# Rocket Math Stuff

## 1 Variables

| Variable | Symbol | Units | Description |
|---|---|---|---|
| Height | $h(t)$ | $m$ | Height off ground |
| Velocity | $v(t)$ | $m/s$ | Velocity of rocket |
| Acceleration | $a(t)$ | $m/s^2$ | Acceleration of rocket |
| Thrust | $T(t)$ | $kg \cdot m/s^2$ | Thrust of rocket |
| Burn rate | $B$ | $m/s$ | Fuel burn rate (time, not energy) |
| Gravity | $g$ | $m/s^2$ | Gravity ($\approx 9.807$) |
| Mass | $m(t)$ | $kg$ | Mass of rocket |
| Core Radius | $R_c$ | $m$ | Initial radius of inner core (function of angle if not cylindrical cor |
| Inner radius | $r(t)$ | $m$ | Radius of core at given time |
| Rocket Length | $L$ | $m$ | Vertical length of rocket |
| Rocket Radius | $R$ | $m$ | Inner rocket radius (e.g. exclude PVC thickness) |
| Outer Radius | $R_o$ | $m$ | Outer rocket radius (measured from center to edge of PVC) |
| Fuel density | $\rho_f$ | $kg/m^3$ | Fuel density |
| Air Density | $\rho_a(h)$ | $kg/m^3$ | Air density (decreases with height) |
| Drag | $C_d$ | None | Drag coefficient (related to Reynold‘s number) |
| Rocket Area | $A$ | $m^2$ | Cross sectional area of rocket (for drag computation) |

## 2 Equations

### 2.1 Velocity/Height

Governing equation:

$$\sum F = T(t) - gm(t) - \gamma v^2 = m(t)a(t) = m(t)\dot{v}$$

$$\dot{v} = \frac{T(t)}{m(t)} - g - \frac{\gamma v^2}{m(t)} \tag{1}$$

Drag is proportional to $v^2$ for turbulent flows (is this turbulent or laminar..? not sure since high speed but moderately aerodynamic object). If it’s laminar, proportional to $v$. Drag coefficient equation (source: Wikipedia):

$$\gamma = \frac{1}{2}\rho_a C_d A = \frac{\pi}{2} C_d R_o^2 \rho_a(h) \tag{2}$$

Gravity (approximated for now to get rid of nonlinearities):

$$g = \frac{GM_\oplus}{R_\oplus + h(t)} \approx 9.807$$

### 2.1.1 Mass

In a given moment, $2\pi r(t) H dr\ m^3$ of fuel is being burned (assumptions: symmetric cylindrical burn).

Assuming the fuel burns outward at a constant rate:

$$r(t) = \begin{cases} Bt + r_c & t \leqslant \frac{R-r_c}{B} \\ r_c & t > \frac{R-r_c}{B} \end{cases} \tag{3}$$

With this, the mass of the rocket is:

$$m(t) = M_0 - \int_0^t 2\pi(B\tau + r_c)H\, d\tau = M_0 - \pi H \left[2r_c t + Bt^2\right] \tag{4}$$