TCS Tutorial Solution_Acceptor

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Turing Machine as an Acceptor

Q1. Design Turing Machine to accept {a^n b^n | n >= 0). Simulate for string "aabb"

Ans: Approach:
Step 1: Replace the first a by Blank
Step 2: Move right till last non-Blank symbol.
Step3: Replace last b by Blank.
Step 4: Move left till first non-Blank symbol of tape.
Step 5: Repeat from Step 1.

7-tuple representation of TM

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

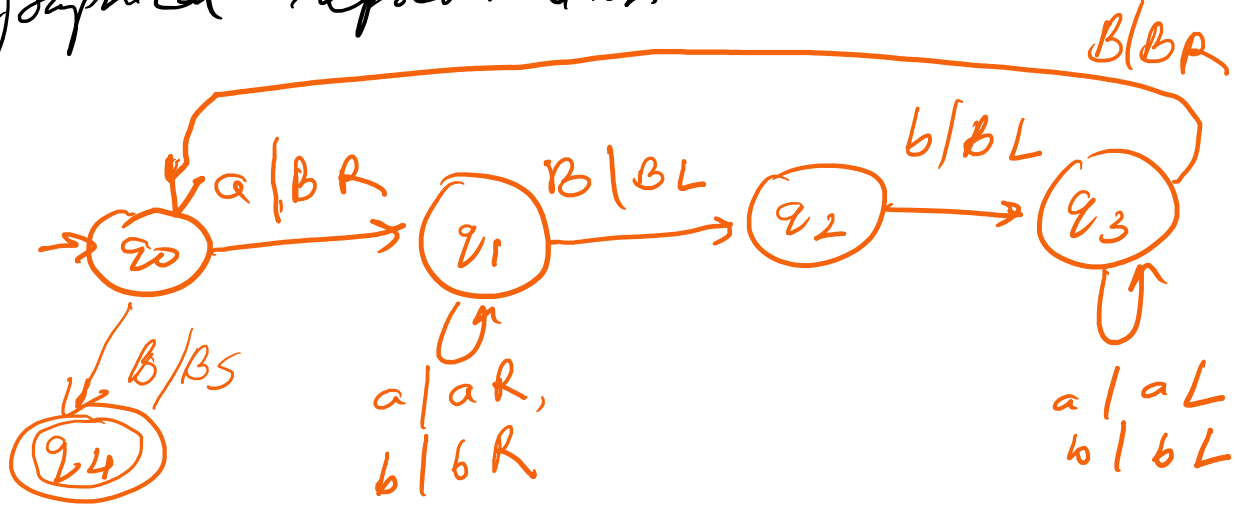
$\Sigma = \{a, b\}$, $\Gamma = \{a, b, B\}$

δ maps $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$

Q \ Γ	a	b	B
q_0	(q_1, B, R)	–	(q_4, B, S)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)
q_2	–	(q_3, B, L)	–
q_3	(q_3, a, L)	(q_3, b, L)	(q_0, B, R)
q_4	–	–	–

$Q = \{q_0, q_1, q_2, q_3, q_4\}$, $F = \{q_4\}$

Graphical representation:



Simulation for "aabb"

⊢ q0 a a b b B		⊢ B q1 a b b B		⊢ B a q1 b b B		⊢ B a b q1 b B
⊢ B a b b q1 B		⊢ B a b q2 b B		⊢ B a q3 b B B		⊢ B q3 a b B B
⊢ q3 B a b B B		⊢ B q0 a b B B		⊢ B B q1 b B B		⊢ B B b q1 B B
⊢ B B q2 b B B		⊢ B q3 B B B B		⊢ B B q0 B B B		⊢ B B q4 B B B

Q2. Design Turing Machine to accept {w c w| w ∈ (a+b)*).

Ans: Approach:
Step 1: Replace the first symbol by Blank. For input a, go to Step 2 and for input b, go to Step 3.
Step 2: Move right till input is c and go to Step 4 (with memory of leftmost as a)
Step 3: Move right till input is c and go to Step 5 (with memory of leftmost as b)
Step 4: Replace the symbol after c by Blank, if the symbol is a (Skip X, if any) and go to Step 6.
Step 5: Replace the symbol after c by Blank, if the symbol is b (Skip X, if any) and go to Step 6.
Step 6: Move left till first non-Blank symbol of tape.
Step 7: Repeat from Step 1.

7-tuple representation of TM

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

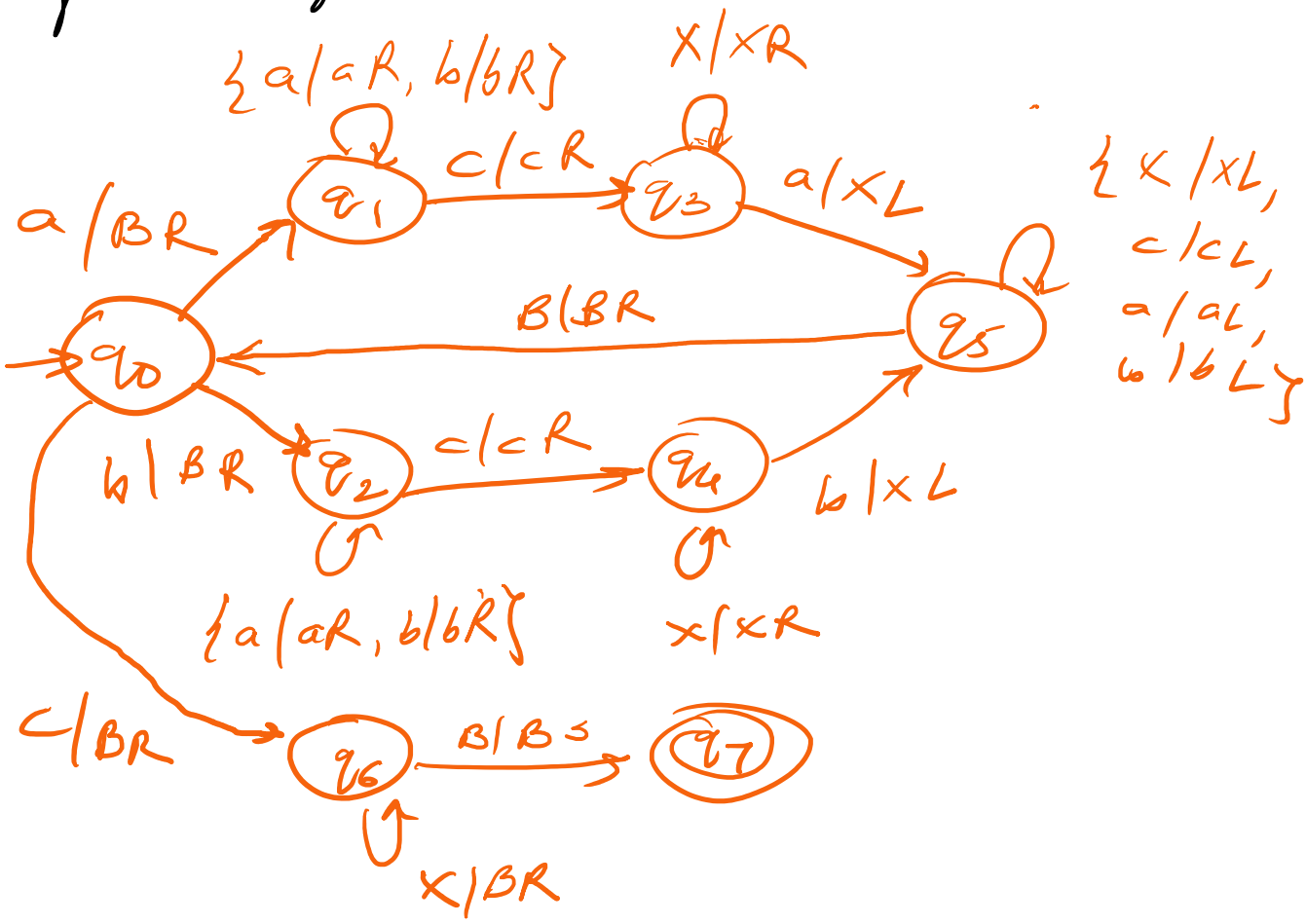
$\Sigma = \{a, b, c\}$, $\Gamma = \{a, b, c, X, B\}$

δ maps $Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$

q \ Γ	a	b	c	X	B
q_0	(q_1, B, R)	(q_2, B, R)	(q_6, B, R)		
q_1	(q_1, a, R)	(q_1, b, R)	(q_3, c, R)		
q_2	(q_2, a, R)	(q_2, b, R)	(q_4, c, R)		
q_3	(q_5, X, L)			(q_3, X, R)	
q_4		(q_5, X, L)		(q_4, X, R)	
q_5	(q_5, a, L)	(q_5, b, L)	(q_5, c, L)	(q_5, X, L)	(q_0, B, R)
q_6				(q_6, B, R)	(q_7, B, S)
q_7	–	–	–	–	–

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$ $F = \{q_7\}$

Graphical representation :



Turing Machine as an Generator

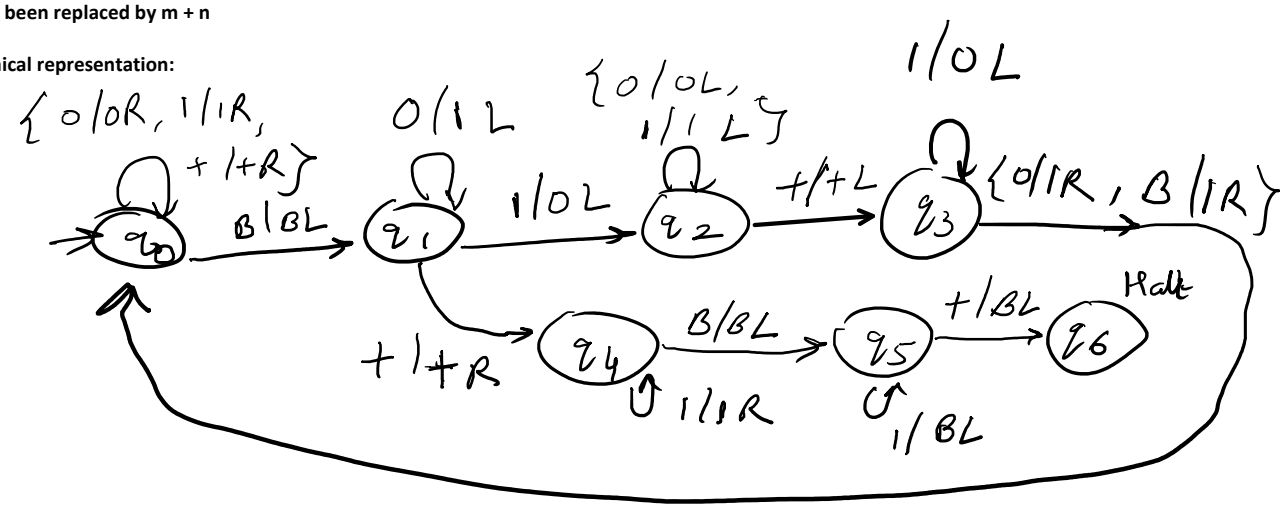
Q1. Design Turing Machine to add 2 binary numbers.

Ans:
Assume 2 way infinite model of TM.
Assume input in form "m + n", where m is first number and n is second number (both in Binary representation)

Approach:
Step 1: Move right till last non-Blank symbol (i.e. LSB of n).
Step 2: Decrement n by 1. If n is 0, go to Step 6
Step 3: Move left till LSB of m (i.e. left of +).
Step 4: Increment m by 1.
Step 5: Repeat from step 1.
Step 6: Blank all symbols from last non-Blank symbol on right till + (plus) and stop/ halt.

m has been replaced by m + n

Graphical representation:



Q2. Design Turing Machine to compute a^2.

Ans:
Assume input is a, followed by infinite Blank symbols.
Assume that a is represented in Unary where alphabet set is {1}

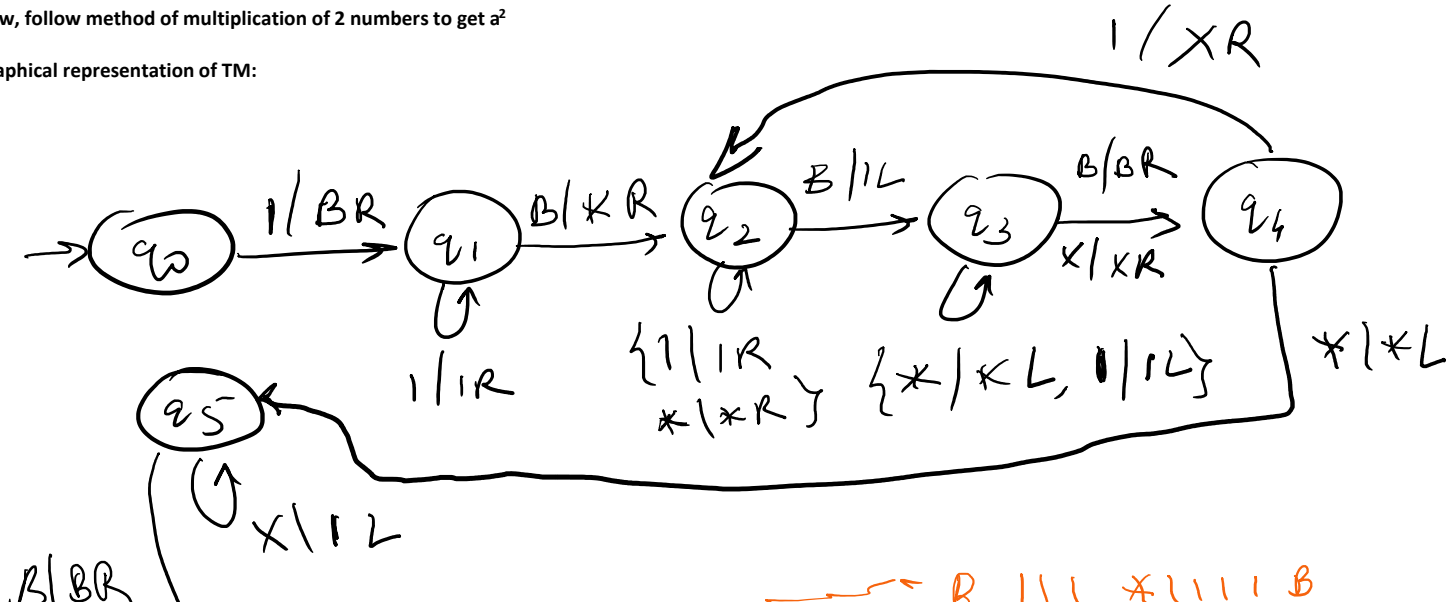
Approach:
Step 1: Decrement a by 1 (replace by B) and for that add a * and 1 at the end.
Step 2: Repeatedly decrement a by 1 (replace 1 by X) and add 1 after *.

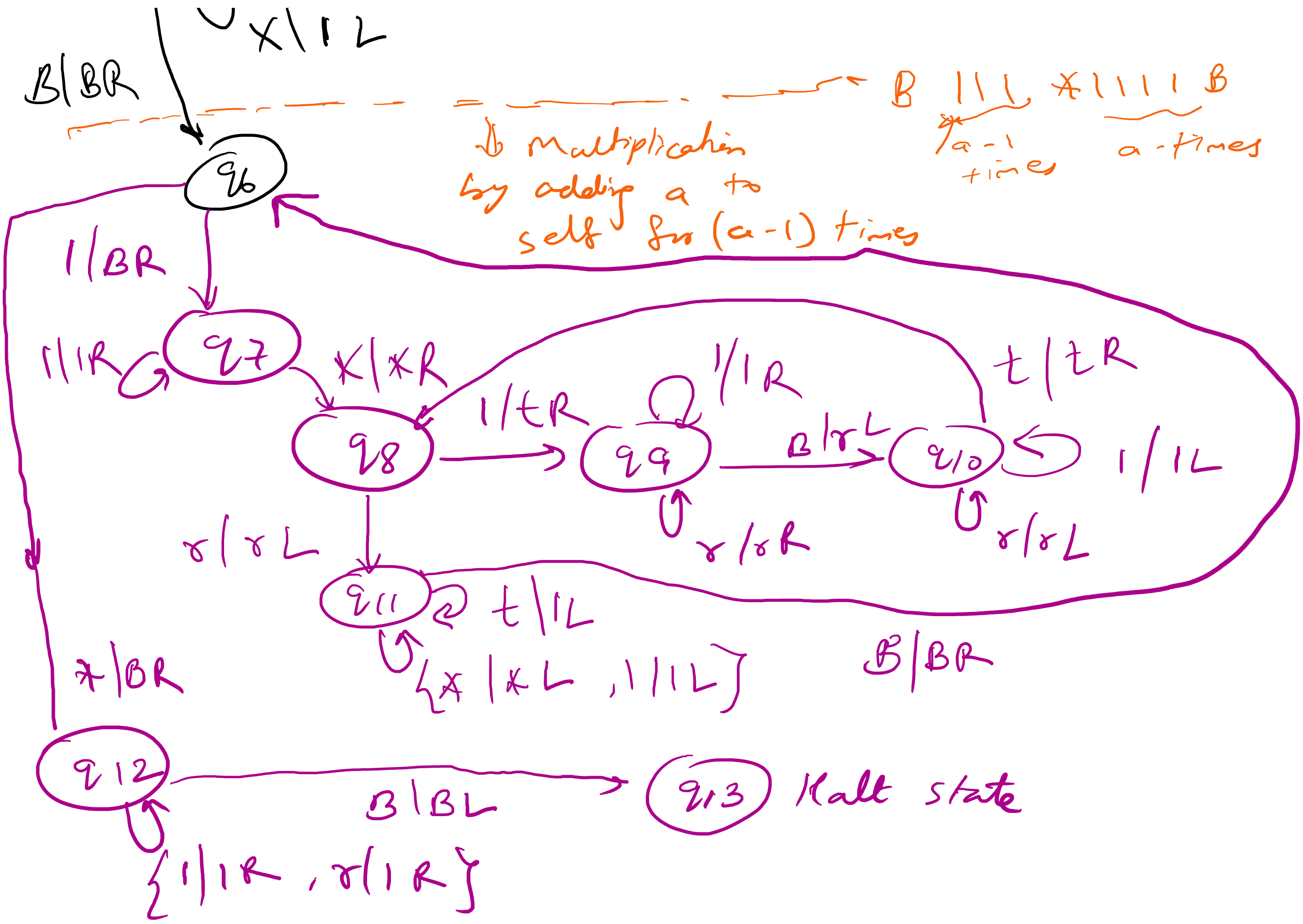
When no more 1s in a, tape has (a-1)-times X, followed by *, followed by a-times 1.

Step 3: Replace all X by 1 to get (a-1) times 1, followed by *, followed by a-times 1.

Now, follow method of multiplication of 2 numbers to get a^2

Graphical representation of TM:




B|BR
x|1L
B 111 *1111 B
a-1 times
a-times
multiplication by adding a to self for (a-1) times
q6
1|BR
q7
1|1R
*|*R
q8
1|tR
q9
1|1R
B|rL
q10
t|tR
1|1L
r|rR
r|rL
r|rL
q11
t|1L
{*|*L , 1|1L}
B|BR
*|BR
q12
B|BL
q13
Halt state
{1|1R , r|1R}