# Estimating Jointly Determined Outcomes: How Minimum Wage Affects Wages and Hours Worked

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**Abstract**

This paper goes beyond the standard single outcome framework for evaluating policy by examining the effects of minimum wage changes on both hourly wages and hours worked. Individuals earning different wages and working different weekly hours are likely affected differently by minimum wage changes. Additionally, while changes to the minimum wage might increase an individual's hourly wage, they might also decrease that individual's number of hours worked. Since wages and hours worked are contemporaneously determined, a conditional effect of a minimum wage change on either wages or hours worked is conditioned on an endogenous variable. Therefore, traditional methods of evaluating heterogeneous outcomes, such as quantile treatment effects, are inappropriate and would not be able to identify effects that vary by both wage and hours worked. This obstacle is overcome by examining the joint distribution of outcomes. Using distribution regressions and an empirical copula to estimate the counterfactual joint distribution of wages and hours worked in the United States, this paper finds that increases to the minimum wage caused both wages and hours worked to increase for both men and women except at the very top of the wage distribution where hours were reduced. As a result of minimum wage increases, some industries saw hours of minimum wage workers increase while others saw them decrease, suggesting that the market structure could be causing these findings.

**Keywords:** Minimum Wages, Wages and Hours Worked, Joint Distribution, Copula, Distribution Regression

**JEL Codes:** J08, J22, J31, C31

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# 1 Introduction

## 1.1 The Minimum Wage Puzzle

The main goal of minimum wage policy is to assist low-wage workers. However, the effectiveness of minimum wage as a poverty or inequality reducing measure is questionable. While several studies conclude that increased minimum wage reduces wage inequality in the United States,[1] and has mild effects on employment,[2] little is known about its effects on hours worked,[3] which might vary for individuals earning different wages and working different hours. By analyzing the joint distribution of wages and hours worked, this paper offers insight into whether increased minimum wage reduces poverty and inequality and what mechanisms cause any such change.

Generally, papers studying the effects of minimum wage on inequality are only concerned with the change in the wage distribution and the change in employment statuses. Such studies usually include an analysis of various indices (e.g. differences between wage quantiles or the Gini coefficient) which address certain properties thought to define inequality or a level of income deemed to represent a level of poverty. However, there are different ways to consider an individual "poorer" or "worse off" besides wages. If, for example, a policy increases the wages of every individual in the population but has a negative effect on the health outcomes, cost of living, or educational attainment of some individuals, it is arguable that the policy increases inequality rather than decreasing it. While it is possible to attribute more or less "weight" (importance) to different measures of well-being at different parts of the distribution (e.g. one unit of health status at the bottom of the wage distribution is worth two units of wages, whereas one unit of health status at the top of the wage distribution is worth three units of wages at that level), such a weighting tends to be arbitrary and controversial. Additionally, Atkinson and Bourguignon (1982) point out another issue with this weighting scheme, namely, that transfers in one measure can change the marginal utility of another measure (e.g. if an individual gains or loses a unit of educational attainment, she might now value an increase in a unit of wages differently).

Although, for example, changes in health status are not likely to be caused by increased minimum wage, changes in educational attainment, cost of living, or hours worked due such increases are likely drastic and vary across the distribution. Consider Figure 1. It is possible for every individual's wage to be

[1] For example, see DiNardo et al., 1996; Lee, 1999; and Autor et al., 2016.

[2] Manning (2021) points out "[t]here is probably no economist who does not believe that there is some point at which higher minimum wages reduce employment." While that may be true, minimum wage increases in the United States at both the Federal and State level have generally been small. As discussed in Section 2, there is a growing consensus that these increases had little or no effect on employment, and future minimum wage research should instead focus on determining when minimum wage would cause a negative employment effect.

[3] Researchers studying the effects of Seattle's 2015 and 2016 minimum wage increases initially found the policies had negative impact on the hours worked of low-wage workers, but in a follow-up paper, the same researchers found little to no impact (Jardim et al., 2017; Jardim et al., 2018).

increased but for some individuals to experience a decrease in hours worked (assuming employment remains constant). Perhaps employers do not adjust for increases in minimum wage by discharging workers or reducing the wages paid to other employees. Instead, they may respond by increasing the responsibilities and hours of high-wage workers who are more productive than low-wage workers and reduce the hours of low-wage workers. Clearly, low-wage individuals value increased hourly wages less when hours worked decrease. Only considering the marginal distribution of hourly wages would lead to the conclusion that inequality was reduced, but considering the joint distribution of hourly wages and hours worked might lead to the conclusion that there was an increase in inequality! More details on comparing welfare of multivariate distributions are discussed in Cahn and Maasoumi (2021).

Of course, the obvious question remains — why not try to estimate quantile treatment effects of wages conditional on hours worked, a more traditional approach? Doing so estimates the *ceteris paribus* effect of the policy on a quantile of wages, assuming the policy does not contemporaneously affect hours worked, an endogenous variable. However, the policy could affect wages and hours worked of individuals very differently across the joint distribution (e.g. low-wage high-hour workers could be affected differently than low-wage low-hour workers). Therefore, the real object of interest for welfare analysis should be some comparison of a functional of the joint distribution of outcomes of interest with a policy and a functional of the counterfactual joint distribution without the policy.[4] Appendix C.1. provides more discussion on the importance of accounting for jointly determined outcomes.

Extending the method of Chernozhukov et al. (2013) for estimating counterfactual distributions using distribution regression, this paper employs a copula method and compares joint distributions of hourly wage and hours worked with and without a minimum wage change. Details on this method are laid out in Section 3. Additional robustness checks can be found in section 5.2.

This paper finds that minimum wage increases caused both wages and hours worked to increase across the joint distribution for both men and women except at the very top of the wage distribution where minimum wage increases induced fewer individuals to work longer hours. This suggests minimum wage increases have large spillover effects and affect individuals working well above the minimum wage.

Increased real Federal minimum wage from \$6.39 in 1989 to \$7.75 in 1992 (in 2019 dollars) caused the percentage of men earning around the median wage and working up to 36 hours per week to increase by 3% while it caused those working up to 40 hours to increase by 11% and those working up to 56 hours per week

[4]Heckman (2010) formulates three classes of problems in policy evaluation: (P1) evaluating the impact of the policy; (P2) forecasting the impacts of the policy (constructing counterfactual states); and (P3) forecasting the impact of the policy never historically experienced (constructing counterfactual states associated with the policy). The class of problems (P1), which makes up a large portion of policy evaluation literature, generally focuses on means or conditional means (i.e. subgroups of the population). The argument here is that class of problems (P1) should also consider on joint distributions.

to increase by 17% (see Table 2). Surprisingly, the effect of increased minimum wage was much weaker for men in the bottom 10th percentile of wages. For those individuals, the minimum wage increase produced only a 5% increase in employees working up to 40 hours per week and a 6% increase in those working up to 56 hours per week. Men earning up to the 90th percentile and working 40 hours or more decreased by 3%. The effects on women, who tend to earn less and work fewer hours, was comparable. Increased real Federal minimum wage from \$6.53 in 2006 to \$8.07 in 2012, and decreased real minimum wage from \$7.63 in 1984 to \$6.39 in 1989 (in 2019 dollars) also had similar effects.[5]

These results could possibly be due to the "backward bending" nature of the wage curve, which describes the labor-leisure trades-offs of workers. As a worker's wages increase, she prefers to work more hours and have less leisure time. However, at a certain wage level, this preference reverses and she would prefer to work fewer hours as her wage increases.[6] In this way, minimum wage's heterogeneous effects are simple — consider Figure 3. Excluding general equilibrium effects, if the markets for hours of minimum wage, medium-wage, and high-wage labor are competitive, all workers should experience an increase in wages. However, medium-wage workers should see an increase in their hours worked, while minimum wage and high-wage workers should see a decrease.

This model of the wage and hour distributional effects due to a minimum wage increase might be an overly simplistic. Indeed, the decrease in minimum wage workers' hours does not seem present in the cases considered. Theoretical reasons why minimum wage workers might not see their hours reduced are the same as the reasons why they might not suffer employment effects laid out in Section 2.1.

A possible explanation for the lack of employment effects due to minimum wage increases is that the markets are not competitive (e.g. Azar et al., 2019). If so, the new minimum wage might still be below a worker's marginal productivity and therefore her hours would would not decrease, but increase. When the effects of minimum wage are separated by industry, minimum wage workers in some industries saw their hours reduced, while the opposite was true in other industries. Accommodation and Food Services and Retail Trade saw hours of their minimum wage workers reduced, while Transportation and Warehousing and Manufacturing saw them increase (see Table 14; these effects were over the period 2003-2019, when there was significant variation in state minimum wage by year — see Figures 4 and 5).[7]

It is not clear whether the industries with positive hour effects are highly concentrated. Figure 10 shows the Herfindahl-Hirschman Index (HHI) by industry in 2017.[8] Since industries are classified so broadly, none of the markets

[5]Some of this could also be due to increased state minimum wages, which is accounted for in how this paper defines the minimum wage variable. However, very few states had minimum wages different than the Federal minimum wage until 1990.

[6]See Figure 2.

[7]In Transportation and Warehousing, hours worked decreased when minimum wage is above \$9.81 for men and \$10.77 for women. In Manufacturing, men saw no change to their hours worked, while women's hours worked decreased when minimum wage was above \$10.27.

[8]Industries are defined by North American Industry Classification System (NAICS) 2-

have a high HHI. Firms' direct competition is likely regional and the data available in this paper are insufficient to determine which industry is concentrated. However, intuitively, it makes sense that Accommodation and Food Services and Retail Trade are competitive in local markets, while Transportation and Warehousing and Manufacturing are not. If so, a simple explanation for the reason a minimum wages increases had near-zero employment and hour effects is that some industries are concentrated while others are competitive.

If the normative assumptions underpinning minimum wage policy are accepted, perhaps minimum wage should be set by industry with competitive industries having lower minimum wages.

Additionally, it is possible minimum wage workers are able to work increased hours at their secondary job or find other part-time work. Since a large portion of minimum wage workers in the United States are paid by the hour and are part-time workers,[9] low-wage workers' increased desire to work more hours is met by work at a secondary job. In Figure 11, after accounting for employment effects, the probability of a worker being part-time decreases with increased minimum wage. This is consistent with the two recent papers on Seattle's increased minimum wage (Jardim et al., 2017; Jardim et al., 2018) which suggested that workers were working additional hours at their secondary jobs.

Furthermore, it might not be market concentration that is causing the different effects of minimum wage on hours by industry, but rather the elasticity of demand for labor in those industries. If the labor costs in an industry are a relatively small percentage of overall expenses and the costs associated with reducing the labor force are high, then an increase in minimum wage is unlikely to result in a reduction to the labor force in that industry.

No other paper has studied the extensive (employment) and intensive (hours and wages) margins for minimum wage policy on the entire distribution of hours and wages. Employment effects are accounted for using a Heckman-type selection modeling. However, unlike the traditional Heckman selection model which models a worker's selection decision into the labor force, this paper models employment selection as a market outcome (i.e. the employer is allowed to select the employee's hours).

The main identification assumptions for these selection models is an exclusion restriction — a variable is included in the labor market participation equation that does not affect the outcome variable. As is common in the literature, number of children under age five is used as an exclusion restriction for wages. Education variables are used as exclusion restrictions for hours worked.[10]

digit codes. Information on HHI for Agriculture, Forestry, Fishing, and Hunting; Public Administration; and Retail Trade are not available.

[9]See Figures 7 and 8.

[10]Having an exclusion restriction is not necessary for identification in a type II Tobit model, see Wooldridge (2001). However, identification would be due to the non-linearity of the inverse Mills ratio, in which case functional form misspecification in the population model could be the main determinant of the inverse Mills ratio.

## 1.2 The Remainder of the Paper

Section 2 reviews the literature on the predictions and empirical effects of minimum wage policy on low-wage workers as well as its distributional effects. Section 3 discusses the methodology of this paper. Section 4 discusses the data used in this paper. Section 5 presents the results. Section 6 concludes.

# 2 Literature Review

## 2.1 Predictions of Minimum Wage Effects on Employment

Stigler (1946) states "...[minimum wage] reduces the earnings of those substantially below the minimum. These are undoubtedly the main allocational effects of a minimum wage in a competitive industry." Indeed, the standard model of a competitive market predicts increased minimum wage will cause an increase in unemployment. However, frictions in the labor market, a small elasticity of demand for labor, or a lack of binding minimum wage can drastically change this prediction.

If there is a large presence of monopsonies, increased minimum wage does not force firm profits to fall below the marginal cost of production (Robinson, 1933; Stigler, 1946; Bhaskar and To, 1999; Manning, 2003; Azar et al., 2019). Hence, there could be little or positive effect on employment.

Flinn (2006) develops a Nash bargaining model in which increased minimum wage intensifies job search and improves employer-employee match quality. In turn, this increases productivity and offsets any negative employment effects.

Another explanation offered is the concept of an "efficiency wage." An efficiency wage is a wage offered by employers that is higher than the market-clearing wage in order to reduce costs associated with turnover (Shapiro and Stiglitz, 1984; Rebitzer and Taylor, 1995). Therefore, firms may be willing to pay higher wages to insure a consistent workforce.

Dessing (2002) suggests workers are "backward-bending" and take jobs below their real reservation wage and productivity level in order to earn some minimal income to feed their families.

Alternatively, unions could affect the labor market. With unions present, firms cannot terminate their employees at will and this generates a cascade effect over the entire wage structure (Lee, 1999; Autor et al., 2016; and Kearney and Harris, 2014).

With many possible alterations to the standard competitive market framework, there is no clear theoretical prediction of the effects of increased minimum wages on employment. Furthermore, some argue market frictions may be negligible and the magnitude of the theorized effects of market frictions has been disputed.

## 2.2 Empirical Results on Low-wage Workers

Traditional empirical work using observational data found an increase in minimum wage led to decreases in employment.[11] However, following the pathbreaking work of Card and Krueger (1994), the empirical consensus has somewhat shifted to supporting the view that an increase in minimum wage does not increase unemployment — at least in the United States, where minimum wages increases have remained modest. Card and Krueger used a "natural experiment" to compute the change in employment due to minimum wage and compare it to a counterfactual "control" state change in employment. Subsequent minimum wage papers generally used two-way fixed effect (e.g. Neumark and Wascher, 2008).

However, Card and Krueger's work was not without criticism and concerns. Using phone survey data of a sample similar to Card and Krueger's, Neumark and Wascher (2000) results led to opposite conclusions. Meer and West (2016) argue that minimum wage impacts happen over time and while immediate relative employment levels might remain stable, the growth rate of job openings likely decreased. Others have shown that the elasticity of employment decreases in the long-run (Sorkin, 2015; Aaronson et al., 2018). Jardim et al. (2017) argue the relevant market as well as the reduction in hours worked was not considered.[12] While most studies—including Card and Krueger—use a proxy for low-wage industries such as teenagers or restaurant workers, they use Seattle data to identify low-wage industries and examine the average treatment effect of a minimum wage increase on both hourly wage and hours worked. They find the increased minimum wage reduced a low-wage worker's monthly earnings by an average of $74 per month. In a follow up paper, Jardim et al. (2018) find the minimum wage increase likely had a more modest or even negligible effect on hours worked citing the possibility that minimum wage workers took up additional outside work or new workers entered the workforce.

Additionally, Card and Krueger (1994) and similar papers have been the subject of methodological concerns. These papers assume "parallel trends" between treatment and control states, which has been criticized especially since the adoption of minimum wage laws appears to be clustered by geographical region (Allegretto et al., 2018). However, there is a large literature that attempts to address these concerns (see Neumark, 2018).

While the empirical effects of minimum wage are disputed, most empirical work does not draw a clear link to a theoretical prediction. If there is indeed no change in employment due to small minimum wage increases, then what theoretical market friction is causing it? Since different theoretical market frictions should have different effects at different parts of the wage distribution, considering distribution effects—discussed in the next section—is critical to understanding the effects of minimum wage.

[11] See Fernández-Villaverde, 2018 for discussion.

[12] Another notable paper that examines how minimum wage affects the average hours worked is Belman et al. (2015).

## 2.3 Distribution Effects of Minimum Wage

Undoubtedly, minimum wage has heterogeneous effects on the distribution of wages. While some might benefit from increased minimum wage, others might see little benefit or even be hurt by it. The relevant policy question is whether increased minimum wage increased or reduced some welfare measure such as poverty or inequality.[13]

Following the work of Lee (1999), traditional analysis of distribution and inequality changes were concerned with "spillovers". Do workers earning below the minimum wage "spillover" into other parts of the wage distribution, earn the new minimum wage, become disemployed, or some combination of the previous possibilities?

Extending DiNardo et al. (1996), which did not account for spillovers, Lee (1999) compared the change in the ratio of 50th to 10th percentile of wage, calculated the reduction in real minimum wage, and concluded that minimum wage increases substantially increased inequality. Autor et al. (2016) considered a longer period of time and included state and time fixed effects and found similar but smaller effects of minimum wage on reducing inequality.

An additional concern to interpreting changes in the wage distribution due to increased minimum wage is the possibility of high-wage workers being substituted for low-wage workers. Cengiz et al. (2019) consider the bottom of the wages lost right below the new minimum wage before a minimum wage increase is implemented and found the new wages created right above the minimum wage after the policy is implemented was equivalent. This "bunching effect" explains the lack of job loss is not due to labor-labor substitutions.

The previous literature on distributional effects of minimum wages does not account for the possibility of firms substituting hours worked by low-wage workers with those of high-wage workers and does not take into account the labor-leisure decisions of the workers whereas this paper does.

# 3 Model and Estimation

## 3.1 Counterfactual Analysis Setting

Suppose we are interested in some outcome $Y$ which has relevant characteristics $X$. In the spirit of Haavelmo (1944), assume $Y$ and $X$ are random variables with supports $\mathcal{Y} \subseteq \mathbb{R}$ and $\mathcal{X} \subseteq \mathbb{R}^{d_x}$, respectively, and have measurable density functions. Observations are therefore realizations that come from the joint probability density function of $Y$ and $X$, $f_{Y,X}(y,x)$, and we are interested in the distribution of $Y$.

By the law of iterated probability

$$F_Y(y) = \int_{\mathcal{X}} F_{Y|X}(y|x) dF_X(x), \tag{1}$$

[13]Kearney and Harris (2014), MaCurdy (2015), and Harasztosi and Lindner (2019) consider the effectiveness of minimum wage relative to other antipoverty programs and the extent to which firm or consumers are paying for the minimum wage changes.

where $F_Y(y)$ is the distribution of $Y$, $F_X(x)$ is the distribution of $X$, and $F_{Y|X}(y|x)$ the conditional distribution of $Y$ given $X$. Now, suppose there are two groups 0 and 1 (e.g. 0 is the control group and 1 is the treatment group, or 0 is one time period and 1 is another time period), then the outcome and relevant characteristics might be different for each group—i.e. the outcome is $Y_t$ and the relevant characteristics are $X_t$ with $t \in \{0,1\}$. However, observations would only be observed from $f_{Y_1,X_1}(x_1,y_1)$ and $f_{Y_0,X_0}(x_0,y_0)$. Equation (1) can be rewritten as

$$F_{Y\langle t,v\rangle}(y) = \int_{\mathcal{X}_v} F_{Y_t|X_t}(y|x)dF_{X_v}(x), \tag{2}$$

$t, v \in \{0,1\}$ and if $t \neq v$ then $F_{Y\langle t,v\rangle}$ is the *counterfactual distribution*—the distribution of the random variable $Y_t$ had it come from the joint distribution $f_{Y_t,X_v}(y_t,x_v)$, for which observations are never made.

To further demonstrate the usefulness of the counterfactual distributions, consider the case in which $X$ is partitioned into two random variables, $X = (X_a, X_b)$.[14] Then by the law of iterated probability

$$F_Y(y) = \int_{\mathcal{X}_b}\int_{\mathcal{X}_a} F_{Y|X_a,X_b}(y|x_a,x_b)dF_{X_a|X_b}(x_a|x_b)dF_{X_b}(x_b). \tag{3}$$

For two groups 0 and 1, the counterfactual distribution is defined by

$$F_{Y\langle t|s,v\rangle}(y) = \int_{\mathcal{X}_{b,s}}\int_{\mathcal{X}_{a,v}} F_{Y|X_{a,t},X_{b,t}}(y|x_a,x_b)dF_{X_{a,s}|X_{b,s}}(x_a|x_b)dF_{X_{b,v}}(x_b), \tag{4}$$

$t, s, v \in \{0,1\}$. The observed difference in distributions of the outcome of interest can be decomposed as follows:

$$F_{Y_{\langle 1|1,1\rangle}} - F_{Y_{\langle 0|0,0\rangle}} = \underbrace{F_{Y_{\langle 1|1,1\rangle}} - F_{Y_{\langle 1|0,1\rangle}}}_{\text{(i)}} + \underbrace{F_{Y_{\langle 1|0,1\rangle}} - F_{Y_{\langle 1|0,0\rangle}}}_{\text{(ii)}} + \underbrace{F_{Y_{\langle 1|0,0\rangle}} - F_{Y_{\langle 0|0,0\rangle}}}_{\text{(iii)}},$$

(i) is the effect of the change from $X_{a,0}$ to $X_{a,1}$ on the distribution of $Y_1$, (ii) is the effect of the change from $X_{b,0}$ to $X_{b,1}$ on the distribution of $Y_1$, and (iii) is the residual effect on the distribution of $Y_1$.[15][16]

There are several proposed methods for estimating the counterfactual distribution. The main practical concern is estimating the conditional distributions.

[14] Note that $X$ can easily be partitioned into as many dimensions as $d_x$.

[15] Note an alternative decomposition is, for example, $F_{Y_{\langle 1|1,1\rangle}} - F_{Y_{\langle 0|0,0\rangle}} = (F_{Y_{\langle 1|1,1\rangle}} - F_{Y_{\langle 0|1,0\rangle}}) + (F_{Y_{\langle 0|1,0\rangle}} - F_{Y_{\langle 0|1,1\rangle}}) + (F_{Y_{\langle 0|1,1\rangle}} - F_{Y_{\langle 0|0,0\rangle}})$ or some decomposition of $F_{Y_{\langle 0|0,0\rangle}} - F_{Y_{\langle 1|1,1\rangle}}$. Therefore, the "sequential" ordering of the decomposition might have important implications and is a major drawback of this kind of decomposition analysis. Accordingly, it is important to check the reverse ordering for robustness (e.g. the effect of a change from $X_{a,0}$ to $X_{a,1}$ on the distribution of $Y_1$ should adhere to the interpretation of a change from $X_{a,1}$ to $X_{a,0}$ on the distribution of $Y_0$).

[16] More generally, for the functional $\phi$ (e.g. Lorenz curve, Gini coefficient, quantile ranges, and more trivially the mean and variance), the observed differences, $\phi(F_{Y_{\langle 1|1,1\rangle}}) - \phi(F_{Y_{\langle 0|0,0\rangle}})$, can be similarly decomposed.

DiNardo et al. (1996) propose an inverse propensity reweighting method. Alternatively, S. Firpo et al. (2009, 2018) use re-centered influence function (RIF) regressions. Chernozhukov et al. (2013) use quantile and distribution regressions and show valid inference can be done with an exchangeable bootstrap.[17]

Suppose now that there are two outcomes of interest, $Y^1$ and $Y^2$.[18] We are interested in the joint distribution of outcomes, $F_{Y^1,Y^2}(y_1, y_2)$. By Sklar's Theorem, if marginal distributions $F_{Y^1}$ and $F_{Y^2}$ are continuous, there exists a unique copula $C$ such that

$$F_{Y^1,Y^2}(y_1, y_2) = C(F_{Y^1}(y_1), F_{Y^2}(y_2)). \tag{5}$$

For the remainder of this section, this paper will provide conditions for identification and use the method of Chernozhukov et al. (2013) to estimate the marginal counterfactual distributions of hourly wages and hours worked and then use an empirical copula to obtain an estimate of the joint counterfactual distribution of hourly wages and hours worked.

## 3.2 Identification

Let $(Y_j^{1*}, Y_j^{2*} : j \in \mathcal{J})$ denote a vector of potential outcome variables for various values of a policy, $j \in \mathcal{J}$, and let $X^1$ and $X^2$ be vectors of covariates for $Y_j^{1*}$ and $Y_j^{2*}$, respectively.[19] Let $J$ be a random variable that denotes the realized policy with $Y^1 := Y_J^{1*}$ and $Y^2 := Y_J^{2*}$ the realized outcome. Let $F_{Y_j^{1*},Y_j^{2*}|J}(y_1, y_2|k)$ denote the joint distribution of the potential outcome $Y_j^{1*}$ and $Y_j^{2*}$ in the population where $J = k \in \mathcal{J}$.

The causal effect of exogenously changing the policy from $\ell$ to $j$ on the distribution of potential outcomes in the population with realized policy $J = k$ is

$$F_{Y_j^{1*},Y_j^{2*}|J}(y_1, y_2|k) - F_{Y_\ell^{1*},Y_\ell^{2*}|J}(y_1, y_2|k).$$

**Assumption 1.** Let $\mathcal{X}_k^1 \subseteq \mathcal{X}_j^1$, $\mathcal{X}_k^2 \subseteq \mathcal{X}_j^2$ for all $(j, k) \in \mathcal{JK}$.

**Assumption 2.** Let the latent variables

$$(Y_j^{1*}, Y_j^{2*} : j \in \mathcal{J}) \perp\!\!\!\perp J|X^1, X^2 \quad \text{a.s.},$$

where $\perp\!\!\!\perp$ denotes independence.

**Assumption 3.** Let $F_{Y^1\langle j|k\rangle}(\cdot)$ and $F_{Y^2\langle j|k\rangle}(\cdot)$ with $j, k \in \mathcal{J}$ be continuous.

**Theorem 3.2.** Under Assumptions 1-3,

$$F_{Y^1,Y^2\langle j|k\rangle}(\cdot) = F_{Y_j^{1*},Y_j^{2*}|J}(\cdot|k), \quad j, k \in \mathcal{J}.$$

[17] See Fortin et al. (2011) for more details on the decomposition method and methods for estimating the counterfactual distribution.

[18] Of course, this can easily be generalized to a case with there are more than two outcomes of interest.

[19] Following the potential outcomes literature, general equilibrium effects are excluded in the definition of potential outcomes. However, since the *ceteris paribus* effects are on the joint distribution of outcomes, general equilibrium effects of the outcomes on each other are not excluded.

**Proof.** See Appendix B.1.

Theorem 3.2 can be generalized to state that as long as the marginal distributions of the latent outcome variables are continuous and identified, the joint distribution of latent outcomes is identified.

## 3.3 Counterfactual Distribution

Let 0 denote a year with lower minimum wage and 1 denote a year with higher minimum wage (e.g. 0 denotes 1989 and 1 denotes 1992) such that $\mathbf{Y}_t = \left(Y_t^1, Y_t^2\right)$ denotes hourly wages and hours worked at year $t \in \{0,1\}$, respectively. Let $\mathbf{X}_v$ denote the job market-relevant characteristics affecting hourly wages and hours worked at year, $v \in \{0,1\}$. For ease of notation, superscripts on the covariates denoting hourly wage or hours worked are dropped. Furthermore, let $\mathbf{X}_v$ be composed of a minimum wage variable $m_v$ and all other characteristics $\mathbf{c}_v$, $\mathbf{X}_v = (m_v, \mathbf{c}_v)$, where

$$m_v = \begin{cases} 1 & \text{earning at or below minimum wage} \\ 0 & \text{otherwise} \end{cases}.$$

Let $F_\eta$, denote the distribution of the random variable $\eta$. The unconditional marginal distribution of $Y^i$, $i \in \{1,2\}$, can be computed by integrating over the conditional distributions as follows:

$$F_{Y^i\langle t|s,v\rangle}(y) := \int_{\mathcal{C}_v} \int_{\mathcal{M}_s} F_{Y_t^i|m_t,\mathbf{c}_t}(y|m,\mathbf{c}) dF_{m_s|\mathbf{c}_s}(m|\mathbf{c}) dF_{\mathbf{c}_v}(\mathbf{c}), \tag{6}$$

where $\mathcal{M}_s \subseteq \mathbb{R}$, $\mathcal{C}_v \subseteq \mathbb{R}^{d_c}$ denote the supports of $m_s$ and $\mathbf{c}_v$, respectively. When $t = v = s$, (6) becomes the unconditional distribution $F_{Y^i\langle t|t,t\rangle} = F_{Y_t^i}$—the observed distribution of $Y^i$ and time $t$—by the law of iterated probabilities. When $t$, $s$, and $v$ are not equal, $F_{Y^i\langle t|s,v\rangle}$ is a counterfactual distribution. For example, When $t = v = 1, s = 0$, the $F_{Y^i\langle 1|0,1\rangle}$ is the distribution of outcomes that would prevail for time 1 had that period had the composition of minimum wage of time 0.

The joint distributions can be obtained using Sklar's Theorem: there exists a copula $C$ such that

$$F_{\mathbf{Y}\langle t|s,v\rangle}(y_1, y_2) = C(F_{Y^1\langle t|s,v\rangle}(y_1), F_{Y^2\langle t|s,v\rangle}(y_2)). \tag{7}$$

If $F_{Y^1\langle t|s,v\rangle}$ and $F_{Y^2\langle t|s,v\rangle}$ are continuous, $C$ is unique.

## 3.4 Estimation of the Counterfactual Distribution

Chernozhukov et al., 2013 propose an algorithm for estimation $F_{Y^i\langle 1|0,1\rangle}$:

1. Estimate $F_{\mathbf{c}_1}(\mathbf{c})$ by empirical CDF to obtain $\hat{F}_{\mathbf{c}_1}(\mathbf{c})$
2. Estimate $F_{m_0|c_0}(m|\mathbf{c})$ by logistic regression to obtain $\hat{F}_{m_0|c_0}(m|\mathbf{c})$

3. Estimate $F_{Y_1^i|m_1,\mathbf{c}_1}(y|m,\mathbf{c})$ by distribution regression (discussed below) to obtain $\hat{F}_{Y_1^i|m_1,\mathbf{c}_1}(y|m,\mathbf{c})$

4. Obtain $\hat{F}_{Y^i\langle 1|0,1\rangle}(y) = \int_{\mathcal{C}_1}\int_{\mathcal{M}_0}\hat{F}_{Y_t^i|m_1,\mathbf{c}_1}(y|m,\mathbf{c})d\hat{F}_{m_0|\mathbf{c}_0}(m|\mathbf{c})d\hat{F}_{\mathbf{c}_1}(\mathbf{c})$

While it is possible to estimate $\hat{F}_{Y_1|m_1,\mathbf{c}_1}$ using quantile regression, Chernozhukov et al. (2013) show quantile regression does not preform well when there are large point masses in the distribution being estimated, such as the wage or hours worked distributions.

Therefore, Chernozhukov et al. (2013) propose the *distribution regression*, a modification of the method proposed by Foresi and Peracchi (1995). For $n$ observations, $F_{Y_1|m_1,\mathbf{c}_1}(y|m,\mathbf{c})$ is estimated by

$$\hat{F}_{Y_1^i|\mathbf{X}_1}(y|\mathbf{X}) = \Lambda(P(\mathbf{X})\hat{\beta}(y)) \quad y \in \mathcal{Y}_i,$$

where $P(\cdot)$ is a vector of transformations of $\mathbf{X}$ (e.g. polynomials or basis splines), $\Lambda$ is the link function, and $\hat{\beta}(y)$ is estimated using maximum likelihood

$$\hat{\beta}(y) = \arg\max_b \sum_{j=1}^{n}\{\mathbb{1}\{Y_j^i \leq y\}\ln(P(\mathbf{X}_j)'b) + \mathbb{1}\{Y_j^i \geq y\}\ln((1-P(\mathbf{X}_j))'b)\}.$$

This paper uses a logit link function.

Once the marginals are estimated, the joint distribution is obtained through

$$\hat{F}_{\mathbf{Y}\langle t|s,v\rangle}(y_1,y_2) = \hat{C}(\hat{F}_{Y^1\langle t|s,v\rangle}(y_1),\hat{F}_{Y^2\langle t|s,v\rangle}(y_2)), \tag{8}$$

where $\hat{C}$ is a consistent estimate of the copula.

This paper uses an "empirical copula" developed by Deheuvels (1979), which is a nonparametric method. However if there are more than two or three outcomes of interest, to avoid the *Curse of Dimensionality*, a parametric copula should be used.

**Empirical Copula Method**: For observations $(Y_i^1, Y_i^2)$, $i = 1,...,n$ we have copula observations $(U_i^1, U_i^2) = (F_{Y^1\langle t|s,v\rangle}(Y_i^1), F_{Y^2\langle t|s,v\rangle}(Y_i^2))$. Therefore $(\hat{U}_i^1, \hat{U}_i^2) = (\hat{F}_{Y^1\langle t|s,v\rangle}(Y_i^1), \hat{F}_{Y^2\langle t|s,v\rangle}(Y_i^2))$ and

$$\hat{C}(u_1,u_2) = \frac{1}{n}\sum_{i=1}^{n}\mathbb{1}\{\hat{U}_i^1 \leq u_1, \hat{U}_i^2 \leq u_2\}$$

is the empirical copula.

A simple simulation of this method can be found in Appendix A.1.

## 3.5 Inference

Let $\hat{\Delta}_{y_1,y_2} := \hat{F}_{\mathbf{Y}\langle 1|0,1\rangle}(y_1,y_2) - \hat{F}_{\mathbf{Y}\langle 1|1,1\rangle}(y_1,y_2)$. The algorithm for uniform bootstrap confidence bands is:

1. Obtain bootstrap draws $\left(\hat{F}^{*(j)}_{\mathbf{Y}\langle 1|0,1\rangle}(y_1,y_2) - \hat{F}^{*(j)}_{\mathbf{Y}\langle 1|1,1\rangle}(y_1,y_2)\right)_{y_1\in T_1, y_2\in T_2}$ for $j = 1, ..., B$
2. For each $y_1 \in T_1$, $y_2 \in T_2$ compute bootstrap variance $\hat{s}^2(y_1,y_2) = B^{-1}\sum_{j=1}^{B}\left(\left(\hat{F}^{*(j)}_{\mathbf{Y}\langle 1|0,1\rangle}(y_1,y_2) - \hat{F}^{*(j)}_{\mathbf{Y}\langle 1|1,1\rangle}(y_1,y_2)\right) - \hat{\Delta}_{y_1,y_2}\right)^2$
3. Compute the critical value $c(1-\alpha) = (1-\alpha)$-quantile of

$$\left\{\max_{y_1\in T_1, y_2\in T_2}\left|\left(\hat{F}^{*(j)}_{\mathbf{Y}\langle 1|0,1\rangle}(y_1,y_2) - \hat{F}^{*(j)}_{\mathbf{Y}\langle 1|1,1\rangle}(y_1,y_2)\right) - \hat{\Delta}_{y_1,y_2}\right| \Big/ \hat{s}(y_1,y_2)\right\}_{j=1}^{B}$$

4. Construct confidence band for $\left(\hat{\Delta}_{y_1,y_2}\right)_{y_1\in T_1, y_2\in T_2}$ as $[L(y_1,y_2), U(y_1,y_2)] = \left[\hat{\Delta}_{y_1,y_2} \pm c(1-\alpha)\hat{s}(y_1,y_2)\right]$

Chernozhukov et al. (2013) show the validity of the Kolmogorov-Smirnov confidence bands obtained through the algorithm above; see Appendix AS of Chernozhukov et al. (2013) for details.

# 4 Data

## 4.1 Data Cleaning

This paper uses the Current Population Survey Merged Outgoing Rotation Group (CPS MORG) for the years 1979 to 2019 and was obtained through the NBER website.[20] CPS MORG is different than the March CPS[21] because survey participants in the CPS MORG extracts were asked about their hourly wage and hours worked from that week (as opposed to imputed weekly hours worked and hourly wage from yearly earnings and usual hours worked). These "point-in-time" measures are arguably more reliable because participants are more likely to accurately remember their hourly wage and hours worked for that week.[22]

The data are cleaned using the specifications of Autor et al. (2016) and state minimum wage data was obtained through Vaghul and Zipperer (2016).[23] Wages are in 2019 dollars and Consumer Price Indexes were obtained from Federal Reserve Bank of Minneapolis.[24] The sample includes individuals ages 18 through 64 and excludes those who are self-employed. Top-coded values are multiplied by 1.5, and the top two wage percentiles for each state, year, and

[20] Available at https://data.nber.org/morg/annual/.

[21] Commonly referred to as IPUM CPS, since it is maintained by the Minnesota Population center at the University of Minnesota.

[22] See Lemieux (2006).

[23] Available at https://github.com/benzipperer/historicalminwage/releases.

[24] Available at https://www.minneapolisfed.org/about-us/monetary-policy/inflation-calculator/consumer-price-index-1913-

sex grouping are "Winsorized" (replaced with the ninety-seventh percentile's value).[25]

However, unlike Autor et al. (2016), this paper uses CPS individual weights and does not multiply these weights by hours worked in the previous week. DiNardo et al. (1996) states, "These 'hours-weighted' estimates put more weight on the wages of workers who supply many hours to the labor market. This gives a better representation of the dispersion of wages for each and every hour worked in the labor market, regardless of who is supplying this hour". While this approach may have originally deviated from using weekly earnings as the dependent variable to account for labor market participation decisions, it makes welfare comparisons more difficult since there is no way of knowing whether individuals actually have higher wage and hours worked bundles (incomes). Additionally, it treats hours as exogenously given. This paper's approach accounts for both of these shortcomings.

Following DiNardo et al. (1996) and Chernozhukov et al. (2013), control variables included in this study are union status, marital status, race, an indicator for part-time worker, educational and experience dummy variables, occupation dummy variables, industry dummy variables, and Standard Metropolitan Statistical Area (SMSA).

Two exclusion restrictions are needed for identification in the selection models. As is common in the literature, this paper uses number of children under age five as an exclusion restriction that potentially affects labor force participation but is uncorrelated with hourly wage. Education variables are used as the exclusion restriction that potentially affects labor force participation but is uncorrelated with hours worked.

HHI data by industry in 2017 was obtained through the US Census Bureau.[26]

## 4.2 Data Visualization

Figure 4 shows real U.S. minimum wage over time. Throughout the 1980's virtually all states shared the declining real Federal minimum wage. Around the turn of the century, many states started adopting a minimum wage that was higher than the Federal minimum wage, causing more variation in minimum wage rates across the country. Figure 5 shows the states' minimum wages in 1979, 1999, and 2019. Table 1 shows the timeline of nominal Federal minimum wage increases.

Figure 6 shows the percentage of workers earning at or below the minimum wage. The percentage of men earning at or below the minimum wage is lower than that of women. Figure 9 shows the evolution of hourly wage and weekly hours worked by five quantile ranges for hourly wage. The gap between the top and bottom quantile ranges of hourly wages in the pooled sample widens over time but seems to be mainly driven by the gap between the top and bottom quantile ranges of hourly wages for women widening. Additionally, Figure 9

[25] See Autor et al. (2016) for more details on the data cleaning.
[26] Available at https://data.census.gov/.

shows the "backward bending" of the labor-leisure curve, i.e. low-wage workers work the fewest hours, mid-wage workers work the most hours, and high-wage workers work fewer hours than mid-wage workers. This is true in all samples, but is most prominent for women. Hours worked seem to be more volatile for workers at the bottom of the wage distribution.

Figure 7 shows the percentage of minimum wage workers who are part-time. Around half of minimum wage workers are part-time. Figure 8 shows the percentage of minimum wage workers who are paid by the hours. After 2000, between 60 and 70 percent of minimum wage workers were paid by the hour.

Figure 10 shows the HHI by industry defined by NAICS 2-digit industry codes. In formation and Utilities are more concentrated industries, while Construction and Health Care and Social Assistance are less concentrated. Figure 11 shows the number of minimum wage workers per industry in the sample, and Figure 12 shows the percentage of workers in Utilities, Information, Construction, and Health Care and Social Assistance.

# 5 Results

## 5.1 Joint Distribution Results

Table 2 shows the effect of the 1992 increase in minimum wage on the distribution of hours and wages for men, when compared to the 1989 minimum wage ($\hat{F}_{\mathbf{Y}_{\langle 1|1,1\rangle}} - \hat{F}_{\mathbf{Y}_{\langle 1|0,1\rangle}}$). Tables 3 through 7 show similar counterfactual effects for both men and women and for different comparison years. Since these tables are differences in CDFs, a way to interpret the tables is, for example in Table 2, there was a 10% increase in men earning up to \$14.12 per hour and working up to 40 hours per week in 1992 assuming a counterfactual minimum wage from 1989.

These tables suggest that, for both men and women, there are considerable spillover effects. Although minimum wage should only directly affect workers at the bottom of the wage distribution, the majority of the effects were on workers working 40 or more hours per week (the bulk of the labor force) and earning wages close to the median of the wage distribution. This would suggest a strong compensating wage differential; when wages of the lowest paid workers increase, employers have to pay all workers more to entice them to remain at their jobs because outside work options have improved.

Additionally, the increase in wages of all workers due to minimum wage increases means hours of medium-wage workers should increase and hours of high-wage workers should decrease.[27] This is present in Figures 2-7 as higher minimum wage caused workers at the top of the wage distribution to work fewer hours. Indeed, for women who generally work lower-paid jobs, there is a smaller negative effect of increased minimum wage on hours worked at the top of the wage distribution.

[27]See discussion of the backward bending nature of the wage curve in Section 1.

## 5.2 Robustness Checks

As stated in the introduction, the conditional mean methods in this section take either hours or wages as exogenously given. While their results cannot capture effects that vary by both wages and hours, they serve as a benchmark.

In this section, the paper redefines the minimum wage variable (MW) as the minimum wage in the region in which an individual works. Hence, MW varies by state and time, and therefore the years 2003 through 2019 with more variation in state minimum wage are used.

Breaking the sample into five buckets of highest to lowest hourly wages, Table 8 shows the results of the fixed effects models

$$\text{hours}_{its} = \alpha_s + \gamma_t + \delta^H X_i^H + \beta^H \text{MW}_{ts} + \epsilon_{its}^H,$$

and

$$\log(\text{wages})_{its} = \alpha_s + \gamma_t + \delta^W X_i^W + \beta^W S\text{MW}_{ts} + \epsilon_{its}^W,$$

where $\alpha_s$ and $\gamma_t$ are state and year fixed effects (respectively), $X_i^H$ and $X_i^W$ are vectors of control variables for hours worked and wages (respectively), and $\epsilon_{its}^H$ and $\epsilon_{its}^W$ are error terms. The results from Table 8 are similar to Tables 2 through 7 in that minimum wages seemed to either increase the hours worked or not be statistically significant at any wage bucket for both men and women. Furthermore, even the statistically significant results are not economically meaningful. For example, this table suggests a one dollar increase in minimum wage yields only a ten-minute increase in hours worked per week by men.

Table 9 estimates the model

$$\text{hours}_{its} = \alpha_s + \gamma_t + \delta X_i^H + \beta_1 \text{MW}_{ts} + \beta_2 \text{MW}_{ts}^2 + \epsilon_{its},$$

which allows for the effect of minimum wages to change based on the level of minimum wage. That is, the marginal effect of minimum wage is $\beta_1 + 2\beta_2 MW_{ts}$, and hence the point at which the effect changes from a negative to a positive or from a positive to a negative is $-\frac{\beta_1}{2\beta_2}$. Overall, it seems that any minimum wage up to \$15.90 increases the number of hours worked for men, whereas for women, any minimum wage above \$3.98 increases their number of hours worked.

Roth and Sant'Anna (2021) formally state criteria for when partitioning the data into buckets and running fixed effects regressions is valid.[28]

## 5.3 Effects on Minimum Wage Workers by industry

Consider the Heckman (1979) selection model (Type II Tobit model) for $i = 1, .., n$ observations

$$\begin{aligned} y_i &= X_i' \beta + \epsilon_i, \\ D_i &= \mathbb{1}\{Z_i' \gamma + \nu_i > 0\}, \end{aligned}$$

[28]They require a "parallel trends" condition on the CDF of the untreated potential outcomes.

with $\begin{pmatrix} \epsilon_i \\ \nu_i \end{pmatrix} \Big| X_i, Z_i \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma \\ \rho\sigma & 1 \end{pmatrix}\right)$, so that the variance of $\epsilon_i$ is $\sigma^2$ and the variance of $\nu_i$ is normalized to 1. Hence, for a random sample of the population $(X_i, Z_i, D_i, D_i y_i)$ with the outcome of interest $y_i$ only observed when $D_i = 1$

$$\begin{aligned} E[\epsilon_i | X_i, \text{sample selection rule}] &= E[\epsilon_i | X_i, D_i = 0] \\ &= E[\epsilon_i | X_i, Z_i'\gamma + \nu_i > 0] \\ &= E[\epsilon_i | X_i, \nu_i > -Z_i'\gamma]. \end{aligned}$$

Therefore,

$$E[y_i | X_i, \text{sample selection rule}] = X_i'\beta + E[\epsilon_i | \nu_i > -Z_i'\gamma].$$

Since $\epsilon_i$ and $\nu_i$ are assumed to be bivariate normal,

$$E[\epsilon_i | \nu_i > -Z_i'\gamma] = \sigma\lambda(Z_i'\gamma),$$

where $\lambda(Z_i'\gamma)$ is the inverse Mills ratio $\frac{\phi(Z_i'\gamma)}{\Phi(Z_i'\gamma)}$, $\phi(\cdot)$ is the PDF of a normal distribution, and $\Phi(\cdot)$ is the CDF of a normal distribution.[29] In that way, the inverse Mills ratio is a monotone decreasing function of the probability that the observation is selected into the sample and including it in the regression of $X_i$ on $y_i$ corrects for the omitted variable bias due to selection (Heckman's two-step procedure).

Table 10 shows the effects of minimum wage on minimum wage workers with state and year fixed effects and without correcting for employment effects. Minimum wage has no statistically significant effects on the hours of minimum wage workers but has a statistically significant positive effect on wages.

Tables 12 and 13 show the effects of minimum wage with and without accounting for employment effects by high- and low-concentrated industries, respectively. There are too few minimum wage workers working in these industries to get statistically significant results. Table 14 shows the effect of minimum wage on industries with many of minimum wage workers accounting for employment effects.

# 6 Conclusion

Considering multivariate outcomes heterogeneously affected by a policy is important since individuals could be affected positively in one dimension and negatively in another. This has been a common criticism of minimum wage literature when applied to US data—while there seems to be little employment effect due to small minimum wage increases and wages increase across the wage distribution, some workers might see their hours reduced. However, this paper finds

[29]See Heckman (1979) for details.

that increased minimum wage had positive effects on both hourly wage and hours worked for all individuals except at the very top of the wage distribution, where hours were reduced. A possible explanation for these findings is that not all industries where minimum wage workers are employed are competitive. This paper finds evidence for such a theory, since the effects of minimum wage policy on the hours of minimum wage workers differs by industry.

With richer data on firms, understanding the effects of minimum wage on different market concentrations would corroborate this papers' findings. Additionally, the method for estimating jointly determined outcomes presented in this paper can help shed light on many controversial policy proposals that cause "winners" and "losers" at different parts of the joint distribution of outcomes.

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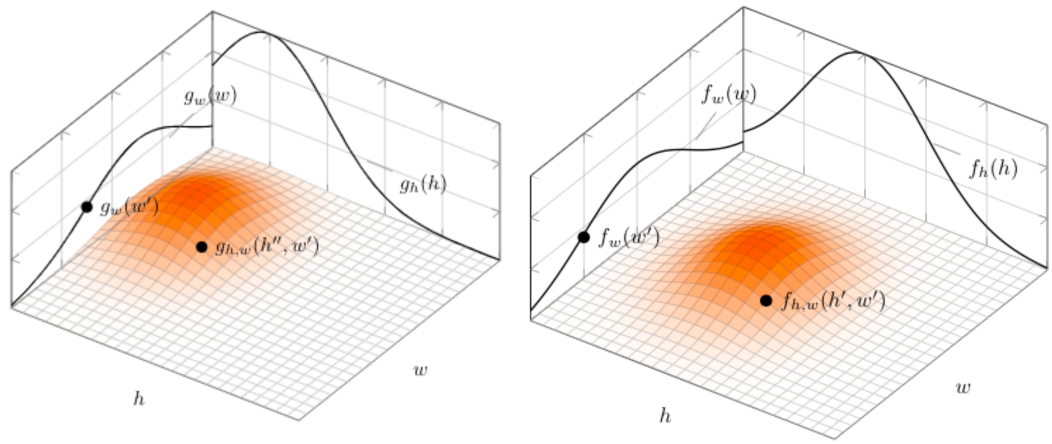
# Figures




(a) With Minimum Wage Change

(b) Without Minimum Wage Change

Figure 1: Let $g_{h,w}$ be the observed frequency distributions after a change in minimum wage for hours worked and hourly wage, with respective marginal distributions $g_h$ and $g_w$. Let $f$ be analogous for the counterfactual distribution had minimum wage not changed. $f_{h,w}(h', w')$ is the number of individuals making wage $w'$ and working $h'$ hours whereas $f_w(w')$ is the number of individuals making wage $w'$ for any number of hours worked. Clearly, just because every individual might earn higher wages with a minimum wage change — the CDFs $G_w(w) < F_w(w)$ for all $w \in \mathbb{R}$ — does not mean $g_{h,w}$ is "preferable" to $f_{h,w}$. Indeed, in this example, while there are fewer individuals working a low wage with the minimum wage change (i.e. $g(w') < f(w')$), those same individuals are working fewer hours (i.e. $f_{h,w}(h', w') > g_{h,w}(h'', w')$).

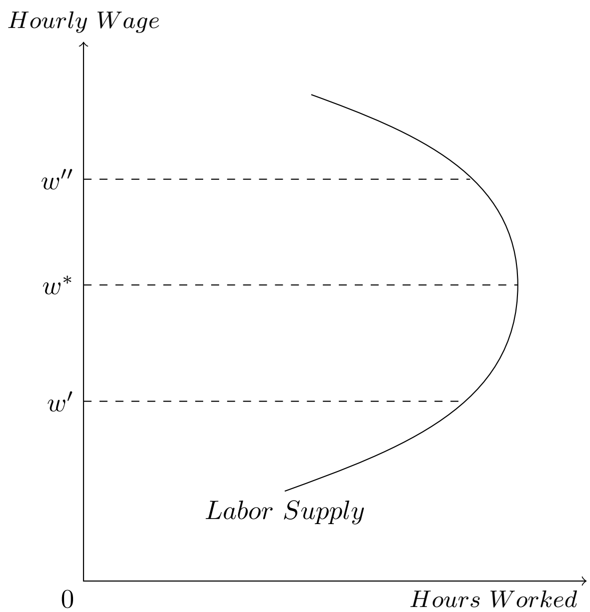
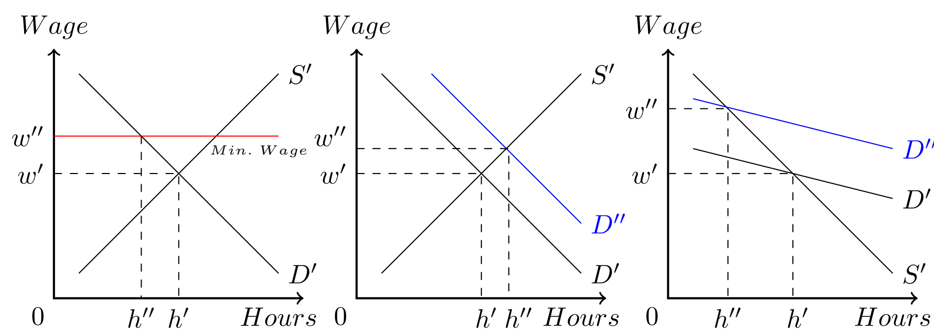



Figure 2: Backwards bending wage curve. As hourly wage increases from $w'$ to $w^*$ to $w''$, a worker's hours increase and then decrease.




(a) Min. Wage Workers. (b) Medium-Wage Workers. (c) High-Wage Workers.

Figure 3: Let $S'$ and $S''$ represent a worker's supply of hours before and after a minimum wage increase, respectively. Let $D'$ and $D''$ represent an employer's demand for hours of work before and after a minimum wage increase, respectively. In panel (a), when a minimum wage increase is binding, wages increase from $w'$ to $w''$, however hours worked decreases from $h'$ to $h''$. In panel (b), an increase in the price of minimum wage labor increases the demand for medium-wage laborers, thereby increasing both their wages and hours worked (assuming the two types of labor are substitutes). In panel (c), similarly, demand increases for high-wage workers, however since their supply is downward-sloping, wages increase but hours decrease.

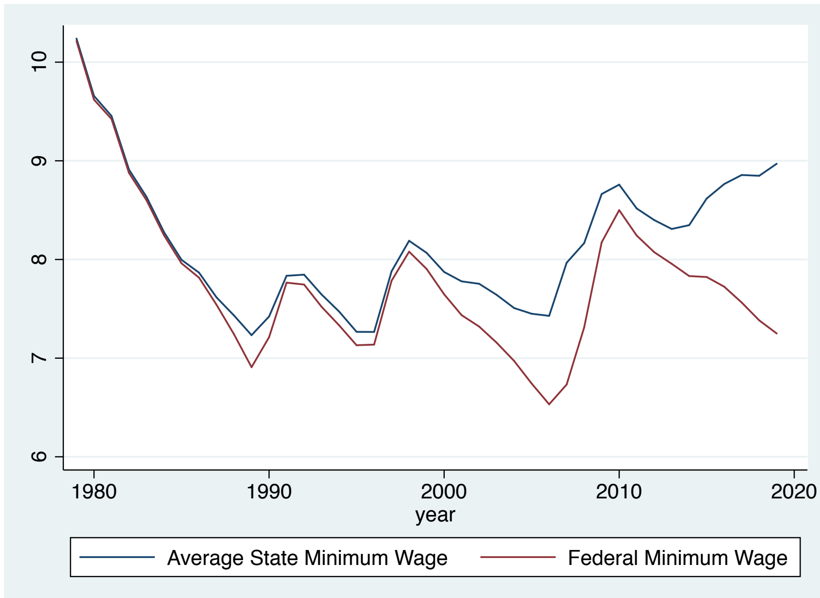
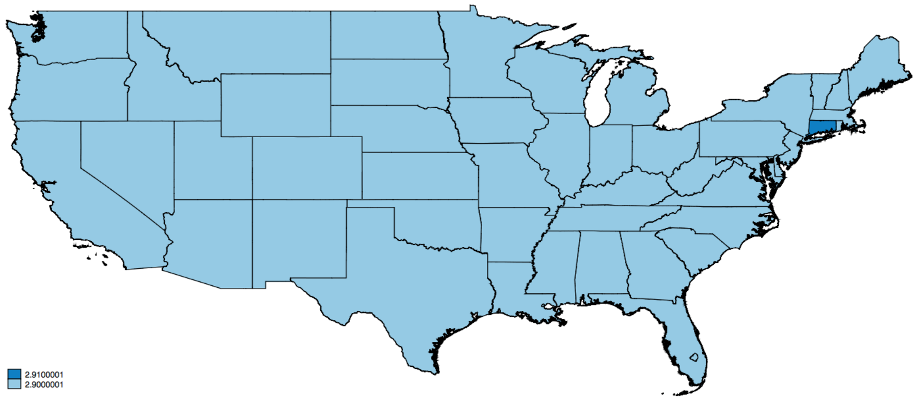
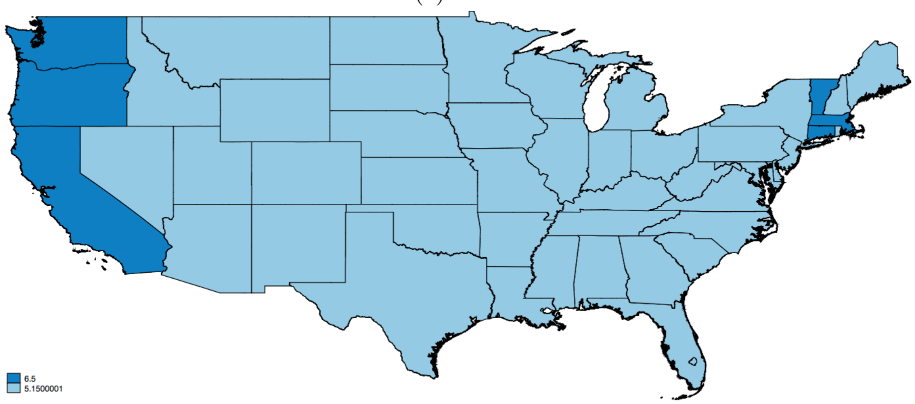


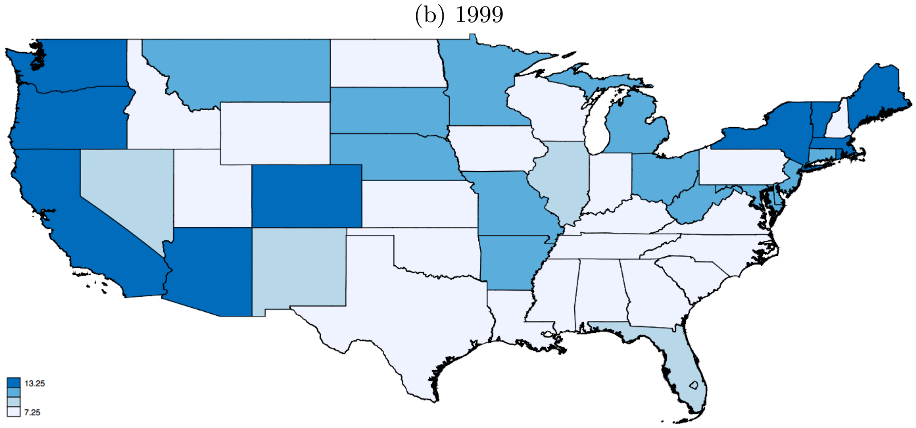

Figure 4: US Minimum Wage 1979-2019 in 2019 dollars




(a) 1979




(b) 1999




(c) 2019

Figure 5: Nominal State Minimum Wages in the US

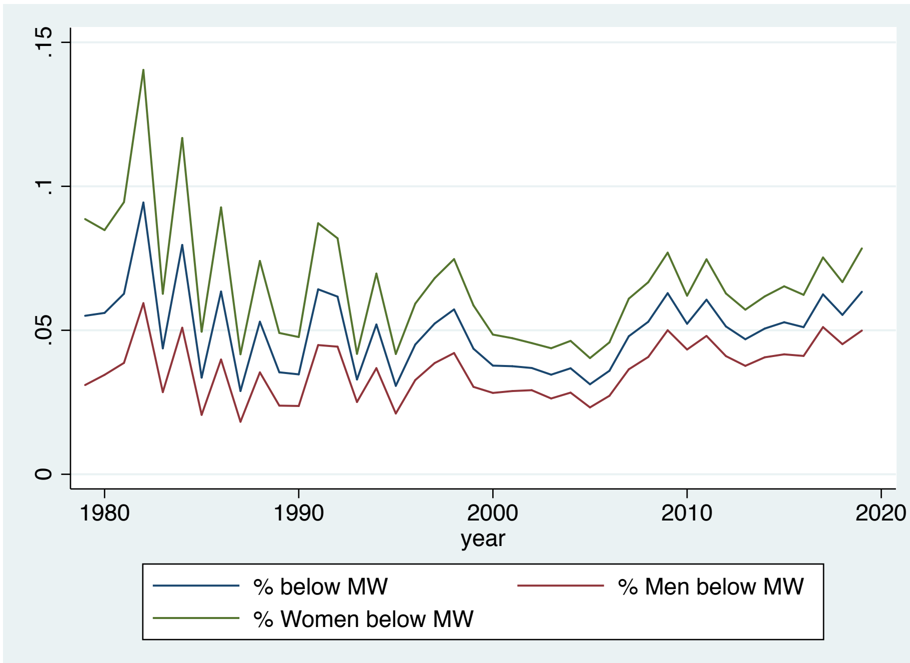



Figure 6: Percent of Worker earning at or Below Minimum Wage 1979-2019

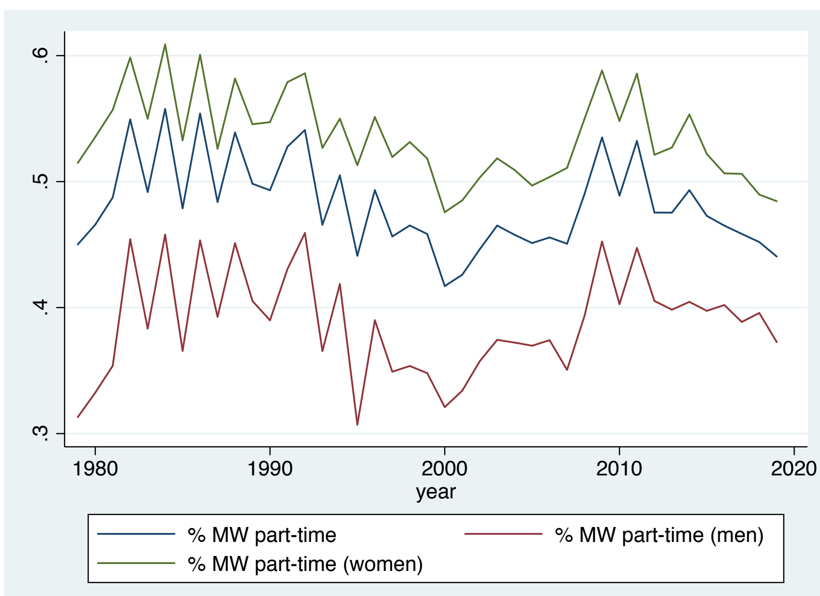



Figure 7: Percent of Minimum Wage Workers who are Part-time

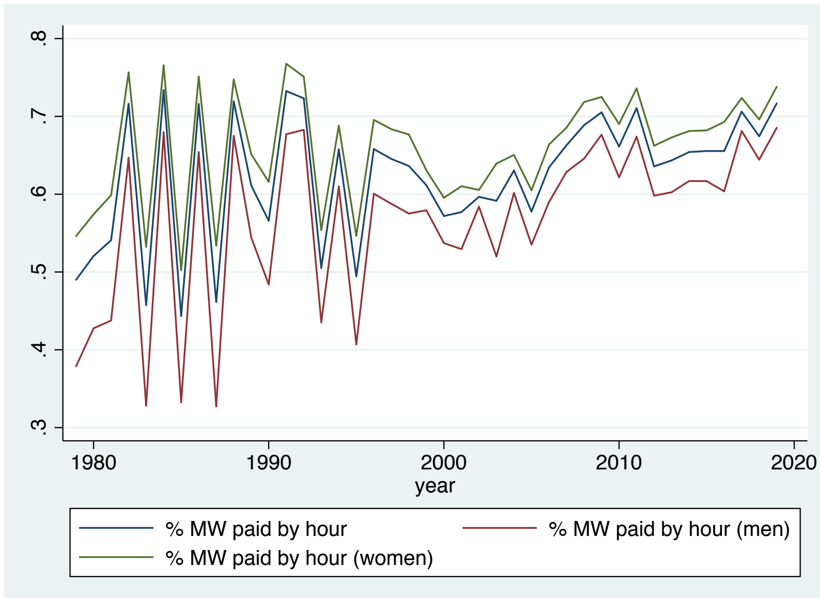
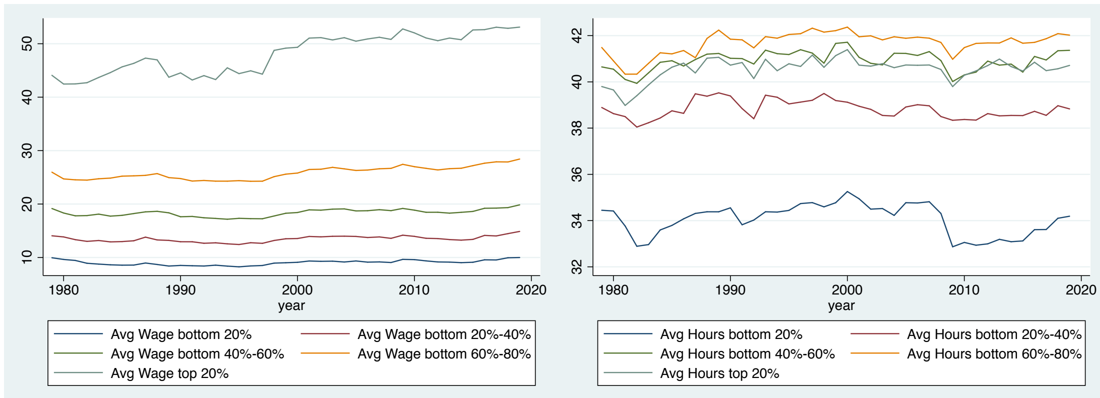
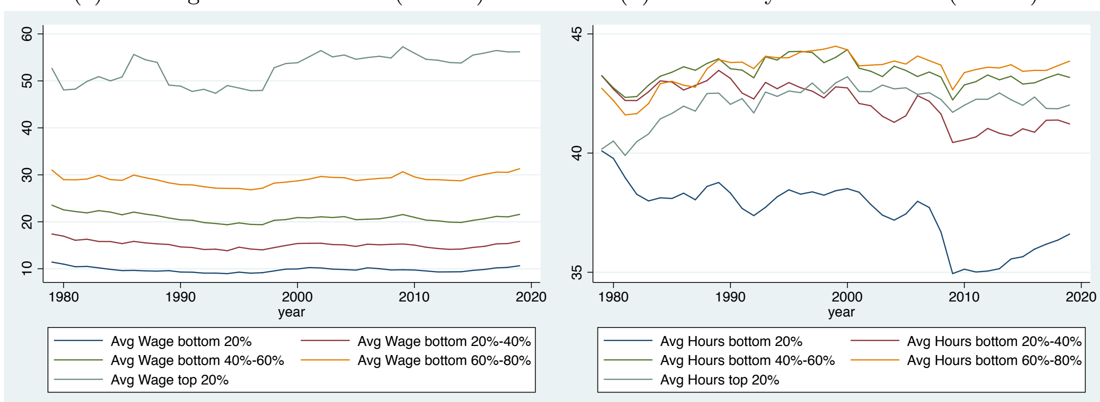



Figure 8: Percent of Minimum Wage Workers paid by the Hour



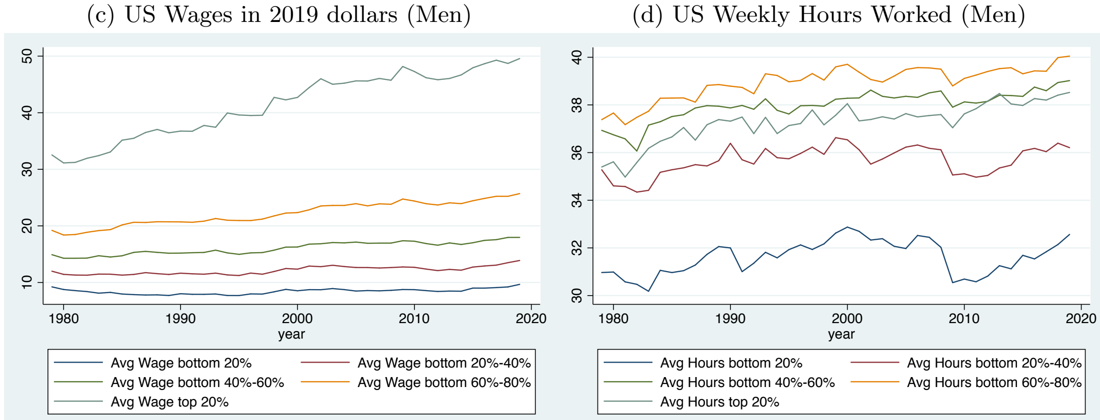

(a) US Wages in 2019 dollars (Pooled)

(b) US Weekly Hours Worked (Pooled)




(c) US Wages in 2019 dollars (Men)

(d) US Weekly Hours Worked (Men)




(e) US Wages in 2019 dollars (Women)

(f) US Weekly Hours Worked (Women)

Figure 9: Average Wages and Hours of 5 Quantiles Ranges of Wages 1979-2019

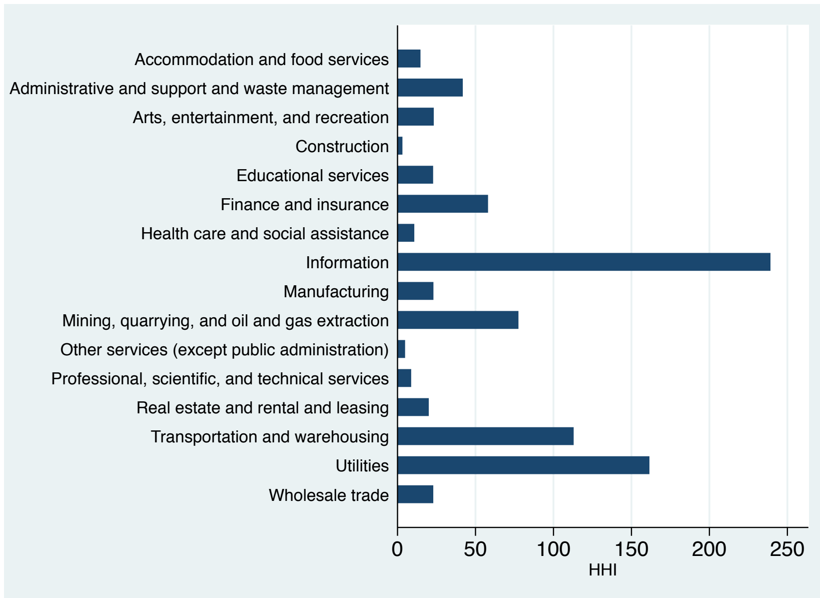
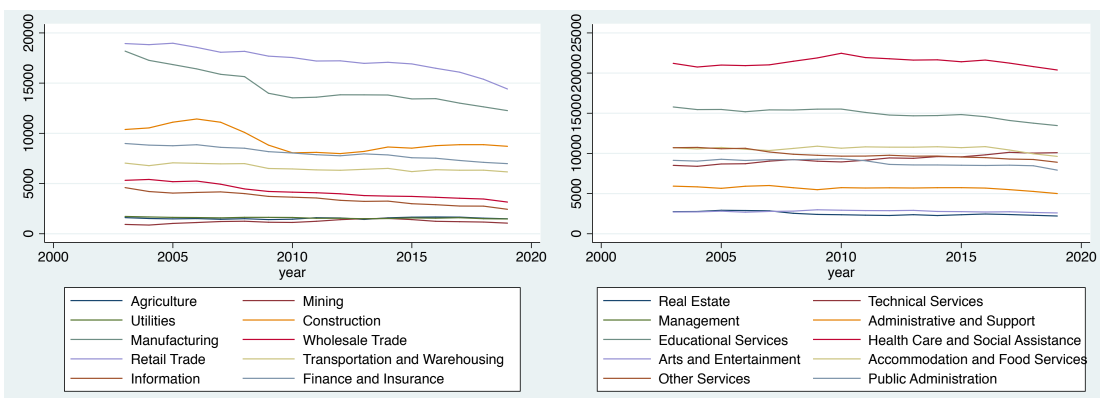
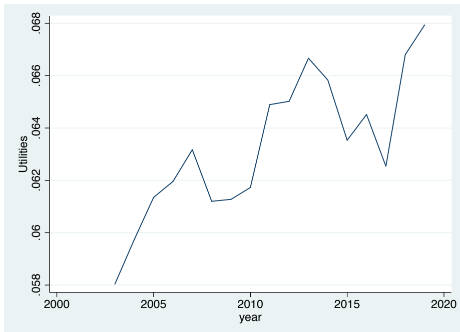


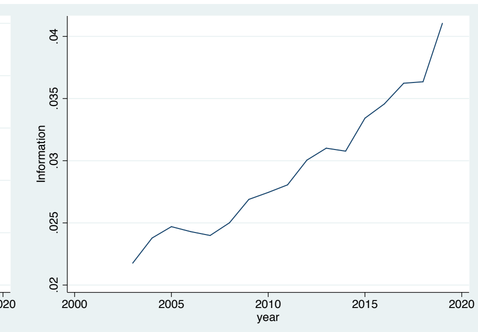

Figure 10: HHI by Industry, 2017



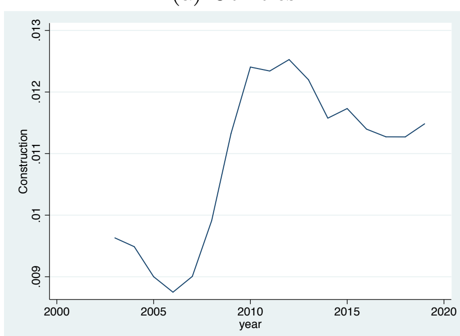

(a) (b)

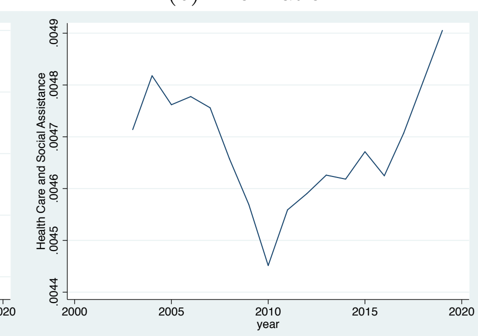
Figure 11: Number of Workers Paid Min. Wage by Industry, 2003-2019



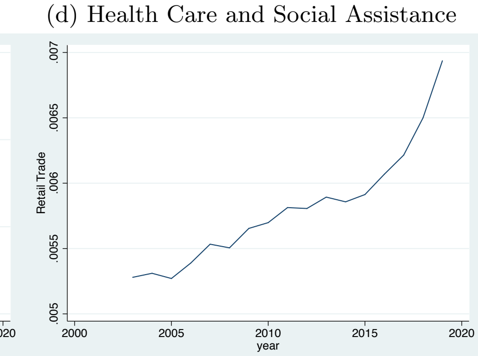




(a) Utilities (b) Information

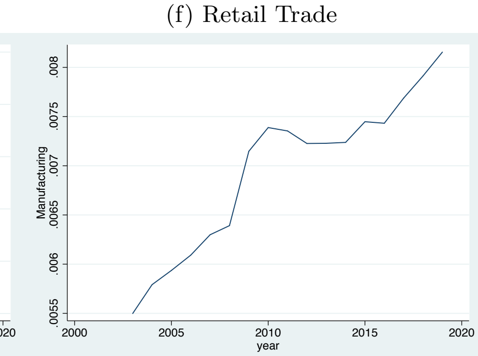



(c) Construction (d) Health Care and Social Assistance


(e) Accommodation and Food Services (f) Retail Trade


(g) Transportation and Warehousing (h) Manufacturing



Figure 12: Percent of Industry Paid Min. Wage, 2003-2019

# Tables

Table 1: Nominal Federal Minimum Wage Timeline

| Date | Minimum wage |
|---|---|
| January 1, 1979 | \$2.90 |
| January 1, 1980 | \$3.10 |
| April 1, 1990 | \$3.80 |
| April 1, 1991 | \$4.25 |
| October 1, 1996 | \$4.75 |
| September 1, 1997 | \$5.15 |
| July 24, 2007 | \$5.85 |
| July 24, 2008 | \$6.55 |
| July 24, 2009 | \$7.25 |

Note: Minimum wages are for for all covered nonexempt workers.
Source: https://www.dol.gov/agencies/whd/minimum-wage/history/chart

Table 2: Men 89-92

| Wages/Hours | 30 | 36 | 40 | 45 | 50 | 56 |
|---|---|---|---|---|---|---|
| 9.11 | 0.02* | 0.03* | 0.05* | 0.05* | 0.05* | 0.06* |
| 11.39 | 0.03* | 0.04* | 0.09* | 0.09* | 0.1* | 0.11* |
| 14.12 | 0.03* | 0.03* | 0.1* | 0.11* | 0.13* | 0.13* |
| 16.60 | 0.03* | 0.03* | 0.11* | 0.12* | 0.15* | 0.15* |
| 19.74 | 0.03* | 0.03* | 0.12* | 0.14* | 0.16* | 0.17* |
| 22.78 | 0.03* | 0.03* | 0.11* | 0.12* | 0.15* | 0.16* |
| 27.34 | 0.02* | 0.02* | 0.06* | 0.07* | 0.09* | 0.1* |
| 32.75 | 0.02 | 0.01 | 0 | 0.01 | 0.01 | 0.01 |
| 42.72 | 0.02 | 0 | −0.03* | −0.03* | −0.03* | −0.03* |

Note: Effect of having 1992's minimum wage on 1989's joint CDF
of wages and hours for men.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 3: Women 89-92

| Wages/Hours | 20 | 28 | 32 | 37 | 40 | 48 |
|---|---|---|---|---|---|---|
| 8.20 | 0.02* | 0.01* | 0.02* | 0.02* | 0.03* | 0.03* |
| 9.57 | 0.03* | 0.02* | 0.04* | 0.04* | 0.06* | 0.06* |
| 11.39 | 0.03* | 0.03* | 0.05* | 0.04* | 0.08* | 0.09* |
| 13.21 | 0.04* | 0.04* | 0.06* | 0.06* | 0.13* | 0.13* |
| 15.16 | 0.04* | 0.03* | 0.06* | 0.05* | 0.11* | 0.12* |
| 17.84 | 0.04* | 0.02* | 0.05* | 0.03* | 0.08* | 0.08* |
| 20.50 | 0.04* | 0.01* | 0.05* | 0.01* | 0.03* | 0.03* |
| 25.06 | 0.04* | 0.01* | 0.05* | 0.01 | 0.01 | 0.01 |
| 32.57 | 0.04* | 0.01* | 0.06* | -0.01 | −0.02* | -0.03* |

Note: Effect of having 1992's minimum wage on 1989's joint CDF of wages and hours for women.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 4: Men 89-84

| Wages/Hours | 30 | 40 | 44 | 49 | 55 |
|---|---|---|---|---|---|
| 9.84 | 0.02* | 0.05* | 0.05* | 0.05* | 0.06* |
| 12.31 | 0.03* | 0.09* | 0.09* | 0.1* | 0.11* |
| 15.38 | 0.03* | 0.12* | 0.13* | 0.14* | 0.16* |
| 18.46 | 0.03* | 0.16* | 0.17* | 0.2* | 0.22* |
| 21.53 | 0.03* | 0.15* | 0.16* | 0.19* | 0.21* |
| 24.61 | 0.02* | 0.14* | 0.15* | 0.18* | 0.19* |
| 28.62 | 0.02* | 0.04* | 0.05* | 0.06* | 0.05 |
| 33.2 | 0.02 | 0.01 | 0.01 | 0.02 | 0 |
| 42.82 | 0.02 | −0.03* | −0.03* | −0.03* | −0.05* |

Note: Effect of having 1984's minimum wage on 1989's joint CDF of wages and hours for men.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 5: Women 89-84

| Wages/Hours | 20 | 26 | 32 | 38 | 40 | 45 |
|---|---|---|---|---|---|---|
| 8.24 | 0.02* | 0.03 | 0.03 | 0.04 | 0.05 | 0.05 |
| 9.84 | 0.03* | 0.02* | 0.03* | 0.05* | 0.07* | 0.07* |
| 11.07 | 0.04* | 0.03* | 0.04* | 0.06* | 0.1* | 0.1* |
| 12.31 | 0.04* | 0.03* | 0.04* | 0.06* | 0.11* | 0.11* |
| 14.77 | 0.05* | 0.03* | 0.05* | 0.08* | 0.15* | 0.17* |
| 16.55 | 0.04* | 0.02* | 0.03* | 0.06* | 0.12* | 0.13* |
| 19.10 | 0.04* | 0.01* | 0.01 | 0.04* | 0.07* | 0.08* |
| 22.76 | 0.04* | 0 | 0 | 0.03* | 0.03* | 0.04* |
| 28.61 | 0.04 | $-0.004^*$ | -0.01 | 0.02* | -0.01 | -0.01 |

Note: Effect of having 1984's minimum wage on 1989's joint CDF of wages and hours for women.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 6: Men 06-12

| Wages/Hours | 28 | 37 | 40 | 48 | 55 |
|---|---|---|---|---|---|
| 9.58 | 0.02* | 0.03* | 0.05* | 0.05* | 0.06* |
| 11.3 | 0.04* | 0.06* | 0.11* | 0.12* | 0.13* |
| 13.92 | 0.03* | 0.06* | 0.15* | 0.16* | 0.17* |
| 16.71 | 0.03* | 0.06* | 0.19* | 0.21* | 0.22* |
| 20.05 | 0.02* | 0.05* | 0.2* | 0.23* | 0.25* |
| 23.56 | 0.02* | 0.04* | 0.19* | 0.22* | 0.24* |
| 28.33 | 0.01* | 0.03* | 0.16* | 0.19* | 0.2* |
| 35.69 | 0.01 | 0.02* | 0.09* | 0.11* | 0.11* |
| 49.42 | 0 | 0.01 | 0 | 0 | -0.02 |

Note: Effect of having 2012's minimum wage on 2006's joint CDF of wages and hours for men.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 7: Women 06-12

| Wages/Hours | 20 | 29 | 35 | 40 | 48 |
|---|---|---|---|---|---|
| 8.91 | 0.02* | 0.02* | 0.04* | 0.05* | 0.05* |
| 10.30 | 0.04* | 0.04* | 0.07* | 0.1* | 0.1* |
| 11.97 | 0.04* | 0.04* | 0.07* | 0.12* | 0.12* |
| 13.95 | 0.05* | 0.04* | 0.08* | 0.15* | 0.15* |
| 16.71 | 0.05* | 0.04* | 0.08* | 0.19* | 0.2* |
| 19.36 | 0.05* | 0.03* | 0.07* | 0.17* | 0.18* |
| 23.36 | 0.04* | 0.02 | 0.06* | 0.13* | 0.14* |
| 28.96 | 0.04* | 0.01 | 0.05* | 0.07* | 0.07* |
| 39.64 | 0.04* | 0 | 0.04* | 0.01 | 0 |

Note: Effect of having 2012's minimum wage on 2006's joint CDF of wages and hours for women.
$*$ = 95% confidence level using 50 bootstrap samples.

Table 8: Effect of Minimum Wage with State and Year Fixed Effects, 2003-2019

| | Hours Worked | | | | log(Wages) | | | |
|---|---|---|---|---|---|---|---|---|
| | Men | | Women | | Men | | Women | |
| Total | 0.168*** | (0.059) | 0.34*** | (0.047) | 0.018*** | (0.003) | 0.015*** | (0.003) |
| Bottom %20 | -0.043 | (0.186) | 0.229** | (0.112) | 0.01 | (0.007) | 0.008 | (0.005) |
| %20-%40 | 0.266* | (0.154) | 0.322*** | (0.103) | 0.003** | (0.001) | 0.001 | (0.001) |
| %40-%60 | 0.142 | (0.132) | 0.334*** | (0.098) | 0.002** | (0.001) | 0.001 | (0.001) |
| %60-%80 | 0.208* | (0.119) | 0.276*** | (0.099) | 0.001 | (0.001) | 0 | (0.002) |
| Top %20 | 0.257** | (0.101) | 0.52*** | (0.113) | 0.001 | (0.003) | 0.005 | (0.004) |

Note: These tables represent regressions of minimum wage on hours or log(wage) with state and year fixed effects for different buckets of wage earners. Standard error in parenthesis. $^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 9: Effect of Minimum Wage on Hours Worked, 2003-2019

| Men | | | | | |
|---|---|---|---|---|---|
| | $\hat{\beta}_1$ | | $\hat{\beta}_2$ | | $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ |
| Total | 0.356 | (0.361) | -0.011 | (0.021) | 15.67 |
| Bottom %20 | 0.338 | (1.26) | -0.023 | (0.075) | 7.404 |
| %20-%40 | 0.188 | (0.999) | 0.005 | (0.06) | -19.933 |
| %40-%60 | -2.162** | (0.907) | 0.141** | (0.055) | 7.652 |
| %60-%80 | 1.565** | (0.744) | -0.083* | (0.045) | 9.455 |
| Top %20 | 1.098* | (0.565) | -0.05 | (0.033) | 11.076 |

| Women | | | | | |
|---|---|---|---|---|---|
| | $\hat{\beta}_1$ | | $\hat{\beta}_2$ | | $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ |
| Total | -0.316 | (0.287) | 0.04** | (0.017) | 3.984 |
| Bottom %20 | -0.387 | (0.814) | 0.038 | (0.05) | 5.073 |
| %20-%40 | -2.129*** | (0.68) | 0.152*** | (0.042) | 7.023 |
| %40-%60 | 1.015 | (0.648) | -0.042 | (0.039) | 12.196 |
| %60-%80 | -1.204** | (0.576) | 0.089*** | (0.034) | 6.775 |
| Top %20 | 0.961 | (0.592) | -0.025 | (0.033) | 19.106 |

Note: These tables represent regressions of minimum wage and minimum wage squared on hours with state and year fixed effects for different buckets of wage earners. Standard error in parenthesis. $^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$

Table 10: Minimum Wage Workers, 2003-2019

| | Hours | | | | log(Wages) | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\hat{\beta}_1$ | 0.5 | 0.38 | 0.13 | -1.45 | 0.11*** | 0.17*** |
| | (0.41) | (0.37) | (0.25) | (1.41) | (0.04) | (0.02) |
| $\hat{\beta}_2$ | - | -0.01 | - | 0.09 | - | - |
| | - | (0.02) | - | (0.08) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 14 | - | 8.13 | - | - |
| State and Year Fixed Effects | Y | Y | Y | Y | Y | Y |
| Additional Controls | Y | Y | Y | Y | Y | Y |
| $R^2$ | 0.57 | 0.42 | 0.6 | 0.6 | 0.09 | 0.07 |
| $N$ | 40157 | 1084675 | 64289 | 64289 | 40157 | 77464 |

Note: These tables represent regressions of minimum wage (and minimum wage squared) on hours or log(wage) with state and year fixed effects for minimum wage workers. Columns 1, 2, and 5 are regressions coefficients when only male observations are used and the remaining columns are regressions coefficients when only female observations are used. $N$ is the number of observations in per state-year group. Standard errors are in parentheses.
$^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$

Table 11: Effect of Minimum Wage on Part-time Worker Status, 2003-2019

| | (1) LPM | (2) Logit | (3) Probit | (4) Heckprobit | (5) LPM | (6) Logit | (7) Probit | (8) Heckprobit |
|---|---|---|---|---|---|---|---|---|
| Min. Wage | 0.001 | 0.011 | 0.007 | $-0.052^{***}$ | $0.01^{***}$ | $0.046^{***}$ | $0.028^{***}$ | $-0.02^{***}$ |
| | (0.002) | (0.01) | (0.006) | (0.006) | (0.002) | (0.007) | (0.004) | (0.005) |
| $\hat{\rho}$ | - | - | - | $0.216^{***}$ | - | - | - | $0.214^{***}$ |
| | - | - | - | (0.045) | - | - | - | (0.034) |
| Add. Controls | Y | Y | Y | Y | Y | Y | Y | Y |
| $R^2$/pseudo-$R^2$ | 0.2 | 0.16 | 0.16 | - | 0.11 | 0.08 | 0.08 | - |
| $N$ | 40157 | 40157 | 40157 | 120184 | 64289 | 64289 | 64289 | 150552 |

Note: These tables represent regressions of minimum wage on part-time worker status for minimum wage workers.Columns 1-4 are for men and Columns 5-8 are work women. LPM stands for linear probability model. $\hat{\rho}$ is an estimate of the employment effect in a Heckman probit selection model. Robust standard errors are in parentheses. $^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$

Table 12: Effects of Min. Wage on Min. Wage Workers by Industry, 2003-2019

| | High-Concentrated Industries | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Information | | | | | | Utilities | | | | | |
| | Hours | | | | log(Wages) | | Hours | | | | log(Wages) | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| $\hat{\beta}_1$ | -6.23 | 191.44*** | -4.04 | -21.16 | 0.38 | 0.8 | -6.7 | -66.67 | 0.95 | 160.16 | -0.13 | 0.21 |
| | (4.2) | (53.21) | (3.87) | (26.9) | (0.5) | (0.5) | (4.88) | (46.18) | (8.05) | (152.63) | (0.27) | (0.5) |
| $\hat{\beta}_2$ | - | −11.12*** | - | 0.82 | - | - | - | 3.79 | - | -9.4 | - | - |
| | - | (2.98) | - | (1.28) | - | - | - | (2.9) | - | (9) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 8.61 | - | 12.88 | - | - | - | 8.8 | - | 8.52 | - | - |
| State and Year Fixed Effects | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Additional Controls | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| $R^2$ | 0.48 | 0.08 | 0.58 | 0.5 | 0.14 | 0.09 | 0.41 | 0.39 | 0.53 | 0.22 | 0.01 | 0.04 |
| $N$ | 720 | 720 | 678 | 678 | 720 | 678 | 1108 | 1108 | 449 | 449 | 1108 | 449 |
| | Low-Concentrated Industries | | | | | | | | | | | |
| | Construction | | | | | | Health Care and Social Assistance | | | | | |
| | Hours | | | | log(Wages) | | Hours | | | | log(Wages) | |
| | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) |
| $\hat{\beta}_1$ | 3 | 25.82** | 11.44 | 10.77 | 0.11 | 0.84 | -1.27 | -9.86 | -1.03 | −10.92*** | -0.29 | 0.21*** |
| | (2.16) | (10.99) | (8.29) | (31.36) | (0.22) | (1.08) | (3.19) | (20.25) | (0.76) | (4.17) | (0.31) | (0.07) |
| $\hat{\beta}_2$ | - | −1.17** | - | 0.04 | - | - | - | 0.46 | - | 0.56** | - | - |
| | - | (0.55) | - | (1.6) | - | - | - | (1.07) | - | (0.23) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 11 | - | -152.1 | - | - | - | 10.7 | - | 9.78 | - | - |
| State and Year Fixed Effects | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| Additional Controls | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| $R^2$ | 0.36 | 0.3 | 0.23 | 0.23 | 0.12 | 0.04 | 0.59 | 0.59 | 0.56 | 0.55 | 0.02 | 0.12 |
| $N$ | 1908 | 1908 | 377 | 377 | 1908 | 377 | 2412 | 2412 | 10153 | 10153 | 2412 | 10153 |

Note: These tables represent regressions of minimum wage (and minimum wage squared) on hours or log(wage) with state and year fixed effects for minimum wage workers in different industries. Columns 1, 2, 5, 7, 8, 11, 13, 14, 17, 19, 20, and 23 are regressions coefficients when only male observations are used and the remaining columns are are regressions coefficients when only female observations are used. $N$ is the number of observations in per state-year group. Standard errors are in parentheses. $^{*}p < 0.10, ^{**} p < 0.05, ^{***} p < 0.01$

Table 13: Effects of Min. Wage on Min. Wage Workers by Industry with Employment Effects, 2003-2019

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | High-Concentrated Industries | | | | | | | | | | | |
| | Information | | | | | | Utilities | | | | | |
| | Hours | | | | log(Wages) | | Hours | | | | log(Wages) | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| $\hat{\beta}_1$ | 0.27 | 0.84 | 0.41 | 4.82** | 0.25*** | 0.21*** | 0.06 | 6.05* | 0.6 | -3.81 | 0.28*** | 0.23*** |
| | (0.27) | (2.45) | (0.26) | (2.15) | (0.05) | (0.04) | (0.32) | (3.7) | (0.42) | (5.24) | (0.03) | (0.05) |
| $\hat{\beta}_2$ | - | -0.03 | - | $-0.24^{**}$ | - | - | - | -0.36 | - | 0.26 | - | - |
| | - | (0.13) | - | (0.11) | - | - | - | (0.22) | - | (0.31) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 13.67 | - | 10.2 | - | - | - | 8.46 | - | 7.25 | - | - |
| Inverse Mills Ratio | -1.23 | -1.22 | 1.58 | 1.56 | 0.44** | 0.4** | 3.29*** | 3.29*** | 4.36*** | 4.35*** | 0.09 | -0.02 |
| | (1.44) | (1.44) | (1.54) | (1.53) | (0.19) | (0.18) | (1.1) | (1.1) | (1.39) | (1.39) | (0.1) | (0.12) |
| $N$ | 2816 | 2816 | 2361 | 2361 | 2314 | 1996 | 4313 | 4313 | 1831 | 1831 | 3502 | 1529 |
| $N$-Censored | 1585 | 1585 | 1304 | 1304 | 1585 | 1304 | 2383 | 2383 | 1060 | 1060 | 2383 | 1060 |
| | Low-Concentrated Industries | | | | | | | | | | | |
| | Construction | | | | | | Health Care and Social Assistance | | | | | |
| | Hours | | | | log(Wages) | | Hours | | | | log(Wages) | |
| | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) |
| $\hat{\beta}_1$ | 0.16 | 0.18 | 0.49 | 6.25** | 0.19*** | 0.24*** | 0.64*** | -0.06 | 0.8*** | 2.6*** | 0.19*** | 0.19*** |
| | (0.17) | (1.8) | (0.34) | (3.03) | (0.02) | (0.05) | (0.15) | (1.26) | (0.07) | (0.57) | (0.02) | (0.01) |
| $\hat{\beta}_2$ | - | -0.001 | - | $-0.31^{*}$ | - | - | - | 0.04 | - | $-0.1^{***}$ | - | - |
| | - | (0.1) | - | (0.16) | - | - | - | (0.07) | - | (0.03) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 75.13 | - | 9.95 | - | - | - | 0.73 | - | 12.87 | - | - |
| Inverse Mills Ratio | 1.66 | 1.66** | 3.16** | 3.31 | 0.11 | 0.06 | 12.2*** | 12.21*** | 11.33*** | 11.31*** | 0.1 | 0.05 |
| | (0.71) | (0.71) | (2.07) | (2.07) | (0.08) | (0.25) | (0.96) | (0.96) | (0.47) | (0.47) | (0.1) | (0.04) |
| $N$ | 15690 | 15690 | 1847 | 1847 | 14791 | 1590 | 8381 | 8381 | 31879 | 31879 | 5302 | 21646 |
| $N$-Censored | 12858 | 12858 | 1208 | 1208 | 12858 | 1208 | 2848 | 2848 | 11217 | 11217 | 2848 | 11217 |

Note: These tables represent regressions of minimum wage (and minimum wage squared) on hours or log(wage) on minimum wage workers with Heckman correction for employment effects by industry. Columns 1, 2, 5, 7, 8, 11, 13, 14, 17, 19, 20, and 23 are regressions coefficients when only male observations are used and the remaining columns are regressions coefficients when only female observations are used. $N$ is the number of observations, and $N - N$-Censored is the number of uncensored observations. Standard errors are in parentheses. $^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$

Table 14: Effects of Min. Wage on Min. Wage Workers by Industry with Employment Effects, 2003-2019

| | Accommodation and Food Services | | | | | | | Retail Trade | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Hours | | | | log(Wages) | | | Hours | | | log(Wages) | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| $\hat{\beta}_1$ | $-0.49^{***}$ | $-2.95^{***}$ | $-0.37^{***}$ | $-1.75^{**}$ | $0.17^{***}$ | $0.19^{***}$ | $-0.24^{***}$ | $3.85^{***}$ | $-0.39^{***}$ | 1.54 | $0.17^{***}$ | $0.15^{***}$ |
| | (0.11) | (1.07) | (0.08) | (0.77) | (0.01) | (0.005) | (0.15) | (1.41) | (0.13) | (1.17) | (0.01) | (0.01) |
| $\hat{\beta}_2$ | | $0.13^{**}$ | | $0.07^{*}$ | | | | $-0.21^{***}$ | | $-0.1^{*}$ | | |
| | - | (0.06) | - | (0.04) | - | - | - | (0.07) | - | (0.06) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 11.52 | - | 12.01 | - | - | - | 9.03 | - | 7.75 | - | - |
| Inverse Mills Ratio | $-8.7^{***}$ | $-8.85^{***}$ | $-5.68^{***}$ | $-5.74^{***}$ | 0.02 | 0.05 | $-7.28^{***}$ | $-7.05^{***}$ | $-6.1^{***}$ | $-5.96^{***}$ | $0.17^{***}$ | $0.17^{***}$ |
| | (0.88) | (0.88) | (0.67) | (0.67) | (0.04) | (0.03) | (0.98) | (0.98) | (0.84) | (0.85) | (0.05) | (0.04) |
| $N$ | 10434 | 10434 | 16491 | 16491 | 10103 | 16067 | 8171 | 8171 | 8887 | 8887 | 7430 | 8346 |
| $N$-Censored | 3494 | 3494 | 4591 | 4591 | 3494 | 4591 | 3662 | 3662 | 3645 | 3645 | 3662 | 3645 |
| | Transportation and Warehousing | | | | | | | Manufacturing | | | | |
| | Hours | | | | log(Wages) | | | Hours | | | log(Wages) | |
| | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) |
| $\hat{\beta}_1$ | 0.26 | $4.12^{***}$ | $0.61^{***}$ | $3.35^{**}$ | $0.16^{***}$ | $0.18^{***}$ | 0.24 | 1.86 | $0.55^{***}$ | $4.91^{***}$ | $0.32^{***}$ | $0.2^{***}$ |
| | (0.16) | (1.46) | (0.15) | (1.44) | (0.01) | (0.01) | (0.16) | (1.5) | (0.17) | (1.73) | (0.03) | (0.02) |
| $\hat{\beta}_2$ | | $-0.21^{***}$ | | $-0.16^{*}$ | | | | -0.09 | | $-0.24^{***}$ | | |
| | - | (0.08) | - | (0.08) | - | - | - | (0.08) | - | (0.09) | - | - |
| $-\frac{\hat{\beta}_1}{2\hat{\beta}_2}$ | - | 9.87 | - | 10.77 | - | - | - | 10.39 | - | 10.27 | - | - |
| Inverse Mills Ratio | 0.36 | 0.43 | $-1.77^{**}$ | $-1.65^{**}$ | $0.14^{***}$ | 0.03 | $8.73^{***}$ | $8.73^{***}$ | $5.37^{***}$ | $5.55^{***}$ | 0.15 | 0.06 |
| | (0.91) | (0.91) | (0.78) | (0.78) | (0.06) | (0.04) | (0.67) | (0.67) | (0.92) | (0.93) | (0.12) | (0.09) |
| $N$ | 10766 | 10766 | 13002 | 13002 | 9625 | 12083 | 11112 | 11112 | 7143 | 7143 | 9622 | 6437 |
| $N$-Censored | 5655 | 5655 | 6690 | 6690 | 5655 | 6690 | 7548 | 7548 | 4441 | 4441 | 7548 | 4441 |

Note: These tables represent regressions of minimum wage (and minimum wage squared) on hours or log(wage) on minimum wage workers with Heckman correction for employment effects by industry. Columns 1, 2, 5, 7, 8, 11, 13, 14, 17, 19, 20, and 23 are regressions coefficients when only male observations are used and the remaining columns are regressions coefficients when only female observations are used. $N$ is the number of observations, and $N - N$-Censored is the number of uncensored observations. Standard errors are in parentheses. $^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

# Appendix

## A.1. Simulation

With a sample size of 3000, let

$$\begin{aligned} x &\sim N(10, 3), \\ z &\sim N(2, 12), \\ \epsilon_1 &\sim N(0, 4), \\ \epsilon_2 &\sim N(0, 4), \\ y^1 &= x + \epsilon_1, \\ y^2 &= x + z + \epsilon_2. \end{aligned}$$

Hence

$$\begin{aligned} y^1 &\sim N(10, 5) \\ y^2 &\sim N(12, 13). \end{aligned}$$

Figures A.1.1 and A.1.2 show histograms of quantiles of $\hat{F}_{y^1}$ and $\hat{F}_{y^1}$ obtained by running distribution regressions with logit link functions of $x$ on $y^1$ and $x, z$ on $y^2$, respectively. Table A.1.1 shows the difference between the estimated joint distribution, $\hat{F}_{y^1,y^2}$, obtained by empirical copula and the empirical CDF. Table A.1.2 shows $c(0.75) - c(0.25)$, i.e. 0.75-quantile-0.25-quantile of

$$\left\{ \max_{y_1 \in T_1, y_2 \in T_2} \left| \left( \hat{F}^{*(j)}_{\mathbf{Y}}(y_1, y_2) - \hat{F}^{*(j)}_{\mathbf{Y}}(y_1, y_2) \right) - \left( \hat{F}^{(j)}_{\mathbf{Y}}(y_1, y_2) - \hat{F}^{(j)}_{\mathbf{Y}}(y_1, y_2) \right) \right| \Big/ \hat{s}(y_1, y_2) \right\}_{j=1}^{B},$$

which should roughly be a table of 2's if the t-ratio is approximately normally distributed.

Both the point estimates and confidence bands seem to be working correctly.

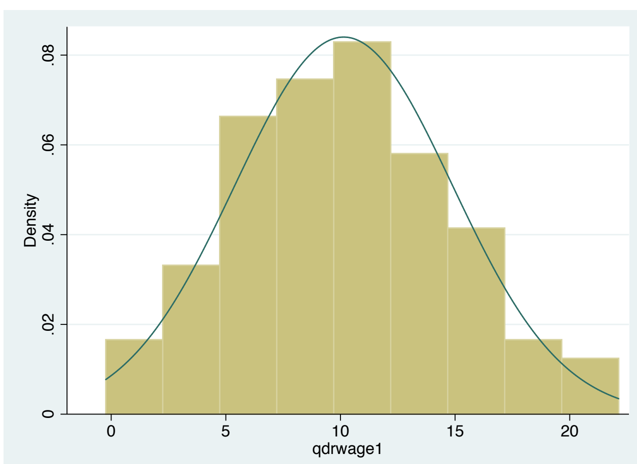



Figure A.1.1

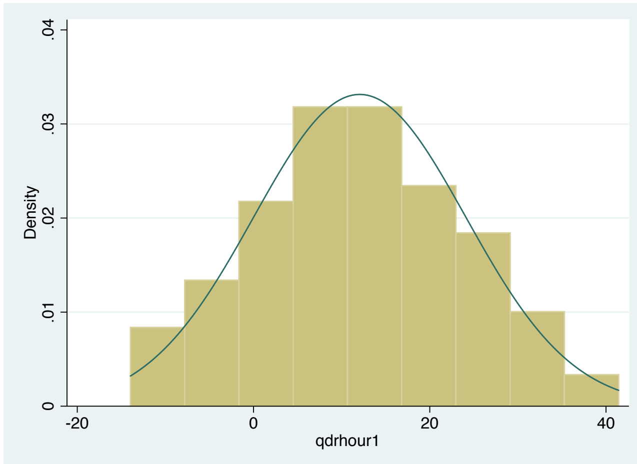



Figure A.1.2

Table A.1.1

| $y^1/y^2$ | -4.65 | 1.07 | 5.26 | 8.52 | 11.39 | 14.91 | 18.33 | 22.64 | 28.38 |
|---|---|---|---|---|---|---|---|---|---|
| 3.72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5.93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7.31 | 0 | 0 | 0 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 |
| 8.62 | 0 | 0 | 0 | 0 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 |
| 9.95 | 0 | -0.01 | -0.01 | -0.01 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 |
| 11.27 | 0 | 0 | 0 | 0 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| 12.53 | 0 | 0 | -0.01 | -0.01 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 |
| 14.08 | 0 | 0 | 0 | -0.01 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 |
| 16.39 | 0 | 0 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |

Note: $*$ = 95% confidence level using 50 bootstrap samples.

Table A.1.2

| $y^1/y^2$ | -4.65 | 1.07 | 5.26 | 8.52 | 11.39 | 14.91 | 18.33 | 22.64 | 28.38 |
|---|---|---|---|---|---|---|---|---|---|
| 3.72 | 2.04 | 1.99 | 1.99 | 2.05 | 1.89 | 1.7 | 1.82 | 1.71 | 1.75 |
| 5.93 | 1.95 | 2.29 | 1.63 | 1.95 | 1.84 | 1.75 | 2 | 2.02 | 2.03 |
| 7.31 | 2.21 | 1.97 | 1.99 | 1.84 | 2.19 | 1.9 | 2.04 | 2.13 | 2.11 |
| 8.62 | 2.61 | 1.84 | 1.88 | 2.26 | 2.14 | 1.8 | 1.81 | 1.83 | 2.14 |
| 9.95 | 2.06 | 2.15 | 1.91 | 2.56 | 2.32 | 2.14 | 2.11 | 2.01 | 1.89 |
| 11.27 | 2.06 | 2.26 | 2.04 | 2.43 | 2.35 | 2 | 2.17 | 1.8 | 2.01 |
| 12.53 | 2.2 | 2.04 | 2.13 | 2.06 | 1.96 | 1.79 | 2.08 | 2.17 | 2.2 |
| 14.08 | 2.02 | 1.99 | 1.84 | 2.09 | 2.01 | 2.31 | 2.03 | 1.97 | 2.06 |
| 16.39 | 1.93 | 1.93 | 2.15 | 1.99 | 2.09 | 2.49 | 1.98 | 1.84 | 2.23 |

## B.1. Proofs

*Proof of Theorem 3.2.* The first part of this proof is essentially identical to the proof of Lemma 2.1 of Chernozhukov et al. (2013). Note that $Y^1 = \mathbb{1}\{J = j\}Y^{1*}$ and $Y^2 = \mathbb{1}\{J = j\}Y^{2*}$. Also, by definition,

$$Y_j^1 := Y^1|J = j \text{ and } X_k^1 := X^1|J = k, \tag{$\star$}$$

and

$$Y_j^2 := Y^2|J = j \text{ and } X_k^2 := X^2|J = k, \tag{$\star\star$}$$

Then, by the law of iterated probability

$$\begin{aligned} F_{Y_j^{1*}|J}(y|k) &= \int_{\mathcal{X}_k^1} F_{Y_j^{1*}|J,X^1}(y|k,x^1)dF_{X^1|J}(x|k) \\ &= \int_{\mathcal{X}_k^1} F_{Y_j^{1*}|J,X^1}(y|j,x^1)dF_{X^1|J}(x|k) \\ &= \int_{\mathcal{X}_k^1} F_{Y_j^{1*}|X_j^1}(y|x^1)dF_{X_k^1}(x). \end{aligned}$$

The second equality follows from Assumption 2 and the last equality follows from ($\star$). Next, by the law of iterated probability

$$\begin{aligned} F_{Y_j^{2*}|J}(y|k) &= \int_{\mathcal{X}_k^2} F_{Y_j^{2*}|J,X^2}(y|k,x^2)dF_{X^2|J}(x|k) \\ &= \int_{\mathcal{X}_k^1} F_{Y_j^{2*}|J,X^2}(y|j,x^2)dF_{X^2|J}(x|k) \\ &= \int_{\mathcal{X}_k^2} F_{Y_j^{2*}|X_j^2}(y|x^2)dF_{X_k^2}(x). \end{aligned}$$

The second equality follows from Assumption 2, and the last equality follows from ($\star\star$).

By Sklar's Theorem, under Assumption 3,

$$F_{Y^{1*},Y^{2*}\langle j,k\rangle}(y^1,y^2) = C(F_{Y^{1*}_{\langle j,k\rangle}}(y^1), F_{Y^{2*}_{\langle j,k\rangle}}(y^2))$$

is unique. ■

## C.1. Jointly Dependent Outcomes

Policy evaluation is chiefly concerned with comparing the observed outcomes after a policy has been implemented with the unobserved potential outcomes had the policy not been implemented.[30] Often, this is done by comparing the

[30] This discussion precludes the case when deep structural parameters conveying mechanisms behind how the data generating process works are of interest. That is, this discussion is centered on "reduced-form" policy evaluation.

estimated mean of an observed outcome against the estimated mean of a counterfactual outcome had the policy not been implemented (i.e. average treatment effect). This can also be done conditioning on some group (e.g. conditional average treatment effect or average treatment effect on the treated). However, if the policy has heterogeneous effects on the distribution of outcomes—for example, a policy affects low-wage workers differently than high-wage workers, then simply comparing means masks the diversity of outcomes a policy maker might be interested in. This is particularly relevant if the policy maker is interested in a policy's effect on inequality or poverty.

While methods of comparing entire distributions[31] and quantiles[32] of outcomes have been employed, many applied researchers simply split the data into groupings of the distribution they are interested in (e.g. splitting the data by high- and low-wage workers) and estimate the mean effect in each grouping. An issue with using informal data splitting to account for outcome heterogeneity is that these groupings can be arbitrary, with results possibly being an artifact of the arbitrary grouping decisions.

Both the sample splitting method and quantile treatment effects methods generally require a "rank invariance" assumption—i.e., the treatment preserves the ordering of individual outcomes—which is not plausible in most cases of interest.[33] Although these two methods estimate different "objects," they are in essence capturing the same idea of heterogeneous outcomes.

However, simple data splitting cannot feasibly take into account multivariate heterogeneous outcomes. If a policy maker is interested in two outcomes—e.g., hourly wage and hours worked—then it is possible some individuals might see their wages increase and at the same time see their hours worked reduced while the opposite can be said about other individuals at a different part of the wage distributions. Splitting the sample into individuals with high-wage high-hours worked, high-wage low-hours worked, low-wage high-hours worked, and low-wage low-hours worked makes it difficult to obtain any meaningful policy conclusions. Moreover, interpreting the results would quickly become infeasible as the number of outcomes of interest increases. Alternatively, comparing multidimensional distributions, discussed in section **??**, leads to interpretable results.

While others have noted the importance of estimating multivariate heterogeneous effects in an interpretable way (e.g. Athey and Imbens, 2015; Wager and Athey, 2018), they focused on conditional means.[34] It might be tempting, for example, to simply condition mean wage on hours worked to capture the heterogeneity of wages. However, that would be taking hours worked as exogenously given whereas the policy might be affecting both wages and hours worked. Therefore, conditional mean methods are not well suited in situations

[31] See, for example, Maasoumi and Wang (2019).

[32] See Angrist and Pischke (2009) for discussion.

[33] A way around this issue is to compare entire distributions, see Maasoumi and Wang (2019).

[34] Carlier et al. (2016) use optimal transport theory to develop vector quantile regression, however their results are difficult to interpret since the results are conditional on covariates.

where the policy affects both outcomes of interest. Additionally, these multivariate heterogeneous effects get their heterogeneity from the covariates and only estimate an effect on the outcome's conditional mean, not a conditional quantile (i.e. the case in which outcomes are affected heterogeneously).

Hence, the main advantages of comparing joint distributions of outcomes proposed in this paper are ease of interpretability of the findings and the fact that this method allows for outcomes to be affected heterogeneously while not relying on exogeneity of any of the outcomes of interest.