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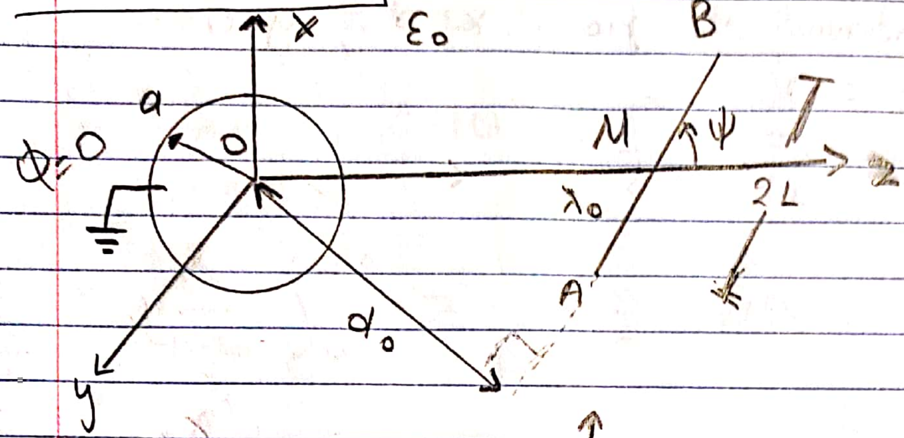
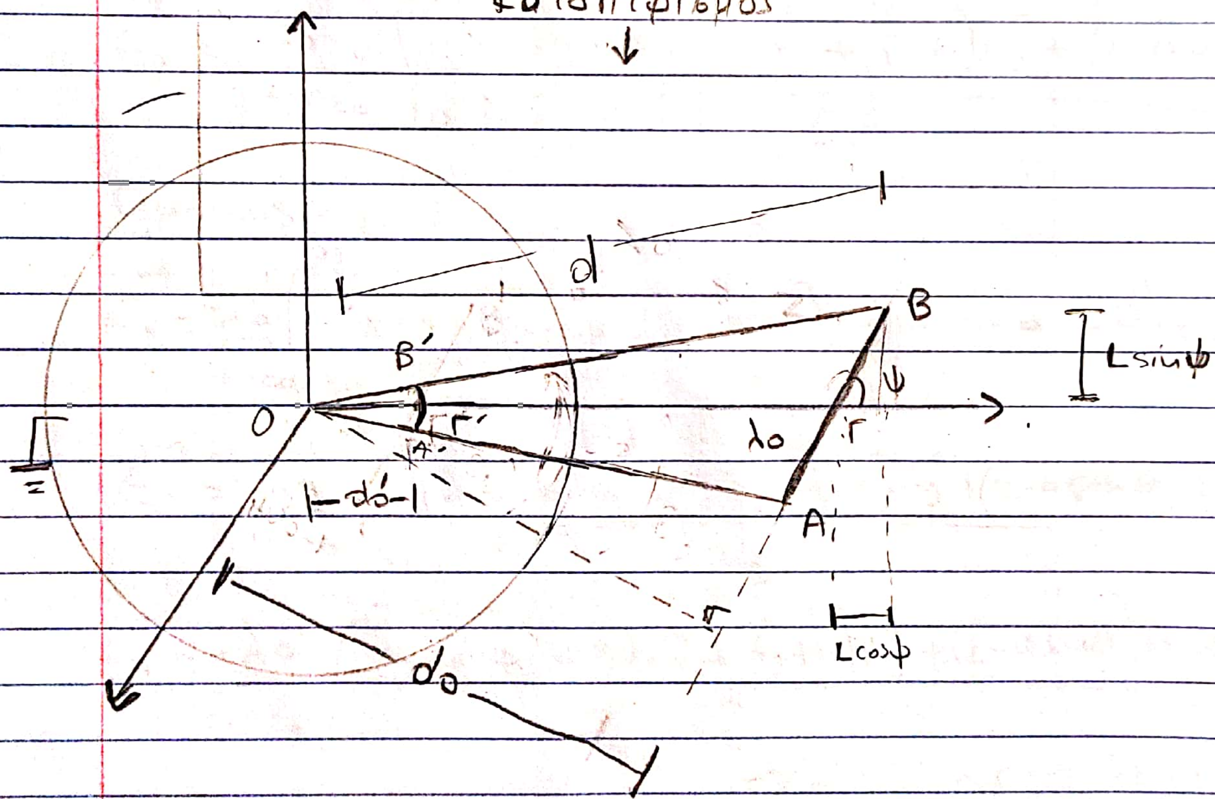
1η Σειρά Ασκήσεων.

Άσκηση 6



↑
Κατοπτρισμός
↓



α) Προφανώς $\phi(x,y,z)=0$
για $R=(x^2+y^2+z^2)^{1/2} \leq a$
αφού έχουμε γειωμένη σφαίρα.

Για $(x^2+y^2+z^2)^{1/2} \geq a$ έχω

$$\phi(x,y,z) = \phi_{\lambda_0}(x,y,z) + \phi_{\lambda_0'}(x,y,z)$$

$$\longrightarrow \phi_{\lambda_0} = \frac{1}{4\pi\varepsilon_0} \lambda_0 \int \frac{1}{R_1} dl$$

$$R_1 = \left[(x-x')^2 + y^2 + (z-z')^2 \right]^{1/2}$$

$$\cos\left(\frac{\pi}{2}-\psi\right) = \frac{d_0}{dM} \Rightarrow dM = \frac{d_0}{\cos(\frac{\pi}{2}-\psi)} = \frac{d_0}{\sin\psi}$$

Άρα οι συντεταγμένες του $M\left(0,0,\frac{d_0}{\sin\psi}\right)$

Χρησιμοποιώ εξίσωση ευθείας : $x' = \tan\psi \cdot z' + C_1$
η οποία διέρχεται απ' το M άρα
$0 = \tan\psi \cdot \frac{d_0}{\sin\psi} + C_1 \Rightarrow C_1 = -\frac{d_0}{\cos\psi}$

με $z' \in \left[\frac{d_0}{\sin\psi} - L\cos\psi , \frac{d_0}{\sin\psi} + L\cos\psi\right]$

Για το dL ισχύει ότι $dL = \sqrt{dx'^2 + dz'^2}$

$$dL = \sqrt{\tan^2\psi\, dz'^2 + dz'^2} \Rightarrow dL = dz'\sqrt{\tan^2\psi + 1}$$

$$\Rightarrow dL = \frac{1}{\cos\psi} dz'$$

Συνεπώς $R_1 = \left[\left(x - \tan\psi \cdot z' + \frac{d_0}{\cos\psi}\right)^2 + y^2 + (z-z')^2 \right]^{1/2}$

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Άρα:

$$\phi_{\lambda o}(x,y,z) = \frac{\lambda o}{4\pi\varepsilon_0}\int_{\frac{do}{\sin\phi}-L\cos\phi}^{\frac{do}{\sin\phi}+L\cos\phi} \frac{dz''/\cos\phi}{\left[\left(x+\frac{do}{\cos\phi}-\tan\phi\, z'\right)^2+y^2+(z-z')^2\right]^{1/2}}.$$

$\longrightarrow$ $\phi_{\lambda o}'$:

$A\left(\tan\phi\, z' - \frac{do}{\cos\phi}, 0, z'\right)$, $z' \in \left[\frac{do}{\sin\phi} - L\cos\phi, \frac{do}{\sin\phi} + L\cos\phi\right]$

$$d = \left[z'^2 + \left(\tan\phi \cdot z' - \frac{do}{\cos\phi}\right)^2\right]^{1/2} =$$

$$\left[z'^2(1+\tan^2\phi) - \frac{do}{\cos^2\phi}(2z'\sin\phi - do)\right]^{1/2}$$

$$\Rightarrow d = \frac{1}{\cos\phi}\left[z'^2 - do\left(2z'\sin\phi - do\right)\right]^{1/2}$$

$$do' = \frac{a^2}{d} = \frac{a^2\cos\phi}{\left[z'^2 - do(2z'\sin\phi - do)\right]^{1/2}}$$

για τη γωνία:

$$\theta = \theta(z') = \tan^{-1}\left(\frac{x'}{z'}\right) = \tan^{-1}\left(\tan\phi - \frac{do}{z'\cos\phi}\right)$$

Για το dq'

$x'' = do'\sin\theta$ $\quad z'' = do'\cos\theta$

και ισχύει $dq' = -dq\frac{a}{d} = -dq\frac{a\cos\phi}{\left[z'^2 - do(2z'\sin\phi - do)\right]^{1/2}}$

και $dq = \lambda o\, dl'$

$$\text{Άρα } dq' = -\frac{a\,\lambda o\, dl'}{\left[z'^2 - do(2z'\sin\phi - do)\right]^{1/2}}$$

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Enigus $dl' = \sqrt{dx''^2 + dz''^2}$

$$R_2 = \left[(x-x'')^2 + y^2 + (z-z'')^2\right]^{1/2}$$

$$dx'' = \frac{d}{dz'} \cdot d_o' \, dz' \sin\theta + d_o' \cos\theta \frac{d\theta}{dz'} \, dz'$$

$$dx'' = dz' \left[\frac{d}{dz'} \left(\frac{a^2 \cos\phi}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{1/2}} \right) \sin\theta + d_o' \cos\theta \frac{d}{dz'} \left(\tan^{-1}\left(\tan\phi - \frac{d_o}{z'\cos\phi}\right)\right)\right]$$

$$= dz' \left[\frac{-a^2\cos\phi}{2[z'^2 - d_o(2z'\sin\phi - d_o)]^{3/2}} \cdot \frac{d}{dz'}\left(z'^2 - d_o(2z'\sin\phi - d_o)\right) \cdot \sin\theta + \frac{a^2\cos\phi}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{1/2}} \cdot \frac{\cos\theta}{1 + \left(\tan\phi - \frac{d_o}{z'\cos\phi}\right)^2} \right.$$

$$\cdot \frac{d}{dz'}\left(\tan\phi - \frac{d_o}{z'\cos\phi}\right) =$$

$$= dz' \left[\frac{-a^2\cos\phi \, (\cancel{2}z' - \cancel{2}d_o\sin\phi) \cdot \sin\theta}{\cancel{2}\,[z'^2 - d_o(2z'\sin\phi - d_o)]^{3/2}} + \frac{a^2\cos\phi\cos\theta}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{1/2}} \frac{1}{\left(1 + \left(\tan\phi - \frac{d_o}{z'\cos\phi}\right)^2\right)} \, d_o / \cos\phi \right.$$

$\Rightarrow$

$$dx'' = dz' \left[\frac{-a^2\cos\phi \, (z' - d_o\sin\phi)\sin\theta}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{3/2}} + \frac{a^2 d_o \cos\theta}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{1/2}} \cdot \frac{1}{z'^2 + \left(z'\tan\phi - \frac{d_o}{\cos\phi}\right)^2} \right.$$

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Για το dz'' ακολουθώ την αντίστοιχη διαδικασία με την διαφορά ότι έχει $\cos\theta$ αντί για $\sin\theta$. Συνεπώς ο πρώτος όρος παραμένει ίδιος και ο δεύτερος έχει αντίθετο πρόσημο και $\sin\theta$

$$dz'' = dz' \left[- \frac{a^2 \cos\phi \,(z' - d_o \sin\phi) \cos\theta}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{3/2}} - \frac{a^2 d_o \sin\theta}{[z'^2 - d_o(2z'\sin\phi - d_o)]^{1/2}} \cdot \frac{1}{z'^2 + \left(z'\tan\phi - \frac{d_o}{\sin\phi}\right)^2} \right]$$

$$dx'' = X_o'' \cdot dz'$$
$$dz'' = Z_o'' \, dz'$$

Άρα $dl' = dz' \sqrt{X_o''^2 + Z_o''^2}$

Τελικά

$$\phi_{\lambda_o'}(x,y,z) = \frac{1}{4\pi\varepsilon_o} \int \frac{dq'}{R_2} \Rightarrow$$

$$\phi_{\lambda_o'} = -\frac{\lambda_o a}{4\pi\varepsilon_o} \int_{\frac{d_o}{\sin\phi} - L\cos\phi}^{\frac{d_o}{\sin\phi} + L\cos\phi} \frac{\sqrt{X_o''^2 + Z_o''^2}}{d \cdot R_2} \, dz'$$

β) $E(x,y,z) = 0$ για $(x^2+y^2+z^2)^{1/2} \leq a$

αφού $\phi = 0$

$E = E_{\lambda_o} + E_{\lambda_o'}$ για $(x^2+y^2+z^2)^{1/2} \geq a$

$$\vec{E}_{\lambda_o} = \frac{1}{4\pi\varepsilon_o} \int \frac{dq}{R_1^2} \hat{i_R} = \frac{1}{4\pi\varepsilon_o} \int_{\frac{d_o}{\sin\phi} - L\cos\phi\ldots}^{\frac{d_o}{\sin\phi} + L\cos\phi} \frac{\lambda_o \cdot dl}{R_1^2} \cdot \frac{\vec{R_1}}{R_1}$$

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$$\vec{E}_{d_0} = \frac{d_0}{4\pi\varepsilon_0}\int_{\frac{d_0}{\sin\phi}-L\cos\phi}^{\frac{d_0}{\sin\phi}+L\cos\phi} \frac{\vec{R_1}}{R_3}\,\frac{1}{\cos\phi}\,dz'$$

όπου $R_1 = |\vec{R_1}| = \left[\left(x-\tan\phi\, z' + \frac{d_0}{\cos\phi}\right)^2 + y^2 + (z-z')^2\right]^{1/2}$

και $\vec{R_1} = \left(x - \tan\phi\, z' + \frac{d_0}{\cos\phi}\right)\hat{i_x} + y\hat{i_y} + (z-z')\hat{i_z}$

Παρόμοια για $\vec{E}_{d_0'}$

$$\vec{E}_{d_0'} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{R_2^2}\,\hat{i}_{R_2} \Rightarrow$$

$$\vec{E}_{d_0'} = -\frac{a\, d_0}{4\pi\varepsilon_0}\int_{\frac{d_0}{\sin\phi}-L\cos\phi}^{\frac{d_0}{\sin\phi}+L\cos\phi} \frac{\vec{R_2}}{R_2^3}\,\frac{\sqrt{x_0''^2+z_0''^2}}{d}\,dz''$$

Με $d = \frac{1}{\cos\phi}\left[(z')^2 - d_0\,(2z'\sin\phi - d_0)\right]^{1/2}$

$\vec{R_2} = (x-x'')\hat{i_x} + y\hat{i_y} + (z-z'')\hat{i_z}$

με $x'' = d_0'\sin\theta$ και $z'' = d_0'\cos\theta$

γ) $\sigma = \hat{i_n}\cdot(\vec{D}^{+} - \cancel{\vec{D}^{-}}) = \varepsilon_0\,\hat{i_n}\cdot\vec{E}(x,y,z)$

με σφαιρικές συντεταγμένες:

$= \varepsilon_0\,\hat{i_n}\,\vec{E}(a\cos\theta,\ a\sin\theta\cos\phi,\ a\sin\theta\sin\phi)$

$= \varepsilon_0\,(\cos\theta\,\hat{i_x} + \sin\theta\cos\phi\,\hat{i_y} + \sin\theta\sin\phi\,\hat{i_z})(E_x\hat{i_x} + E_y\hat{i_y} + E_z\hat{i_z})$

$= \varepsilon_0\,(\cos\theta\, E_x + \sin\theta\cos\phi\, E_y + \sin\theta\sin\phi\, E_z)$

και E_x, E_y, E_z οι συνιστώσες του ηλ. πεδίου στο καρτεσιανό σύστημα.

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askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```
%askhsh 6
xmin = -3;
xmax = 3;
zmin = -3;
zmax = 4.5;
x0 = xmin: (xmax-xmin)/100 : xmax;
z0 = zmin: (zmax-zmin)/100 : zmax;
[Z0,X0] = meshgrid(z0,x0);
a = 1;
d = 2;
L = 1;
y = pi/4;
thita_min = d/sin(y)-L*cos(y);
thita_max = d/sin(y)+L*cos(y);

for i = 1:length(z0)
    for j = 1:length(x0)
        z = Z0(i,j);
        x = X0(i,j);
        F(i,j) = integral(@(zf)pot(zf,z,x),thita_min,thita_max);
        Ez(i,j) = integral(@(zf)Ezintf(zf,z,x),thita_min,thita_max);
        Ex(i,j) = integral(@(zf)Exintf(zf,z,x),thita_min,thita_max);
    end
end

%c
```

Command Window

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```matlab
figure(1)
hold off
c = [0.01, 0.025, 0.05:0.05:1, 1.1:0.1:1.5,2,3,4,5];
[C,H] = contour(Z0,X0,F,c);
clabel(C,H,c);
hold on;
zr(1) = thita_min;
xr(1) = tan(y)*zr(1)-d/cos(y);
zr(2) = thita_max;
xr(2) = tan(y)*zr(2)-d/cos(y);
plot(zr,xr);
[r,f] = meshgrid(a,0:pi/50:2*pi);
[zt,xt] = pol2cart(f,r);
fill(zt,xt,[1 1 1]);
xlabel('z')
ylabel('x')
title('Normalized Equipotential Lines')
axis equal
grid on

figure(2)
hold off
surface(Z0,X0,F);
shading interp;
colorbar;
xlabel('z')
```

Command Window

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```
    ylabel('x')
    title('normalized potential')
    axis equal
    axis tight
    grid on

    %d
    figure(3)
    hold off
    En = sqrt(((Ez).^2+(Ex).^2));
    quiver(Z0,X0,Ez./En,Ex./En);
    hold on
    E = streamslice(Z0,X0,Ez,Ex,2);
    set(E,'Linewidth', 1, 'color', 'b');
    plot(zr,xr,'Linewidth',2,'color',[1 0.5 0]);
    fill(zt,xt,[1 1 1]);
    xlabel('z')
    ylabel('x')
    title('normalized Electric Field Lines')
    axis equal
    axis tight
    grid on

    %functions-----------------------------------------------------------------
    function F = pot(z,z0,x0)
    a=1;
```

Command Window

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```
    d=2;
    y = pi/4;
    xt = tan(y)*z-d/cos(y);
    Rt = sqrt((x0-xt).^2+(z0-z).^2);
    Ft = 1./(Rt.*cos(y));
    dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
    dt = (a.^2)./dl;
    th = atan(tan(y)-d./(z.*cos(y)));
    xtt = dt.*sin(th);
    ztt = dt.*cos(th);
    Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
    h1 = -(a^2.*cos(y).*(z-d*sin(y)))./((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
    h2 = (a^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./(z.^2+(z*tan(y)-d/cos(y)).^2);
    Xtt = h1.*sin(th)+h2.*cos(th);
    Ztt = h1.*cos(th)-h2.*sin(th);
    Ftt = -a.*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt);
    F = Ft + Ftt;
end

function Ex = Exintf(z,z0,x0)
    a=1;
    d=2;
    y = pi/4;
    xt = tan(y)*z-d/cos(y);
    Rt = sqrt((x0-xt).^2+(z0-z).^2);
    Ext = (x0-xt)./((Rt.^3).*cos(y));
```

Command Window

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```matlab
    dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
    dt = (a.^2)./dl;
    th = atan(tan(y)-d./(z.*cos(y)));
    xtt = dt.*sin(th);
    ztt = dt.*cos(th);
    Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
    h1 = -(a^2.*cos(y).*(z-d*sin(y)))./((z.^2-2.*d.*sin(y).*z+d^2).^1.5);
    h2 = (a^2*d)./sqrt(z.^2-2.*d.*sin(y).*z+d^2).*1./(z.^2+(z*tan(y)-d/cos(y)));
    Xtt = h1.*sin(th)+h2.*cos(th);
    Ztt = h1.*cos(th)-h2.*sin(th);
    Extt = -a.*(x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*(Rtt).^3);
    Ex = Ext + Extt;
end

function Ez = Ezintf(z,z0,x0)
    a=1;
    d=2;
    y = pi/4;
    xt = tan(y)*z-d/cos(y);
    Rt = sqrt((x0-xt).^2+(z0-z).^2);
    Ezt = (z0-z)./((Rt.^3).*cos(y));
    dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
    dt = a./dl;
    th = atan(tan(y)-d./(z.*cos(y)));
    xtt = dt.*sin(th);
    ztt = dt.*cos(th);
```

Command Window

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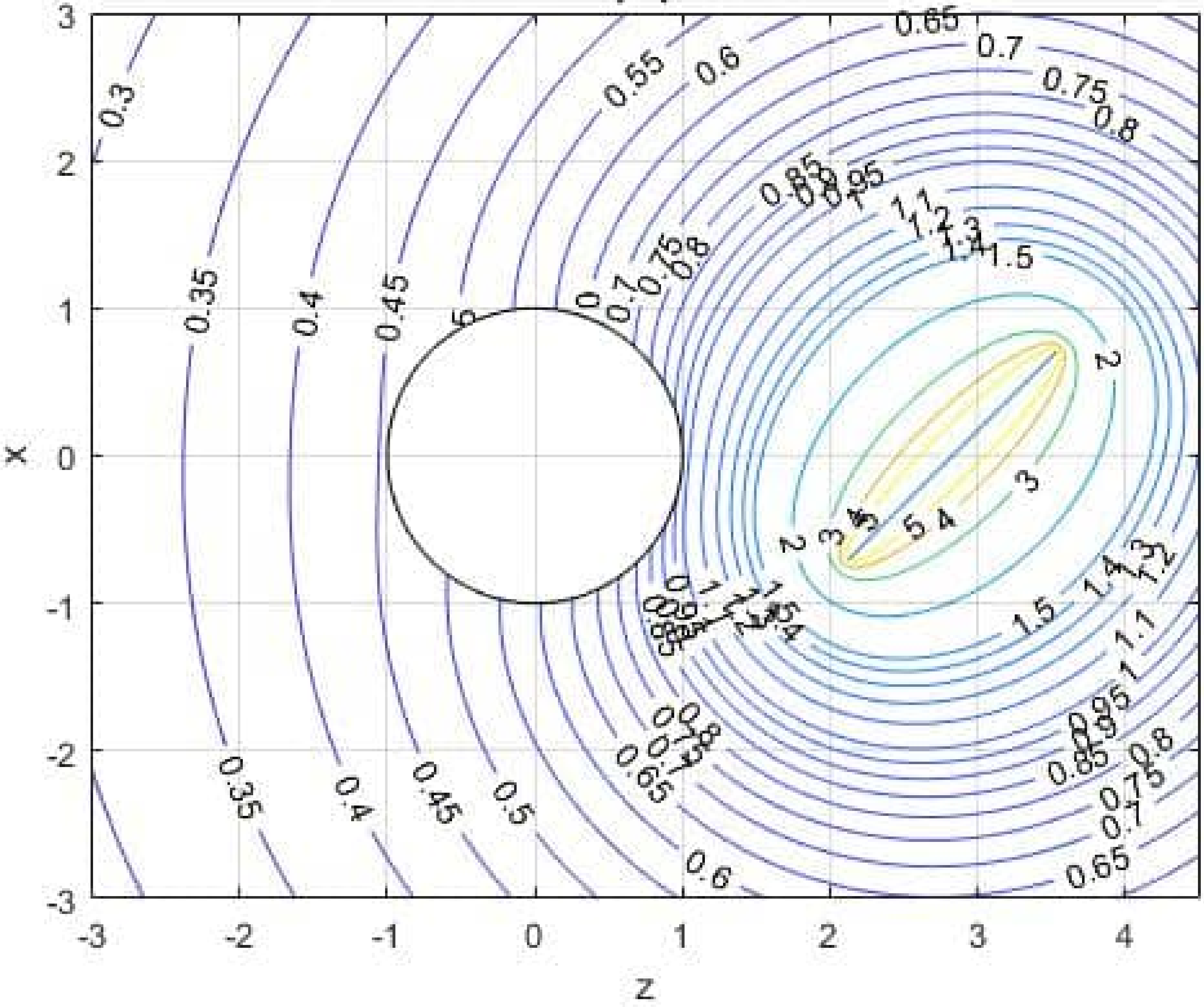
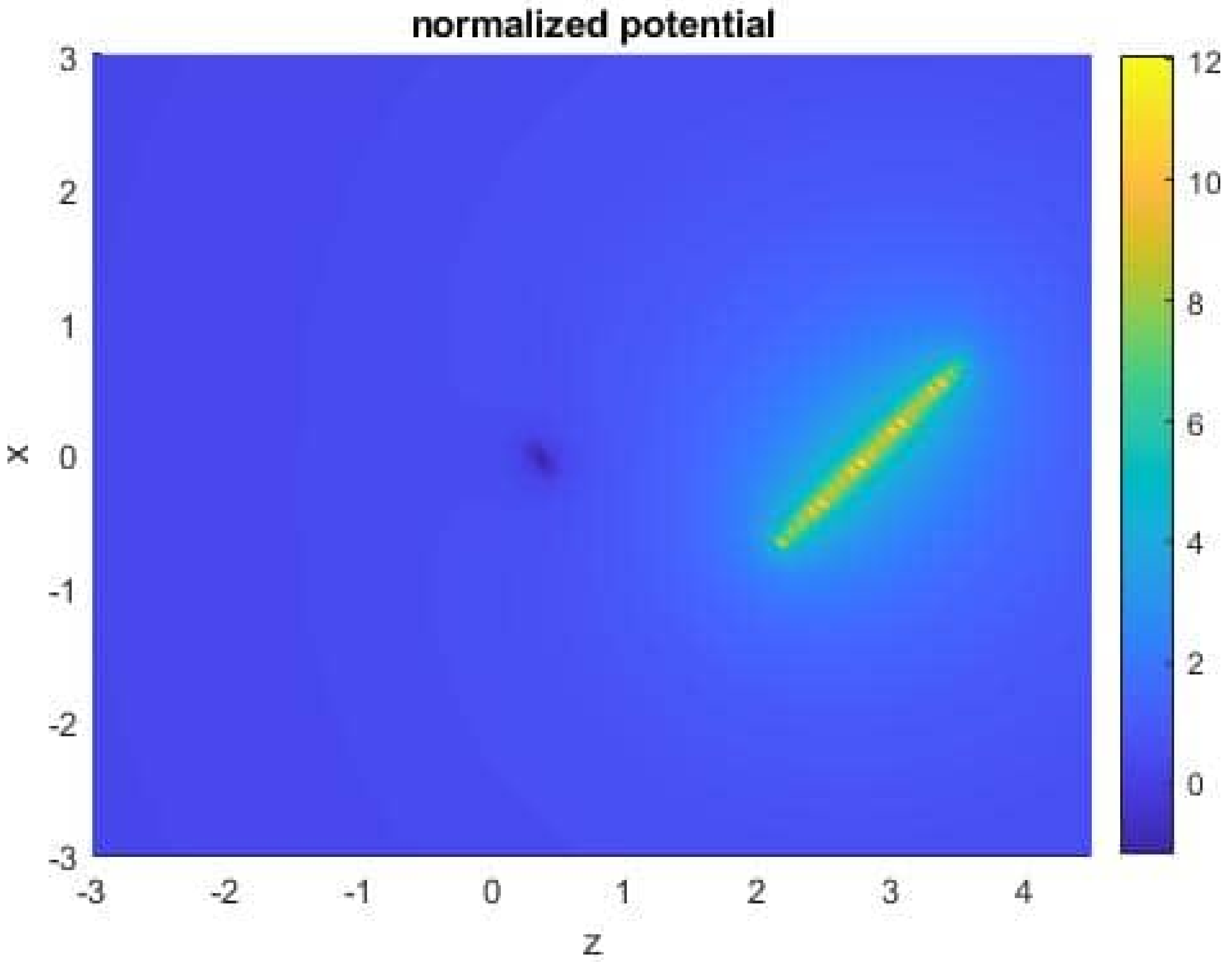
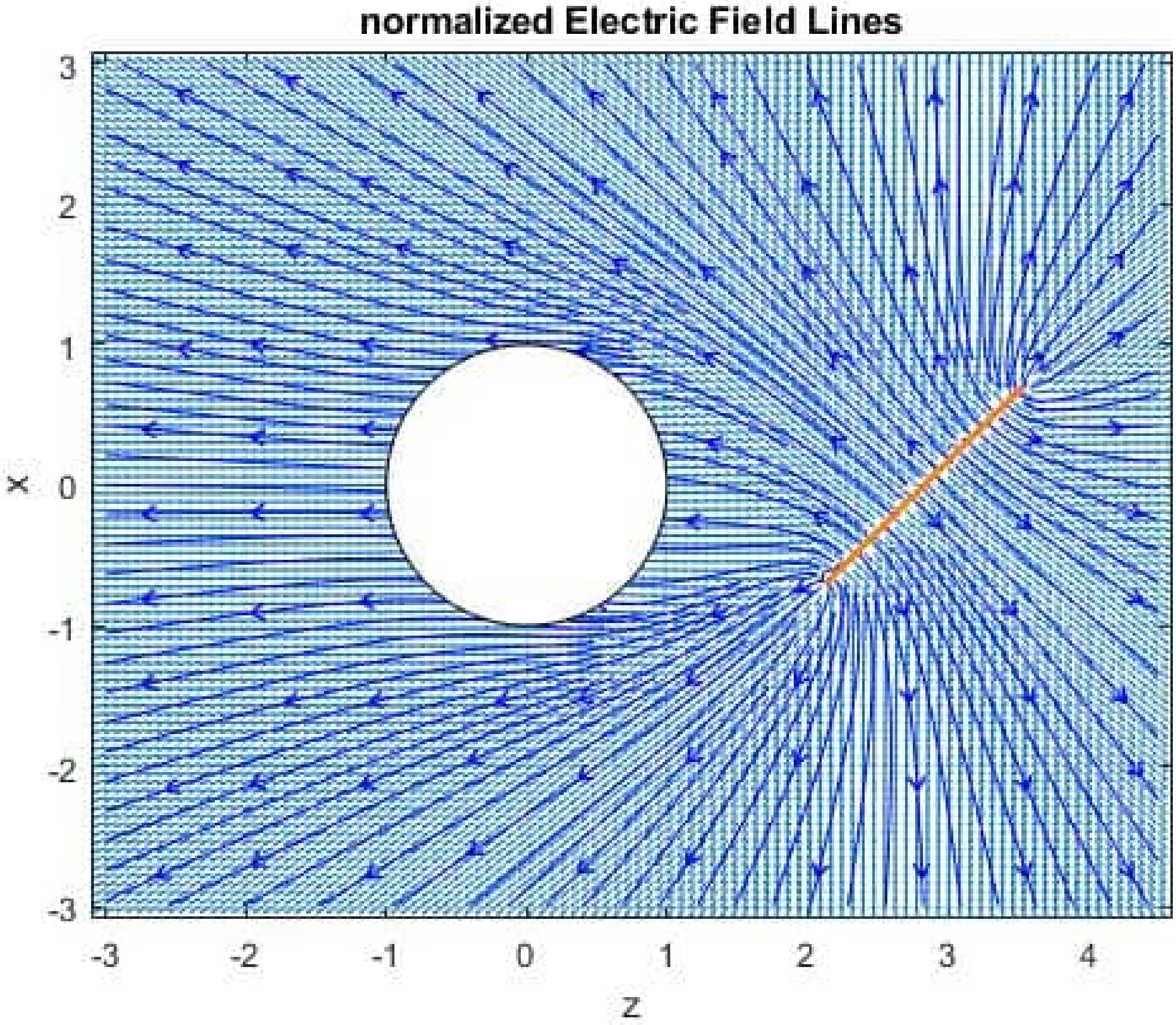
```
    Xtt = h1.*sin(th)+h2.*cos(th);
    Ztt = h1.*cos(th)-h2.*sin(th);
    Extt = -a.*(x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*(Rtt).^3);
    Ex = Ext + Extt;
end

function Ez = Ezintf(z,z0,x0)
a=1;
d=2;
y = pi/4;
xt = tan(y)*z-d/cos(y);
Rt = sqrt((x0-xt).^2+(z0-z).^2);
Ezt = (z0-z)./((Rt.^3).*cos(y));
dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
dt = a./dl;
th = atan(tan(y)-d./(z.*cos(y)));
xtt = dt.*sin(th);
ztt = dt.*cos(th);
Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
h1 = -(a^2.*cos(y).*(z-d*sin(y)))./((z.^2-2.*d.*sin(y).*z+d^2).^1.5);
h2 = (a^2*d)./sqrt(z.^2-2.*d.*sin(y).*z+d^2).*1./(z.^2+(z*tan(y)-d/cos(y)));
Xtt = h1.*sin(th)+h2.*cos(th);
Ztt = h1.*cos(th)-h2.*sin(th);
Eztt = -a.*(z0-ztt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*(Rtt).^3);
Ez = Ezt + Eztt;
end
```

Command Window

Normalized Equipotential Lines




normalized potential
x
z
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4
12
10
8
6
4
2
0


normalized Electric Field Lines
x
z
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4

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askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```matlab
%e
a=1;
d=2;
L=1;
y=pi/4;
thita_min = d/sin(y)-L*cos(y);
thita_max = d/sin(y)+L*cos(y);
thita0 = 0:0.001:2*pi;

Ex = integral(@(z)Exint(z,thita0),thita_min,thita_max,'ArrayValued',true);
Ez = integral(@(z)Ezint(z,thita0),thita_min,thita_max,'ArrayValued',true);

e0 = 8.8541878176203 9*10^(-12);
charge1 = e0.*Ex.*cos(thita0);
charge2 = e0.*(Ez.*sin(thita0));
charge=charge1 + charge2;
plot(thita0, charge, 'm');
xlim([0 2*pi]);
grid on;
xlabel('Angle (rad)');
ylabel('s');
title('Normalized Surface Charge')
%functions--------------------------------------------------------------

function Exx=Exint(z,thita)
a=1;
```

Command Window

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```
    d=2;
    y=pi/4;
    x0=a.*cos(thita);
    z0=a.*sin(thita);
    xt = tan(y)*z-d/cos(y);
    Rt = sqrt((x0-xt).^2+(z0-z).^2);
    Ext = (x0-xt)./((Rt.^3).*cos(y));
    dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
    dt = (a.^2)./dl;
    thita = atan(tan(y)-d./(z.*cos(y)));
    xtt = dt.*sin(thita);
    ztt = dt.*cos(thita);
    Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
    h1 = -(a^2.*cos(y).*(z-d*sin(y)))./((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
    h2 = (a^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./(z.^2+(z*tan(y)-d/cos(y)).^2);
    Xtt = h1.*sin(thita)+h2.*cos(thita);
    Ztt = h1.*cos(thita)-h2.*sin(thita);
    Extt = -a.*(x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
    Exx = Ext + Extt;
end

function Ezz=Ezint(z,thita)
    a=1;
    d=2;
    y=pi/4;
    x0=a.*cos(thita);
```

Command Window

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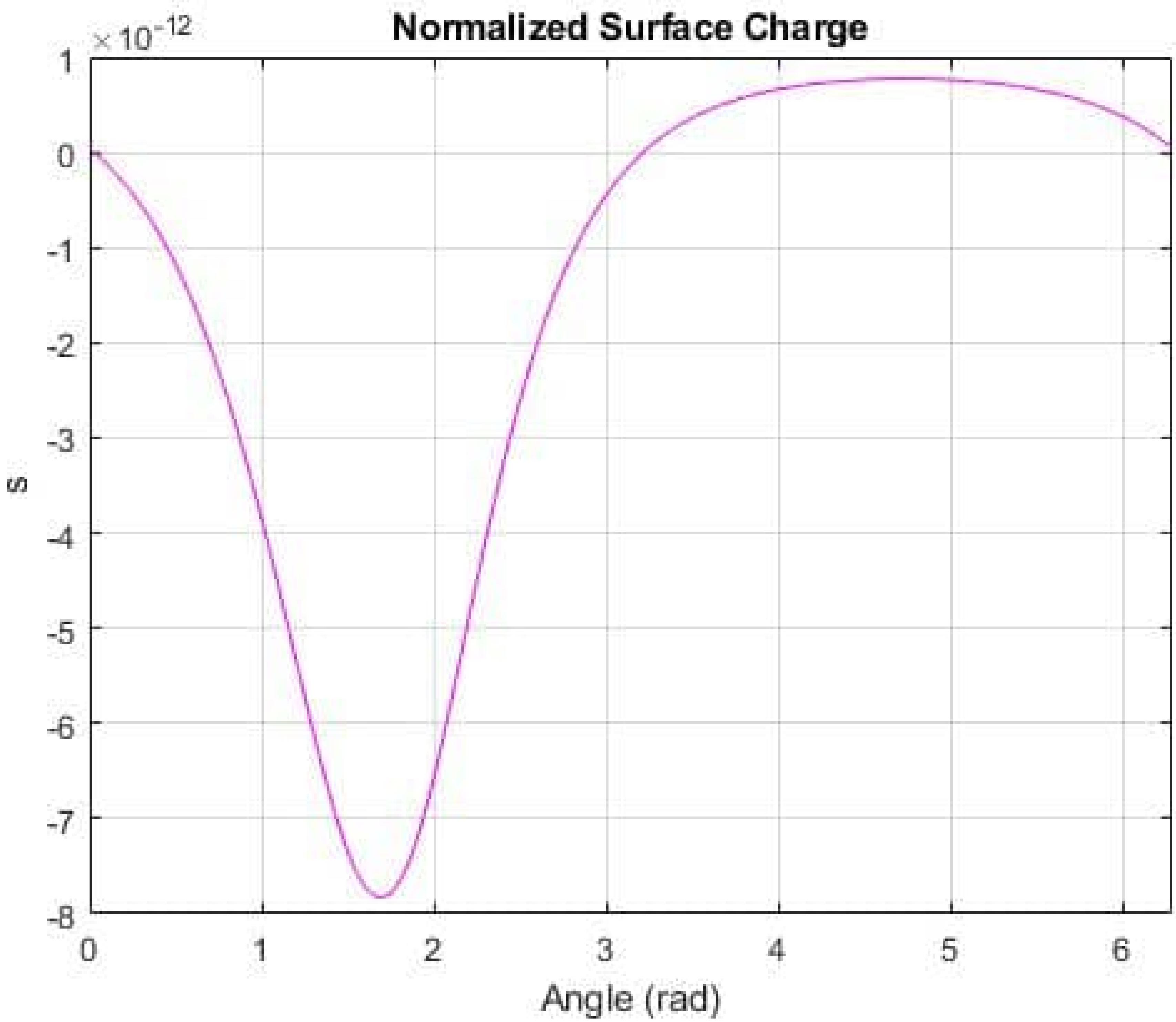
```matlab
    Extt = -a.*(x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
    Exx = Ext + Extt;
end

function Ezz=Ezint(z,thita)
    a=1;
    d=2;
    y=pi/4;
    x0=a.*cos(thita);
    z0=a.*sin(thita);
    xt = tan(y)*z-d/cos(y);
    Rt = sqrt((x0-xt).^2+(z0-z).^2);
    Ezt = (z0-z)./((Rt.^3).*cos(y));
    dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d^2);
    dt = (a.^2)./dl;
    thita = atan(tan(y)-d./(z.*cos(y)));
    xtt = dt.*sin(thita);
    ztt = dt.*cos(thita);
    Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
    h1 = -(a^2.*cos(y).*(z-d*sin(y)))./((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
    h2 = (a^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./(z.^2+(z*tan(y)-d/cos(y)).^2);
    Xtt = h1.*sin(thita)+h2.*cos(thita);
    Ztt = h1.*cos(thita)-h2.*sin(thita);
    Eztt = -a.*(z0-ztt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
    Ezz = Ezt + Eztt;
end
```

Command Window


Normalized Surface Charge
× 10^-12
1
0
-1
-2
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
Angle (rad)
s

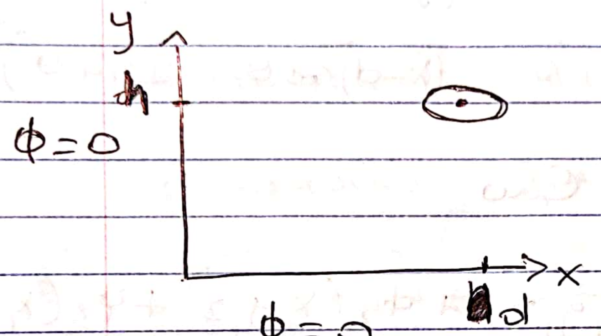
Ιωάννης Μποσκοβίτς 03119640

1η Σειρά Ασκήσεων

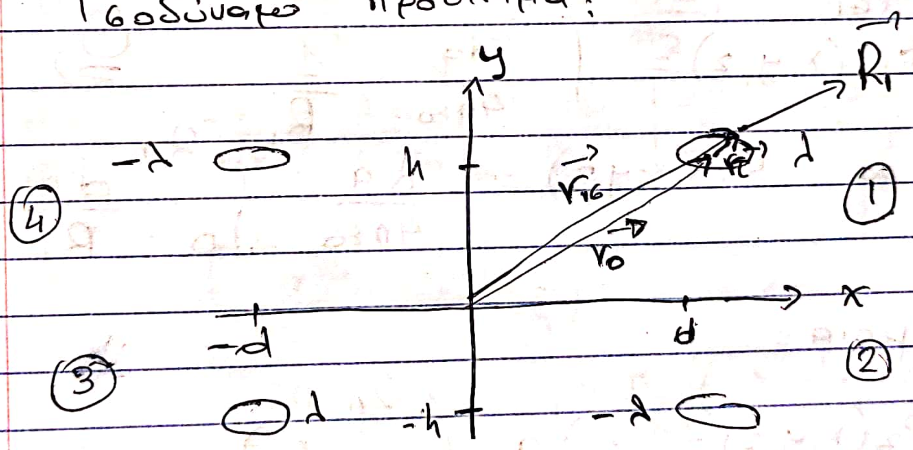
Άσκηση 7

Σύμφωνα με τις διαφάνειες: Μέθοδος κατοπτρισμού

Αρχικό πρόβλημα



Ισοδύναμο πρόβλημα:



$$\left.\begin{aligned} dq &= \lambda\, dl \\ dl &= a\, d\theta \end{aligned}\right\} \Rightarrow dq = a\lambda\, d\theta \quad \text{με } 0 \le \theta \le 2\pi$$

(1)

Επίσης $\vec{r_0} = d\,\hat{i_x} + h\,\hat{i_y}$

και $\vec{r_\tau} = a\cos\theta\,\hat{i_x} + a\sin\theta\,\hat{i_z}$

Άρα $\vec{r_{i\sigma}} = \vec{r_0} + \vec{r_\tau} = (d + a\cos\theta)\hat{i_x} + h\hat{i_y} + a\sin\theta\,\hat{i_z}$

$$\vec{R_{o\lambda}} = x\hat{i_x} + y\hat{i_y} + z\hat{i_z} = \vec{r_{i6}} + \vec{R_1}$$

και

$$\vec{R_1} = (x-d-a\cos\theta)\hat{i_x} + (y-h)\hat{i_y} + (z-a\sin\theta)\hat{i_z}$$

$$|\vec{R_1}| = R_1 = \left[(x-d)^2 + (y-h)^2 + z^2 + a^2 - 2a\left((x-d)\cos\theta + z\sin\theta\right)\right]^{1/2}$$

Για $x>0$, $y>0$ έχω

$$\Phi(x,y,z) = \Phi_1(x,y,z) + \Phi_2(x,y,z) + \Phi_3(x,y,z) + \Phi_4(x,y,z)$$

Έτσι:

$$\Phi_1(x,y,z) = \int \frac{dq}{4\pi\varepsilon_0}\frac{1}{R_1} \overset{(1)}{\Rightarrow}$$

Άρα $\Phi_1(x,y,z) = \dfrac{\lambda a}{4\pi\varepsilon_0}\displaystyle\int_0^{2\pi}\frac{d\theta}{R_1}$

Όμοια:

$$\Phi_2(x,y,z) = \frac{-\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{d\theta}{R_2}$$

όπου $R_2 = \left[(x-d)^2 + (y+h)^2 + a^2 + z^2 - 2a\left((x-d)\cos\theta + z\sin\theta\right)\right]^{1/2}$

$$\Phi_3(x,y,z) = \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{d\theta}{R_3}$$

όπου $R_3=\left[(x+d)^2+(y+4)^2+z^2+a^2-2a\left((x+d)\cos\theta+z\sin\theta\right)\right]^{1/2}$

Τέλος $\Phi_{4}(x,y,z) = \frac{-\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{d\theta}{R_4}$

όπου $R_4=\left[(x+d)^2+(y-4)^2+z^2+a^2-2a\left((x+d)\cos\theta+z\sin\theta\right)\right]^{1/2}$

Προφανώς για $x<0,\ y<0$ $\quad \Phi=0$

β) $\vec{E}_{o\lambda}=\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4$

$$\vec{E}_1 = -\frac{d\Phi_1}{dx} - \frac{d\Phi_1}{dy} - \frac{d\Phi_1}{dz}$$

$$\rightarrow -\frac{d\Phi_1}{dx} = \frac{-d}{dx}\left[\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{d\theta}{R_1}\right] =$$

$$-\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta \cdot \frac{d}{dx}\left(\frac{1}{R_1}\right) =$$

$$-\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta\,\frac{1}{R_1^3}\left(-\frac{1}{2}\right)\cdot\frac{d}{dx}\left[\left(x-d-\cos\theta\cdot a\right)^2\right]$$

$$= \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{x-d-a\cos\theta}{R_1^3}\,d\theta$$

$$\rightarrow -\frac{d\Phi_1}{dy} = \frac{-\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta\,\frac{d}{dy}\left(\frac{1}{R_1}\right)$$

$$= -\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta\,\frac{1}{R_1^3}\left(-\frac{1}{2}\right)\cdot\frac{d}{dy}\left[(y-4)^2\right]$$

$$= \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{y-4}{R_1^3}\,d\theta$$

Scanned with CamScanner

$$\rightarrow \quad \frac{-d\Phi_1}{dz} = -\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \, \frac{d}{dz}\left(\frac{1}{R_1}\right)$$

$$= -\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \left(-\frac{1}{2}\right)\cdot \frac{d}{dz}\left[(z-a\sin\theta)^2\right]$$

$$= \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{z-a\sin\theta}{R_1^3}\, d\theta$$

Άρα

$$\vec{E_1} = \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{(x-d-a\cos\theta)\hat{i_x} + (y-4)\hat{i_y} + (z-a\sin\theta)\hat{i_z}}{R_1^3}\, d\theta$$

Ομοίως

$$\vec{E_2} = -\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{(x-d-a\cos\theta)\hat{i_x} + (y+4)\hat{i_y} + (z-a\sin\theta)\hat{i_z}}{R_2^3}\, d\theta$$

$$\vec{E_3} = \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{(x+d-a\cos\theta)\hat{i_x} + (y-4)\hat{i_y} + (z-a\sin\theta)\hat{i_z}}{R_3^3}\, d\theta$$

$$\vec{E_4} = -\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{(x+d-a\cos\theta)\hat{i_x} + (y+4)\hat{i_y} + (z-a\sin\theta)\hat{i_z}}{R_4^3}\, d\theta$$

γ) $$\sigma(x,z) = \varepsilon_0 \hat{y}\left[\vec{E}^+ - \cancel{\vec{E}^-}\right] = \varepsilon_0 \hat{y}\left[\vec{E_1}+\vec{E_2}+\vec{E_3}+\vec{E_4}\right]$$

$$= \varepsilon_0\left[\frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{y-4}{R_1^3}d\theta - \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{y+4}{R_2^3}d\theta + \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{y+4}{R_3^3}d\theta\right.$$

$$\left. - \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{y-4}{R_4^3}d\theta\right] \quad \xRightarrow{y=0}$$

$$\sigma(x,z) = \frac{\lambda}{4\pi}\left[\int_0^{2\pi}\left(-\frac{4}{R_1^3} - \frac{4}{R_2^3} + \frac{4}{R_3^3} + \frac{4}{R_4^3}\right)d\theta\right.$$

Προφανώς για $x<0$, $y<0$

$\vec{E} = \vec{0}$

Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```
function [X,Y] = askisi7Fol
% Define computational parameters
a=0.1;
d=1;
h=1;
white = [1 1 1];
black = [0 0 0];
red = [192 0 0]/255;
green = [119 172 48]/255;
darker_green = [ 26 108 28]/255;
purple = [125 46 143]/255;
magenta_light = [255 102 255]/255;
nfig=1;
Npoints=133;
xmin = 0;
xmax = 2;
ymin = 0;
ymax = 2;
xx = xmin:(xmax-xmin)/Npoints:xmax;
yy = ymin:(ymax-ymin)/Npoints:ymax;
[X, Y] = meshgrid(xx,yy);
% point by point calculation numerical integration
tic
for ix = 1:length(xx)
 for iy = 1:length(yy)
 x0 = X(ix,iy);
```

Editor - D:\πολυτεχνειο\matlab\askisi7FoI.m

askisi7FoI.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```
 y0 = Y(ix,iy);
 FA(ix,iy) = integral(@(thita)potential(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
 end
end
disp('Point-by-point numerical integration time:')
toc
FB=0;
% point by point calculation numerical integration
tic
for ix = 1:length(xx)
 for iy = 1:length(yy)
 x0 = X(ix,iy);
 y0 = Y(ix,iy);
 ExA(ix,iy) = integral(@(thita)Ex_comp(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
 end
end
disp('Point-by-point array_X numerical integration time:')
toc
tic
for ix = 1:length(xx)
 for iy = 1:length(yy)
 x0 = X(ix,iy);
 y0 = Y(ix,iy);
 EyA(ix,iy) = integral(@(thita)Ey_comp(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
 end
end
```

Command Window

Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

```matlab
disp('Point-by-point array_Y numerical integration time:')
toc
EyB=0;
ExB=0;
% Logical Variables
CircleA=(X>=0) & (Y>=0);
CircleB=(X<0)|(Y<0);
FC=FA.*CircleA;
FD=FB.*CircleB;
F=FC+FD;
ExC=ExA.*CircleA;
EyC=EyA.*CircleA;
ExD=ExB.*CircleB;
EyD=EyB.*CircleB;
Ex=ExC+ExD;
Ey=EyC+EyD;
% Plot Potential
xr(1) = 0.9;
yr(1) = 1;
xr(2) = 1.1;
yr(2) = 1;
Phi_max = max(max(F));
Phi_min = min(min(F));
Pm = 0.8*Phi_max;
figure(nfig);
hold off
```

Command Window

Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

```
surface(X,Y,F), shading interp
hold on
pr = plot(xr,yr,'Linewidth',2);
set(pr, 'Color', red);
set(pr,'ZData',Phi_max+1+zeros(size(xr)))
% Move the line charge to Z = Phi_max + 1
set(gca,'Fontsize',12,'Fontweight','bold')
xlabel('x','Fontsize',12,'FontWeight','bold')
ylabel('y','Fontsize',12,'FontWeight','bold')
title('Normalized Potential','Fontsize',10,'FontWeight','bold','Color','b')
axis equal
caxis([0 Pm])
colorbar
% ------------------------------
% Plot Lines
% ------------------------------
xr(1) = 0.9;
yr(1) = 1;
xr(2) = 1.1;
yr(2) = 1;
nfig = nfig + 1;
figure(nfig);
hold off
cont = [ 0.01, 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2, 2.5, 3,3.5, 4, 5, 6,7.5];
[CS,H] = contour(X,Y,F,cont,'Linewidth',1,'Color','b');
clabel(CS,H,cont);
```

Command Window

Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

```
hold on
pr = plot(xr,yr,'Linewidth',2);
set(pr, 'Color', red);
set(gca,'Fontsize',12,'Fontweight','bold')
xlabel('x','Fontsize',12,'FontWeight','bold')
ylabel('y','Fontsize',12,'FontWeight','bold')
title('Normalized Lines', 'Fontsize',10,'FontWeight','bold','Color','b')
axis equal
grid on
% Plot Electric Field (streamslice)
xr(1) = 0.9;
yr(1) = 1;
xr(2) = 1.1;
yr(2) = 1;
nfig = nfig + 1;
figure(nfig)
hold off
LL = sqrt(((Ex).^2 + (Ey).^2));
quiver(X,Y,Ex./LL,Ey./LL,0.5)
hold on
hs = streamslice(X,Y,Ex,Ey,2);
set(hs,'Color','m','Linewidth',1.0);
cont = [ 0.01, 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2, 2.5, 3,3.5, 4, 5, 6, 7.5];
contour(X,Y,F,cont,'Linewidth',1,'Color','b');
pr = plot(xr,yr,'Linewidth',2);
set(pr, 'Color', red);
```

Command Window

Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```
set(gca,'Fontsize',12,'Fontweight','bold')
xlabel('x','Fontsize',12,'FontWeight','bold')
ylabel('y','Fontsize',12,'FontWeight','bold')
title('Normalized Electric Field','Fontsize',10,'FontWeight','bold','Color','b')
axis equal
grid on
% Functions
function Phi = potential(thita,x0,y0)
a=0.1;
d=1;
h=1;
PhiA= a./sqrt((x0-d).^2+(y0-h).^2+a.^2-2*a.*(x0-d).*cos(thita));
PhiB=(-a)./sqrt((x0-d).^2+(y0+h).^2+a.^2-2*a.*(x0-d).*cos(thita));
PhiC= a./sqrt((x0+d).^2+(y0+h).^2+a.^2-2*a.*(x0+d).*cos(thita));
PhiD= (-a)./sqrt((x0+d).^2+(y0-h).^2+a.^2-2*a.*(x0+d).*cos(thita));
Phi=PhiA+PhiB+PhiC+PhiD;
%==========================================================================
function dEx = Ex_comp (thita,x0,y0)
a=0.1;
d=1;
h=1;
R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
dEx_compA=(a*d).*(x0-d-a.*cos(thita))./R1.^3;
```

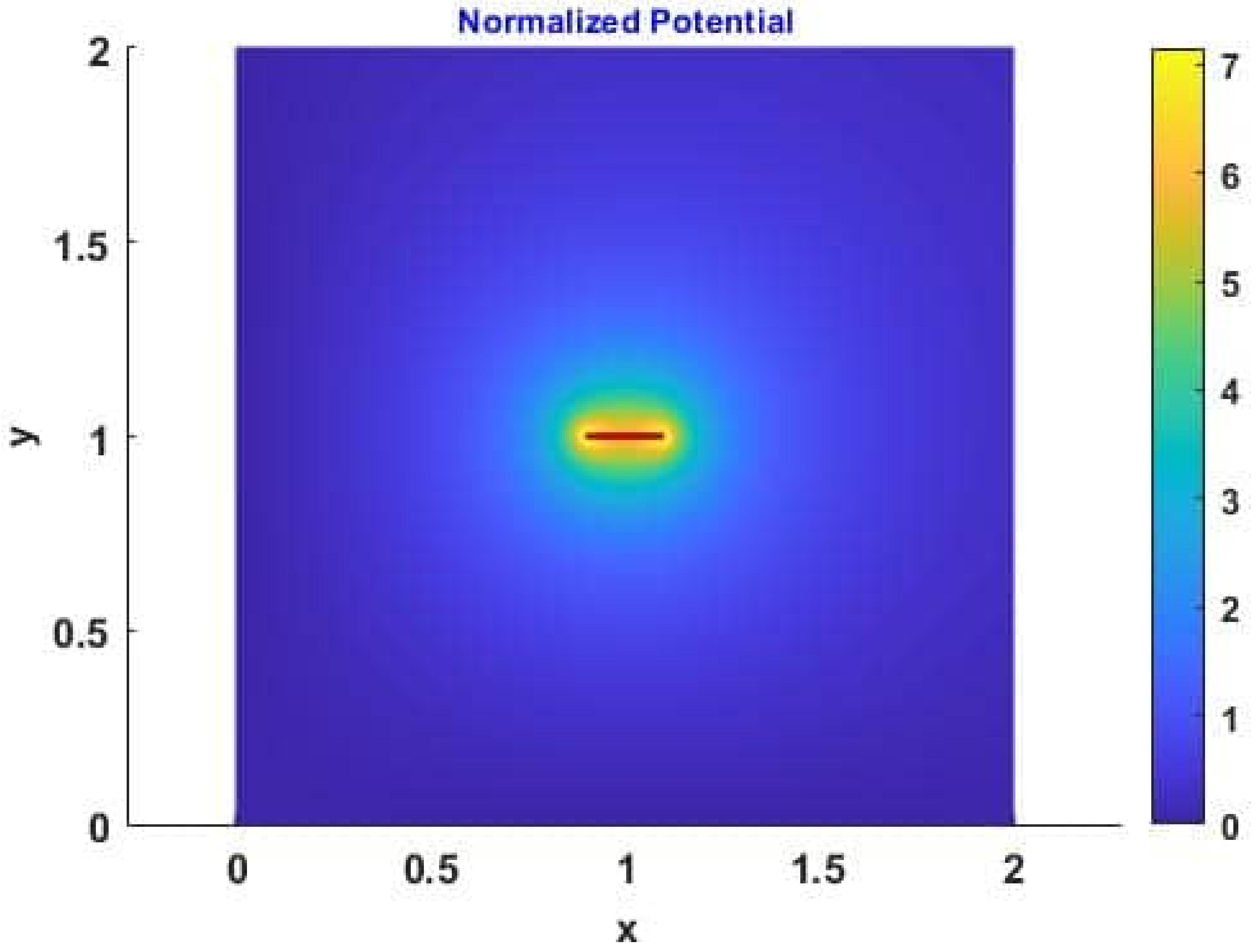
Command Window

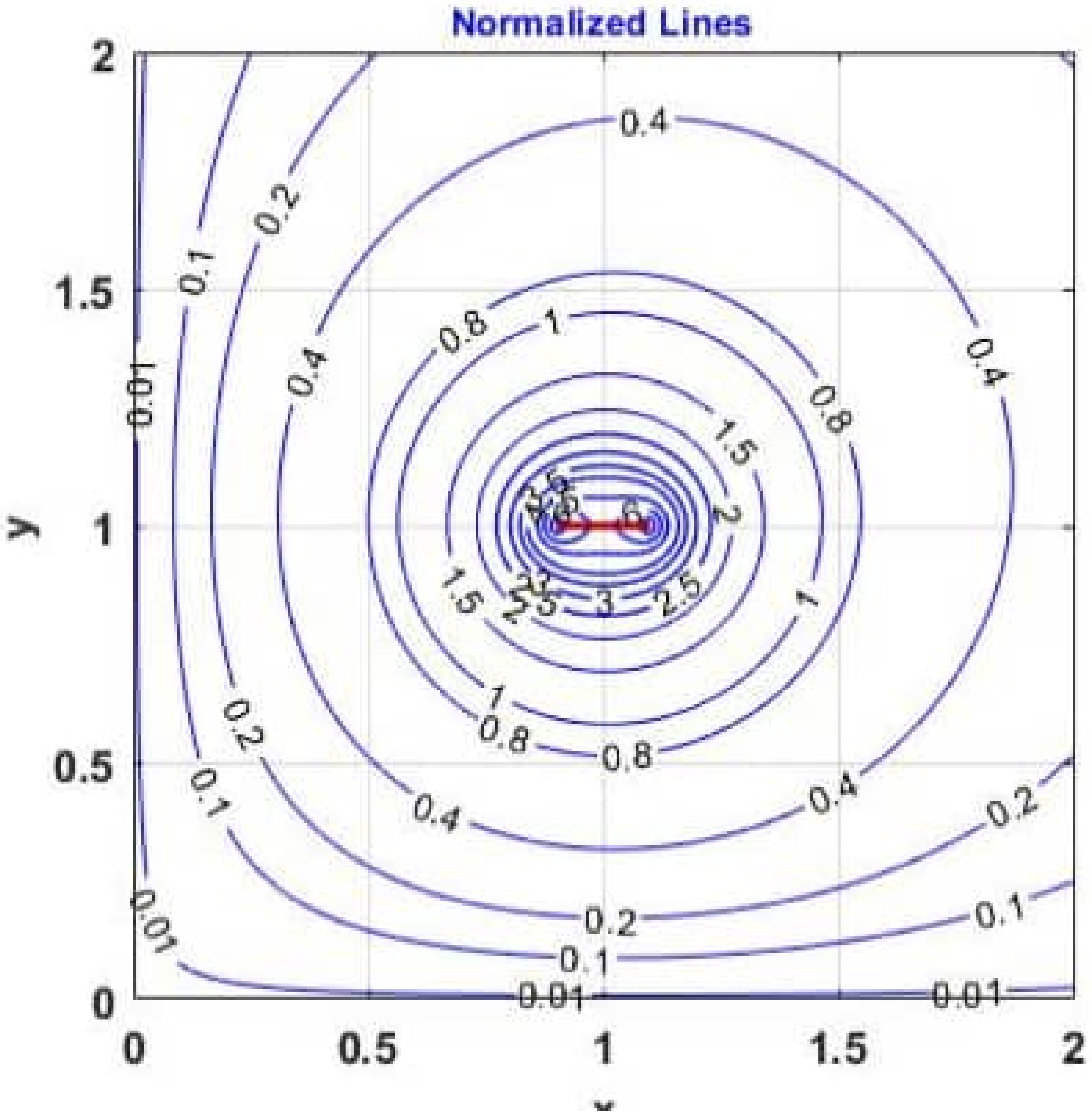
Editor - D:\πολυτεχνειο\matlab\askisi7Fol.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

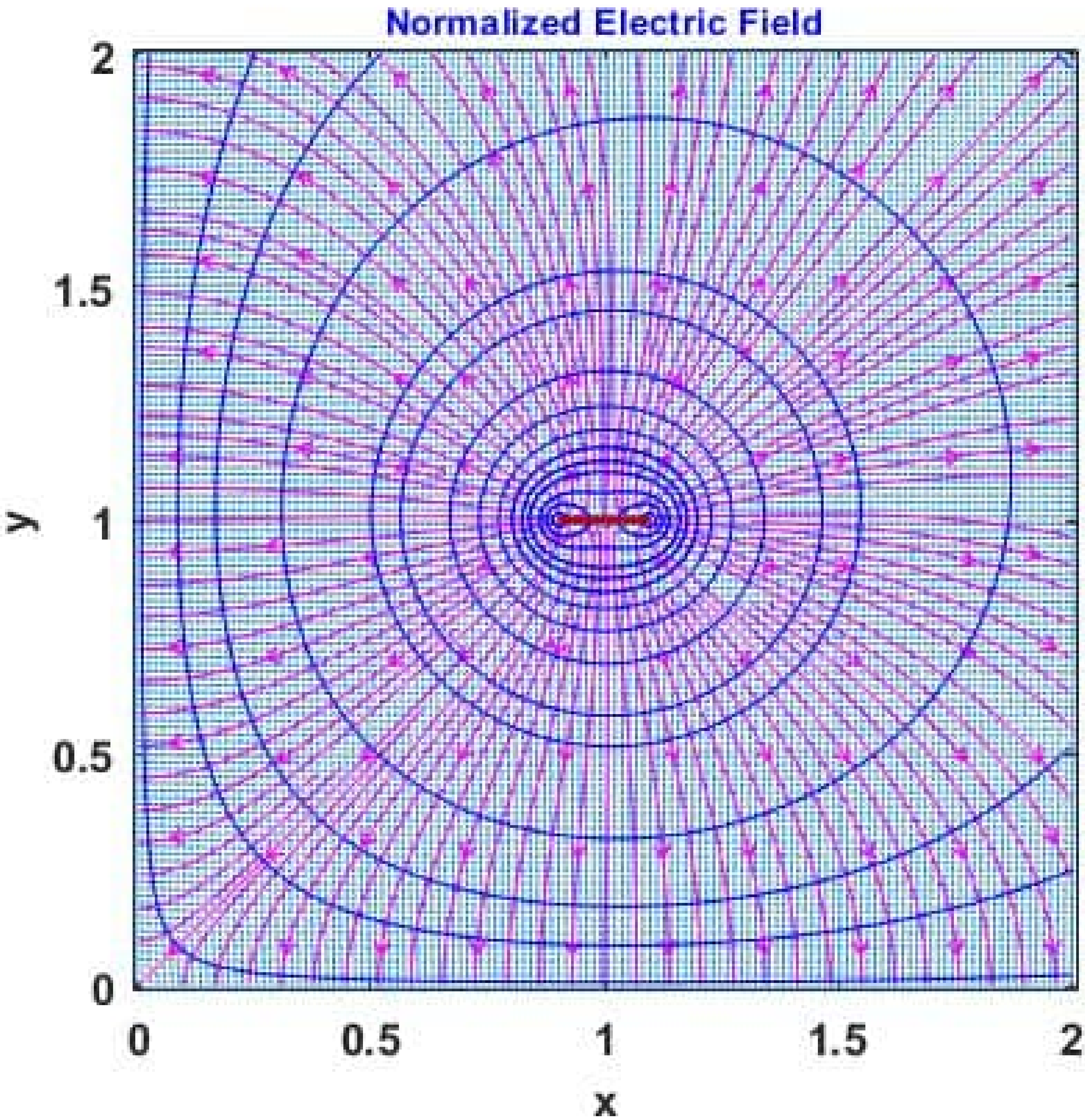
```
a=0.1;
d=1;
h=1;
R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
dEx_compA=(a*d).*(x0-d-a.*cos(thita))./R1.^3;
dEx_compB=((-a)*d).*(x0-d-a.*cos(thita))./R2.^3;
dEx_compC=(a*d).*(x0+d-a.*cos(thita))./R3.^3;
dEx_compD=((-a)*d).*(x0+d-a.*cos(thita))./R4.^3;
dEx=dEx_compA+dEx_compB+dEx_compC+dEx_compD;
%=====================================================================
function dEy = Ey_comp (thita,x0,y0)
a=0.1;
d=1;
h=1;
R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).*(x0-d).*cos(thita));
R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).*(x0+d).*cos(thita));
dEy_compA=(a*d).*(y0-h)./R1.^3;
dEy_compB=((-a)*d).*(y0+h)./R2.^3;
dEy_compC=(a*d).*(y0+h)./R3.^3;
dEy_compD=((-a)*d).*(y0-h)./R4.^3;
dEy=dEy_compA+dEy_compB+dEy_compC+dEy_compD;
```

Command Window


Normalized Potential
y
x
0
0.5
1
1.5
2
0
1
2
3
4
5
6
7



Normalized Lines
y
x
0
0.5
1
1.5
2
0.01
0.1
0.2
0.4
0.8
1
1.5
2
2.5
3



Normalized Electric Field
2
1.5
1
0.5
0
y
0
0.5
1
1.5
2
x

Editor - D:\πολυτεχνειο\matlab\askisi77G.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

```
function [X,Z] = askisi7G
% Define computational parameters
nfig=1;
Npoints=133;
xmin = 0;
xmax = +2;
zmin = -2;
zmax = 2;
xx = xmin:(xmax-xmin)/Npoints:xmax;
zz = zmin:(zmax-zmin)/Npoints:zmax;
[X, Z] = meshgrid(xx,zz);
% point by point calculation numerical integration
tic
for ix = 1:length(xx)
 for iz = 1:length(zz)
 x0 = X(ix,iz);
 z0 = Z(ix,iz);
 SA(ix,iz) = integral(@(thita)charge(thita,x0,z0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
 end
end
toc
SB=0;
% -----------------------------
% Logical Variables
% -----------------------------
CircleA=(X>=0)&(Z>=-2)&(X<=2)&(Z<=2);
```

Command Window

Editor - D:\πολυτεχνειο\matlab\askisi77G.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m | +

```
CircleB=(X<0)|(Z<-2)|(X>2)|(Z>2);
SC=SA.*CircleA;
SD=SB.*CircleB;
S=SC+SD;
% Plot Lines Of Charge
nfig = nfig + 1;
figure(nfig);
hold off
[CS,H] = contour(X,Z,S,'Linewidth',1,'Color','b');
clabel(CS,H)
hold on
set(gca,'Fontsize',12,'Fontweight','bold')
xlabel('x','Fontsize',12,'FontWeight','bold')
ylabel('z','Fontsize',12,'FontWeight','bold')
title('Normalized Charge','Fontsize',10,'FontWeight','bold','Color','b')
axis equal
grid on
% Functions
function S = charge(thita,x0,z0)
a=0.1;
d=1;
h=1;
e0=8.85418782e-12;
R1=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
R2=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
R3=sqrt((x0+d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(thita)+z0.*sin(thita)));
```

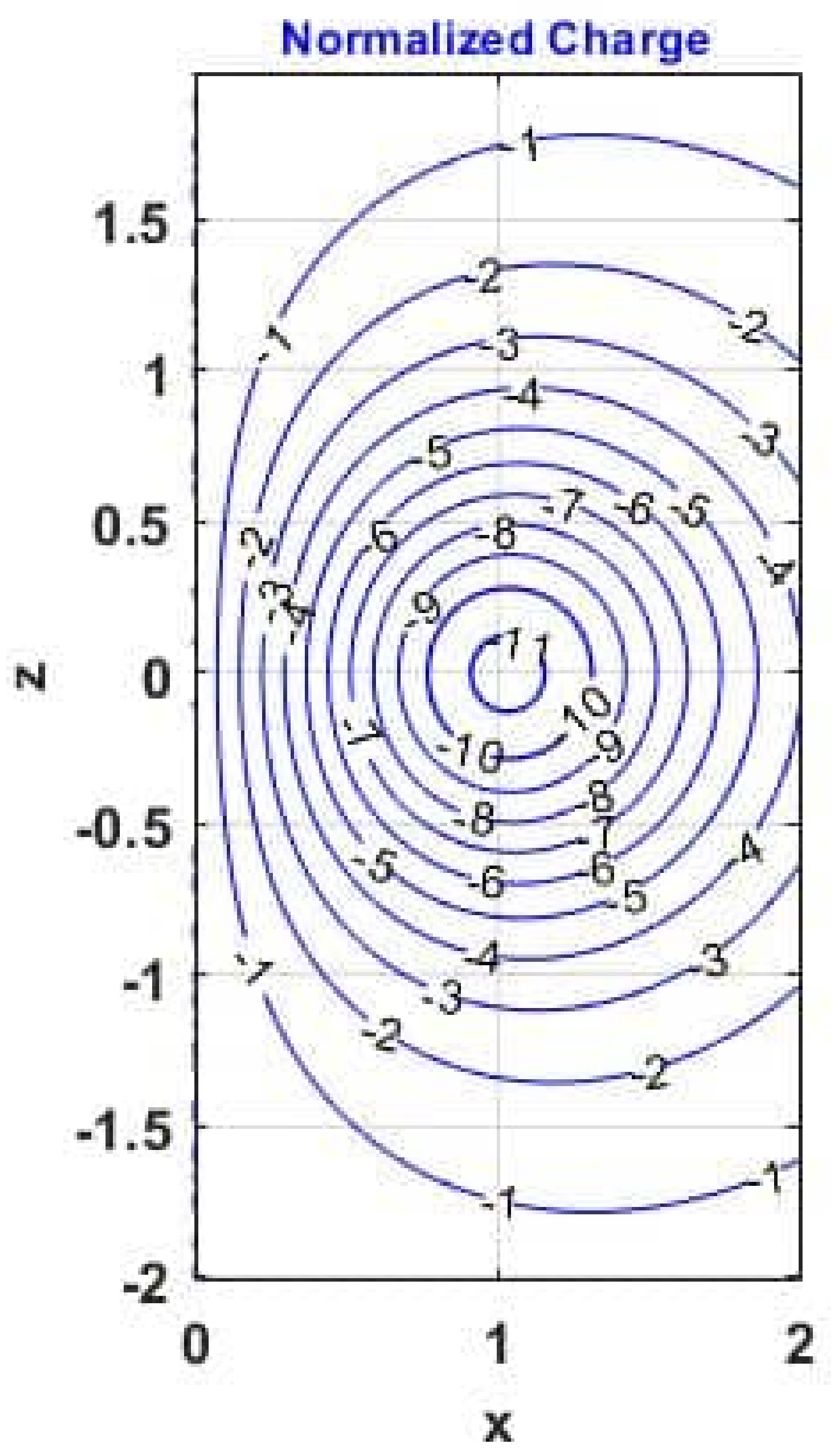
Command Window

Editor - D:\πολυτεχνειο\matlab\askisi77G.m

askisi7Fol.m | askisi77G.m | askisi6.m | ask6cd.m | askisi6e.m

```
    figure(nfig);
    hold off
    [CS,H] = contour(X,Z,S,'Linewidth',1,'Color','b');
    clabel(CS,H)
    hold on
    set(gca,'Fontsize',12,'Fontweight','bold')
    xlabel('x','Fontsize',12,'FontWeight','bold')
    ylabel('z','Fontsize',12,'FontWeight','bold')
    title('Normalized Charge','Fontsize',10,'FontWeight','bold','Color','b')
    axis equal
    grid on
    % Functions
function S = charge(thita,x0,z0)
    a=0.1;
    d=1;
    h=1;
    e0=8.85418782e-12;
    R1=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
    R2=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
    R3=sqrt((x0+d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(thita)+z0.*sin(thita)));
    R4=sqrt((x0+d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(thita)+z0.*sin(thita)));
    SA= ((-h))./R1.^3;
    SB= ((-h))./R2.^3;
    SC= ((h))./R3.^3;
    SD= ((h))./R4.^3;
    S=SA+SB+SC+SD
```

Command Window


Normalized Charge
z
x