Boat Design Report

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1 Executive Summary

For this project, we were tasked with designing a boat, made primarily out of pink insulation foam, that would float level with a payload of about 720g. Furthermore, our boat needed to have an angle of vanishing stability between 120° and 140°. We also needed to include a .5m mast and between 0g to 300g of ballast. The boat was also supposed to be as fast as possible, but we chose not to focus on that requirement.

This report discusses the different elements–conceptual and mathematical–that we had to consider while designing a boat to meet the given requirements. It also explains the design choices we made.

Our final product was a boat with a hull defined by $z = |y|^2 - 1$, a length of 40 cm, a width of 20 cm, and a depth of 10 cm. We correctly estimated its angle of vanishing stability to be around 132°.

Below is a summary of this report's sections and their contents.

1. **Executive Summary**

   Contains an overview of the project and this report.

2. **Background and Terminology**

   Defines relevant boating terms.

3. **Design Considerations**

   Analyzes the 3 main considerations necessary to fulfill the requirements.

   I. **To Float a Boat**

      Design Consideration 1, making the boat float. Looks at forces of gravity and buoyancy

   II. **To Float a Boat Flat**

      Design Consideration 2, making the boat float level. Looks at center of mass and center of buoyancy.

III. **Angle of Vanishing Stability**

Design Consideration 3, determining the boat's AVS. Looks at the moment created by the forces of gravity and buoyancy.

4. **Proposed Design and Justification**

   Presents our boat design and explains how it fulfills all the requirements.

   I. **Defining the Hull**

   Choosing the equation $z = |y|^n - 1$ to define our hull.

   II. **Placing the Center of Mass**

   Placing the mast, ballast and payload in appropriate locations to achieve the desired AVS between 120° and 140°.

5. **Performance**

   Analyzing whether our boat performed to our expected standard.

6. **Appendix**

   Contains the detailed math we used to calculate our boat's center of buoyancy as the boat tips.

2 Background and Terminology

In order to understand our design choices, it is necessary to understand some basic boat terminology, as well as physics concepts associated with buoyancy. Below we will define the key terms and concepts that we will reference throughout this paper.

Hull

The hull is the outer, curved surface of the boat. It is often defined by equation driven curves.

Keel

The keel is an extension of the hull, which can help lower the center of mass of a boat. It protrudes from the bottom of the hull and has extra weight in its lower end.

Deck

The deck is the top surface of a boat; it is where you would stand if you were on a boat.

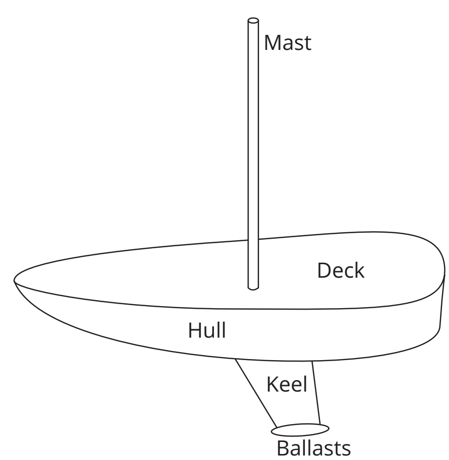


Figure 1: Boat Diagram: Visualizing key boat terminology

Mast

the Mast is a long post extending vertically from the deck of the boat, used to attach sails and rigging to.

Ballast

Ballast is extra weight added in order to lower a boat's center of mass, often attached to the bottom of a boat or the end of a keel.

Center of Mass (COM)

The unique point that can be used to represent where external forces and torques on an object act. The center of mass is, for example, the point upon which the gravitational force acts. see Figure 2 for a visualization of this concept.

Center of Buoyancy (COB)

The center of buoyancy is the center of mass of the submerged part of an object; this is the point that the sum of all the buoyant forces acts upon. See Figure 2 for a visualization of this concept.

Angle of Vanishing Stability (AVS)

The AVS is the angle at which a boat will capsize when tipped to one side. If the boat is tipped to any angle less than the AVS, it will right itself. If the boat is tipped to an angle larger than its AVS, it will capsize.

Waterline

The waterline is the line that results from the intersection of the water and a boat; this line separates the submerged part of the boat from the non-submerged part.

Righting Moment/Righting Arm

This is the moment that attempts to right a boat–or capsize it–when it has been tipped. It occurs when the center of buoyancy is out of line from the center of mass. If the righting moment is positive, the boat will right itself; if the righting moment is negative, the boat will capsize.

Payload

A boat's payload is whatever the boat is carrying that is not part of the boat itself (freight, people, etc.).

3 Design Considerations

For this project, we were tasked with designing a boat that would float level with a payload of about 720g, and have an AVS between 120° and 140°. The goal is also to design a boat that is as fast as possible (while adhering to the requirements stated above). However, we went with a simpler design that let us to prioritize our boat's stability and AVS at the unfortunate cost of its speed.

Our conceptual approach to these design restraints can be broken down into 3 sub sections:

1. Ensuring the boat will float
2. Ensuring that the boat will float level with the water
3. Giving the boat an AVS between 120° and 140°

I. *To Float a Boat*

The first design consideration is to make sure the boat will float. In order for that to be true, the density of the boat must be less than the density of water (1000 kg/m^3). The density of the boat can be described as such:

$$M_{boat} = m_1 + m_2 + m_3 \tag{1}$$

$$V_{boat} = \int\int_{Domain} \text{dzdy} * L \tag{2}$$

$$D_{boat} = M_{boat}/V_{boat} \tag{3}$$

where $m_1 = Mass\ of\ Soda\ Cans$, $m_2 = Mass\ of\ Mast$, $m_3 = Mass\ of\ Ballast$ and $L = Length\ of\ Boat$.
For our purposes, the mass of the foam is negligible, so we will only take into account the mass of the payload (2 soda cans), the mast, and ballasts.

If the boat had the same density as water, the top of the boat would float flush with the water surface. Figure 1 shows a free body diagram of the forces acting on the boat: gravity and buoyancy. As the boat's density decreases, the waterline moves down the boat and the deck of the boat rises higher above the water's surface.

Since the density of our primary material (pink insulation foam) is extremely low, and the density of our payload is roughly the same as water, it is highly unlikely that our boat will sink.

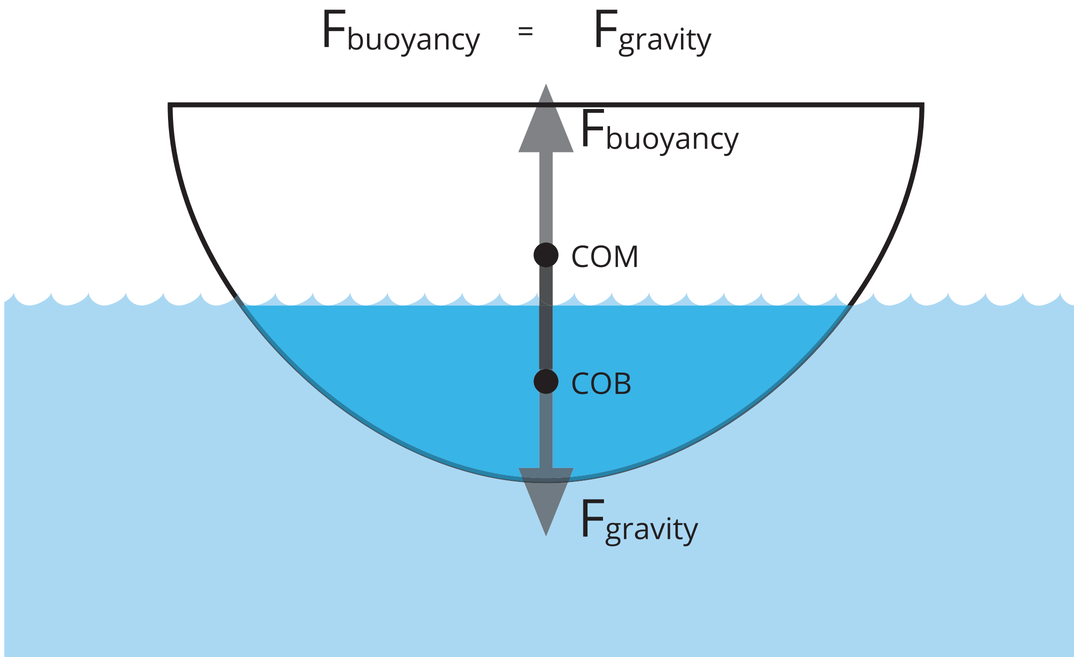


Figure 2: Boat Free Body Diagram: A boat floats where the sum of the buoyant and gravitational forces equals zero

II. *To Float a Boat Flat*

In order for a boat to float flat, the buoyant force must act directly above or below the boat's center of mass when the deck is parallel to the waterline. For example, in Figure 3, the boat on the left would float flat, since the the center of buoyancy is below the center of mass when the boat is level. However, the boat on the right would not float flat, because there is a moment acting on the boat when the deck and the waterline are parallel, due to the center of buoyancy not being in line with the center of mass.

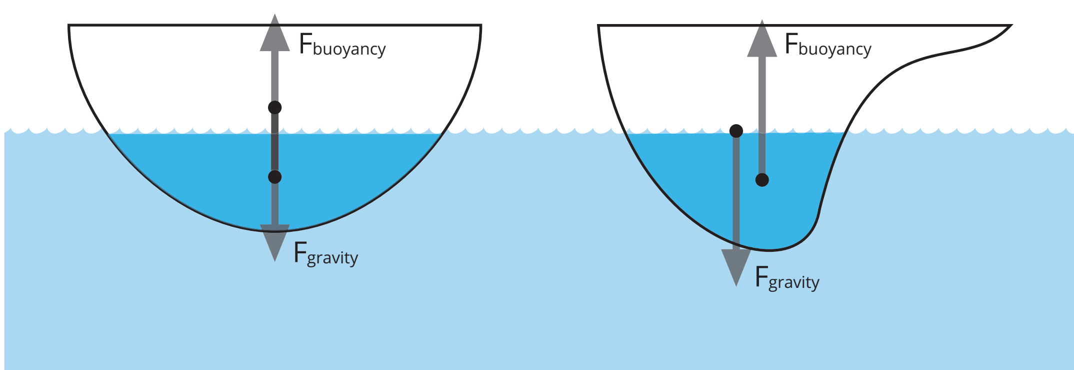


Figure 3: Boat Equilibrium: The boat on the left would float flat, since it has a net moment of zero when the waterline and deck are parallel. The boat on the right, however, would not float flat, since it has a positive net moment when it is in the given position.

To make sure our boat floats flat, we will first define our hull with an equation, $f(y)$, that is symmetric across the z-axis (see Figure 5 for coordinate set up). We will also be careful to place our mast, ballast, and payload along this same line of symmetry when we fabricate our boat.

III. *Angle Of Vanishing Stability*

In order to calculate the Angle of Vanishing Stability (AVS), or the angle at which the boat will capsize, one needs to be able to calculate the righting arm, or the moment about the boat's center of mass caused by the buoyant force. As the boat tips, the center of buoyancy will change location relative to the center of mass, creating a moment–the righting arm. If the righting arm is positive, the boat will right itself; if it is negative, the boat will capsize. In general, as seen in Figure 4, the lower a boat's center of mass, the higher its AVS.

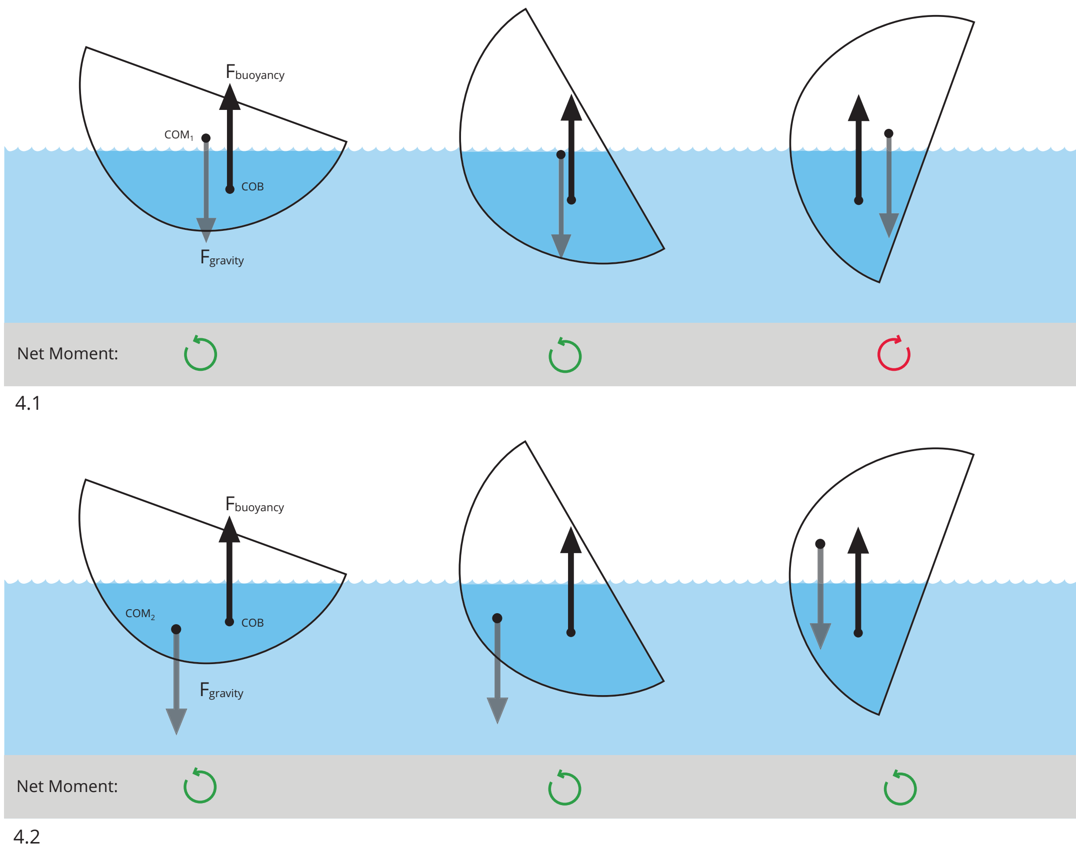


Figure 4: Tipping Cases: The cases above show how the center of buoyancy changes as the boat is tipped and how the righting moment depends on the center of mass's location. As seen in Figure 4.1, once the center of buoyancy moves to the left of the center of mass, the direction of the moment changes, and the boat capsizes. If the center of mass is closer to the bottom of the boat, as in Figure 4.2, the center of buoyancy will stay to the right of the COM for larger angles, giving the boat a higher AVS.

Equations 4-6 below give the equations necessary to find the righting moment:

$$\text{Submerged Area} = \int\int_{\text{Domain}} \text{dzdy} \tag{4}$$

$$\vec{COB} = \frac{\int\int_{\text{Domain}} y\hat{\imath} + z\hat{\jmath}\ \text{dzdy}}{Submerged\ Area} \tag{5}$$

$$\vec{M} = \vec{r}_{COB} \times \vec{F_b} + \vec{r}_{COM} \times \vec{F_g} \tag{6}$$

where $\vec{r}_{COM}$ is the vector from the deck's center to the center of mass, $\vec{r}_{COB}$ is the vector from the deck's center to the COM, $\vec{F_b}$ is the buoyant force, and $\vec{F_g}$ is the gravitational force. The domain used for the integral in Equation 4 is the bounds of the cross-sectional area of the submerged part of the boat.

4 Proposed Design and Justification

Having introduced you to the general concepts behind boats and the three main tasks we must consider given our requirements, we will now present our boat design and explain how it fulfills all the requirements. This is broken down into two steps:

1. Defining the hull's equation

2. Placing the center of mass by placing the payload, mast and ballast

I. *Defining the Hull*

When deciding what equation to use to define our boat's hull, we chose to use the basic equation $z = |y|^n - 1$ to simplify our later calculations. As can be seen in Figure 5, we used a (y,z) coordinate system, where the deck of the boat is coincident with the y axis.

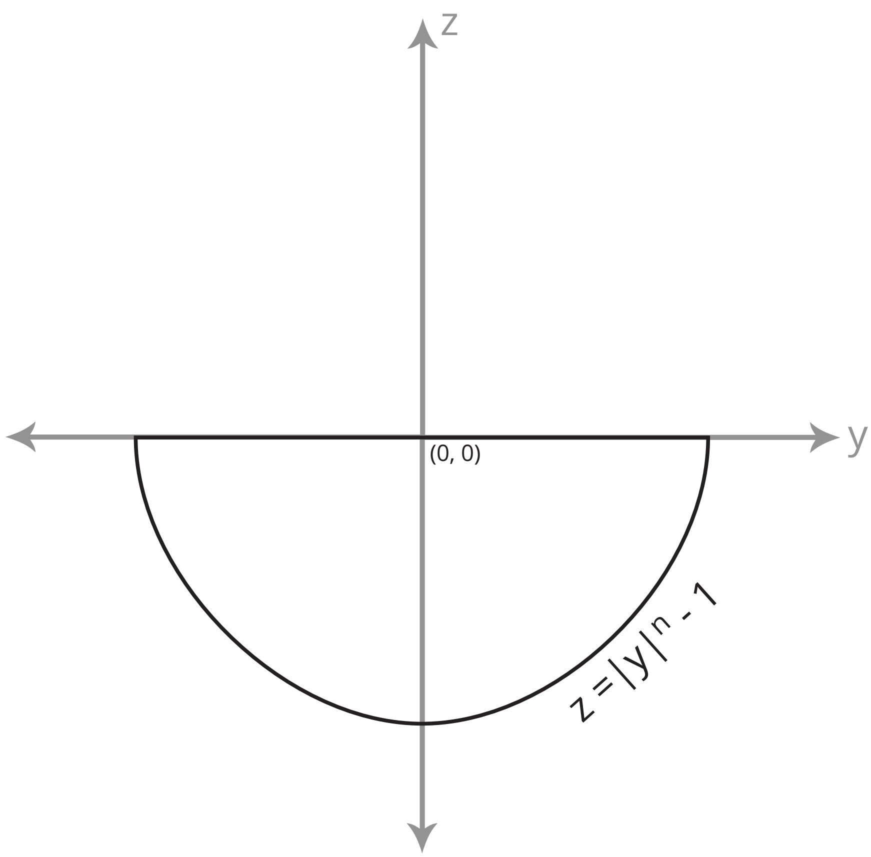


Figure 5: Defining the Hull: the general equation we used to define our hull, $z = |y|^n - 1$, with a (y,z) coordinate system.

Given the general equation $z = |y|^n - 1$, as the value of n changes, the cross-sectional shape of the boat will also change. As can be seen in Figure 6, as the value of n increases, the boat's cross-section becomes more and more rectangular, and the area of the lower portion of the boat increases. This causes the boat to float higher in the water. As a result, as the value of n increases, the boat's center of mass moves further above the waterline, making the boat less stable. With this in mind, we chose n=2, since $z = |y|^2 - 1$ results in a boat with a center of mass closer to the waterline, as well as a rounded hull.

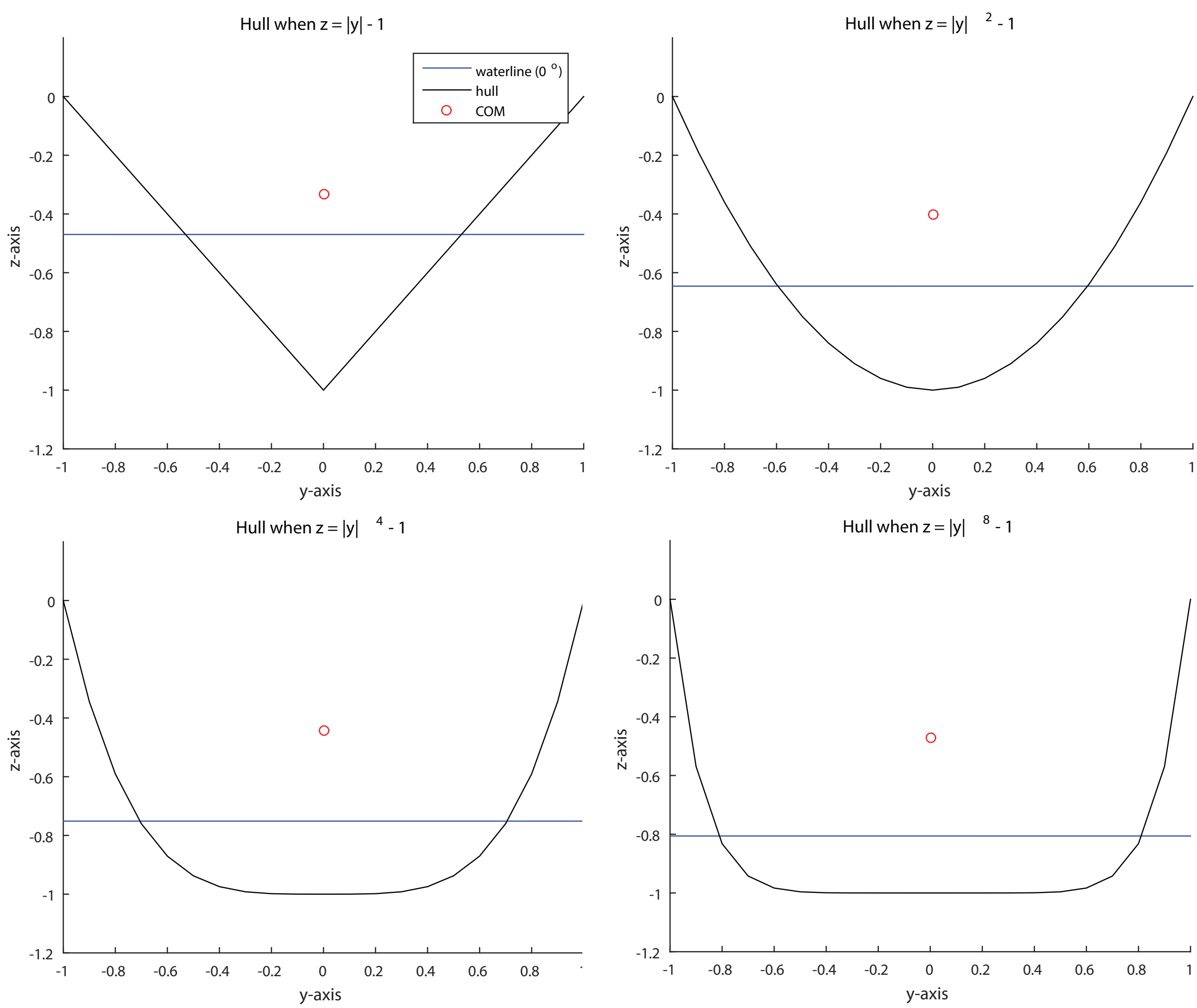


Figure 6: Relationship between n and COM: For a hull defined by the general equation $z = |y|^n - 1$, as the value of n increases, the hull shape becomes increasingly rectangular. Furthermore, as n increases, the boat's center of mass (represented by a red circle) moves further above the waterline, decreasing the boat's overall stability

II. *Placing the Center of Mass*

After deciding on an equation for the hull, and knowing the total mass of our boat, the one thing left to decide is the location of the cans, ballast and mast. As these three things are moved up and down relative to the boat's deck, the boat's center of mass will also move up and down, thus affecting the boat's stability/AVS.

This section will be divided into two sub-sections to analyze how, specifically, the center of mass's location affects (A) the center of buoyancy, and (B) the righting arm.

In this section, r_x refers to the vertical distance between the deck's center and the COM of either the payload, mast or ballast, where r_1 is associated with the payload, r_2 the mast, and r_3 the ballast.

A. *Center of Buoyancy*

As our boat tips, its center of buoyancy will move out of line with the boat's center of mass, as seen in Figure 7. Therefore, in order to later calculate the righting arm, it is necessary to calculate the COB's location as ϑ changes.

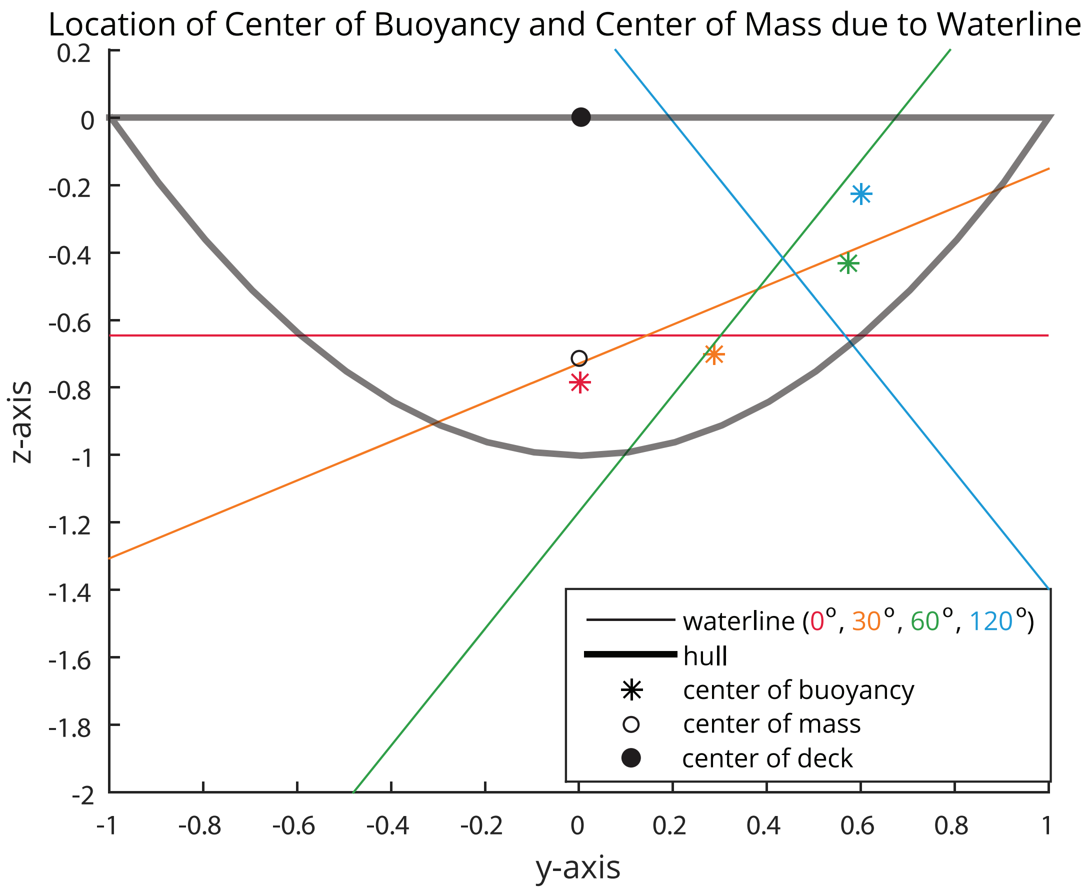


Figure 7: When our boat is tipped 0°, 30°, 60° and 120°, the waterline and center of buoyancy (represented by the colored lines and colored stars, respectively) change to reflect the new value of ϑ. This figure looks specifically at a boat where the center of mass is 0.7227 m below the deck ($r_1 = -0.033m$, $r_2 = 0m$, $r_3 = -0.25m$). The resulting AVS is around 132°

As mentioned in the Angle of Vanishing Stability portion of the Design Considerations section, the boat's center of buoyancy can be calculated using these equations:

$$\text{Submerged Area} = \int\int_{\text{Domain}} \text{dzdy} \tag{7}$$

$$C\vec{O}B = \frac{\int\int_{\text{Domain}} y\hat{\imath} + z\hat{\jmath}\ \text{dzdy}}{Submerged\ Area} \tag{8}$$

For more detail on the specific domains of the integrals in Equations 7 and 8, especially the changes that occur as the boat tilts, please refer to the Appendix.

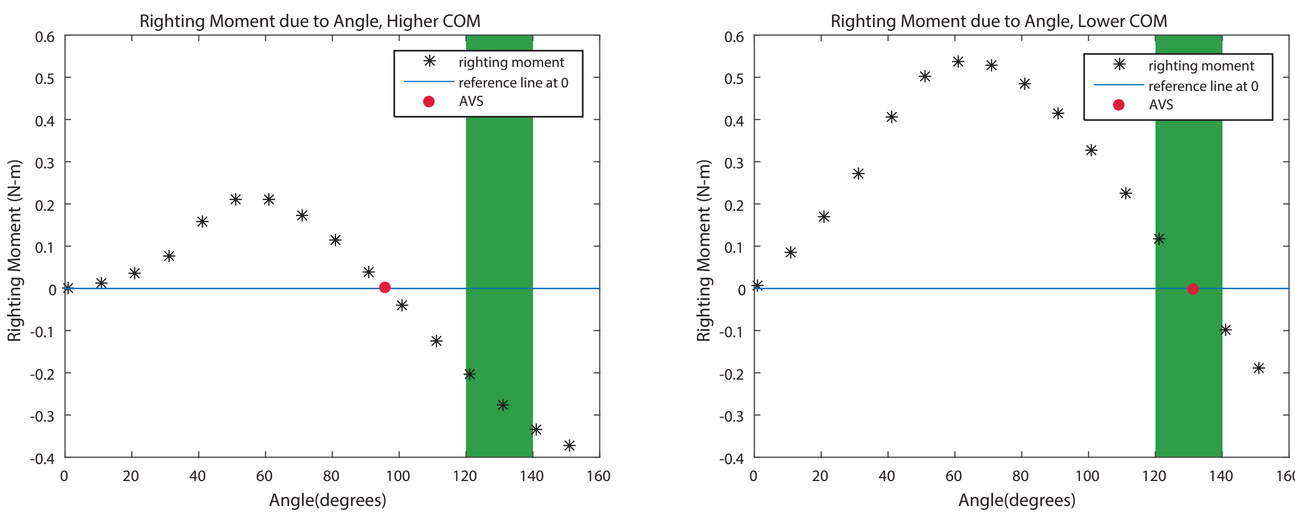
B. *Righting Arm*

As the boat's center of mass moves down, the righting arm is positive for more values, causing the AVS to become larger. As mentioned in the Angle of Vanishing Stability portion of the Design Considerations section, the moment generated by the buoyant and gravitational forces can be calculated using this equation:

$$\vec{M} = \vec{r}_{COB} \times \vec{F_b} + \vec{r}_{COM} \times \vec{F_g} \tag{9}$$

where $\vec{r}_{COM}$ is the vector from the deck's center to the center of mass, $\vec{r}_{COB}$ is the vector from the deck's center to the COM, $\vec{F_b}$ is the buoyant force, and $\vec{F_g}$ is the gravitational force.

Figure 8 shows that, if our boat had a center of mass .3470 m below the deck ($r_1 = 0.4\ m$, $r_2 = 0\ m$, $r_3 = -0.25\ m$), the AVS would be about 98°. On the other hand, if the center of mass is 0.7227 m below the deck ($r_1 = -0.033\ m$, $r_2 = 0\ m$, $r_3 = -0.25\ m$), it results in an AVS of about 132°. As this is right in the target range of 120° to 140°, this is where we chose to place our center of mass. A visualization of this set up can be seen in Figure 9. Note that, when the boat has a higher center of mass (as in Figure 8.1), the boat tips over at smaller angles then when the center of mass is lower (as in Figure 8.2), where the boat will right itself when tipped to larger angles.



8.1 8.2

Figure 8: Righting Moments: Figure 8.1 shows the righting moments for a boat with a center of mass 0.3470 m below the deck ($r_1 = 0.4\ m$, $r_2 = 0\ m$, $r_3 = -0.25\ m$), which results in an AVS of around 98°. Figure 8.2 shows the righting moments for a boat with a center of mass 0.7227 m below the deck ($r_1 = -0.033\ m$, $r_2 = 0\ m$, $r_3 = -.25\ m$) resulting in an AVS around 132° – right in our target zone of 120° to 140° (represented by the green stripe). The inflection point of each graph (occurring somewhere around 50° to 70°) are due to how, as seen in Figure 4, when the boat is tipped, the COB initially moves further away from the COM and the righting arm increases. Then, the COB starts moving back towards the COM and the righting arm decreases, eventually becoming negative.

C. *Final Product*

Once we had done all of our boat research and related calculations, we had to implement our findings and create our boat. The final dimensions we chose can be seen in Figure 9, and our final boat being tested can be seen in Figure 10 Our final boat is 0.4 m long, 0.2 m wide and 0.1 m deep. We chose these dimensions specifically because we wanted a compact boat, and because it was possible with these dimensions to adjust the center of mass to give us our desired AVS. Once we chose the boat's dimensions, we adjusted the depth of the mast, ballast and payload to adjust the COM to obtain our desired AVS. The front profile of the boat is defined by $z = |y|^2 - 1$ and $z = 0$, and to give our boat 3-dimensions we extruded this shape 0.4 m. This cross-section was chosen because it helps keep the COM low and closer to the waterline, making it easier to stabilize the boat. The mast's center of mass is level with the deck, the payload's center of mass is 0.033 m below the deck, and the ballast's center of mass is 0.25 m below the deck. This gives the boat an overall center of mass 0.7227 m below the deck, which results in an estimated AVS of about 132°.

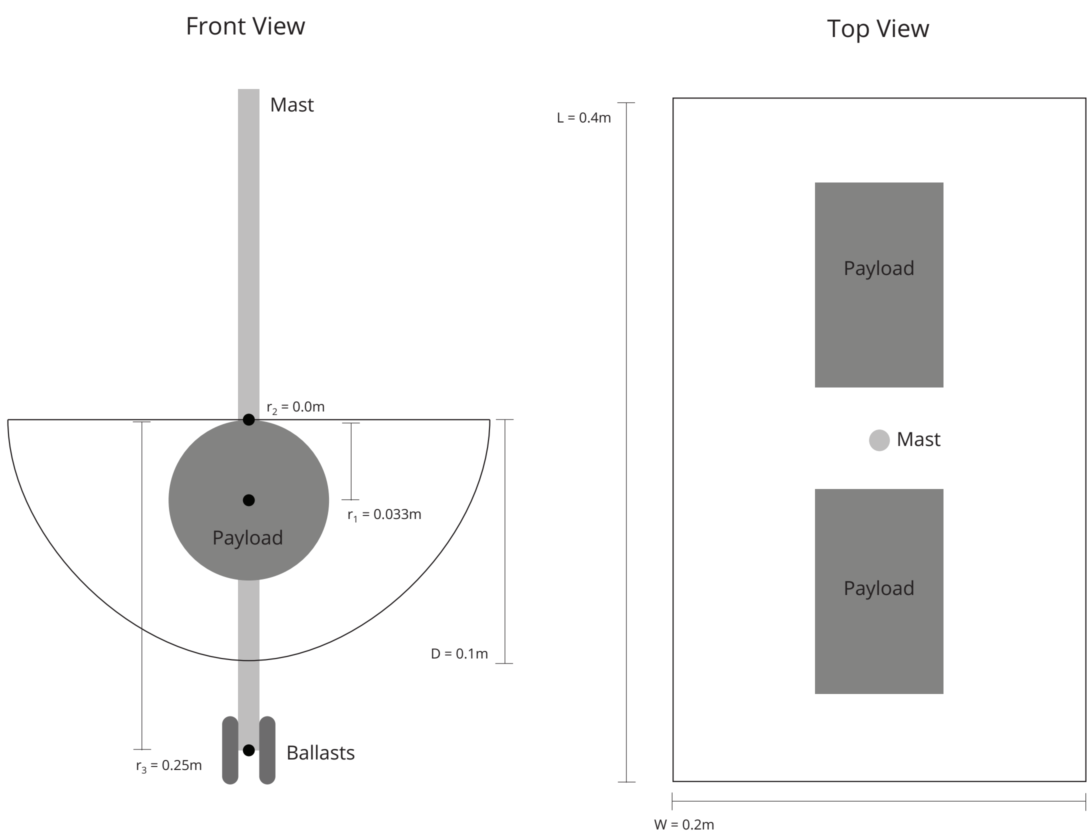


Figure 9: Final Boat Layout: This figure shows two cross-sectional view and a top view of our boat with its final design, where $r_1 = -0.33\ m$, $r_2 = 0\ m$, and $r_3 = -.25\ m$. This results in a COM 0.7227 m below the deck and an AVS of about 132°. The boat is 0.2 m wide, 0.1 m deep, and 0.4 m long. *Note: This figure is not to scale.*

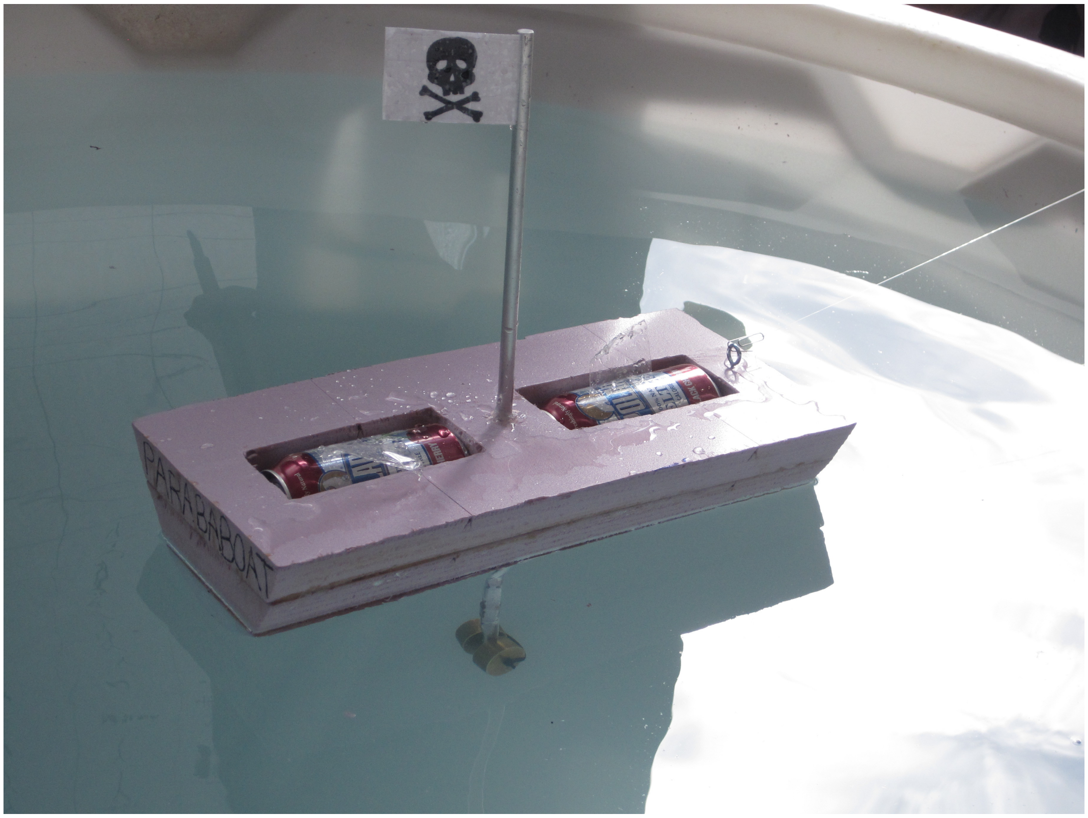


Figure 10: The finished boat in testing

5 Performance

Our boat's performance was within a reasonable threshold of our calculations. As predicted, due to our boat's density and center of mass, our boat floated and the deck was level with the water's surface. Furthermore, as seen in Figure 8.2, our calculations predicted that the boat would have an AVS of 132°. When tested, we found that the actual AVS was 135°. This 3° discrepancy was likely due to human error in the fabrication process as well as the addition of adhesive, a flag, and zip-ties. Although we did not come up with an estimate for the speed of our boat (due to time constraints), we expected the boat to move slowly since we chose to focus on designing our boat for stability rather than speed. Thus, it was no surprise that our boat's top speed was only .286 m/s.

6 Appendix

In order to calculate the center of buoyancy for the boat (and thus the righting arm) for varying values of ϑ, it is easiest to split up the calculations into 3 different cases. Case 1 is when $\vartheta < 90$ and the waterline does not intersect the deck, Case 2 is when $\vartheta < 90$ and the waterline does intersect the deck, and Case 3 is when $\vartheta > 90$.

In all of the following math, $|y|^n - 1$ is the equation that defines the boat's hull, $z = (tan\vartheta)y - d$ defines the waterline, d is the distance in the z-direction between the boat's deck and the waterline, and $\frac{d}{tan\vartheta}$ is the intersection between the waterline and the boat's deck. Intersections 1 and 2 are explained in the next paragraph. Finally, this is all being defined on a y-z coordinate plane with its origin at the center of the boat's deck.

In all three scenarios, it is necessary to know the intersection point(s) between the waterline and the hull. In all of the math to follow (as well as all of the diagrams), the intersection of these two equations to the left of the z-axis is referred to as Intersection 1, and the intersection to the right of the z-axis is referred to as Intersection 2.

To solve for these points of intersection, we set $|y|^n - 1$ and $z = (tan\vartheta)y - d$ equal to each other and solved for y (when given a value of d), as shown below in Equation 10. We did this using the fzero root finding function in Matlab. Again, Intersection 1 is the negative resulting value of y, while Intersection 2 is the positive resulting value of y. Figure 11 depicts these cases.

$$tan(\vartheta)y + d + 1 - y^n = 0 \tag{10}$$

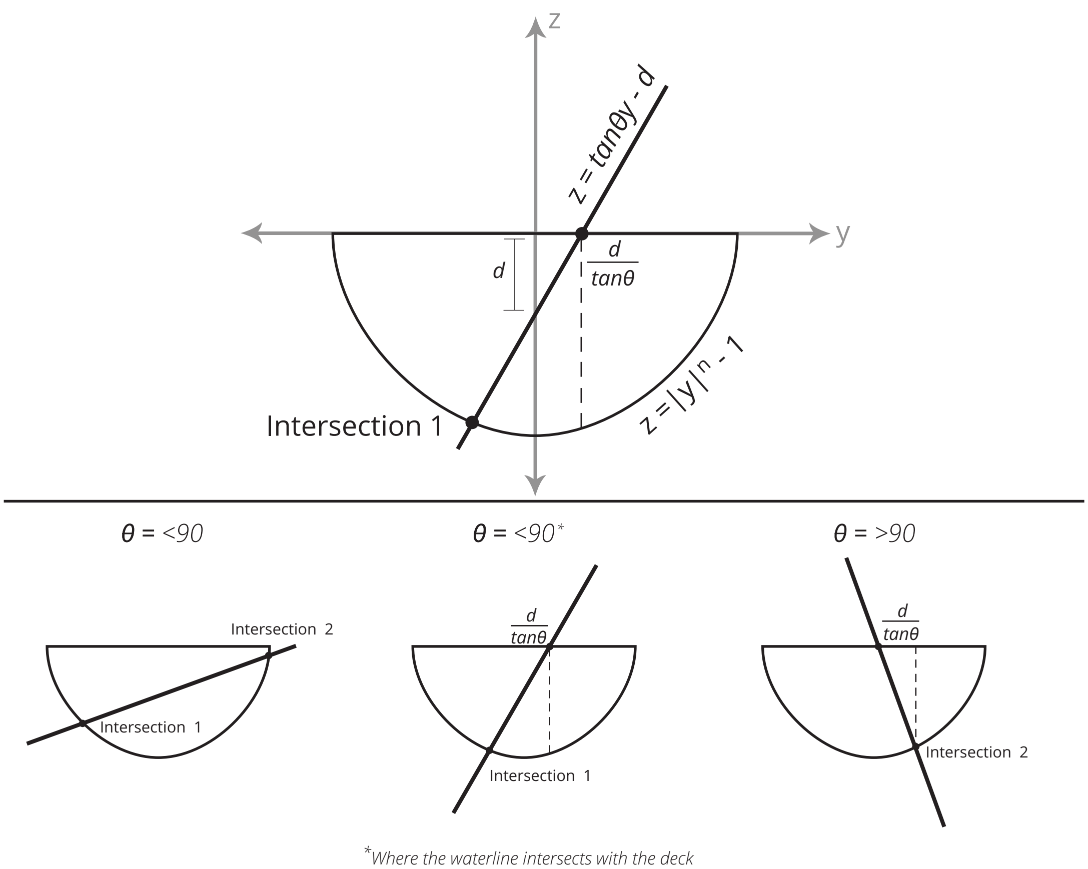


Figure 11: This figure shows $|y|^n - 1$ as the equation defining the boat's hull, $z = (tan\vartheta)y - d$ defining the waterline, d as the distance in the z-direction between the boat's deck and the waterline, $\frac{d}{tan\vartheta}$ as the intersection between the waterline and the boat's deck, Intersection 1 as the negative intersection point between the waterline and the hull, and Intersection 2 as the positive intersection point. The three different cases show how the definition of the domain changes as the angle of the waterline changes.

Case 1: $\vartheta < 90$, Waterline does not intersect Deck

$$\text{Submerged Area} = \int_{intersection1}^{d/tan(\vartheta)} \int_{|y|^n-1}^{tan(\vartheta)y+d} 1 \text{ dzdy} + \int_{d/tan(\vartheta)}^{1} \int_{|y|^n-1}^{0} 1 \text{ dzdy} \tag{11}$$

$$C\vec{O}B_1 = \frac{\int_{intersection1}^{intersection2} \int_{|y|^n-1}^{tan(\vartheta)y+d} y\hat{i} + z\hat{j} \text{ dzdy}}{\int_{intersection1}^{intersection2} \int_{|y|^n-1}^{tan(\vartheta)y+d} 1\text{dzdy}} \tag{12}$$

Case 2: $\vartheta < 90$**, Waterline intersects Deck**

$$\text{Submerged Area} = \int_{d/tan(\vartheta)}^{intersection2} \int_{tan(\vartheta)y+d}^{0} 1 \text{ dzdy} + \int_{intersection2}^{1} \int_{|y|^n-1}^{0} 1 \text{ dzdy} \tag{13}$$

$$C\vec{O}B_2 = \frac{\int_{intersection1}^{d/tan(\vartheta)} \int_{|y|^n-1}^{tan(\vartheta)y+d} y\hat{i} + z\hat{j} \text{ dzdy}}{\int_{intersection1}^{d/tan(\vartheta)} \int_{|y|^n-1}^{tan(\vartheta)y+d} 1\text{dzdy}} \tag{14}$$

Case 3: $\vartheta > 90$

$$\text{Submerged Area} = \int_{intersection1}^{intersection2} \int_{|y|^n-1}^{tan(\vartheta)y+d} 1 \text{ dzdy} \tag{15}$$

$$C\vec{O}B_3 = \frac{\int_{d/tan(\vartheta)}^{intersection2} \int_{tan(\vartheta)y+d}^{0} y\hat{i} + z\hat{j} \text{ dzdy}}{\int_{d/tan(\vartheta)}^{intersection2} \int_{tan(\vartheta)y+d}^{0} 1\text{dzdy}} \tag{16}$$