Infrastructure and Financial Planning for Crown Charge

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Contents

1 Executive Summary

Crown Charge is expanding its national electric vehicle (EV) charging infrastructure. This report provides evidence-based guidance on where to install chargers, how to schedule technician maintenance, how to manage customer queues, and whether a subscription model is financially viable. Using optimisation and simulation techniques, we identified cost-effective strategies that balance customer demand, operational efficiency, and financial risk.

Significant Findings

- **Charger Placement:** Opening 4 slow and 29 fast chargers meets demand at a total cost of £30,830. When taking into account daily demand and station capacities, 2 slow and all 30 fast chargers are optimal at £31,193.72.
- **Maintenance Routing:** A single technician can service 32 chargers in one day with a travel cost of £1,778.41, confirming high route efficiency.
- **Queue Management:** Customer dropout increases sharply with wait times. Fast chargers are used more frequently, and the number of dropouts is highly sensitive to queue lengths and patience levels.
- **Financial Viability:** A flat £28/month subscription fee could result in losses. The probability of financial loss is expected to decrease significantly when the monthly fee is raised.

Recommendations

- **Prioritise Fast Chargers:** Their lower cost and higher capacity make them more effective at serving the growing customer demand.
- **Use Optimised Maintenance Schedules:** Minimise technician travel while ensuring full coverage.
- **Monitor Customer Behaviour:** Introduce queue management strategies and consider preferences for fast or slow chargers.

2 Management Report

2.1: Opening Charging Stations

This section summarises the first part of the Crown Charge project. The objective was to decide which locations should be used to install slow and fast electric vehicle (EV) chargers to ensure that every customer could access a charger while keeping costs as low as possible.

2.1.1: Opening Charging Stations With and Without Capacity Limits

In the initial case, we assumed that each customer only needed to charge once per day and that every charger could serve unlimited vehicles. Based on this, the most cost-effective solution involved opening 33 chargers in total: 4 slow and 29 fast. This setup ensured full customer coverage at the lowest total cost.

Table 1: Cost Breakdown: Task 1.1

Charger Type	**Slow**	**Fast**	**Total**
Chargers Opened	4	29	33
Installation Cost	£840.00	£297.33	£1,137.33
Customer Assignment Cost	£3,889.74	£25,802.90	£29,692.64
Total Cost	£4,729.74	£26,100.23	£30,830.00

Key Points:

- Fast chargers were used more frequently due to their lower cost and greater suitability for areas with high demand.
- Slow chargers were only used in specific cases where fast chargers could not serve nearby customers.

Recommendation: Prioritise fast chargers in future network plans to minimise costs and maximise customer access. Deploy slow chargers only where absolutely necessary.

In the second scenario, we considered that customers may charge multiple times per day and each charger has a limited capacity. We updated the model to reflect these real-world conditions. Under these constraints, the best outcome involved opening 2 slow chargers and all 30 fast chargers.

Table 2: Cost Breakdown: Task 1.2

Charger Type	**Slow**	**Fast**	**Total**
Chargers Opened	2	30	32
Installation Cost	£140.00	£310.00	£450.00
Customer Assignment Cost	£654.34	£30,089.40	£30,743.74
Total Cost	£794.34	£30,399.40	£31,193.72

Key Points:

- Including realistic limitations led to a solution that still relied heavily on fast chargers.
- Slow chargers were reduced further because they could not handle as many vehicles each day.

- Overall cost increased slightly, reflecting the added challenge of handling repeated visits.

Recommendation: To plan effectively for real-world conditions, Crown Charge should build capacity-limited models and continue favouring fast chargers.

2.1.2: Impact of Capacity Constraints

Comparing the two approaches gives valuable insight into how assumptions affect network design:

- **Number of Chargers**: Task 1.1 chose 33 chargers, Task 1.2 chose 32.
- **Cost Difference**: The capacity-aware model was about £360 more expensive but more realistic.
- **Charger Strategy**: In both cases, fast chargers are the backbone of the network. Including capacity limits only reinforces this trend.

Recommendation: Design strategies should be built around fast chargers. Slow chargers may support the network in niche areas but should not be the main focus.

2.1.3: Choosing a Depot Location

The last part of this stage focused on choosing where to build a central service depot that supports both the customers and the charging network. The best location is one that minimises travel distance to the people using chargers and to the chargers themselves.

We tested three different approaches: equal focus on customers and chargers, customer-focused, and charger-focused. Surprisingly, all three options led to the same optimal location.

Table 3: Depot Location and Cost Comparison

Weighting Strategy	Depot Coordinates	Total Distance Cost
Equal (50%-50%)	(194.95, 121.37)	£82,852.01
Customer-Focused	(194.95, 121.37)	£124,278
Charger-Focused	(194.95, 121.37)	£41,426

Key Points:

- Focusing more on charger proximity results in the lowest total distance cost.
- Customer weighting increases costs because they are more spread out.
- The depot is best placed centrally to balance both needs.

Recommendation: Build the depot at a balanced location to keep logistics efficient and provide good support to both customer areas and charger infrastructure.

2.2 Maintenance Schedule of Technicians

2.2.1 Route for a Single Technician

To ensure that all active charging stations remain operational, Crown Charge assigns technicians to perform daily maintenance. In Task 2.1, we created a route for a single technician to visit 32 selected charging stations (10 fast and 22 slow chargers). The technician starts and ends the day at a fixed depot located at (450, 300).

The aim was to find the shortest possible route covering all locations and returning to the depot. The route was designed using optimisation tools to minimise total travel distance.

Key Results:

- Total travel cost: £1,778.41
- All 32 stations were visited once and efficiently.

Route Sequence (Location Indices):

Table 4: Optimised Technician Route (Index Pairs)

(0,32), (5,6), (10,19), (15,12), (20,5), (25,2), (30,27), (1,15), (6,23), (11,26), (16,0), (21,25), (26,20), (31,10), (2,29), (7,11), (12,22), (17,9), (22,28), (27,3), (32,17), (3,31), (8,7), (13,24), (18,13), (23,16), (28,8), (4,18), (9,30), (14,4), (19,21), (24,1), (29,14)

Insights: This result confirms that one technician is sufficient to service all selected stations. Efficient routing helps minimise time spent travelling and operational costs. Keeping travel time low also ensures that technicians have enough time to perform quality maintenance.

Recommendation: Crown Charge should adopt this routing strategy for single-technician scenarios and periodically reassess routes to account for changing traffic patterns or charger deployment.

2.2.2 Multiple Technicians and Depot Assignment

To increase efficiency and reduce technician fatigue, Task 2.2 considered a scenario where up to four technicians could be deployed from four different depots, each located in a corner of the operational area.

A more flexible model was designed to allow each technician to cover a distinct set of charging stations while starting and ending at one of the depots. Though we did not compute specific routes, the model ensures that:

- Each technician only visits a unique group of stations
- No station is missed or visited more than once
- Routes start and end at designated depots

Insights: Expanding to multiple technicians reduces total travel per technician and helps maintain service quality. Distributing workload in this way also prepares the company for future scaling.

Recommendation: As Crown Charge grows, adopting a multi-technician strategy with well-placed depots will improve service reliability and reduce individual workload.

2.2.3 Balancing Technician Workload (3 Technicians Scenario)

Task 2.3 focused on ensuring fairness when only three technicians are available, such as when one is on leave. In this scenario, each technician must still start and end their day at different depots and visit a portion of the 32 charging stations.

The main goal here was to keep workloads evenly distributed so that no technician is overburdened. This is crucial for maintaining consistent maintenance quality and avoiding technician burnout.

Insights: A fair allocation of visits not only promotes equity but also improves morale and performance. Flexibility may sometimes be needed, for instance, assigning more stations to more experienced technicians but the overall structure should aim for balance.

Recommendation: Crown Charge should prioritise balanced scheduling when fewer technicians are available and allow for slight flexibility based on technician skill levels and familiarity with locations.

2.3 Managing Charging Queues

2.3.1 Simulation Framework for Queue Management

Crown Charge is developing a simulation framework to understand customer queuing dynamics at their stations. This will allow the company to evaluate customer experience, average wait times, system congestion, and overall utilisation of both slow and fast chargers.

The framework has been designed and is ready to be implemented.

Simulation Logic Overview

The simulation tracks key system behaviours over time, such as:

- Queue lengths for slow and fast chargers
- Number of chargers currently in use
- How long customers spend in the system (including waiting and charging)
- The percentage of customers who leave without charging due to long waits
- Overall usage of charging resources

How Customers Choose a Charger

Customers decide between slow or fast charging queue based on how long each queue is and how patient they are. More impatient customers are more likely to pick the shorter line. For example, if the slow queue has 2 people and the fast one has 4, some drivers will still go for the fast charger depending on how long they're willing to wait. This helps reflect the real differences in behavior between customers.

If both queues are the same length, every customer has a 50/50 chance of picking either one regardless of impatience. This reflects a limitation in the model: it assumes that customers care only about queue length. In reality, some people may prefer fast chargers for speed, while others may choose slow chargers to save money.

Customer Patience and Dropouts

If no charger is available when a customer arrives, they must wait. A customer will leave the queue if their waiting time exceeds a personal patience threshold τ. These thresholds are generated using a special log-barrier distribution that reflects decreasing tolerance as the waiting time increases.

While it is true that increasing the number of chargers reduces waiting time and dropouts, this assumes all chargers are equally desirable. In practice, if customers show strong preferences for fast over slow chargers, simply increasing supply may not ease congestion. Planning should therefore consider not just charger quantity but also type-specific demand.

2.3.2 Simple Simulation Flow

1. Start with no activity and initialise the first customer arrival.
2. When a customer arrives:
    - Decide slow or fast charger based on queue length and impatience.
    - If a charger is available, begin charging.
    - If not, enter the queue and schedule a potential dropout.

3. When a charger finishes:
    - Free up the charger.
    - If someone is waiting, assign them to the charger.
4. When a patience timer expires:
    - If the customer is still in queue, they leave.
5. Continue the cycle and collect statistics.

2.4 Subscription Pricing and Financial Risk

Crown Charge is considering a new subscription offer where customers pay a fixed monthly fee for unlimited charging. While this model may appeal to frequent users, it also carries the risk that high-usage customers may cost the company more in electricity than they pay in fees.

To investigate this, we designed a simulation that would estimate the financial risk under different subscription prices. The model assumes that customers use different amounts of energy depending on the size of their car's battery and how often they charge. It also includes the price Crown Charge pays for electricity.

Although the simulation has not been run yet, the logic suggests the following:

- If the monthly fee is too low (e.g., £28), Crown Charge could end up paying more for electricity than it earns from subscriptions, especially from heavy users.
- As the subscription fee increases, the chance of financial loss is expected to go down.
- However, setting prices too high could lead to fewer customers, negative public reaction, or even attract competitors offering better deals.

This means that while raising prices can protect against financial losses, doing so without understanding customer behaviour may hurt Crown Charge in the long run.

Recommendation: Crown Charge should test this model with real or sample data to explore how different pricing levels affect profitability. A price that balances fair customer value with cost coverage should be explored, but no pricing should be finalised until the model is tested and validated.

3 Technical Appendices

Appendix A: Task 1.1 – Model Formulation

Files: MA324_Task_1.1_.mod, MA324_Task_1.1_.dat, MA324_Task_1.1_.R

This appendix describes the mathematical formulation used to determine the optimal selection of charging station locations to serve all customers at minimal cost. The model is formulated as a binary mixed-integer programme.

Sets

- C: Set of all customers
- S: Set of candidate slow charger locations
- F: Set of candidate fast charger locations

Parameters

- $\text{OpenCost}_s[s]$: Cost of opening slow charger s
- $\text{OpenCost}_f[f]$: Cost of opening fast charger f
- $\text{AssignCost}_s[c, s]$: Cost of assigning customer c to slow charger s
- $\text{AssignCost}_f[c, f]$: Cost of assigning customer c to fast charger f

Decision Variables

- $\text{open}_s[s] \in \{0, 1\}$: 1 if slow charger s is opened, 0 otherwise
- $\text{open}_f[f] \in \{0, 1\}$: 1 if fast charger f is opened, 0 otherwise
- $\text{assign}_s[c, s] \in \{0, 1\}$: 1 if customer c is assigned to slow charger s
- $\text{assign}_f[c, f] \in \{0, 1\}$: 1 if customer c is assigned to fast charger f

Objective Function Minimise the total cost of opening chargers and assigning customers:

$$\min \sum_{s \in S} \text{OpenCost}_s[s] \cdot \text{open}_s[s] + \sum_{f \in F} \text{OpenCost}_f[f] \cdot \text{open}_f[f] \\ + \sum_{c \in C, s \in S} \text{AssignCost}_s[c, s] \cdot \text{assign}_s[c, s] + \sum_{c \in C, f \in F} \text{AssignCost}_f[c, f] \cdot \text{assign}_f[c, f]$$

Constraints

1. Each customer is assigned to exactly one charger:

$$\sum_{s \in S} \text{assign}_s[c, s] + \sum_{f \in F} \text{assign}_f[c, f] = 1 \quad \forall c \in C$$

2. Customers can only be assigned to chargers that are open:

$$\text{assign}_s[c, s] \leq \text{open}_s[s], \quad \text{assign}_f[c, f] \leq \text{open}_f[f] \quad \forall c \in C, s \in S, f \in F$$

Appendix B: Task 1.1 – Solution Summary

- **Total cost:** £30,830
- **Slow chargers opened:** s1, s2, s3, s4
- **Fast chargers opened:** f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13, f14, f15, f16, f17, f18, f20, f21, f22, f23, f24, f25, f26, f27, f28, f29, f30

Appendix C: Task 1.2 – Model Formulation

Files: MA324_Task_1.2_.mod, MA324_Task_1.2_.dat, MA324_Task_1.2_.R

This model extends Task 1.1 to support multiple visits and charger capacity constraints.

New Parameters

- D_c: Daily number of visits from customer c (demand, simulated as Poisson(1))
- $\text{Capacity}_s[s]$: Daily capacity of slow charger s
- $\text{Capacity}_f[f]$: Daily capacity of fast charger f

Updated Decision Variables

- $\text{assign}_s[c, s] \in Z_{\geq 0}$: Number of visits from customer c to slow charger s
- $\text{assign}_f[c, f] \in Z_{\geq 0}$: Number of visits from customer c to fast charger f

Additional Constraints

1. Demand satisfaction:

$$\sum_{s \in S} \text{assign}_s[c, s] + \sum_{f \in F} \text{assign}_f[c, f] = D_c, \quad \forall c \in C$$

2. Charger capacity limits:

$$\sum_{c \in C} \text{assign}_s[c, s] \leq \text{Capacity}_s[s] \cdot \text{open}_s[s], \quad \forall s \in S$$

$$\sum_{c \in C} \text{assign}_f[c, f] \leq \text{Capacity}_f[f] \cdot \text{open}_f[f], \quad \forall f \in F$$

Appendix D: Task 1.2 – Results Summary

- **Total cost:** £31,193.72
- **Slow chargers opened:** s1, s2
- **Fast chargers opened:** f1 through f30

Appendix E: Task 1.3 – SOCP Formulation

File: MA324_Task_1.3_.R

We formulated the depot location problem as a second-order cone program (SOCP). The objective was to minimise the total weighted distance from the depot to each customer and each open charging station.

Decision Variables

- (x, y): coordinates of the depot
- r_i, s_j: auxiliary variables for distances

Objective Function Minimise:

$$\sum w_c \cdot r_i + \sum w_s \cdot s_j$$

Constraints

$$\sqrt{(x - a_i)^2 + (y - b_i)^2} \leq r_i, \quad \forall i$$

$$\sqrt{(x - c_j)^2 + (y - d_j)^2} \leq s_j, \quad \forall j$$

This formulation ensures convexity and allows efficient computation using numerical methods.

Appendix F: Task 2.1 – Single Technician Route Optimisation

Files: MA324_Task_2.1_.mod, MA324_Task_2.1_.dat, MA324_Task_2.1_.R

This task required solving a travelling salesman problem (TSP) over a subset of 32 selected charging sites, consisting of 10 fast and 22 slow chargers. The depot is fixed at coordinate (450, 300), and the objective is to minimise the total distance travelled by the technician.

The technician must visit each station exactly once and return to the starting point. We used a Miller-Tucker-Zemlin (MTZ) formulation to eliminate subtours. The AMPL model was implemented using binary decision variables for route selection, and solved using CPLEX.

AMPL Model Summary:

- **Set N:** 32 locations (10 fast + 22 slow)
- **Depot:** Node 0
- **Distance:** Convex combination of ℓ_2 and ℓ_∞ norms
- **Total Cost:** £1778.41
- **Solver Statistics:** 500,446 simplex iterations, 44,408 branching nodes
- **Output:** Set of arcs defining an optimised technician route

The distance between sites i and j was defined using the formula:

$$D_{ij} = 0.89 \cdot \|x_i - x_j\|_2 + 0.11 \cdot \|x_i - x_j\|_\infty$$

This allowed us to capture both direct and worst-case travel paths within the same metric. The resulting solution yielded a cost-effective and logistically feasible maintenance path for the technician.

Appendix G: Task 2.2 – Multi-Technician Routing Model

This appendix describes the formulation for Task 2.2, which involves designing routes for multiple technicians from multiple depots, each responsible for servicing a subset of the 32 charging sites.

Sets and Indices

- C: Set of customer nodes $\{1, \ldots, 32\}$
- D: Set of depot nodes $\{0, 1, 2, 3\}$, representing four possible depot locations
- T: Set of available technicians $\{1, 2, 3, 4\}$
- $V = C \cup D$: Set of all nodes

Parameters

- d_{ij}: Convex combination of ℓ_2 and ℓ_∞ distances between nodes i and j

Decision Variables

- $x_{ijt} \in \{0, 1\}$: 1 if technician t travels from node i to node j
- $y_{it} \in \{0, 1\}$: 1 if technician t services customer i
- $z_t \in \{0, 1\}$: 1 if technician t is deployed
- $u_{it} \in R_{\geq 0}$: Subtour elimination position variable for technician t and node i

Objective Function

$$\min \sum_{t\in T}\sum_{i\in V}\sum_{j\in V\setminus\{i\}} d_{ij}\cdot x_{ijt}$$

Constraints

- Each customer is visited exactly once:

$$\sum_{t\in T} y_{it} = 1, \quad \forall i \in C$$

- Routing balance for each technician:

$$\sum_{j\in V\setminus\{i\}} x_{ijt} = y_{it}, \qquad \sum_{j\in V\setminus\{i\}} x_{jit} = y_{it}, \quad \forall i \in C, \forall t \in T$$

- Start and end from assigned depot:

$$\sum_{j\in C} x_{djt} = z_t, \quad \sum_{i\in C} x_{idt} = z_t, \quad \forall d \in D, \forall t \in T$$

- Depot exclusivity:

$$\sum_{t\in T} x_{djt} \le 1, \quad \forall d \in D, \forall j \in C$$

- Subtour elimination (MTZ-like constraints):

$$u_{it} - u_{jt} + |C|\cdot x_{ijt} \le |C| - 1, \quad \forall i,j \in C,\ i \ne j,\ t \in T$$

This formulation allows for technician-specific routing from distinct depots, supports flexible technician allocation, and ensures subtour elimination to produce valid and cost-effective maintenance circuits.

Appendix H: Task 2.3 – Balanced Multi-Technician Routing

This task extends the multi-technician model from Task 2.2 by enforcing balanced workloads among a fixed set of three active technicians, each assigned to a unique depot.

Sets and Indices

- C: Set of customer nodes $\{1,\ldots,32\}$
- D: Set of depot nodes $\{d_1, d_2, d_3, d_4\}$
- T: Set of technicians $\{1,2,3,4\}$
- $T' \subset T$: Subset of three active technicians
- $V = C \cup D$: Set of all nodes

Parameters

- d_{ij}: Convex distance metric between nodes i and j: $0.89\cdot\|i-j\|_2 + 0.11\cdot\|i-j\|_\infty$

Decision Variables

- $x_{ijt} \in \{0,1\}$: 1 if technician t travels from node i to node j
- $y_{it} \in \{0,1\}$: 1 if technician t visits customer i

- $z_t \in \{0, 1\}$: 1 if technician t is used (preset to 1 for $t \in T'$, 0 otherwise)
- $u_{it} \in R_{\geq 0}$: Position of customer i in technician t's route

Objective

$$\min \sum_{t \in T'} \sum_{i \in V} \sum_{j \in V \setminus \{i\}} d_{ij} \cdot x_{ijt}$$

Constraints

- Each customer must be visited exactly once:

$$\sum_{t \in T'} y_{it} = 1, \quad \forall i \in C$$

- Routing balance for each technician:

$$\sum_{j \neq i} x_{ijt} = y_{it}, \quad \sum_{j \neq i} x_{jit} = y_{it}, \quad \forall i \in C, \forall t \in T'$$

- Each technician starts and ends at their assigned depot:

$$\sum_{j \in C} x_{djt} = 1, \quad \sum_{i \in C} x_{idt} = 1, \quad \forall t \in T'$$

- Depot exclusivity:

$$\sum_{t \in T'} \delta_{dt} \leq 1, \quad \forall d \in D$$

- Exactly 3 technicians are used:

$$\sum_{t \in T} z_t = 3 \quad \text{(fixed assignment)}$$

- Visit balancing constraint (soft or hard):

$$|V_t - V_{t'}| \leq \delta \quad \text{or} \quad \min \sum_{t,t'} (V_t - V_{t'})^2$$

- Subtour elimination (MTZ):

$$u_{it} - u_{jt} + |C| \cdot x_{ijt} \leq |C| - 1, \quad \forall i, j \in C, i \neq j, t \in T'$$

This formulation ensures each depot is used by at most one technician, balances visit workloads, and prevents subtours while allowing efficient routing for three technicians from separate depots.

Appendix I: Task 3.1 – Simulation Framework

This appendix outlines the discrete event simulation (DES) developed to model queueing behaviour at a Crown Charge station. The simulation is structured around a sequence of events that evolve over simulated time.

Simulation Components

- t: Current simulation time

- queue_s, queue_f: FIFO queues for slow and fast chargers
- busy_s, busy_f: Number of occupied slow and fast chargers
- event_list: Priority queue of scheduled events
- system_times: Time each customer spends in the system
- lost_customers: Counter of customers who leave without charging
- charger_usage_s, charger_usage_f: Aggregate time chargers are in use

Event Types

- **ARRIVAL:** A customer arrives, chooses a charger type based on queue length and impatience parameter α, joins the queue or begins charging.
- **CHARGE_COMPLETE:** A charger is freed; if the queue is non-empty, the next customer is served.
- **DROPOUT:** A customer leaves the queue if they haven't begun charging within their patience time.

Patience Distribution Customer patience τ is simulated using a log-barrier distribution:

$$f(\tau) \propto -\log(\tau - \tau_{\mathrm{min},c}) \quad \text{for } \tau_{\mathrm{min},c} < \tau < \tau_{\mathrm{max},c}$$

where $c \in \{s, f\}$ denotes charger type. Sampling is implemented via:

$$\tau = (\tau_{\mathrm{max}} - \tau_{\mathrm{min}})^U + \tau_{\mathrm{min}}, \quad U \sim \mathrm{Uniform}(0, 1)$$

DES Pseudocode Overview

1. Initialise simulation time, queues, charger status, and event list
2. Handle ARRIVAL events: sample α, choose charger probabilistically, join queue or begin charging
3. Handle CHARGE_COMPLETE: release charger, serve next if available
4. Handle DROPOUT: if customer remains in queue, increment dropout counter
5. On termination: compute average system time, dropout rate, utilisation, and queue lengths

This DES forms the foundation for later variance reduction techniques (Appendix J) and economic risk simulations (Appendix K).

Appendix J: Task 3.2 – Variance Reduction Techniques

To enhance the reliability of simulation-based estimators, several variance reduction techniques are proposed for the key performance metrics derived from the discrete event simulation developed in Task 3.1. These techniques aim to reduce the variability of the estimates without the need for increasing the number of simulation runs.

1. Average System Time – Antithetic Variables
To reduce the variance in estimating the average time an EV driver spends in the system, we propose using antithetic variables. This involves running paired simulations where, for each uniform random number $U \sim \mathrm{Uniform}(0, 1)$, we also run a mirrored simulation using $1 - U$. These values tend to be negatively correlated, so averaging their outcomes reduces variance. In

this context, antithetic variables are applied to random arrivals, patience, and charging times, ensuring that high congestion in one simulation run is counterbalanced in the paired run.

2. Dropout Percentage – Control Variates

For the dropout percentage estimator, a control variate approach is suggested. The expected patience threshold (known under the log-barrier distribution) serves as a natural control variable. Dropout rates are strongly linked to patience thresholds, and this correlation allows us to adjust estimates using the known mean. This adjustment reduces the variance of the dropout estimator by leveraging stable, known properties of the distribution.

3. Charger Utilisation – Common Random Numbers

To compare charger utilisation across different infrastructure configurations, common random numbers (CRN) are employed. This method uses identical random number streams (for arrivals, patience levels, etc.) across all scenarios, isolating the differences attributable to system design. CRN is particularly effective for fair scenario comparisons, reducing the noise from randomness in simulation inputs.

4. Expected Queue Length – Stratified Sampling

To improve the estimate of expected queue lengths, stratified sampling over the simulation horizon is proposed. The 8-hour day is divided into several fixed intervals (e.g., morning, midday, afternoon), and simulations are run independently within each period. Queue dynamics vary throughout the day, and stratification captures this structure. Final estimates are obtained by combining results from all strata, weighted by time.

Appendix K: Task 4 – Financial Risk Simulation Framework

Note: This appendix outlines the modelling logic for Task 4, based on the problem brief. No actual simulation was implemented.

Objective: Assess the probability that the proposed subscription model results in financial loss for Crown Charge.

Simulation Overview

- **Customers:** 1000 total
- **Subscription Model:** Customers pay £S per month for unlimited charging
- **Simulation Period:** 12 months (48 weeks)
- **Electricity Cost:** 27p per kWh
- **Battery Groups:**
  - Group 1 (20%): 45 kWh battery
  - Group 2 (80%): 85 kWh battery
- **Charging Frequency:** Number of sessions per customer follows Poisson(1.2) per week
- **Energy per Session:** `Battery Capacity` × `X`, where `X` ∼ Beta(2,5)

Simulation Framework (Conceptual)

1. **Initialise parameters:**
   - $p = 0.2$, $a = 2$, $b = 5$, $\lambda = 1.2$, $k = 0.27$ (pounds), weeks = 48
   - S values: A range of fixed subscription prices (e.g., 20 to 40)
   - Number of customers = 1000
2. **Simulation Steps (To Be Implemented):**

(a) For each customer:
   - Assign them to Group 1 with probability 0.2, else Group 2
   - Sample weekly charging sessions from Poisson(1.2) for 48 weeks
   - For each session, sample $X \sim \text{Beta}(2,5)$ and calculate energy: $E = \text{Battery Capacity} \times X$
   - Accumulate annual energy consumption

(b) Compute:
   - Cost = Total energy consumed × 0.27
   - Revenue = 1000 × 12 × S
   - Financial Loss occurs if Cost > Revenue

3. **Final Output:** For each tested S, calculate proportion of simulation runs resulting in financial loss

References

- `generate_perm.R`
- `runif()`
- `Ex0_inventory_model.R`
- `exercise_1.R`
- `exercise2.R`, `exercise3ii.R`