Networks: Fall 2023	**Homework 6**
David Easley and Yian Yin	**Due 3:30pm, Thursday, Nov 2, 2023**

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope be sure to assign your answers to the correct question.

To be eligible for full credit, your homework must come in by 3:30pm Thursday. We will also accept late homeworks after 3:30pm Thursday until 3:30pm Friday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 3:30pm Friday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and **approved by the TA in charge of this homework prior to the due date of the homework**. (These include illness, family emergencies, and travel associated with university activities.)

The TA in charge of this homework is Jonathan Aimuyo (oja7@cornell.edu).

Reading: The questions are primarily based on the material in Chapter 16.

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(1) [8 points] Recall the experiment with marbles in an urn from lecture and from Sections 16.2 and 16.4 of the textbook. The students line up and, one by one, each perform the following steps:

1. Draw a marble from the urn and observe its color.
2. Put the marble back in the urn.
3. Make a guess about the type of urn (this guess is heard by everyone).

At the end, everyone who guesses correctly is given a monetary reward of $1. We will use the same setup for the urns as in class:

- The MB urn has 2 blue marbles and 1 red marble.
- The MR urn has 2 red marbles and 1 blue marble.

We also assume that if a student is indifferent about which guess to make, they will break the tie by guessing that the urn contains a majority of the marble color that they observed.

In class, we discussed the problem of information casecades. Let us consider the case where Player 1 and 2 drew red marbles, and Player 3 and 4 drew blue marbles (which we denote as (R, R, B, B) for simplicity). Even if the same number of red and blue marbles were drawn, all four players will make a guess of "MR".

(a) [3 points] Let's keep the same setup, except that every person can only observe the choice made by the previous person, i.e., Player n can only observe the choice made by Player $(n-1)$, but not the choices made by Players $1, 2, \cdots, (n-2)$. Player 1 does not observe any choice made by others. In the case of (R, R, B, B), what would be the guesses made by Players 1 - 4?. Give an explanation for your answer.

(b) [2 points] Now let's see how these results may be generalized. Under the setup in **(a)**, describe the strategy for players to make guesses, and give an explanation for your answer.

(c) [3 points] Suppose each person can only observe the choices made by the previous two people, i.e., Player n can only observe the choices made by Players $(n-2)$ and $(n-1)$, but not the choices madem by Players $1, 2, \cdots, (n-3)$. Player 1 does not observe any choice made by others. Player 2 only observes the choice made by Player 1. In the case of (R, R, B, B), what would be the guesses made by Players 1 - 4?

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(2) [15 points] Let's consider a similar experiment with marbles in an urn. The students line up and, one by one, each perform the following steps:

1. Draw a marble from the urn and observe its color.
2. Put the marble back in the urn.
3. Make a guess about the type of urn (this guess is heard by everyone).

At the end, everyone who guesses correctly is given a monetary reward of \$1. We will use the same setup for the urns as in class:

- Urn A has an red marbles and $(1-a)n$ blue marbles.
- Urn B has bn red marbles and $(1-b)n$ blue marbles.

where $0 \le a < b \le 1$. an, $(1-a)n$, bn, and $(1-b)n$ are all integer numbers.

We now want to explore the role of the prior probability on urns; the probability that the experimenter selected each urn. The person running the experiment picks (at the start of the experiment) Urn A with probability $0 \le p \le 1$ and Urn B with probability $1-p$. The students know these probabilities. They are the students' prior probabilities on the types of urns. Before the experiment is run, the students will be given a value for p, but we want to explore the effect of that value on herding.

Given a value for p each student can guess the composition of the urn that has the highest conditional probability based on what they have observed (both their own marble and the guesses of others). They guess the urn that is most likely given what they know. If the two urns have exactly the same conditional probabilities, we assume students break the tie by guessing that the urn contains the majority of the marble color that they observed.

(a) [3 points] If $p = 1$, Player 1 will guess Urn A regardless of the color of the marble they draw or if $p = 0$ they will guess Urn B regardless of the color of the marble they draw.

So Player 1's guess obviously depends on the value of p. For what range of p will Player 1 guess Urn A regardless of the color of the marble they draw? It's fine to express your answer as a function of a and b. Give an explanation for your answer.

(b) [3 points] Suppose that p is in the range which causes Player 1 to guess Urn A regardless of the color of the marble they draw. Suppose also that Player 1 drew a red marble. Now Player 2 hears the guess of Player 1 (but as usual doesn't see what marble they actually drew); Player 2 then draws a marble and guesses. How should the Player 2 guess if they draw a Red marble; what if they draw a Blue marble? Give an explanation for your answer.

(c) [3 points] Suppose $p = 0.5$, $a = 0.25$, and $b = 0.5$. Suppose Player 1 has guessed Urn A, while Player 2 has guessed Urn B. What guess should Player 3 make if they have seen a Blue marble? Give an explanation for your answer by calculating the conditional probability.

(d) [3 points] Suppose $p = 0.4$, $a = 0.25$, and $b = 0.5$. Suppose Player 1 has guessed Urn A, while Player 2 has guessed Urn B. What guess should Player 3 make if they have seen a Blue marble? Give an explanation for your answer by calculating the conditional probability.

(e) [3 points] Suppose $p = 0.5$. Player 1 has observed a Red marble. They will get \$1 for a correct guess or \$0 for an incorrect guess. Alternatively, they may also choose not to guess and get \$$q$. For what range of q will Player 1 choose not to guess? It's fine to express your answer as a function of a and b. Give an explanation for your answer.

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(3) [9 points] Suppose you throw a 4-sided die twice. The 4 sides are numbered 1-4 respectively and have equal chance to be selected from a throw.

(a) [2 points] How many potential outcomes do you have in the sample space? Give an explanation for your answer.

(b) [3 points] Let's denote:

- A as the event that the numbers from the two throws differ by -1, 0, or 1.
- B as the event that the number from the first throw is an odd number.

As discussed in class, each event can be represented as a subset of the sample space. Write the subsets corresponding to events A, B, and $A \cap B$ respectively.

(c) [4 points] Calculate $P(A), P(B), P(A|B), P(B|A)$ respectively.