Optimal Trajectories for Kite-Power Generation

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***Abstract*—The looming threat of climate change prompts us to adopt renewable energy as fast as possible, so researchers and companies, around the world are pursuing kites as a simple and affordable means of harnessing wind power. One such approach involves tethering a kite to a generator on the ground. When the kite pulls the cable, the generator generates power. When the generator reels in the kite, it spends power. The kite is controlled to maximize the net power generated. The kite and power-generating turbine are an underactuated dynamical system with nonlinear dynamics, input and state constraints, and a clear optimization objective: maximize amortized power output. In this project we employ optimization to find orbits which maximize the power output of the system, and then track those target trajectories using model predictive control.**

I. Introduction

Wind power has the potential to provide a tremendous amount of clean energy to the world's power grids. Wind already accounts for almost 10% of power generated in the United States, but the heavy, expensive turbines present obstacles to adoption. Wind is more consistent at higher altitude, favoring tall bases that support the turbine blades several hundred feet in the air. This supporting structure must be very heavy, making installation more expensive. The turbine blades generate a substantial amount of their power from wind near the tip, in effect wasting a lot of energy that goes into keeping the rest of the blade turning. These deficiencies have led to a variety of proposed kite-based wind-harvesting designs such as the one depicted in Figure 1. Such systems would be lighter weight and more cost-efficient.

Unlike a conventional wind turbine, in order to generate power the kite must fly a nontrivial pattern. Identifying a dynamically feasible, optimal trajectory presents a challenge as the kite system is underactuated; the kite's 6-dimensional state must be controlled with only two control inputs – the kite's roll orientation and its tether length. We describe a two-stage approach to the challenge of controlling a power-generating kite. First, a power-maximizing orbit is computed offline with nonlinear optimization. The resulting reference trajectory is tracked with a model predictive controller.

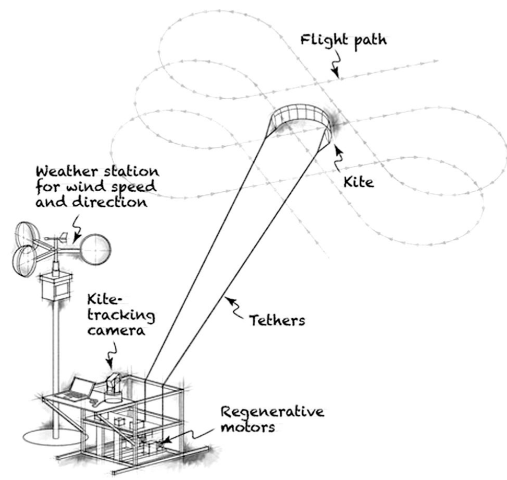

Fig. 1. One implementation of a kite power system uses a ground-based weather station to estimate the wind vector, and a ground based camera to estimate the kite state. The kite is connected to a generator/motor by two lines. Pulling differentialy on the lines allows for roll control. Figure adapted from https://www.popsci.com/science/article/2012-11/blueprint-kite-captures-energy/

II. Related Work

There have been many previous examinations of the kite control problem. Our kite dynamics model is based on the one presented in [1]. That paper examines a figure-eight periodic orbit for the kite and shows it is unstable when open loop. It then demonstrates stabilization of the desired trajectory with an MPC formulation. We extend the control problem in that paper to allow for a dynamic tether length. In [2], the authors analyze a two-line kite in even more depth, investigating the stability of several orbits and investigating chaotic behavior of the dynamics. They show that a kite can be designed to have passively stable periodic orbits.

In [3], the authors develop a full system model of the joint kite / generator system. They examine the two-step process of generating and stabilizing a kite trajectory that maximizes the power output of the generator. [4] takes a similar approach to the kite power generation model, but examines the kite dynamics in more depth and compares model predictions to data collected from physical kite experimentaion.

Interest in power generation with kites has not been limited to the academic sphere. Several companies have emerged in this field as well. Ampyx Power[1] has developed a large airframe to generate power on a ground-based generator in a similar manner to the aformentioned academic work. KiteGen Research[2] explored Gigawatt-scale offshore kite power using swarms of kites tethered to a single anchor. Makani Power

[1] www.ampyxpower.com
[2] http://www.kitegen.com/en/

[3] also has developed a flying power generating machine, although it generates electricity onboard the airframe via an array of propellers, using its tether as a means for sending the electricity back to the ground. The substantial interest in kite control problems from both academia and industry shows that these control problems are a burgeoning field worth studying.

III. Dynamics Model

For this project, we use a simple model of kite dynamics developed in [1].

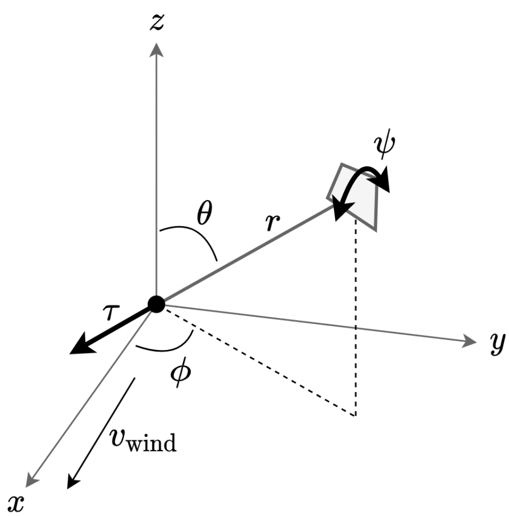

Fig. 2. Kite state $q = (\theta, \phi, r)$. Kite control $u = (\psi, \tau)$

We describe the state of the kite with spherical coordinates (θ, ϕ, r) where the origin is located at base station. θ is the angle between vertical and the kite. ϕ describes the lateral angle from the kite anchor's "forward" direction. r is the kite tether length. The kite's orientation is described by its two principal directions: the longitudinal and transverse axes. Control lines connected to the lateral tips of the kite give control of the kite's roll around its longitudinal axis, ψ. We have no direct control over the kite's motion along the longitudinal axis. The kite passively rotates such that this axis is always aligned with kite-relative airspeed. The wind acts on the kite with a lift force

$$F_l = \frac{1}{2}\rho \|w_e\|^2 AC_l \tag{1}$$

and drag force

$$F_d = \frac{1}{2}\rho \|w_e\|^2 AC_d \tag{2}$$

where ρ represents air density, w_e denotes the kite's airspeed vector, A is the characteristic area, and C_l and C_d are the coefficients of lift and drag respectively. We make the simplifying assumption of a time-invariant, uniform wind field.

The lift force acts perpendicular to the plane of the kite, and the drag force acts in the plane of the kite.

[3]makanipower.com

The electric generator on the ground controls the tension on the kite's tether, denoted τ. It can reel the kite in or let it out. The generator's instantaneous power equals the tether velocity times the applied torque, $-\dot{r}\tau$. Ideally, the kite's trajectory will result in a net generation of power. As shown by [1], these assumptions lead to the system dynamics

$$f(x,u,w) = \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{r} \\ \frac{F_\theta^{\text{aer}}(x,u)}{rm} + \sin(\theta)(\frac{g}{r} + \cos(\theta)\dot{\phi}^2) - \frac{2\dot{r}\dot{\theta}}{r} \\ \frac{F_\phi^{\text{aer}}(x,u)}{rm} - 2\cot(\theta)\dot{\phi}\dot{\theta} - \frac{2\dot{r}\dot{\phi}}{r} \\ \frac{F_r^{\text{aer}}(x,u)+\tau}{m} - g\cos(\theta) + r\dot{\theta}^2 + r\sin^2(\theta)\dot{\phi}^2 \end{bmatrix} \tag{3}$$

$F_\theta^{\text{aer}}(x,u)$, $F_\phi^{\text{aer}}(x,u)$ and $F_r^{\text{aer}}(x,u)$ map the lift and drag force from Equations 1 and 2 into the spherical coordinate system.

IV. Finding an Optimal Orbit

When deployed in the real world, we'd like our kite to fly indefinitely and generate as much power as possible. A greedy policy with respect to power is likely to crash the kite. We could use MPC with a suitably long planning horizon to find a better policy, but this requires an expensive optimization online. Instead, we perform the expensive optimization offline to search for a dynamically feasible orbit which maximizes amortized power. The kite can then fly this orbit indefinitely, and robustly track the desired trajectory using a more efficient controller.

To optimize for this orbit offline, we solve a constrained nonlinear optimization problem using SNOPT. Our orbit is discretized into a fixed number of steps, T (we experimented between 100 and 800 steps)

A. Optimization Variables

We optimize a trajectory of states and controls, as well as the timestep, h

- $q = \{q_1 \dots q_{T+1}\}$, where $q_t = (\theta_t, \phi_t, r_t)$
- $\dot{q} = \{\dot{q}_1 \dots \dot{q}_{T+1}\}$, where $\dot{q}_t = (\dot{\theta}_t, \dot{\phi}_t, \dot{r}_t)$
- $\ddot{q} = \{\ddot{q}_1 \dots \ddot{q}_T\}$, where $\ddot{q}_t = (\ddot{\theta}_t, \ddot{\phi}_t, \ddot{r}_t)$
- $u = \{u_1 \dots u_T\}$ is a vector of controls, $u_t = (\psi_t, \tau_t)$
- h is the timestep for the discrete dynamics

For simplified notation, we define the state $x_t = (q_t, \dot{q}_t)$

B. Constraints

Our trajectory is constrained to be dynamically feasible using Backward Euler integration

- $(\dot{q}_{t+1}, \ddot{q}_t) = f((q_{t+1}, \dot{q}_{t+1}), u_t, w)$
- $q_{t+1} = q_t + h\dot{q}_{t+1}$
- $\dot{q}_{t+1} = \dot{q}_t + h\ddot{q}_t$

To prevent the kite from hitting the ground we impose a constraint on the angle of the tether

$$0 \le \theta_t \le \theta_{\text{max}} \ \forall t$$

We also constrain the length of the length of the tether (in meters)

$$10 \leq r_t \leq 60 \ \forall t$$

Our dynamics model is only valid when the tension on the line is non-negative, so we constrain the tension control $\tau_t \geq 0 \ \forall t$. We also impose roll constraint to prevent the kite from tipping over $|\psi_t| \leq \psi_{\text{max}} \ \forall t$.

The duration of the orbit, hT, can be optimized by SNOPT, and we constrain it to between 5 and 60 seconds

$$\frac{5}{T} \leq h \leq \frac{60}{T}$$

We initially experimented with optimizing a separate h_t for each timestep, but we found that the optimal solutions would exploit the discretized dynamics approximation to produce dynamically infeasible trajectories by placing the largest timesteps where the velocity was highest. Optimizing a uniform timestep allowed us to reduce the number of optimization variables, which in turn meant that we could increase T and discretize our trajectory more finely.

Finally we impose the the trajectory must be an orbit, by constraining the last state to be equal to the first: $x_1 = x_{T+1}$.

C. Objective

The optimal orbit is the orbit which produces the most power. The instantaneous power is $-\tau \dot{r}$, and we optimize the average power along the whole trajectory. In order to ensure that the resulting trajectories are smooth, we impose a control smoothness cost by penalizing the squared difference between successive successive control inputs. We use the notation $\|y\|_W^2 = y^T W y$, where W is a diagonal matrix, to denote the weighted L_2 norm of y

The trajectory optimization is then

$$\begin{aligned} \min_{x,u} \ & \frac{1}{T} \sum_{t=1}^{T} \tau_t \dot{r}_t + \sum_{t=1}^{T-1} \|u_t - u_{t+1}\|_{W_u}^2 \\ \text{s.t.} \quad & \dot{x} = f(x, u, w) \\ & x_{i+1} = x_i + h\dot{x}_i \\ & x_1 = x_T \\ & \tau \leq 0 \\ & |\psi| \leq \psi_{max} \\ & 0 \leq \theta \leq \pi/2 \\ & h_{\text{min}} \leq h \leq h_{\text{max}} \end{aligned} \tag{4}$$

D. Symmetric Trajectory Assumption

The kite dynamics are symmetric in ϕ, so if we constrain the optimal orbit to be symmetric across the $\phi = 0$ plane, then it suffices to optimize only half of the trajectory. This significantly reduces the number of optimization variables, and allows us to find much longer power-generating trajectories.

To accomplish this we modify several constraints. Rather than enforcing the last state to equal the first $x_1 = x_{T+1}$, we flip the ϕ and $\dot{\phi}$ components:

$$(\theta_1, -\phi_1, r_1, \dot{\theta}_1, -\dot{\phi}_1, \dot{r}_1) = x_{T+1} \tag{5}$$

This ensures that the last state is the same as the first when we mirror the trajectory. We must also ensure that when transitioning from one half the trajectory to the other, we do not teleport across the ϕ axis. This requires that $\phi_{T+1} = 0$.

Additionally, our control smoothness constraint must be modified to ensure the last control input, u_t is close to the first control input of the mirrored second half of the trajectory. The roll is mirrored across $\phi = 0$, but the tension is not, so we add the cost

$$c_u \|u_T - (-\psi_1, \tau_1)\|_2^2 \tag{6}$$

V. MPC

The reference trajectory trajectory is unstable, so we use a receding horizon Model Predictive Controller (MPC) to stabilize it. The MPC formulation allows us to account for asymmetric constraint on tether tension (i.e. that we can pull as much as we want, but cannot push) more easily than other techniques like LQR.

With the reference trajectory denoted $\hat{x}$, we can write the MPC optimization as

$$\begin{aligned} \min_{x,u} \ & \|x_T - \hat{x}_T\|_{W_f}^2 + \sum_{i}^{T-1} \|x_i - \hat{x}_i\|_W^2 \\ \text{s.t.} \quad & \dot{x}_i = f(x_i, u_i) \\ & x_{i+1} = x_i + h\dot{x}_i \\ & |u| \leq u_{max} \end{aligned} \tag{7}$$

where are W and W_f are the diagonal weighting matrices for the deviation costs for the first $T-1$ timesteps and final timestep respectively. We found that making the weights in W_f slightly larger than those in W was beneficial. The nonlinear program was solved with SNOPT.

Linearized Dynamics MPC

Even when only optimizing 10 timesteps into the future, the nonlinear optimization runs too slowly to be used in real time. To address this problem, we linearized the kite dynamics around the reference trajectory to allow for efficient solving as a quadratic program. We attempted to find symbolic Jacobians for the kite dynamics using both Sympy and PyDrake's symbolic tools, but both approaches failed to produce a result. We expect that the issue stems from a cross product that is buried in the kite dynamics. The cross product term in the aerodynamic forces resulted in extraordinarily large symbolic expressions for the dynamics. Inside the dynamics model, we compute a change of bases from polar to euclidean coordinates to calculate aerodynamic forces, and then back to polar coordinates to calculate the resulting accelerations. This lengthy expression is not compactly unravelled by the chain rule. We elected compute a simple numerical linearization instead. The linearization results in two matrices for each timestep, A_i and B_i, such that we can write the state evolution constraint as

$$\dot{x_i} = A_i x_i + B_i u_i$$

The linearized MPC formulation ran much faster than the nonlinear version, as we expected. Unfortunately, its performance declined substantially. While we were able to very roughly track some simple orbits, the performance was quite sensitive to tuning parameters and we were never able to achieve satisfactory results.

VI. Methods

We begin by finding a desired energy-generating orbit with Drake's mathematical program framework. We use the default SQP-based SNOPT optimizer to deal with the nonlinear dynamics constraints. The resulting trajectory is then used as the reference for a model predictive controller. The kite system can then be simulated in Drake as shown in Figure 3.

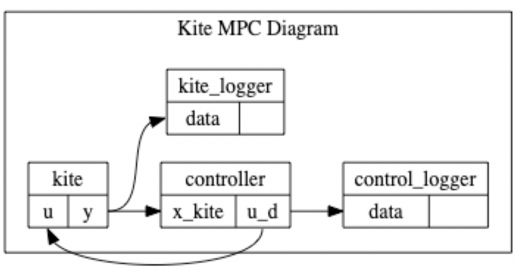

Fig. 3. Diagram of the MPC system generated by `DiagramBuilder`

VII. Results

We successfully optimized energy-producing orbits in SNOPT, and managed to stabilize some of them with a nonlinear MPC implementation. We begin by introducing the range of trajectories that were expected to be feasible by the trajectory optimization process, and then discuss to what extent these were able to be controlled with nonlinear MPC.

A. *Finding Orbits*

It took considerable fine-tuning to convince SNOPT to produce valid trajectories. Typically the optimization would fail with numerical errors. We found that placing limits on ϕ and θ to prevent the kite from flying behind the anchor was necessary to produce reasonable orbits. We also found that the optimization was very sensitive to the relative scaling of the quadratic smoothness and power objectives.

We used a time-parameterized lemniscate in ϕ and θ (and it's time derivatives) as an initial guess for the optimization. The initial guess was augmented with small Gaussian noise to avoid singularities in our model. To find longer trajectories,

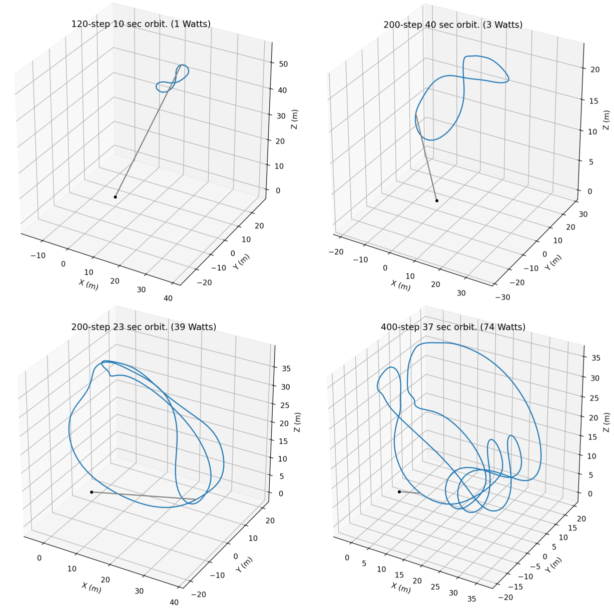

Fig. 4. Orbits produced by optimizing for power.

we had greater success initializing the optimization with interpolations of shorter trajectories.

Initially, we limited $\tau \leq 10$ and disabled the power objective. This produced the top-left orbit shown in Figures 4 and 5.

Although optimizing short trajectories seemed to produce symmetric orbits or otherwise simple orbits, as we increased the number of timesteps the resulting trajectories became convoluted and power output did not improve significantly. Introducing the symmetry constraint described in section IV.*D* allowed us to optimize only half the trajectory and resulted in more performant orbits (top-left 4, 5). We increased the tension limit to $\tau \leq 100$, allowing the generator to pull much harder, and found orbits with significant power output (bottom two in 4, 5).

We were excited to find that under our dynamics model, and with aggressive costs on control smoothness, a 0.5m^2 kite in a 6m/s wind could theoretically be controlled produce an average power output of 74 Watts.

B. *Understanding Orbits*

By coloring each step along the orbit location by the instantaneous power output at that time, we can gain some understanding of the behavior of the system (Figures 5, 6).

As can be seen in Figure 6, the orbit is composed of two main phases (which are repeated twice per full cycle due to the symmetric constraint). In the power generating phase, the kite flies low figure-eights, creating significant tension on the tether which allows the generator to slowly reel out the kite and generate up to 400 Watts. After several loops, the tether stops reeling out and the kite flies up to the zenith, where it is largely aligned with the wind and there is minimal tension on the tether. Here, the generator expends some power to reel the kite back in and the cycle repeats.

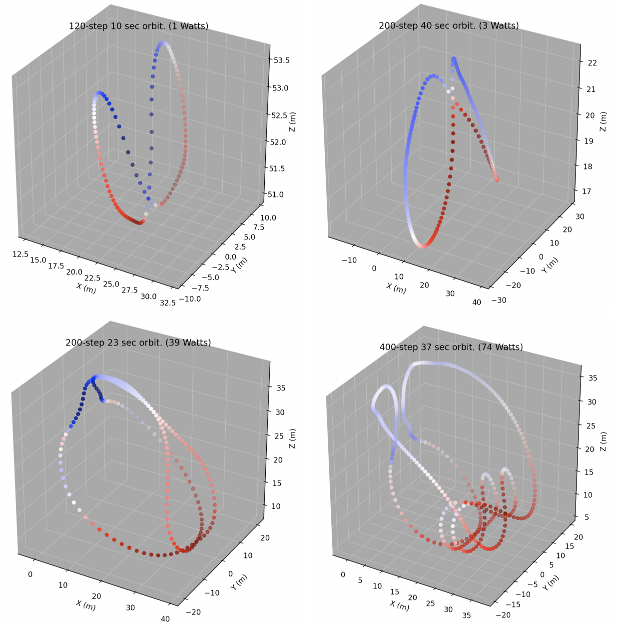

Fig. 5. The same orbits as in Figure 4 showing power generation along the trajectory. Red indicates positive power output, and blue indicates when the generator expends power to reel the kite in.

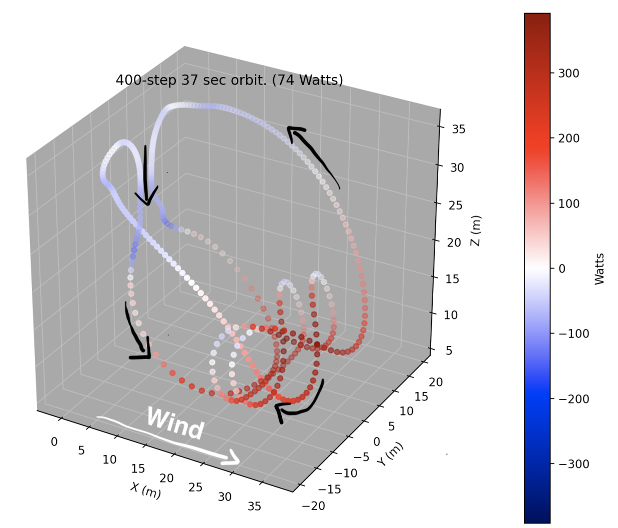

Fig. 6. An orbit showing the two-phase power generating cycle.

This cycle of low figure-eights and then flying high to reel in is widely discussed in the literature [3], [5] and is said to be optimal. Most previous work parameterizes the orbit by different phases of the cycle or adds constraints to force the optimal trajectory to include figure-eights, though our approach is able to recover the behavior directly.

C. MPC

We had the most success with stabilizing the simple lemniscate orbit shown in Figure 7. The MPC closely tracked the reference trajectory and successfully generated power. As we were only able to achieve stabilization with the nonlinear

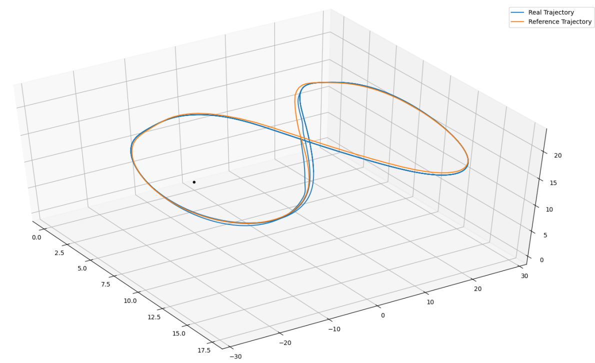

Fig. 7. Reference trajectory and simulated trajectory stabilized by MPC.

MPC (and not the linearized version), it was not quite fast enough to run in real time. On more aggressive trajectories that were found from the trajectory optimization, the nonlinear solver seemed to get stuck in certain parts of the trajectory, unable to find solutions. We expect that this shortcoming is due to SNOPT or its default settings not being a good fit for this tracking problem and not an inherent limitation of nonlinear optimization for this problem. Similar trajectory tracking approaches, with similar kite models, have successfully used nonlinear MPC approaches (such as [3]).

We found that the most reasonable tradeoff between solution times and tracking accuracy was a MPC update update rate of about 5 Hz, with a 10-step lookahead. Figure 8 shows how well the MPC was able to track the reference trajectory from Figure 7. The resulting instantaneous power output is shown in Figure 9. The average power generation for this trajectory is 3.8 Watts, slightly better than the reference trajectory's 3 Watts. We expect that tweaks to the nonlinear optimizer (such as further tuning of SNOPT's settings or switching to an interior point method) would allow us to increase power generation by tracking the more aggressive trajectories.

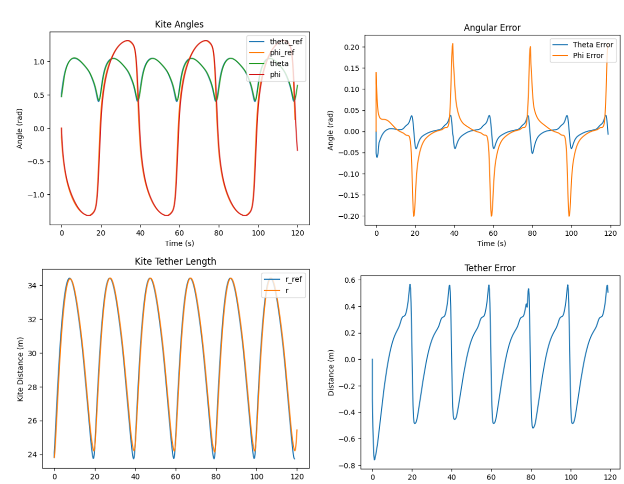

Fig. 8. MPC tracking performance of desired kite state.

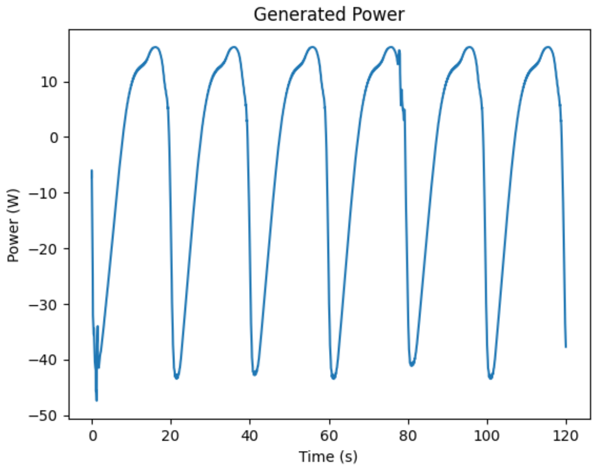

Fig. 9. Instantaneous power generation

VIII. Conclusion

Climate change creates an urgent need for cost effective renewable energy, and kite power is one such technology which is on the cusp of readiness for deployment at scale. Generating power with kites involves optimal control of an underactuated dynamical system. This paper describes how to use nonlinear constrained optimization to identify dynamically feasible orbits which maximize the power output of the kite, and then track those trajectories using model predictive control. Our approach was able to find an orbit which produces an average output of 75 watts given a a 0.5m^2 kite in a 6m/s wind, enough to power LED lighting in a small home. In future work, we'd like to address wind and state uncertainty in our orbits using robust optimization techniques. We'd also like to directly optimize the stability of the orbits, rather than leaning heavily on MPC to recover stability.

Code for this project is available on GitHub at https://github.com/IzzyBrand/kiteTrajOpt.

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Keyword: underactuated