Steel Section Flexural Buckling Resistance Calculator

In structural engineering, steel columns are frequently used to support axial loads. However, when a member is subject to compressive forces, they can fail due to buckling rather than yielding. Flexural buckling refers to the column deforming laterally, becoming misaligned with the axial forces line of action, leading to further deformation. This implies that the capacity of the section could be lower than anticipated, a crucial factor that needs to be accounted for. From Eurocode 3 (European Committee for Standardization [CEN], 2005a), the compression resistance of a cross-section can be found through the equation,

$$N_{c,Rd} = \frac{A\, f_y}{\gamma_{M0}} \tag{1}$$

Where,

- $N_{c,Rd}$ = Design resistance of the cross-section for uniform compression (kN)
- A = Cross-sectional area (mm^2)
- f_y = Yield strength of steel (N/mm^2)
- γ_{M0} = Partial factor for cross-sectional resistances

This refers to the basic compressive strength, when the steel would yield, ignoring any effects of buckling. This is derived from the expected force, pressure and area equation, where the product of yield stress and the area it acts over leads to the resistance of the cross-section, however, a partial factor is included to overengineer the design. One of the many checks done by the engineer is to ensure that $N_{c,Rd}$ is greater than the load applied to the structure.

While good as an initial estimate, this is only relevant to small portions of the section, an idealised and maximum resistance that the column could bear. In reality, buckling poses a significant risk. To account for this, an alternate partial factor is used, along with a reduction factor based on the slenderness of the member. In this case, the equation follows:

$$N_{b,Rd} = \frac{\chi_{min} A\, f_y}{\gamma_{M1}} \tag{2}$$

Where,

- $N_{b,Rd}$ = Design buckling resistance of a compression member (kN)
- γ_{M1} = Partial factor for member instability resistance
- χ_{min} = Buckling reduction factor

Buckling is capable of happening in either axis, both major and minor. Each side of the column could be restrained differently, which affects the effective slenderness in that direction. The slenderness of the section is partially responsible for the buckling reduction factor, greatly affecting the section capacity. The resistance of the member is determined by the weaker axis, the more slender one, since they will buckle first. Due to this, it is crucial that slenderness is checked on both axes and calculating the strength based on the lowest buckling reduction factor.

To get the buckling reduction factor, various intermediate values need to be determined. This factor is dependent on the slenderness of the member, material properties and imperfections.

Firstly, Eq 3 and 4 are material-dependent parameters independent of the axis.

$$\varepsilon = \sqrt{\frac{235}{f_y}} \tag{3}$$

$$\lambda_1 = 93.9\varepsilon \tag{4}$$

Where,

- ε = Strain factor based on f_y (N/mm^2)
- λ_1 = intermediate value for non-dimensional slenderness

Since the steel member can buckle in either axis, its slenderness must be evaluated separately for both directions,

$$\lambda_y = \frac{L_{cr,y}}{i_y} \tag{5}$$

$$\lambda_z = \frac{L_{cr,z}}{i_z} \tag{6}$$

Where,

- λ_y = Slenderness ratio in Major axis
- λ_z = Slenderness ratio in Minor axis
- i_y = Radius of gyration in Major axis (mm)
- i_z = Radius of gyration in Minor axis (mm)
- $L_{cr,y}$ = Length between lateral restraints in Major axis (mm)
- $L_{cr,z}$ = Length between lateral restraints in Minor axis (mm)

The radius of gyration is a cross-sectional property, easily found in data tables, while the restraint distances are to be assessed by the design. Together, the slenderness ratio is representative of a section's geometric stability. A high slenderness implies the member is narrow relative to its length, meaning it would be less stable. The more critical slenderness should not be chosen now to progress calculations as other factors affecting the reduction factor could pose greater influence.

Using the few previous variables, the next intermediate calculation is,

$$\overline{\lambda_y} = \frac{L_{cr,y}}{i_y}\frac{1}{\lambda_1} = \frac{\lambda_y}{\lambda_1} \tag{7}$$

$$\overline{\lambda_z} = \frac{L_{cr,z}}{i_z}\frac{1}{\lambda_1} = \frac{\lambda_z}{\lambda_1} \tag{8}$$

Where

- $\overline{\lambda_y}$ = non-dimensional slenderness in Major axis
- $\overline{\lambda_z}$ = non-dimensional slenderness in Minor axis

Flaws in the section are accounted into the buckling reduction factor through the intermediate value below, and further emphasise the effects of slenderness,

$$\Phi_y = 0.5\ \left(1 + \alpha_y\left(\overline{\lambda_y} - 0.2\right) + \overline{\lambda_y}^2\right) \tag{9}$$

$$\Phi_z = 0.5\ \left(1 + \alpha_z\left(\overline{\lambda_z} - 0.2\right) + \overline{\lambda_z}^2\right) \tag{10}$$

Where,

- Φ_y = Intermediate value for buckling reduction factor in Major axis
- Φ_z = Intermediate value for buckling reduction factor in Minor axis

- α_y = imperfection factor in Major axis
- α_z = imperfection factor in Minor axis

The imperfection factor itself is dependent on a large number of properties that vary case by case, such as the cross-section shape, dimensions, welds, steel type and strength, as well as the axis of buckling. These combine leading to different variations of the buckling reduction factor with non-dimensional slenderness, shown below, in Figure 1.

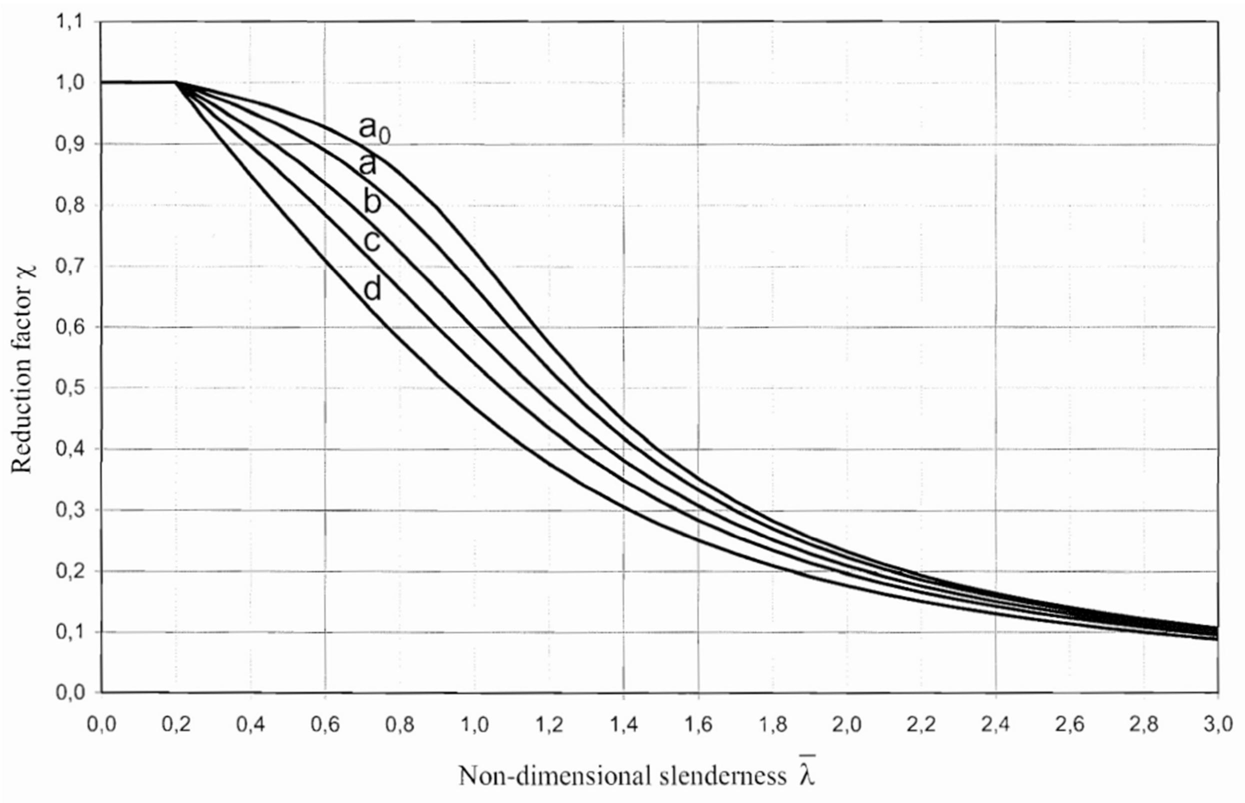


Figure 1 Effect of Bucking curves on reduction factor (European Committee for Standardization [CEN], 2005)

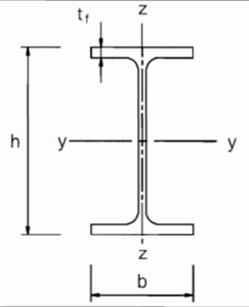
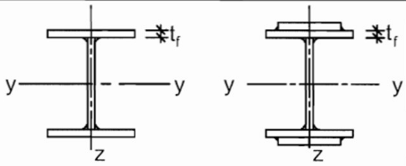
Cross section		Limits		Buckling about axis	Buckling curve: S 235, S 275, S 355, S 420	Buckling curve: S 460
Rolled sections		$h/b > 1{,}2$	$t_f \leq 40$ mm	y – y	a	a_0
				z – z	b	a_0
			40 mm $< t_f \leq 100$	y – y	b	a
				z – z	c	a
		$h/b \leq 1{,}2$	$t_f \leq 100$ mm	y – y	b	a
				z – z	c	a
			$t_f > 100$ mm	y – y	d	c
				z – z	d	c
Welded I-sections		$t_f \leq 40$ mm		y – y	b	b
				z – z	c	c
		$t_f > 40$ mm		y – y	c	c
				z – z	d	d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c

Table 1 Buckling Curve Selection (European Committee for Standardization [CEN], 2005)

The appropriate buckling curve is to be determined by the engineer, using Table 1, and then the imperfection factor can be chosen, using Table 2.

Buckling Curve	**a_0**	**a**	**b**	**c**	**d**
Imperfection factor α	0.13	0.21	0.34	0.49	0.76

Table 2 Imperfection Factor Selection (European Committee for Standardization [CEN], 2005)

Now, the buckling reduction factors for both axes can be calculated using the following expressions,

$$\chi_y = \frac{1}{\Phi_y + \sqrt{{\Phi_y}^2 - \overline{\lambda_y}^2}} \tag{11}$$

$$\chi_z = \frac{1}{\Phi_z + \sqrt{{\Phi_z}^2 - \overline{\lambda_z}^2}} \tag{12}$$

Where,

- χ_y = Buckling reduction factor in Major axis
- χ_z = Buckling reduction factor in Minor axis

The governing axis for buckling, χ_{min}, can be selected using the lowest of the two, then using Eq 2, the design buckling resistance of the member can be finally found.

$$N_{b,Rd} = \frac{\chi_{min} A\, f_y}{\gamma_{M1}} \tag{2}$$

Where variables take the same meaning and units as previously mentioned.

Assumptions and Limitations

Summary of Key Assumptions and Limitations:

- Load is static, purely axial compression with no eccentricity
- Perfect linear elasticity
- Cross-section is uniform
- Material properties are uniform
- Section is Class 1, 2 or 3. Not class 4
- Flexural buckling is limiting failure mode
- Can be modelled as an isolated column

This calculator is capable of finding the flexural buckling resistance of a steel section, however, several assumptions and limitations restrict its scope. Firstly, the method assumes that the applied load is purely static and does not vary with time, meaning that dynamic loading and fatigue effects are not considered. These aspects fall outside the scope of this tool and are instead governed by EN 1993-1-9 (European Committee for Standardization [CEN], 2005b).

Additionally, the load is also assumed to be purely axial, acting perpendicular to the cross-section with no eccentricity. If the load were to be applied off-centre, additional bending moments would be induced, which could lead to flexural-torsional buckling, beyond the scope of the calculation.

The calculation also assumes that the cross-sectional area of the column is uniform along its length. This means that key geometric properties like area and radius of gyration, are constant with no sort of variation from tapering or curvature. If this were to be the case, more advanced structural analysis methods like finite element modelling would be suitable to more accurately represent the effects. In addition, material properties are assumed to be uniform throughout the member, meaning local variations in the steel that could affect yield strength are ignored.

This method also evaluates a column in isolation, ignoring effects of load distribution and interactions between adjacent structural members. In structural systems, columns, beams, connections, etc, all interact with each other, which influence their stability, implying that the method may be less accurate for these scenarios.

Furthermore, flexural buckling is assumed to be the only relevant failure mode. Other possible buckling failure mechanisms such as local, torsional and lateral-torsional buckling are neglected. This assumption is built into the equations as many of them are only valid for sections classed from 1 to 3, where local buckling is not expected to be the limiting factor. The classification of the section refers to the section's ability to form plastic hinges and is determined by the strain factor as well as its geometry. Sections labelled as class 4 are expected to experience local buckling before the cross-section can fully mobilise its yield strength. Other possible failures use different calculations and are outside the scope of the calculator. This links to another key assumption; that the material behaves as perfectly elastic or plastic-elastic up to the point of buckling. The steel is taken to remain linearly elastic, with no sort of plastic deformation or strain hardening before it buckles.

References

European Committee for Standardization (CEN). (2005a). *Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings.* EN 1993-1. Brussels. European Committee for Standardization.

European Committee for Standardization (CEN). (2005b). *Eurocode 3: Design of steel structures - Part 1-9: Fatigue*. EN 1993-1-9. Brussels. European Committee for Standardization.