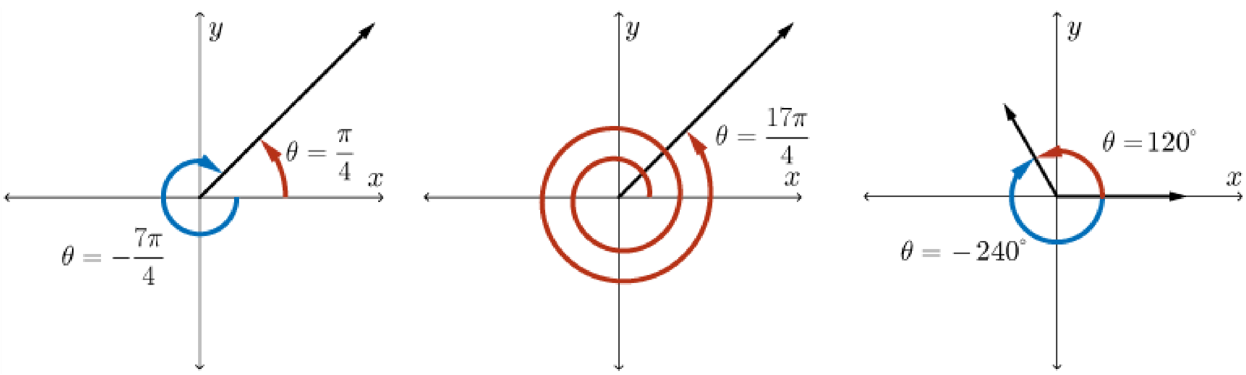


Draw an angle with the given measure in standard position.

a) 115^o b) $-\frac{3\pi}{4}$ c) $\frac{28\pi}{9}$

Coterminal Angles

Two angles in standard position are said to be **coterminal** if they share the same terminal arm.



Note: In general, two angles in standard position are coterminal if their difference is a non-zero integer multiple of 2π or 360^0.

Find a coterminal angle between 0 and 2π for each given angle.

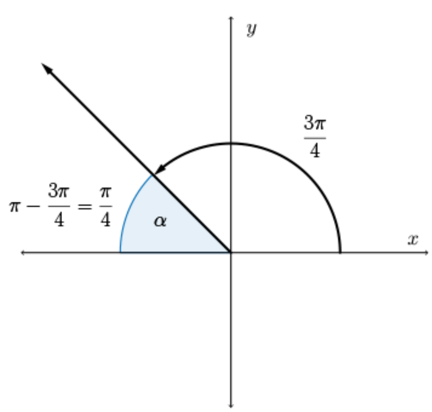
a) $\frac{11\pi}{4}$

b) $-\frac{41\pi}{36}$

c) $\frac{17\pi}{3}$

d) $-\frac{11\pi}{36}$

Reference Angles

	A **reference angle** is the acute angle formed between the terminal arm of a standard position angle and the *x*-axis. A reference angle is also referred to as a related acute angle.

Find the reference angle (related acute angle) for each angle $\boldsymbol{\theta}$.

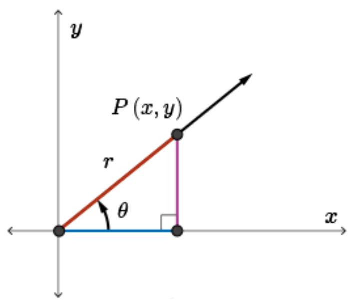
a) $\theta = -390^o$

b) $\theta = 247^0$

c) $\theta = \frac{11\pi}{5}$

d) $\theta = -\frac{8\pi}{7}$

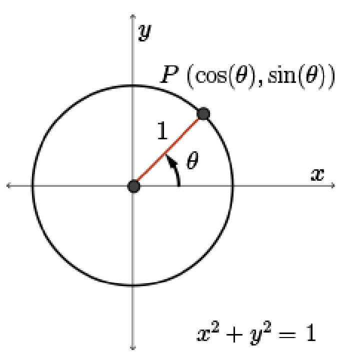
Defining Trigonometric Ratios

	sin(θ) = cos(θ) = tan(θ)=

The reciprocal trigonometric ratios are defined as follows:

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

What is a unit circle?

	sin(θ) = cos(θ) = tan(θ) =

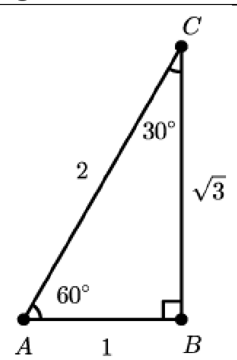
Find sin(θ), cos(θ), tan(θ) if

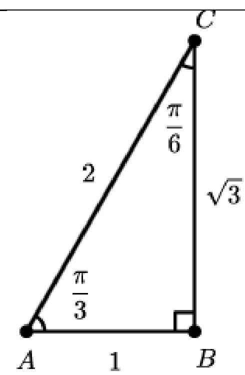
a. $\theta = \frac{\pi}{2}$ b. $\theta = \frac{3\pi}{2}$ c. $\theta = \pi$ d. $\theta = 2\pi$ e. $\theta = 0$

Example: The point $P(-6, 3)$ is on the terminal arm of an angle θ in standard position where $0 \leq \theta \leq 2\pi$. Determine the exact values of the six trigonometric ratios.

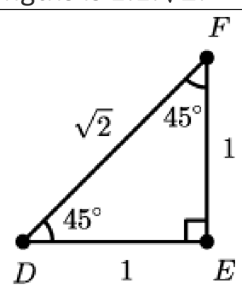
Special Triangles

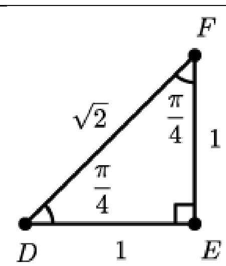
The first of the triangles is a **30^0-60^0-90^0** triangle. The ratio of the corresponding opposite side lengths is 1: $\sqrt{3}$: 2.





The second triangle is a **45^0 -45^0 -90^0** triangle. The ratio of the corresponding opposite side lengths is 1:1:$\sqrt{2}$.





Using radian measure, the first triangle is a $\pi/6$ - $\pi/3$ - $\pi/2$ triangle and the second triangle is a $\pi/4$ - $\pi/4$ - $\pi/2$ triangle.

Determine the exact values of the six trigonometric ratios for each of the angles $\frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$.

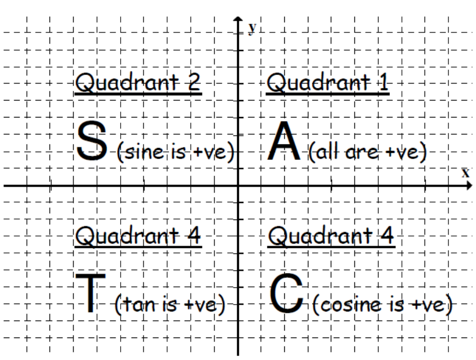
Angle(θ)	sin(θ)	tan(θ)	tan(θ)
$\frac{\pi}{6}$ or 30°			
$\frac{\pi}{3}$ or 60°			
$\frac{\pi}{4}$ or 45°			

Example:

a) A unit circle is shown with OP as the terminal arm of a $\frac{\pi}{3}$ radian standard position angle. Determine the coordinates of P.

b) A unit circle is shown with OQ as the terminal arm of a $\frac{-4\pi}{3}$ radian standard position angle. Determine the coordinates of Q.

Determining the Sign of a Trigonometric Ratio



Example: Determine the exact value of the following:

a. $\cos(300^0)$

b. $\tan(-855^0)$

c. $\sec(\frac{17\pi}{3})$

d. $\tan(2018\pi)$

e. $\csc(\frac{17\pi}{2})$

f. $\cot(-\frac{25\pi}{4})$

Example:

a) If $\cos(\theta) = -\frac{1}{4}$, determine the possible values of θ such that $-180^0 \le \theta \le 180^0$.

b) If $\tan(\theta) = -\sqrt{3}$, determine the possible values of θ such that $0 \le \theta \le 2\pi$.

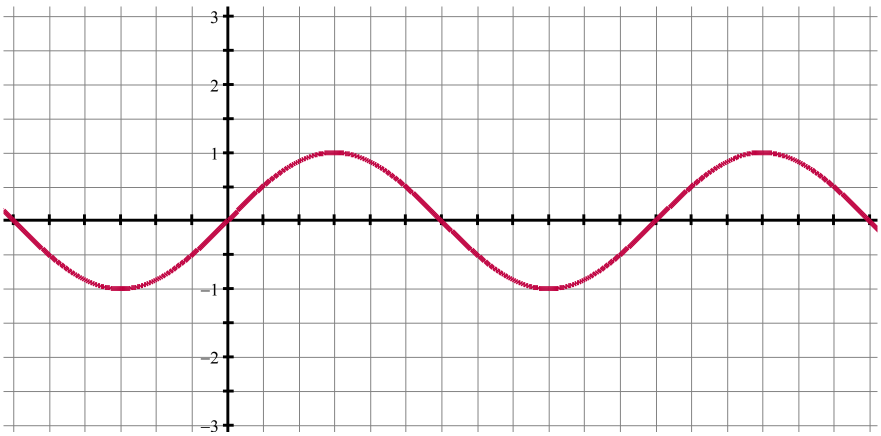
c) If $\sin(\theta) = -\frac{\sqrt{2}}{2}$, determine the possible values of θ such that $-\pi \le \theta \le 3\pi$.

d) For $\tan\theta = -\frac{5}{24}$, where $0^o \leq \theta \leq 360^o$, determine the values of other trig ratios.

Graphing $\sin x$, $\cos x$, and $\tan x$

For each of the following trigonometric graphs, identify the function, mark on the scale, and highlight one cycle of the graph. State the amplitude, period, max/min values, domain, range, and end behaviours.

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

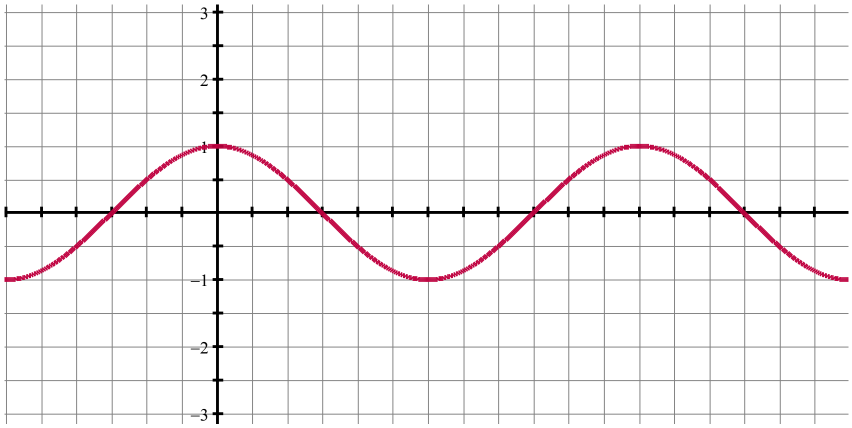
Minimum: __________

Domain: _____________________________

End Behaviour:

Range: _____________________________

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

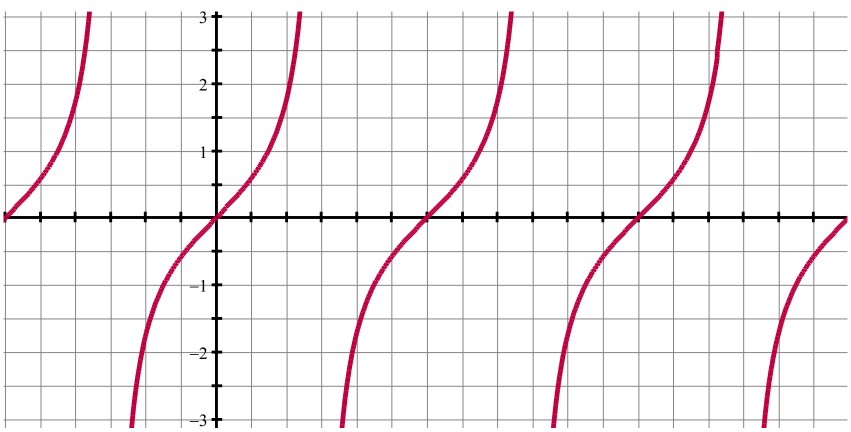
Minimum: __________

Domain: _____________________________

Range: _____________________________

End Behaviour:

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

Minimum: __________

Domain: _____________________________

Range: _____________________________

End Behaviour:

Warm UP!

On the same grid, sketch two cycles of *y* = sin *x* and *y* = cos *x*.

Rewrite *y* = sin *x* as a cosine function:

Rewrite *y* = cos *x* as a cosine function:

Summary

	$y = \sin(x)$	$y = \cos(x)$	$y = \tan(x)$
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$	$\left\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\right\}$
Range	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
Maximum	$y = 1$	$y = 1$	none
Minimum	$y = -1$	$y = -1$	none
Period	2π	2π	π
Amplitude	1	1	not defined
Vertical Asymptotes	none	none	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
y-intercept	0	1	0
x-intercepts	$x = n\pi, \in \mathbb{Z}$	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	$x = n\pi, n \in \mathbb{Z}$