1.

Now, let's consider the equilibrium conditions:

(a)

0 will be considered an equilibria as it is considered a special condition in order to find the other equilibria we do r(x)f(z)=p and solve for z:

z(9-9z)=2

(b) To determine which of the equilibria found in part (a) are stable, we need to consider the reaction of consumers to changes in z. An equilibrium is stable if, when the population fraction z deviates slightly from the equilibrium fraction, consumers do not have an incentive to change their behavior.

Let's consider the equilibria for z:

1. z = 2/9: At this equilibrium, the utility for consumers is just equal to the price (U(x, z) = 2 for all x). If z decreases slightly from 2/9, consumers will have a lower utility than the price, and they will not want to purchase the good. If z increases slightly, consumers will have a higher utility than the price, and they will want to purchase the good. So, this equilibrium is not stable.
2. z > 2/9: Any value of z greater than 2/9 will also be an equilibrium because consumers' utilities will always be higher than the price, and they will want to purchase the good. These equilibria are stable.

In summary, the only stable equilibrium is when z is greater than 2/9. Consumers will purchase the good when the fraction of the population using the product is greater than 2/9.

2.

To find the equilibrium fractions of the population purchasing the good, we need to determine the fractions of the population for which the reservation price is equal to or greater than the price of the good, which is 3/2.

The reservation price for consumer x when a fraction z of the population uses the product is given by the formula r(x)f(z), where: r(x) = 6(1 - x) f(z) = 2z

So, the reservation price for consumer x when z fraction of the population uses the product is: P(x, z) = r(x)f(z) = 6(1 - x) \* 2z = 12z(1 - x)

(a) Equilibrium fractions of the population purchasing the good: We want to find z such that P(x, z) ≥ 3/2 for all consumers x.

P(x, z) ≥ 3/2 12z(1 - x) ≥ 3/2

Now, let's consider different values of x:

For x = 0: 12z(1 - 0) ≥ 3/2 12z ≥ 3/2 z ≥ (3/2) / 12 z ≥ 1/8

For x = 1: 12z(1 - 1) ≥ 3/2 0z ≥ 3/2 This is not relevant since x = 1 is not in the equilibrium range.

So, the equilibrium fraction of the population purchasing the good is z ≥ 1/8.

(b) If the fraction of potential users who are actually using the product is 1/2, it means z = 1/2. To understand what will happen over time, we can look at the change in the fraction of users. We will use the formula P(x, z) = 12z(1 - x).

Let's calculate P(x, 1/2) for different values of x:

For x = 0: P(0, 1/2) = 12 \* (1/2) \* (1 - 0) = 6

For x = 1/2: P(1/2, 1/2) = 12 \* (1/2) \* (1 - 1/2) = 3

For x = 1: P(1, 1/2) = 12 \* (1/2) \* (1 - 1) = 0

The fraction of users will decrease over time. As x approaches 1, fewer consumers will find the product worth the price, and eventually, the fraction of users will converge to 0.

(c) To find the maximum price, p\*, the company could charge and still have a positive equilibrium fraction of users, we need to consider the reservation price formula P(x, z) = 12z(1 - x) and determine at what price this value is equal to zero. In other words, we need to find P(x, z) = 0:

12z(1 - x) = 0

This equation has a solution when either z = 0 (meaning no users) or x = 1 (meaning everyone uses it). We want a positive fraction of users, so we consider z = 1 and solve for x:

12(1)(1 - x) = 0 1 - x = 0 x = 1

So, the maximum price p\* the company could charge to have a positive equilibrium fraction of users is when x = 1, and in that case, the equilibrium fraction is 1/2.

Whether it's a good idea for the company to increase the price to p\* depends on their goals and market strategy. Increasing the price to p\* will result in half of the potential users continuing to use the product, but it may also deter new users from joining. The company should carefully consider the price elasticity of demand and potential consequences on its revenue and market share when making this decision.

4.

(a) We have been given that there are 8000 articles receiving 25 views and 2000 articles receiving 50 views. The number of articles each day that receive k views is assumed to be c/k^α.

Using this information, we can set up two equations:

1. 8000 = c/25^α
2. 2000 = c/50^α

Let's solve for α:

From equation 1: 8000 = c/25^α c = 8000 \* 25^α

Now, substitute the value of c into equation 2:

2000 = (8000 \* 25^α) / 50^α

Now, simplify and solve for α:

2000 = (8000 \* (25/50)^α) 2000 = 8000 \* (1/2)^α

Dividing both sides by 8000: 1/4 = (1/2)^α

Taking the logarithm of both sides (base 2): log₂(1/4) = log₂(1/2)^α -2 = -α

So, α = 2.

(b) To estimate the number of articles that receive 200 views, we can use the formula c/k^α:

c/200^2 = 8000/25^2 = 2000/50^2

So, the number of articles receiving 200 views would be c/200^2.

(c) The popularity index p is defined as p = v/2 when v is even, and p = (v-1)/2 when v is odd. Given the number of articles each day that receive k views is c/k^α, we can write h(p), the fraction of articles with a popularity index value p, as follows:

h(p) = (Number of articles with v = 2p) / (Total number of articles each day) h(p) = (c/(2p)^α) / (∑(c/k^α) for all k values)

The denominator is a sum over all possible k values, but we can approximate it as an integral since it's a continuous distribution:

h(p) ≈ (c/(2p)^α) / ∫(c/k^α) dk (from 1 to ∞)

Now we need to integrate the power law c/k^α:

h(p) ≈ (c/(2p)^α) / (c/(1-α) \* (k^(1-α))) |(from 1 to ∞) h(p) ≈ (1-α) \* (1/(2p)^α) / (1/(1-α))

Simplifying: h(p) ≈ (1-α) \* (2p)^α

(d) Given that h(p) approximately follows another power law, d/p^β, we can equate the two expressions:

(1-α) \* (2p)^α ≈ d/p^β

Now, let's find the parameters d and β:

d = (1-α) β = -α

So, the parameters are d = 1-2 = -1 and β = -2.