Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope be sure to assign your answers to the correct question.

To be eligible for full credit, your homework must come in by 3:30pm Thursday. We will also accept late homeworks after 3:30pm Thursday until 3:30pm Friday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 3:30pm Friday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and **approved by the TA in charge of this homework prior to the due date of the homework**. (These include illness, family emergencies, SDS accommodations and travel associated with university activities.)

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Reading: The questions below are primarily based on the material in Chapter 15.

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(1) [6 points] Suppose a search engine has three ad slots that it can sell. Slot a has a clickthrough rate of 7, slot b has a clickthrough rate of 9 and slot c has a clickthrough rate of 2. There are three advertisers who are interested in these slots. Advertiser x values clicks at 4 per click, advertiser y values clicks at 2 per click, and advertiser z values clicks at 1 per click.

(a) Set up this problem of allocating slots to advertisers as a matching market in which the values per click, as well as the clickthrough rates, are known. Find market clearing prices and the perfect matching between advertisers and slots occurs when you use these market clearing prices? [You do not need to show the steps of the procedure. It's enough to display the final prices and the perfect matching.]

(b) Now assume that the search engine does not know advertisers' values per click. Suppose that the search engine runs the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay?

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(2) [5 points] In lecture we discussed the relationship between the VCG Principle and second price auctions. In particular, we saw that the VCG Principle is a generalization of the idea behind second price auctions to a setting in which the seller has more than one object to sell. In this problem we will explore this relationship in an example. Suppose that a seller has one item, let's call it item a. There are three buyers, let's call them x, y, and z. The values that these buyers (x, y, and z) have for the item are 8, 10, and 6, respectively.

(a) Suppose that the seller runs a second price auction for the item. Which buyer will win the auction and how much will this buyer pay?

(b) Now let's suppose that the seller uses the VCG procedure to allocate the item. Remember that the first step in the running the VCG procedure when there are more bidders than items is to create fictional items, which each buyer values at 0, so that the number of items to be allocated is the same as the number of bidders. Let's call these additional items b and c. Find the allocation that results from running the VCG procedure. What are the prices charged to each buyer for the item that they receive? The main principle of the VCG procedure is that the price paid by the buyer who wins an item equals the harm that he causes to the remaining bidders by taking the item he is assigned. Explain briefly how this is true in the prices you just computed.

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(3) [7 points]

In our application of VCG to allocating ad slots we created advertisers' values for slots from their values for clicks and the clickthrough rates of the slots. This resulted in advertisers agreeing about the ranking of slots; each advertiser prefers a slot with a higher clickthrough rate to one with a lower clickthrough rate. However, our proof that truth-telling is a dominant strategy when the VCG procedure is used does not make use of this uniformity of rankings. In fact, our proof shows that if the VCG procedure is used to allocate items and set prices then announcing values truthfully is a dominant strategy for any values.

Let's apply this idea to a setting in which we have three items, a, b, and c, to allocate to four individuals, w, x, y and z. Each individual wants at most one item. Person w has values 10, 4, and 2 for items a, b and c, respectively. Person x has values 7, 8 and 6 for items a, b and c, respectively. Person y has values 1, 3 and 5 for these items; and person z has values 4, 4 and 4 for these items.

(a) Compute the socially optimal allocation (one that maximizes total valuation) and the VCG prices for the items.

(b) You should notice that several of the items have the same price. Provide a brief explanation of why the prices of the these items are the same.

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(4) [10 points] Suppose a search engine has three ad slots that it can sell. Slot a has a clickthrough rate of 5, slot b has a clickthrough rate of 4 and slot c has a clickthrough rate of 3. There are three advertisers who are interested in these slots. Advertiser x values clicks at 5 per click, advertiser y values clicks at 3 per click, and advertiser z values clicks at 2 per click.

(a) Suppose that the search engine runs the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay?

(b) Now a new advertiser, w, enters the market. Suppose that the search engine again runs the VCG procedure and after the procedure is run advertiser x is assigned the same slot as in part (a) but the price advertiser x has to pay for this slot has increased by 3. Suppose that advertiser x does not precisely know any other advertiser's value per click. But he does know that the new advertiser, w, has the smallest value per click of any advertiser. Let's represent this value per click for advertiser w by v. [Assume that nothing has changed other than the entry of a new advertiser and that each advertiser follows their dominant strategy.]

1. Since there are more advertisers than slots create a fictitious slot, d, with a 0 clickthrough rate and run the VCG procedure again. What slot will be assigned to advertiser w?

2. Can advertiser x determine the value of v (the value per click for advertiser w) from the information provided? If you think that the answer is yes what is this value. If you think that the answer is no explain why not.

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(5) [12 points] Suppose a search engine has three ad slots (a, b and c) that it can sell. Slot a has a clickthrough rate of 6, slot b has a clickthrough rate of 4 and slot c has a clickthrough rate of 2. There are three advertisers (x, y and z) who are interested in these slots. Advertiser x values clicks at 10 per click, advertiser y values clicks at 9 per click, and advertiser z values clicks at 6 per click.

Suppose the search engine uses a generalized second price auction in which each advertiser is asked to announce a bid per click. We will assume that all the advertisers know their own value per click as well as the values per click for the other two advertisers. The advertiser with the highest bid per click gets the top slot (a) and pays per click the bid announced by the second highest bidder, the advertiser with the second highest bid per click gets the second slot (b) and pays per click the bid announced by the third highest bidder, the advertiser with the third highest bid per click gets the third slot (c) and pays 0 per click for slot (as there is no lower bid).

(a) In this bidding game is it a Nash equilibrium for each advertiser to bid his true value per click? That is, is it a Nash equilibrium for advertiser x to bid 10 per click, advertiser y to bid 9 per click, and advertiser z to bid 6 per click? If your answer is No then demonstrate a profitable deviation by some advertiser. If your answer is Yes then either show that no profitable deviation exists or use some principle from class or the book to argue that these bids form a Nash equilibrium..

(b) In the generalized second price auction is it a Nash equilibrium for advertiser x to bid 9 per click, advertiser y to bid 6 per click and advertiser z to bid 5 per click? If your answer is No then demonstrate a profitable deviation by some advertiser. If your answer is Yes then either show that no profitable deviation exists or use some principle from class or the book to argue that these bids form a Nash equilibrium.

(c) In the generalized second price auction is it a Nash equilibrium for advertiser x to bid 6 per click, advertiser y to bid 5 per click and advertiser z to bid 3 per click? If your answer is No then demonstrate a profitable deviation by some advertiser. If your answer is Yes then either show that no profitable deviation exists or use some principle from class or the book to argue that these bids form a Nash equilibrium.