Rocket Math Stuff

1 Variables

Variable	Symbol	Units	Description
Height	$h(t)$	m	Height off ground
Velocity	$v(t)$	m/s	Velocity of rocket
Acceleration	$a(t)$	m/s^2	Acceleration of rocket
Thrust	$T(t)$	$kg \cdot m/s^2$	Thrust of rocket
Burn rate	B	m/s	Fuel burn rate (time, not energy)
Gravity	g	m/s^2	Gravity (≈ 9.807)
Mass	$m(t)$	kg	Mass of rocket
Core Radius	R_c	m	Initial radius of inner core (function of angle if non-cylinder)
Inner radius	$r(t)$	m	Radius of core at given time
Rocket Length	L	m	Vertical length of rocket
Rocket Radius	R	m	Inner rocket radius (e.g. exclude PVC thickness)
Outer Radius	R_o	m	Outer rocket radius (measured from center to edge of PVC)
Fuel density	ρ_f	kg/m^3	Fuel density
Air Density	$\rho_a(h)$	kg/m^3	Air density (decreases with height)
Drag	C_d	None	Drag coefficient (related to Reynold‘s number)
Rocket Area	A	m^2	Cross sectional area of rocket (for drag computation)
Burn End Time	t_e	s	The time at which the burn ends

2 Equation Derivations

2.1 Velocity/Height

2.1.1 Governing equation

Note: Currently just used $F = ma$, but since it‘s a mass varying system, need to integrate this equation: Mass Varying System. Shouldn't be too much of an issue since we find $m(t)$ explicity below and the derivative is simple. It just adds another constant, exhaust rate, that we'll need to figure out or approximate as the speed of sound.

$$\sum F = T(t) - gm(t) - \gamma v^2 = m(t)a(t) = m(t)\dot{v}$$

$$\dot{v} = \frac{T(t)}{m(t)} - g - \frac{\gamma v^2}{m(t)} \tag{1}$$

2.1.2 Drag

Drag is proportional to v^2 for turbulent flows (is this turbulent or laminar..? not sure since high speed but moderately aerodynamic object). If it's laminar, proportional to v. Drag coefficient equation (source: Wikipedia):

$$\gamma = \frac{1}{2}\rho_a C_d A = \frac{\pi}{2} C_d R_o^2 \rho_a(h) \tag{2}$$

2.1.3 Gravity

Gravity (approximated for now to get rid of nonlinearities):

$$g = \frac{GM_\oplus}{R_\oplus + h(t)} \approx 9.807$$

2.1.4 Mass

In a given moment, $2\pi r(t)H \cdot \rho_f dr$ (kg) of fuel is being burned (assumptions: symmetric cylindrical burn).

Assuming the fuel burns outward at a constant rate (it doesn't, it's dependent on chamber pressure. Need to integrate these equations: Pressure related burn rate. However, assuming it is linear:

$$r(t) = \begin{cases} Bt + R_c & t \leqslant \frac{R-R_c}{B} \\ R & t > \frac{R-R_c}{B} \end{cases} \tag{3}$$

Let $t_e = \frac{R-R_c}{B}$, when all the fuel has burned. With this, the mass of the rocket is:

$$m(t) = \begin{cases} M_0 - \int_0^t 2\pi(B\tau + R_c)H \cdot \rho_f \, d\tau = M_0 - \pi H \rho_f \left[2R_c t + Bt^2\right] & t \leqslant t_e \\ M_0 - \pi H \rho \left(\frac{(R-R_c)^2}{B} + \frac{2R_c(R-R_c)}{B}\right) & t > t_e \end{cases} \tag{4}$$

3 Summary

Variable	Equation	Comments
t_e	$\frac{R-R_c}{B}$	End time of symmetric cylindrical burn
m for $t \leqslant t_e$	$M_0 - \pi H\rho[2R_c t + Bt^2]$	Must use mass-variable system for this
m for $t > t_e$	$M_0 - \pi H \rho \left(\frac{(R-R_c)^2}{B} + \frac{2R_c(R-R_c)}{B}\right)$	Final weight
$\dot{m}$ for $t \leqslant t_e$	$-2\pi H\rho[R_c + Bt]$	Strictly negative
$\dot{m}$ for $t > t_e$	0	No mass change (burn has ended)