# <u>Reinforced Concrete Bending Moment Capacity Calculator</u>

Reinforced concrete is a widely used material, sometimes used for beams and slabs that are subject to bending. Loading causes internal bending moments inside the section, and it is crucial to understand its capabilities. The bending moment resistance of a reinforced concrete section is the maximum internal bending moment before the concrete fails. This is determined by using force equilibrium and taking the moment about the tension rebar. To do this, the internal forces in the section need to be determined. These include the tensile and compressive forces from the steel reinforcements, and the compressive force from the concrete, as shown in the figure below.

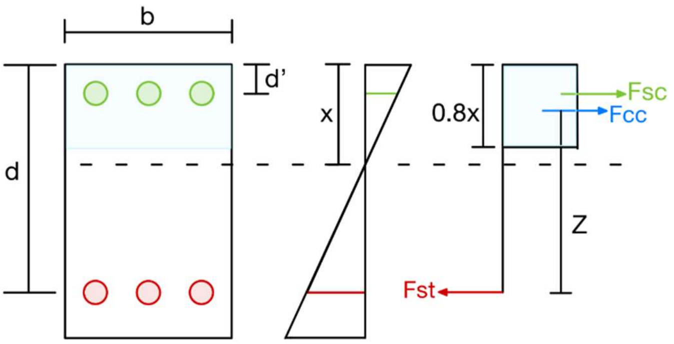


*Figure 1 Concrete Stress Block Diagram*

However, before that, the amount of concrete actually in compression needs to be found by using the depth of the neutral axis. Firstly, it is assumed that the rebars have yielded, allowing for the calculations below for the forces exerted by the steel rebar.

$$F_{st} = \frac{f_{yk}}{\gamma_s} A_s \tag{1}$$

$$F_{sc} = \frac{f_{yk}}{\gamma_s} A'_s \tag{2}$$

Where,

- $F_{st}$ = Tensile rebar force
- $F_{sc}$ = Compressive rebar force
- $f_{yk}$ = Steel Yield Strength (N/mm$^2$)
- $\gamma_s$ = Steel Partial factor

- $A_s$ = Cross-sectional Area of tension rebar
- $A'_s$ = Cross-sectional Area of compression rebar

In the case that there are no compressive reinforcements, the area of the compressive rebar can be set to zero. The force from the concrete is also found by using a simplified stress block shown in Figure 5.

$$F_{cc} = \frac{0.85\, f_{ck}}{\gamma_c} \times bs \tag{3}$$

Where,

- $F_{cc}$ = Concrete compressive force
- $f_{ck}$ = Concrete Characteristic Strength (N/mm$^2$)
- $\gamma_c$ = Partial Concrete Factor
- $b$ = Section Width (mm)
- $s$ = Depth of concrete stress block

Assuming that the steel has yielded, the only unknown is the concrete force. By applying equilibrium between the forces from the concrete stress block, the tension and compression rebar, if present, the concrete force, and consequently the neutral axis depth can be found (Gopalakrishnan, 2022). The sum of tensile forces is balanced with compressive, so,

$$F_{st} = F_{cc} + F_{sc} \tag{4}$$

Substituting the previous force equations, Eq 1 to 3, then rearranging for the depth of the concrete stress block leads to Eq 5, and then a simple relationship links this to the neutral axis depth (European Committee for Standardization [CEN], 2004a).

$$s = \frac{\gamma_c}{\gamma_s \times 0.85} \times \frac{f_{yk}\,(A_s - A_s')}{f_{ck} \times b} \tag{5}$$

$$x = \frac{s}{0.8} \tag{6}$$

Where,

- $x$ = Depth of Neutral Axis

When there is only tension rebar, to ensure that the steel yields, the neutral axis can not be below a certain depth, as shown in Eq 7, and if it is, compressive reinforcements would be needed.

$$x < 0.45\, d \tag{7}$$

Where,

- $d$ = Depth of Tensile rebar

If this is satisfied then the bending moment capacity can be found by taking the moment about the centre of the tension rebar.

$$Z_{cc} = d - \frac{s}{2} \tag{8}$$

$$M = F_{cc} Z_{cc} \tag{9}$$

Where,

- $Z_{cc}$ = Concrete stress block lever arm (m)

If the neutral axis depth is outside the boundary in Eq 7, a limited bending moment resistance would be calculated by setting the neutral axis depth to the permissible maximum, and repeating the calculations, leading to the equation below.

$$M = 0.167\, f_{ck}\, b\, d^2 \tag{10}$$

When there are compression reinforcements too, they also need to be confirmed to be yielding. If they are not, then the force, and moment, they supply will be reduced. The minimum strain needed for them to yield is found by (Plevris et al.,2013),

$$\varepsilon_y = \left(\frac{f_{yk}}{\gamma_s}\right) / E_s \tag{11}$$

Where,

- $\varepsilon_y$ = Yield Strain
- $E_s$ = Steel Young's Modulus (N/mm$^2$)

The strains in the rebar are calculated using Eq 12 and 13.

$$\varepsilon_{st} = 0.0035 \left(\frac{d - x}{x}\right) \tag{12}$$

$$\varepsilon_{sc} = 0.0035 \left( \frac{x - d'}{x} \right) \tag{13}$$

Where,

- $\varepsilon_{st}$ = Tensile rebar strain
- $\varepsilon_{sc}$ = Compressive rebar strain
- $d'$ = Depth of Compressive rebar

If either strain is below the yield point, then their respective forces need to be recalculated using the equations below (Plevris et al.,2013).

$$F_{st} = E_s \, \varepsilon_{st} \, A_s \tag{14}$$

$$F_{sc} = E_s \, \varepsilon_{sc} \, A'_s \tag{15}$$

The neutral axis depth would then need to be recalculated using force equilibrium but with the new, non-yielded forces. Shown below is the new equation after resubstituting these values as well as rearranging for $s$.

$$s = \frac{F_{st} - F_{sc}}{(0.85 \, f_{ck} \, b)/\gamma_c} \tag{16}$$

Using the stress block depth, after recalculating if needed, the final bending moment resistance of doubly reinforced concrete can be found using the equations below and Eq 8 (Plevris et al.,2013).

$$Z_{sc} = d - d' \tag{17}$$

$$M = F_{cc} Z_{cc} + F_{sc} Z_{sc} \tag{18}$$

## <u>Assumptions and Limitations</u>

Summary of Key Assumptions and Limitations

- Rectangular Section
- Singly or doubly reinforced section
- Single layer of reinforcement each
- Concrete carries compression only
- Concrete in tension is ignored

- Concrete strength is below 50 MPa
- Steel is assumed to yield, otherwise behaves linearly elastic
- Only flexural loading

The method used for calculating the bending moment resistance is based on several simplifying assumptions that limit its range of application. Firstly, the concrete section is assumed to be rectangular, with a fixed width and depth. This makes the method unsuitable for T-shaped or flanged sections with complex geometry. The sections are only able to handle singly or doubly reinforced concrete, with a single layer of rebar each. Multiple layers would have multiple moment lever arms at different depths, so to simplify the analysis, only one depth is used, measured from the top of the section to the centre of the reinforcements.

Furthermore, the concrete is assumed to only provide compressive resistance, where any concrete in tension is ignored (Gopalakrishnan, 2022). This method is only suitable for concretes with a strength below 50 MPa, as it assumes that the ultimate compressive strain of the concrete is 0.0035, after which the concrete would have failed (European Committee for Standardization [CEN], 2004a). This strength limitation also is imposed on the link between the concrete stress block and the neutral axis depth.

The steel reinforcements are initially assumed to have yielded. If, after the strain check, it is found to be false, then the steel is considered to behave linearly elastic. This means that the internal stress and force are directly proportional to the strain.

Additionally, the bending moment resistance is calculated under the assumption that there is only flexural loading, with no dynamic effects. Influence from any shear, torsion, or axial forces are not accounted for, as well as pre-stresses in the rebar. Furthermore, the material behaviour is idealised so that there is no sort of creep, shrinkage, or strain hardening.

## References

European Committee for Standardization (CEN). (2004a). *Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings*. EN 1992-1-1. Brussels: European Committee for Standardization.

Gopalakrishnan, N. (2022) ‘Application of the Flexure Theory for Reinforced Concrete’, *International Journal of Innovative Research in Computer Science & Technology*, Volume 10, pp. 60–65. Presidency University, Bangalore: Innovative Research Publication.

Plevris, V., Papazafeiropoulos, G. and Papadrakakis, M. (2013). DESIGN OF RC SECTIONS IN THE ULTIMATE LIMIT STATE UNDER BENDING AND AXIAL FORCE ACCORDING TO EC2. *Proceedings of the 3rd South-East European Conference on Computational Mechanics – SEECCM III*, pp.520–547. DOI:10.7712/seeccm-2013.2159.