Rocket Math Stuff

1 Variables

Variable	**Symbol**	**Units**	**Description**
Height	$h(t)$	m	Height off ground
Velocity	$v(t)$	m/s	Velocity of rocket
Acceleration	$a(t)$	m/s^2	Acceleration of rocket
Thrust	$T(t)$	$kg \cdot m/s^2$	Thrust of rocket
Burn rate	B	m/s	Fuel burn rate (time, not energy)
Gravity	g	m/s^2	Gravity (≈ 9.807)
Mass	$m(t)$	kg	Mass of rocket
Core Radius	R_c	m	Initial radius of inner core (function of angle if non-cylinder
Inner radius	$r(t)$	m	Radius of core at given time
Rocket Length	L	m	Vertical length of rocket
Rocket Radius	R	m	Inner rocket radius (e.g. exclude PVC thickness)
Outer Radius	R_o	m	Outer rocket radius (measured from center to edge of PVC)
Fuel density	ρ_f	kg/m^3	Fuel density
Air Density	$\rho_a(h)$	kg/m^3	Air density (decreases with height)
Drag	C_d	None	Drag coefficient (related to Reynold's number)
Rocket Area	A	m^2	Cross sectional area of rocket (for drag computation)
Burn End Time	t_e	s	The time at which the burn ends
Air Temperature	$T_a(h)$	K	Temperature of air at height h
Air Pressure	$p_a(h)$	kPa	Pressure of atmosphere at height h
Lapse rate	L	K/m	Temperature lapse rate (≈ 0.0065)
Ideal Gas Constant	R	$J/\text{mol} \cdot K$	Ideal Gas Constant
Air Molar Mass	M_m	kg/mol	Dry air molar mass

2 Equation Derivations

2.1 Velocity/Height

2.1.1 Governing equation

Note: Currently just used $F = ma$, but since it's a mass varying system, need to integrate this equation: Mass Varying System. Shouldn't be too much of an issue since we find $m(t)$ explicity below and the derivative is simple. It just adds another constant, exhaust rate, that we'll need to figure out or approximate as the speed of sound.

$$\sum F = T(t) - gm(t) - \gamma v^2 = m(t)a(t) = m(t)\dot{v}$$

$$\dot{v} = \frac{T(t)}{m(t)} - g - \frac{\gamma v^2}{m(t)} \tag{1}$$

2.1.2 Drag

Drag is proportional to v^2 for turbulent flows (is this turbulent or laminar..? not sure since high speed but moderately aerodynamic object). If it's laminar, proportional to v. Drag coefficient equation (source: Wikipedia):

$$\gamma = \frac{1}{2}\rho_a C_d A = \frac{\pi}{2} C_d R_o^2 \rho_a(h) \tag{2}$$

For air density, assumptions are: 1) Troposphere, 2) No humidity. From Wikipedia, with $T_0 = 288.15$ and $p_0 = 101.325$:

$$T(h) = T_0 - Lh \tag{3}$$

$$p(h) = \rho_0 \left(1 - \frac{Lh(t)}{T_0}\right)^{\frac{gM_m}{RL}} \tag{4}$$

$$\rho_a(h) = \frac{p(h)M_m}{RT(h)} \tag{5}$$

For fun, let's approximate $C_D \approx 0.4$ (relatively close I think).

2.1.3 Gravity

Gravity (approximated for now to get rid of nonlinearities):

$$g = \frac{GM_\oplus}{R_\oplus + h(t)} \approx 9.807$$

2.1.4 Mass

In a given moment, $2\pi r(t)H \cdot \rho_f dr$ (kg) of fuel is being burned (assumptions: symmetric cylindrical burn).

Assuming the fuel burns outward at a constant rate (it doesn't, it's dependent on chamber pressure. Need to integrate these equations: Pressure related burn rate: $\frac{dr}{dt} = B \implies dr = Bdt$, so $2\pi H\rho_f r(t)dt$ (kg) of fuel is burned.

$$r(t) = \begin{cases} Bt + R_c & t \leqslant \frac{R-R_c}{B} \\ R & t > \frac{R-R_c}{B} \end{cases} \tag{6}$$

Let $t_e = \frac{R-R_c}{B}$, when all the fuel has burned. With this, the mass of the rocket is:

$$m(t) = \begin{cases} M_0 - \int_0^t 2\pi H\rho_f B(Bt + R_c)\, dt = M_0 - \pi H\rho_f B(Bt^2 + 2R_c t) & t \leqslant t_e \\ M_0 - \pi H\rho_f (R^2 - R_c^2) & t > t_e \end{cases} \tag{7}$$

$$\dot{m} = \begin{cases} -2\pi H\rho_f B(Bt + R_c) & t \leqslant t_e \\ 0 & t > t_e \end{cases} \tag{8}$$

2.2 Thrust

Need to figure this bit out.