Production and Costs

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Production Function

- $q = f(L, A, K, R, E, \text{etc.})$

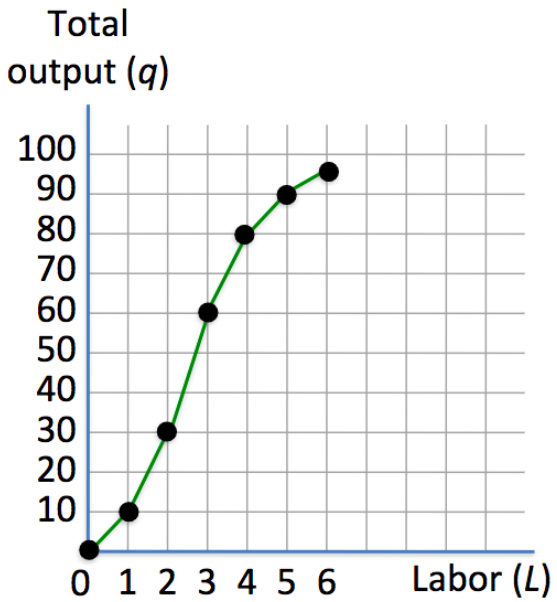
Simple Productivity and Costs Model

(A1) There is a single output q

(A2) There is a single variable input L

(A3) The price (wage) of L is $\$w$ per unit

Production



Labor	Total output	APL (q/L)	MPL ($\Delta q/\Delta L$)
0	0	-	-
1	10	10	10
2	30	15	20
3	60	20	30
4	80	20	20
5	90	18	10
6	95	15.8	5

Average and Marginal

TEST #	TOTAL	AVERAGE	MARGINAL
1	80	80	80
2	180	90	100
3	240	80	60

- When the average is increasing,

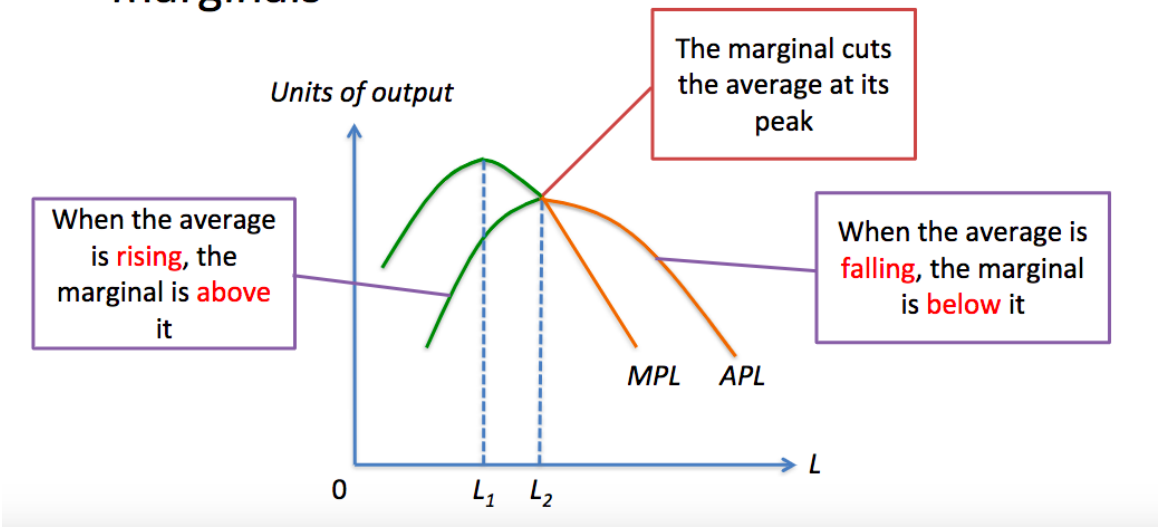
  The marginal is above the average

- When the average is decreasing,

  The marginal is below the average

APL and MPL

- There is a relationship between averages and marginals



Review

- Production concepts
  - *Total product*: q
  - *Average product*: $APL = q/L$
  - *Marginal product*: $MPL = \Delta q/\Delta L$
- Cost concepts
  - *Total cost*: $TC = TFC + TVC$
  - *Average cost*: $AC = TC/q$
  - *Marginal cost*: $MC = \Delta TC/\Delta q = \Delta TVC/\Delta q$

How Costs and Production are Related

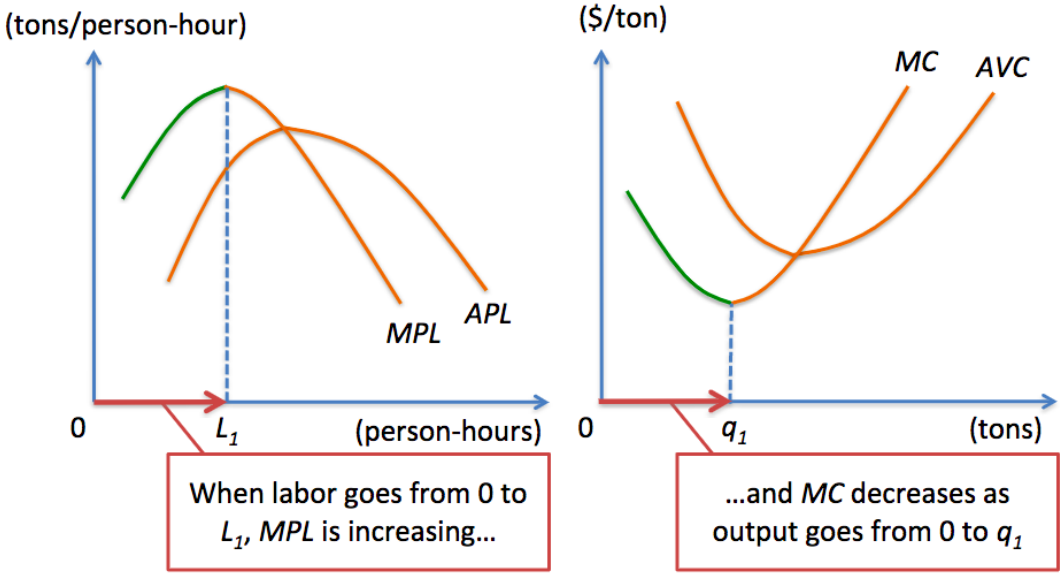
- $\text{MC} = \frac{w}{\text{MPL}}$, because

$$\text{MC} = \frac{\Delta \text{TC}}{\Delta q} = \frac{w \Delta L}{\Delta q} = \frac{w}{\frac{\Delta q}{\Delta L}} = \frac{w}{\text{MPL}}.$$

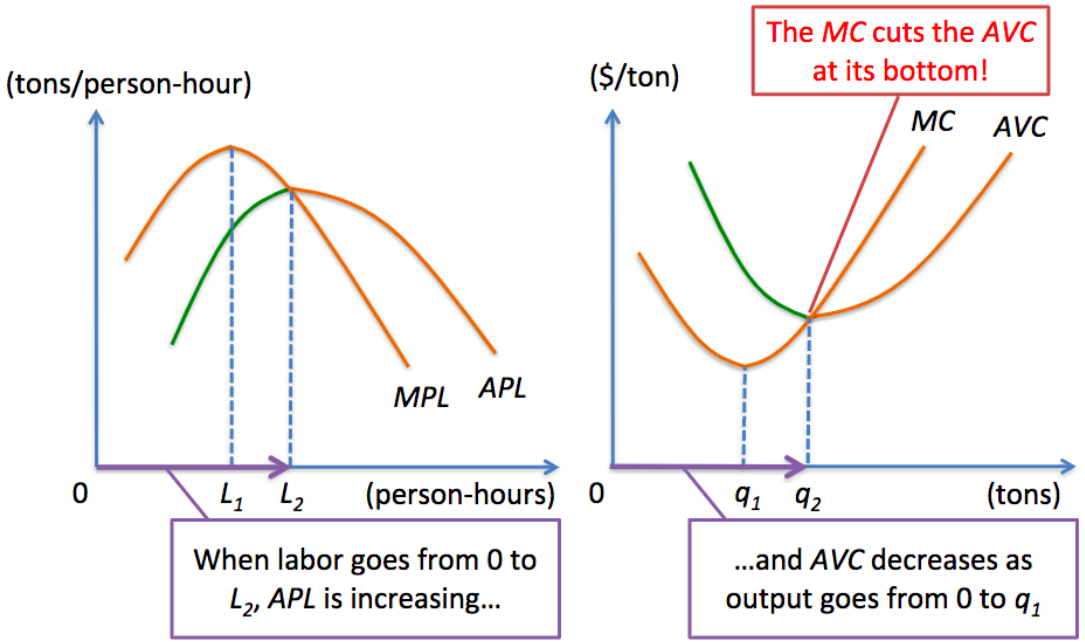
- As MPL goes up, MC go down. And, as MC go up, MPL goes down.
- $\text{AVC} = \frac{w}{\text{APL}}$, because

$$\text{AVC} = \frac{\text{TVC}}{q} = \frac{wL}{q} = \frac{w}{\frac{q}{L}} = \frac{w}{\text{APL}}.$$

Costs and Production



Costs and Production



Practice Problem

Labor	TP (q)	APL	MPL
1			4
2	10		
3		5	
4			3
5		4	

The price of wage, w, is \$180. The marginal cost of producing the 18th unit of output is

(a) \$30

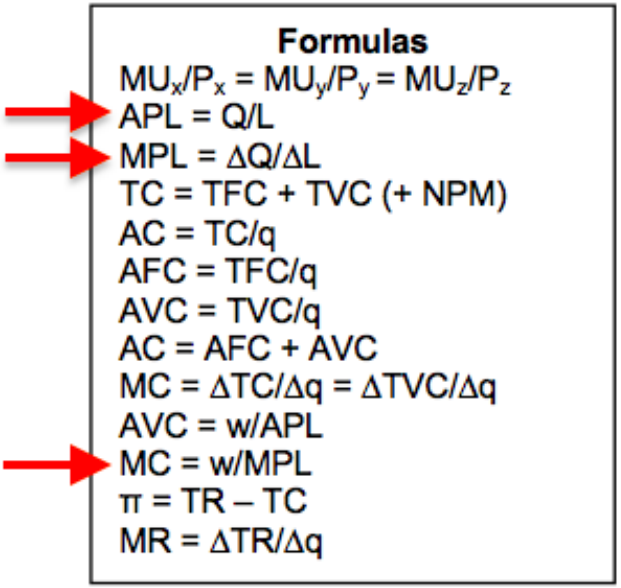
(b) \$36

(c) \$45

(d) \$60

(e) \$90

Answer


Formulas
$MU_x/P_x = MU_y/P_y = MU_z/P_z$
APL = Q/L
MPL = ΔQ/ΔL
TC = TFC + TVC (+ NPM)
AC = TC/q
AFC = TFC/q
AVC = TVC/q
AC = AFC + AVC
MC = ΔTC/Δq = ΔTVC/Δq
AVC = w/APL
MC = w/MPL
π = TR – TC
MR = ΔTR/Δq

Answer

Labor	TP (q)	APL	MPL	MC
1	4	4	4	45
2	10	5	6	30
3	15	5	5	36
4	**18**	4.5	3	**60**
5	20	4	2	90

(d) \$60