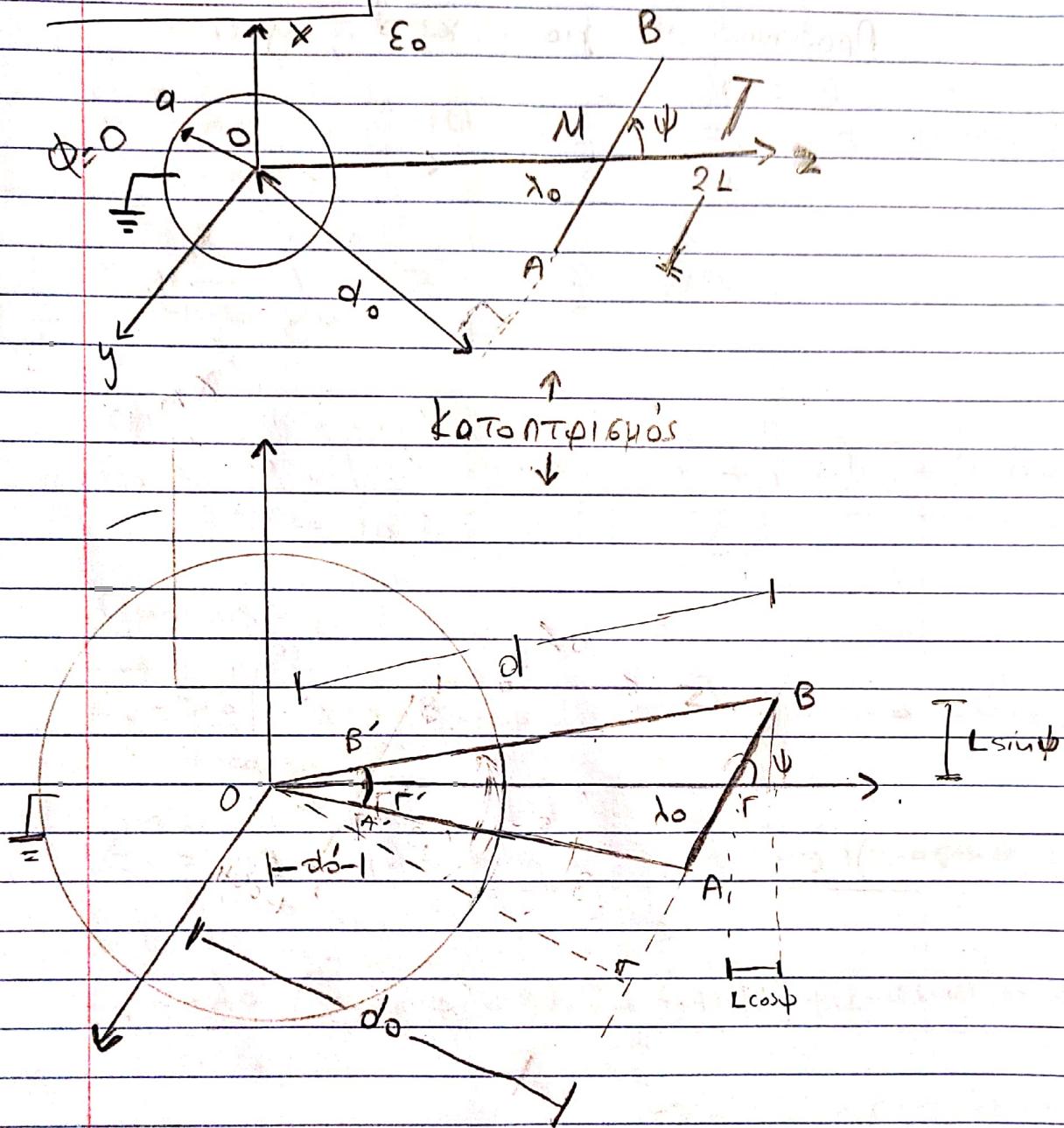


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1<sup>η</sup> Σειρά Ασκήσεων.

Άσκηση 6



a) Προβλήματος  $\phi(x, y, z) = 0$

$$\text{Για } R = \sqrt{x^2 + y^2 + z^2} \leq a$$

αφού είναι με διεύθυνση σφαιρικό

$$\text{Για } (x^2 + y^2 + z^2)^{1/2} \geq a, \text{ είνω}$$

$$\phi(x, y, z) = \phi_{\lambda_0}(x, y, z) + \phi_{\lambda_0^0}(x, y, z)$$

$$\rightarrow \phi_{\lambda_0} = \frac{1}{4\pi\epsilon_0} \lambda_0 \int \frac{1}{R_1} dL$$

$$R_1 = \left[ (x - x')^2 + y^2 + (z - z')^2 \right]^{1/2}$$

$$\cos\left(\frac{\pi}{2} - \psi\right) = \frac{d\phi}{dM} \Rightarrow dM = \frac{d\phi}{\cos\left(\frac{\pi}{2} - \psi\right)} = \frac{d\phi}{\sin\psi}$$

Από τις γεννιέται για την  $M(0, 0, \frac{d\phi}{\sin\psi})$

Χρησιμοποιώ εξισώσεις θετικών:  $x' = \tan\psi \cdot z' + c_1$

η οποία διερχεται από την  $M$  από

$$0 = \tan\psi \cdot \frac{d\phi}{\sin\psi} + c_1 \Rightarrow c_1 = -\frac{d\phi}{\cos\psi}$$

$$\psi \in z' \in \left[ \frac{d\phi}{\sin\psi} - L \cos\psi, \frac{d\phi}{\sin\psi} + L \cos\psi \right]$$

$$\text{Για τη } dL \text{ λέγεται } dL = \sqrt{dx^2 + dz'^2}$$

$$dL = \sqrt{\tan^2\psi dz'^2 + dz'^2} \Rightarrow dL = dz' \sqrt{\tan^2\psi + 1}$$

$$\Rightarrow dL = \frac{1}{\cos\psi} dz'$$

$$\text{Συνεπώς } R_1 = \left[ (x - \tan\psi \cdot z' + \frac{d\phi}{\cos\psi})^2 + y^2 + (z - z')^2 \right]^{1/2}$$

A<sub>po</sub>:

$$\Phi_{do}(x, y, z) = \frac{do}{4\pi\epsilon_0} \left( \frac{\frac{do}{\sin\phi} + L \cos\phi}{\sqrt{(x + \frac{do}{\cos\phi} - \tan\phi z')^2 + y^2 + (z - z')^2}} \right)^{1/2}$$

→  $\Phi_{do}'$ :

$$A \left( -\tan\phi z' - \frac{do}{\cos\phi}, 0, z' \right), z' \in \left[ \frac{do - L \cos\phi}{\sin\phi}, \frac{do + L \cos\phi}{\sin\phi} \right]$$

$$d = \left[ z'^2 + \left( \tan\phi \cdot z' - \frac{do}{\cos\phi} \right)^2 \right]^{1/2}$$

$$\left[ z'^2 (1 + \tan^2\phi) - \frac{do}{\cos^2\phi} (2z' \sin\phi - do) \right]^{1/2}$$

$$\Rightarrow d = \frac{1}{\cos\phi} \left[ z'^2 - do (2z' \sin\phi - do) \right]^{1/2}$$

$$do' = \frac{a^2}{d} = \frac{a^2 \cos\phi}{[z'^2 - do (2z' \sin\phi - do)]^{1/2}}$$

πα τη γωνία:

$$\theta = \Theta(z') = \tan^{-1} \left( \frac{x'}{z'} \right) = \tan^{-1} \left( -\tan\phi - \frac{do}{z' \cos\phi} \right)$$

πα τη  $dq'$

$$x'' = do' \sin\theta \quad z'' = do' \cos\theta$$

$$\text{και } 1 \times \omega \quad dq' = -dq \frac{a}{d} = -dq \frac{a \cos\phi}{[z'^2 - do (2z' \sin\phi - do)]^{1/2}}$$

$$\text{και } dq = \lambda do \ell'$$

$$\text{Αφε } dq' = -\frac{a do \ell'}{[z'^2 - do (2z' \sin\phi - do)]^{1/2}}$$

$$\text{Enigus } dL' = \sqrt{dx''^2 + dz''^2}$$

$$R_2 = [(x-x'')^2 + y^2 + (z-z'')^2]^{1/2}$$

$$dx'' = \frac{d}{dz'} \cdot d_0' dz' \sin\theta + d_0' \cos\theta \frac{d\theta}{dz'} dz'$$

$$dx'' = dz' \left[ \frac{d}{dz'} \left( \frac{\alpha^2 \cos\phi}{[z'^2 - d_0(2z' \sin\phi - d_0)]^{1/2}} \right) \sin\theta + \frac{d_0' \cos\theta}{dz'} \left( \tan^{-1} \left( \frac{\tan\phi - \frac{d_0}{z' \cos\phi}}{1 + (\tan\phi - \frac{d_0}{z' \cos\phi})^2} \right) \right) \right]$$

$$= dz' \left[ \frac{-\alpha^2 \cos\phi}{2[z'^2 - d_0(2z' \sin\phi - d_0)]^{3/2}} \cdot \frac{d}{dz'} (z'^2 - d_0(2z' \sin\phi - d_0)) \right. \\ \left. + \frac{\alpha^2 \cos\phi}{[z'^2 - d_0(2z' \sin\phi - d_0)]^{1/2}} \cdot \frac{\cos\theta}{1 + (\tan\phi - \frac{d_0}{z' \cos\phi})^2} \right]$$

$$\cdot \frac{d}{dz'} \left( \tan\phi - \frac{d_0}{z' \cos\phi} \right) =$$

$$= dz' \left[ \frac{-\alpha^2 \cos\phi (z'^2 - d_0 \sin\phi) \sin\theta}{2[z'^2 - d_0(2z' \sin\phi - d_0)]^{3/2}} + \right. \\ \left. \frac{\alpha^2 \cos\phi \cos\theta}{[z'^2 - d_0(2z' \sin\phi - d_0)]^{1/2}} \cdot \frac{1}{(1 + (\tan\phi - \frac{d_0}{z' \cos\phi})^2)} \frac{d_0 / \cos\phi}{z' \cos\phi} \right]$$

$\Rightarrow$

$$dx'' = dz' \left[ \frac{-\alpha^2 \cos\phi (z' - d_0 \sin\phi) \sin\theta}{2[z'^2 - d_0(2z' \sin\phi - d_0)]^{3/2}} + \right.$$

$$\left. \frac{\alpha^2 d_0 \cos\theta}{[z'^2 - d_0(2z' \sin\phi - d_0)]^{1/2}} \cdot \frac{1}{z'^2 + (2' \tan\phi - \frac{d_0}{\cos\phi})^2} \right]$$

Γιατί το  $d_2''$  ακολουθώντας την αντίστοιχη διαδικασία  
με την προφορά στην έκπι θα γίνει  $\sin\theta$  αντί την  $\cos\theta$ .  
Συνεπώς ο πρώτος όπος παρατίθεται είδος ήση  
ο δεύτερος. Έχει αντίθετο πρόσημο ήση  $\sin\theta$

$$d_2'' = d_{2'} \left[ -\frac{\alpha^2 \cos\phi (z' - d_0 \sin\phi) \cos\theta}{[z'^2 - d_0(z' \sin\phi - d_0)]^{3/2}} - \frac{\alpha^2 d_0 \sin\phi}{[z'^2 - d_0(z' \sin\phi - d_0)]^{1/2}} \cdot \frac{1}{z'^2 + (z' \tan\phi - \frac{d_0}{\sin\phi})^2} \right]$$

$$dx'' = X_0'' \cdot d_2' \quad \text{Από } d_1' = d_2' \sqrt{X_0''^2 + z_0''^2}$$

$$d_2'' = z_0'' d_2'$$

Τελικά

$$\Phi_{\lambda_0'}(x, y, z) = \frac{1}{4\pi\epsilon_0} \int \frac{dq'}{R_2} \Rightarrow \Phi_{\lambda_0'} = \frac{q'}{4\pi\epsilon_0 R_2}$$

$$\Phi_{\lambda_0'} = -\frac{d_0 \alpha}{4\pi\epsilon_0} \int \frac{\frac{dq}{\sin\phi} + L \cos\phi}{\sqrt{X_0''^2 + z_0''^2}} \cdot \frac{d_2'}{d \cdot R_2}$$

b)  $E(x, y, z) = 0 \quad \text{με } (x^2 + y^2 + z^2)^{1/2} \leq a$   
 $\text{αλλα } \phi = 0$

$$E = E_{d_0} + E_{\lambda_0'} \quad \text{με } (x^2 + y^2 + z^2)^{1/2} \geq a$$

$$E_{d_0} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{R_1^2} \stackrel{iR}{=} \frac{1}{4\pi\epsilon_0} \int \frac{d_0 \cdot d_1'}{R_1^2} \cdot \frac{R_1}{R_1} \stackrel{\frac{d_0}{\sin\phi} + L \cos\phi}{\frac{\sin\phi}{\sin\phi} - L \cos\phi}$$

$$\vec{E}_{d_0} = \frac{do}{4\pi\epsilon_0} \begin{cases} \frac{do}{\sin\phi} + L \cos\phi \\ \frac{\vec{R}_1}{R_3} \frac{1}{\cos\phi} d_2' \\ \frac{do}{\sin\phi} - L \cos\phi \end{cases}$$

όπου  $R_1 = |\vec{R}_1| = \left[ (x - t \sin\phi z' + \frac{do}{\cos\phi})^2 + y^2 + (z - z')^2 \right]^{1/2}$

και  $\vec{R}_1 = (x - t \sin\phi z' + \frac{do}{\cos\phi}) \hat{i}_x + y \hat{i}_y + (z - z') \hat{i}_z$

Λαρνάκωση για  $\vec{E}_{d_0}$

$$\vec{E}_{d_0} = \frac{1}{4\pi\epsilon_0} \int \frac{dz'}{R_2^2} \hat{i}_{R_2} \rightarrow$$

$$\vec{E}_{d_0} = - \frac{\alpha do}{4\pi\epsilon_0} \int \frac{\frac{do}{\sin\phi} + L \cos\phi}{R_2^3} \frac{\sqrt{x_0'' + z_0''}}{d} dz''$$

με  $d = \frac{1}{\cos\phi} [(z')^2 - do (2z' \sin\phi - do)]^{1/2}$

$$\vec{R}_2 = (x - x'') \hat{i}_x + y \hat{i}_y + (z - z'') \hat{i}_z$$

με  $x'' = do \sin\theta$ , και  $z'' = do \cos\theta$

γ)  $G = i_n \cdot (\vec{D}^+ - \vec{D}^-) = \epsilon_0 \cdot i_n \cdot \vec{E}(x, y, z)$

με  $G$  απίκες συνεπειώσεις:

$$= \epsilon_0 i_n \cdot E(\alpha \cos\theta, \alpha \sin\theta \cos\phi, \alpha \sin\theta \sin\phi)$$

$$= \epsilon_0 (\cos\theta \hat{i}_x + \sin\theta \cos\phi \hat{i}_y + \sin\theta \sin\phi \hat{i}_z) (E_x \hat{i}_x + E_y \hat{i}_y + E_z \hat{i}_z)$$

$$= \epsilon_0 (\cos\theta E_x + \sin\theta \cos\phi E_y + \sin\theta \sin\phi E_z)$$

και  $E_x, E_y, E_z$  οι λυγισμές των γεν. πεδίων  $E_{x0}$   
καρεγιανός εντύπων.

askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisib.e.m +

```
1 %askhsh 6
2 - xmin = -3;
3 - xmax = 3;
4 - zmin = -3;
5 - zmax = 4.5;
6 - x0 = xmin: (xmax-xmin)/100 : xmax;
7 - z0 = zmin: (zmax-zmin)/100 : zmax;
8 - [Z0,X0] = meshgrid(z0,x0);
9 - a = 1;
10 - d = 2;
11 - L = 1;
12 - y = pi/4;
13 - thita_min = d/sin(y)-L*cos(y);
14 - thita_max = d/sin(y)+L*cos(y);
15
16 - for i = 1:length(z0)
17 -   for j = 1:length(x0)
18 -     z = Z0(i,j);
19 -     x = X0(i,j);
20 -     F(i,j) = integral(@(zf)pot(zf,z,x),thita_min,thita_max);
21 -     Ez(i,j) = integral(@(zf)Ezintf(zf,z,x),thita_min,thita_max);
22 -     Ex(i,j) = integral(@(zf)Exintf(zf,z,x),thita_min,thita_max);
23 -   end
24 - end
25
26 %C
```

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askisi7Fol.m X askisi77G.m X askisi6.m X ask6cd.m X askisi6e.m X +

```
27 - figure(1)
28 - hold off
29 - c = [0.01, 0.025, 0.05:0.05:1, 1.1:0.1:1.5, 2, 3, 4, 5];
30 - [C,H] = contour(Z0,X0,F,c);
31 - clabel(C,H,c);
32 - hold on;
33 - zr(1) = thita_min;
34 - xr(1) = tan(y)*zr(1)-d/cos(y);
35 - zr(2) = thita_max;
36 - xr(2) = tan(y)*zr(2)-d/cos(y);
37 - plot(zr,xr);
38 - [r,f] = meshgrid(a,0:pi/50:2*pi);
39 - [zt,xt] = pol2cart(f,r);
40 - fill(zt,xt,[1 1 1]);
41 - xlabel('z')
42 - ylabel('x')
43 - title('Normalized Equipotential Lines')
44 - axis equal
45 - grid on
46
47 - figure(2)
48 - hold off
49 - surface(Z0,X0,F);
50 - shading interp;
51 - colorbar;
52 - xlabel('z')
```

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askisi7Fol.m    askisi77G.m    askisi6.m    ask6cd.m    askisi6e.m    +

```
53 -     ylabel('x')
54 -     title('normalized potential')
55 -     axis equal
56 -     axis tight
57 -     grid on
58
59 %d
60 figure(3)
61 hold off
62 En = sqrt(((Ez).^2+(Ex).^2));
63 quiver(Z0,X0,Ez./En,Ex./En);
64 hold on
65 E = streamslice(Z0,X0,Ez,Ex,2);
66 set(E,'Linewidth', 1, 'color', 'b');
67 plot(zr,xr,'Linewidth',2,'color',[1 0.5 0]);
68 fill(zt,xt,[1 1 1]);
69 xlabel('z')
70 ylabel('x')
71 title('normalized Electric Field Lines')
72 axis equal
73 axis tight
74 grid on
75
76 %functions-----
77 function F = pot(z,z0,x0)
78 a=1;
```

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askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisi6e.m +

```
79 - d=2;
80 - y = pi/4;
81 - xt = tan(y)*z-d/cos(y);
82 - Rt = sqrt((x0-xt).^2+(z0-z).^2);
83 - Ft = 1./ (Rt.*cos(y));
84 - dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);
85 - dt = (a.^2)./dl;
86 - th = atan(tan(y)-d./ (z.*cos(y)));
87 - xtt = dt.*sin(th);
88 - ztt = dt.*cos(th);
89 - Rtt = sqrt((x0-xt).^2+(z0-ztt).^2);
90 - h1 = -(a^2.*cos(y).* (z-d.*sin(y)))./ ((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
91 - h2 = (a^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./ (z.^2+(z*tan(y)-d/cos(y)).^2);
92 - Xtt = h1.*sin(th)+h2.*cos(th);
93 - Ztt = h1.*cos(th)-h2.*sin(th);
94 - Ftt = -a.*sqrt(Xtt.^2+Ztt.^2)./ (dl.*Rtt);
95 - F = Ft + Ftt;
96 - end
97
98 - function Ex = Exintf(z,z0,x0)
99 - a=1;
100 - d=2;
101 - y = pi/4;
102 - xt = tan(y)*z-d/cos(y);
103 - Rt = sqrt((x0-xt).^2+(z0-z).^2);
104 - Ext = (x0-xt)./ ((Rt.^3).*cos(y));
```

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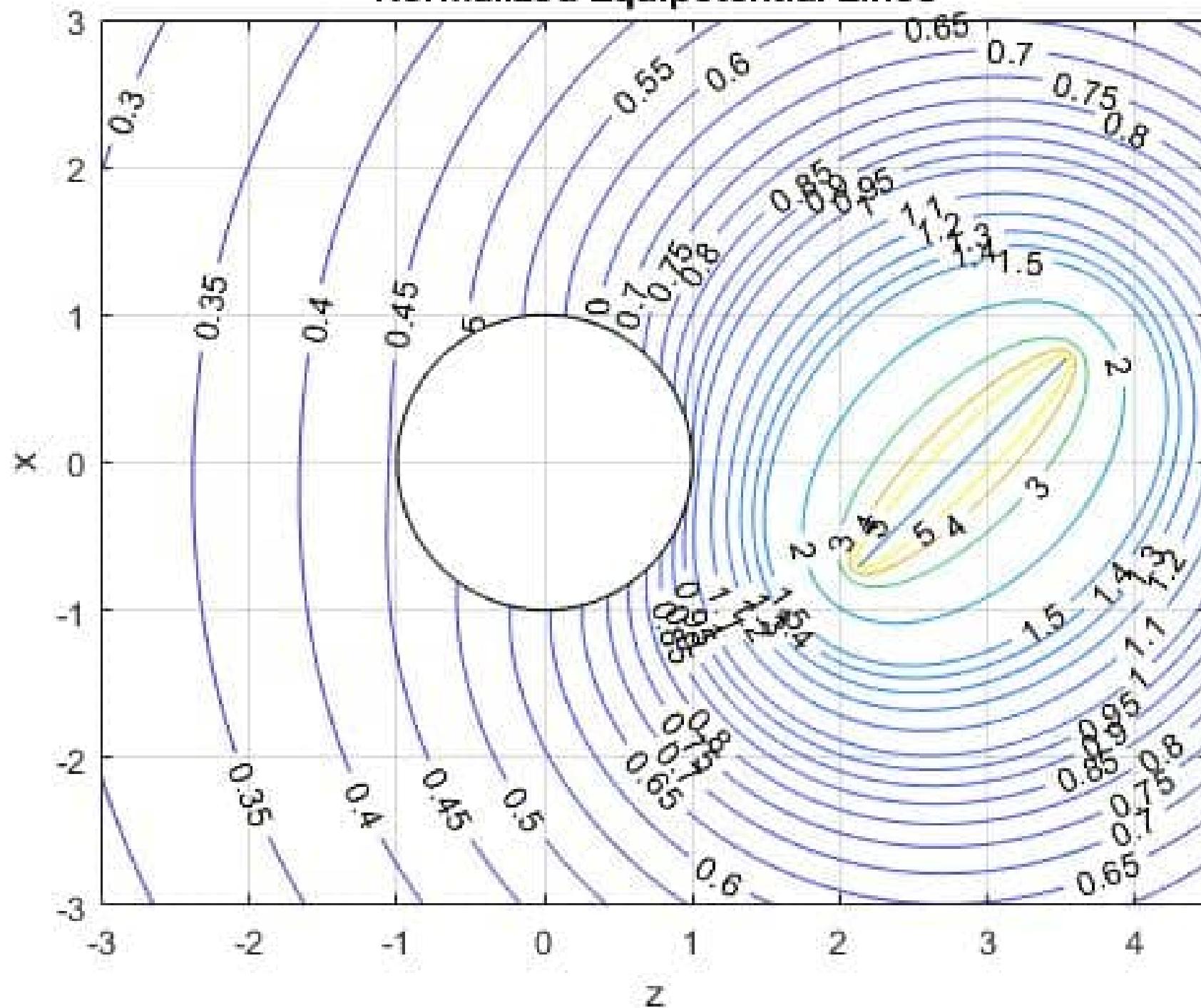
askisi7Fol.m X askisi77G.m X askisi6.m X ask6cd.m X askisi6e.m X +

```
105 - dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);
106 - dt = (a.^2)./dl;
107 - th = atan(tan(y)-d./ (z.*cos(y)));
108 - xtt = dt.*sin(th);
109 - ztt = dt.*cos(th);
110 - Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
111 - h1 = -(a.^2.*cos(y).* (z-d.*sin(y)))./ ((z.^2-2.*d.*sin(y).*z+d.^2).^1.5);
112 - h2 = (a.^2*d)./sqrt(z.^2-2.*d.*sin(y).*z+d.^2).*1./ (z.^2+(z*tan(y)-d/cos(y)));
113 - Xtt = h1.*sin(th)+h2.*cos(th);
114 - Ztt = h1.*cos(th)-h2.*sin(th);
115 - Extt = -a.* (x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./ (dl.* (Rtt).^3);
116 - Ex = Ext + Extt;
117 - end
118
119 [-function Ez = Ezintf(z,z0,x0)
120 - a=1;
121 - d=2;
122 - y = pi/4;
123 - xt = tan(y)*z-d/cos(y);
124 - Rt = sqrt((x0-xt).^2+(z0-z).^2);
125 - Ezt = (z0-z)./((Rt.^3).*cos(y));
126 - dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);
127 - dt = a./dl;
128 - th = atan(tan(y)-d./ (z.*cos(y)));
129 - xtt = dt.*sin(th);
130 - ztt = dt.*cos(th);
```

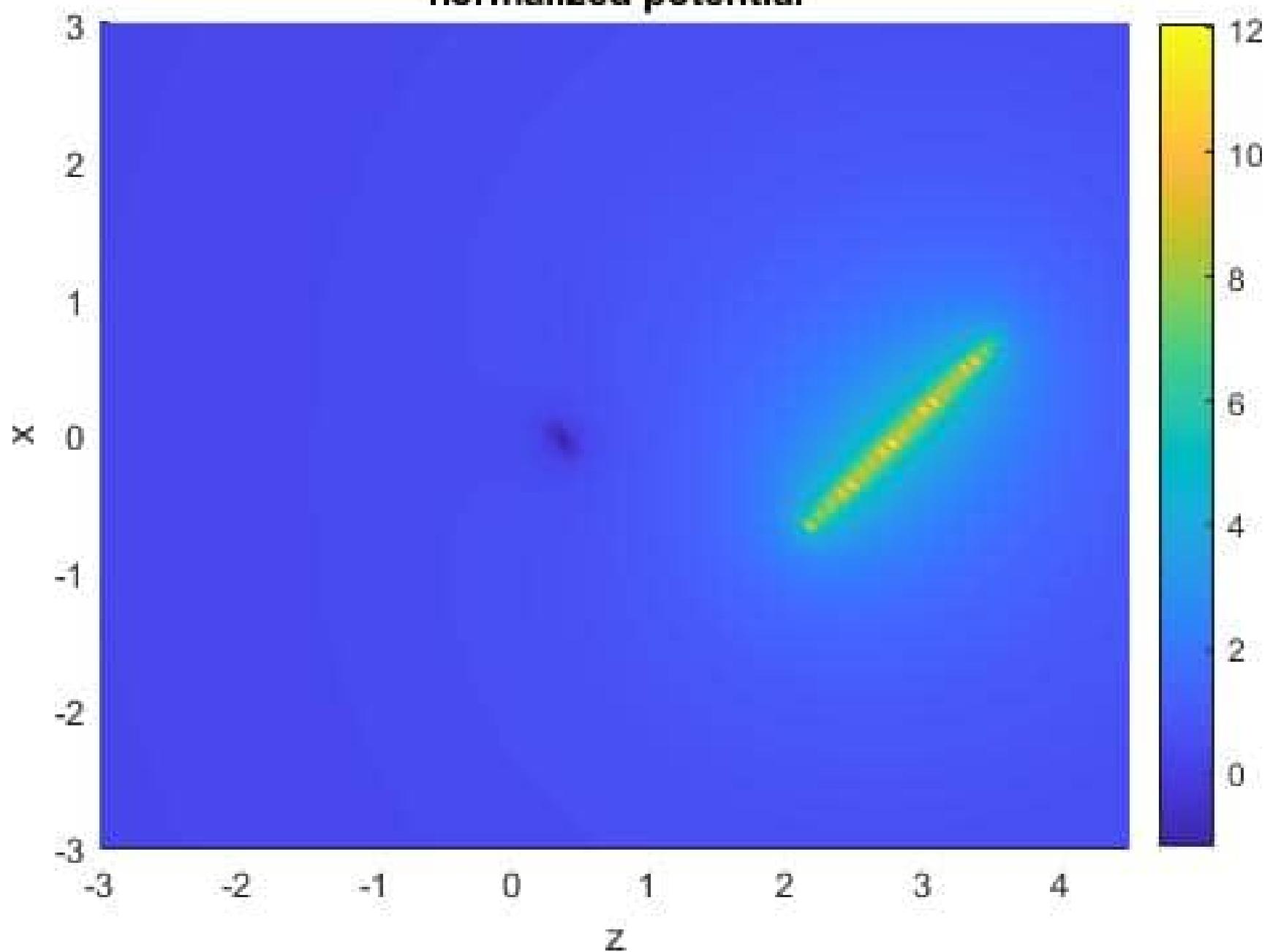
Command Window

```
askisi7Fol.m X askisi77G.m X askisi6.m X askbcd.m X askisi6e.m X +  
113 - Xtt = h1.*sin(th)+h2.*cos(th);  
114 - Ztt = h1.*cos(th)-h2.*sin(th);  
115 - Extt = -a.* (x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.* (Rtt).^3);  
116 - Ex = Ext + Extt;  
117 - end  
118  
119 function Ez = Ezintf(z,z0,x0)  
120 - a=1;  
121 - d=2;  
122 - y = pi/4;  
123 - xt = tan(y)*z-d/cos(y);  
124 - Rt = sqrt((x0-xt).^2+(z0-z).^2);  
125 - Ezt = (z0-z)./(Rt.^3).*cos(y);  
126 - dl = (1/cos(y)).*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);  
127 - dt = a./dl;  
128 - th = atan(tan(y)-d./ (z.*cos(y)));  
129 - xtt = dt.*sin(th);  
130 - ztt = dt.*cos(th);  
131 - Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);  
132 - h1 = -(a.^2.*cos(y).* (z-d.*sin(y)))./ ((z.^2-2.*d.*sin(y).*z+d.^2).^1.5);  
133 - h2 = (a.^2*d)./sqrt(z.^2-2.*d.*sin(y).*z+d.^2).*1./ (z.^2+(z*tan(y)-d/cos(y)));  
134 - Xtt = h1.*sin(th)+h2.*cos(th);  
135 - Ztt = h1.*cos(th)-h2.*sin(th);  
136 - Eztt = -a.* (z0-ztt).*sqrt(Xtt.^2+Ztt.^2)./(dl.* (Rtt).^3);  
137 - Ez = Ezt + Eztt;  
138 - end
```

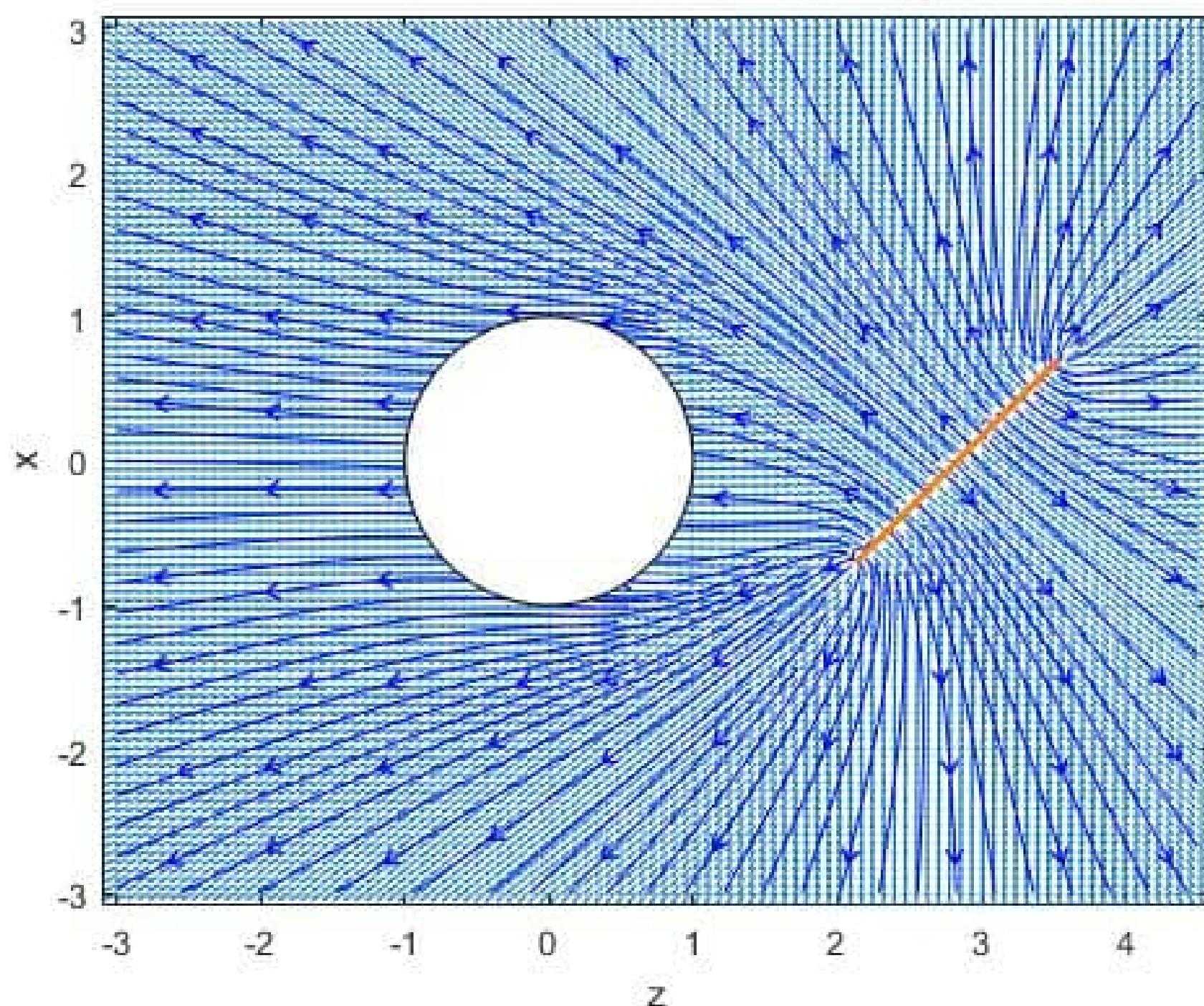
## Normalized Equipotential Lines



**normalized potential**



# normalized Electric Field Lines



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askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisi6e.m +

```
1 %e
2 a=1;
3 d=2;
4 L=1;
5 y=pi/4;
6 thita_min = d/sin(y)-L*cos(y);
7 thita_max = d/sin(y)+L*cos(y);
8 thita0 = 0:0.001:2*pi;
9
10 Ex = integral(@(z)Exint(z,thita0),thita_min,thita_max,'ArrayValued',true);
11 Ez = integral(@(z)Ezint(z,thita0),thita_min,thita_max,'ArrayValued',true);
12
13 e0 = 8.85418781762039*10^(-12);
14 chargel = e0.*Ex.*cos(thita0);
15 charge2 = e0.* (Ez.*sin(thita0));
16 charge=chargel + charge2;
17 plot(thita0, charge, 'm');
18 xlim([0 2*pi]);
19 grid on;
20 xlabel('Angle (rad)');
21 ylabel('s');
22 title('Normalized Surface Charge')
23 %functions-
24
25 function Exx=Exint(z,thita)
26 a=1;
```

Command Window

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askisi7Fol.m askisi7G.m askisi6.m ask6cd.m askisi6e.m +

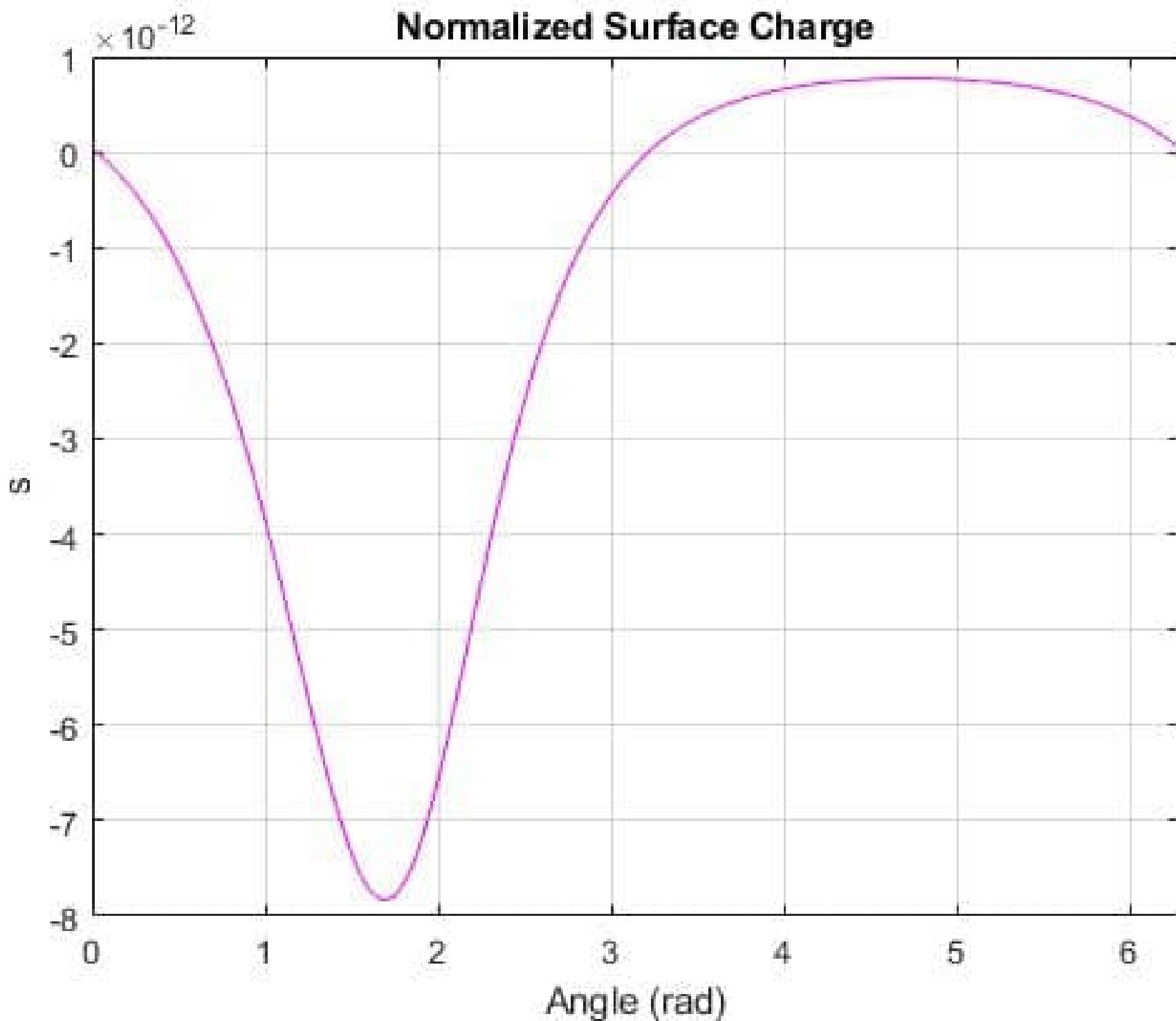
```
27 - d=2;
28 - y=pi/4;
29 - x0=a.*cos(thita);
30 - z0=a.*sin(thita);
31 - xt = tan(y) *z-d/cos(y);
32 - Rt = sqrt((x0-xt).^2+(z0-z).^2);
33 - Ext = (x0-xt)./((Rt.^3).*cos(y));
34 - dl = (1/cos(y))*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);
35 - dt = (a.^2)./dl;
36 - thita = atan(tan(y)-d./ (z.*cos(y)));
37 - xtt = dt.*sin(thita);
38 - ztt = dt.*cos(thita);
39 - Rtt = sqrt((x0-xtt).^2+(z0-ztt).^2);
40 - h1 = -(a.^2.*cos(y).* (z-d.*sin(y)))./ ((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
41 - h2 = (a.^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./ (z.^2+(z*tan(y)-d/cos(y)).^2);
42 - Xtt = h1.*sin(thita)+h2.*cos(thita);
43 - Ztt = h1.*cos(thita)-h2.*sin(thita);
44 - Exttt = -a.* (x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
45 - Exx = Ext + Exttt;
46 - end
47
48 - function Ezz=Ezint(z,thita)
49 - a=1;
50 - d=2;
51 - y=pi/4;
52 - x0=a.*cos(thita);
```

Command Window

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askisi7Fol.m    askisi7G.m    askisi6.m    ask6cd.m    askisi6e.m    +

```
44 - Extt = -a.* (x0-xtt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
45 - Exx = Ext + Extt;
46 - end
47
48 - function Ezz=Ezint(z,thita)
49 - a=1;
50 - d=2;
51 - y=pi/4;
52 - x0=a.*cos(thita);
53 - z0=a.*sin(thita);
54 - xt = tan(y)*z-d/cos(y);
55 - Rt = sqrt((x0-xt).^2+(z0-z).^2);
56 - Ezt = (z0-z)./((Rt.^3).*cos(y));
57 - dl = (1/cos(y)).*sqrt(z.^2-2.*d.*sin(y).*z+d.^2);
58 - dt = (a.^2)./dl;
59 - thita = atan(tan(y)-d./ (z.*cos(y)));
60 - xtt = dt.*sin(thita);
61 - ztt = dt.*cos(thita);
62 - Rtt = sqrt((x0-xt).^2+(z0-ztt).^2);
63 - h1 = -(a.^2.*cos(y).* (z-d.*sin(y)))./((z.^2-d.*2.*sin(y).*z+d.^2).^1.5);
64 - h2 = (a.^2*d)./sqrt(z.^2-d.*2.*sin(y).*z+d.^2).*1./ (z.^2+(z*tan(y)-d/cos(y)).^2);
65 - Xtt = h1.*sin(thita)+h2.*cos(thita);
66 - Ztt = h1.*cos(thita)-h2.*sin(thita);
67 - Eztt = -a.* (z0-ztt).*sqrt(Xtt.^2+Ztt.^2)./(dl.*Rtt.^3);
68 - Ezz = Ezt + Eztt;
69 - end
```



Ιωάννης Μπογδόπουλος

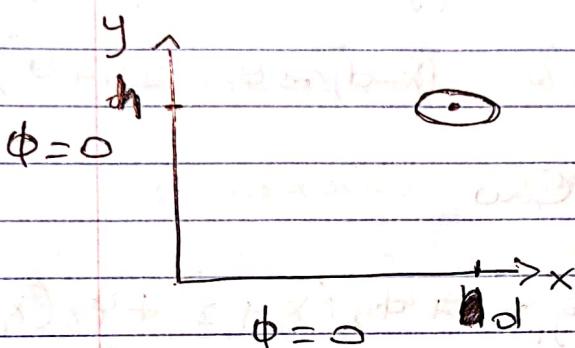
03119640

1<sup>η</sup> Σειρα Ασκήσεων

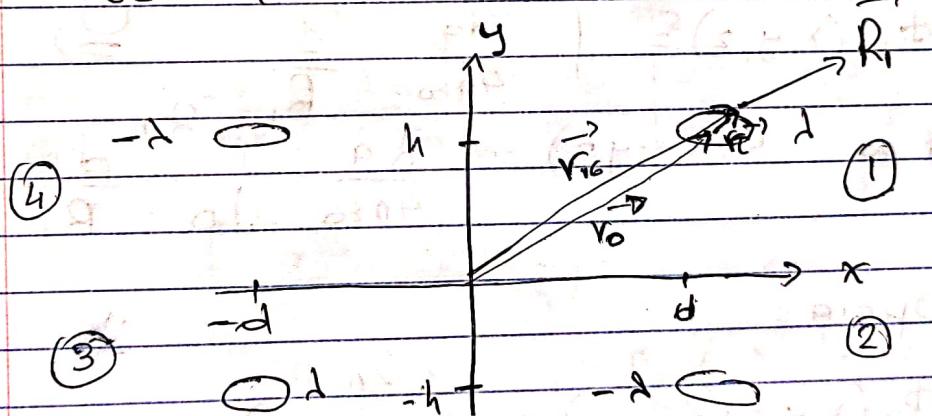
### Ασκηση 7

Συρθωνα με τις βασικαιεις: Μεθόδος κατρούγκας

Αρχική πρόβλημα



Εγονική πρόβλημα:



$$\begin{aligned} dq &= \alpha dl \\ dl &= \alpha d\theta \end{aligned} \quad \left. \begin{array}{l} \Rightarrow dq = \alpha \alpha d\theta \quad \text{ψε } 0 \leq \theta \leq 2\pi \\ \end{array} \right\}$$

$$\text{Ενίσια } \vec{v}_0 = d\hat{i}_x + h\hat{i}_y$$

$$\text{και } \vec{v}_T = \alpha \cos \theta \hat{i}_x + \alpha \sin \theta \hat{i}_y$$

$$\text{Άρα } \vec{v}_{t0} = \vec{v}_0 + \vec{v}_T = (d + \alpha \cos \theta) \hat{i}_x + h \hat{i}_y + \alpha \sin \theta \hat{i}_z$$

$$\vec{R}_{\text{obj}} = x \hat{i_x} + y \hat{i_y} + z \hat{i_z} = \vec{r}_{\text{obj}} + \vec{R},$$

$$\vec{R}_{\text{obj}} = (x - d - a \cos \theta) \hat{i_x} + (y - b) \hat{i_y} + (z - a \sin \theta) \hat{i_z}$$

$$|\vec{R}_{\text{obj}}| = R_{\text{obj}} = \left[ (x - d)^2 + (y - b)^2 + z^2 + a^2 - 2a((x - d) \cos \theta + z \sin \theta) \right]^{1/2}$$

for  $x > 0, y > 0$  case

$$\Phi(x, y, z) = \Phi_1(x, y, z) + \Phi_2(x, y, z) + \Phi_3(x, y, z) + \Phi_4(x, y, z)$$

$\Phi_1$ :

$$\Phi_1(x, y, z) = \int \frac{dq}{4\pi\epsilon_0} \frac{1}{R_{\text{obj}}} \quad \textcircled{1}$$

$$\text{And } \Phi_1(x, y, z) = \frac{dq}{4\pi\epsilon_0} \int_0^{2\pi} \frac{d\theta}{R_{\text{obj}}}$$

$\Phi_2$ :

$$\Phi_2(x, y, z) = -\lambda q \int_0^{2\pi} \frac{d\theta}{R_2}$$

$$\text{Now } R_2 = [(x - d)^2 + (y + b)^2 + z^2 - 2a((x - d) \cos \theta + z \sin \theta)]^{1/2}$$

$$\Phi_3(x, y, z) = \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{d\theta}{R_3}$$

$$\text{όπου } R_3 = \left[ (x+d)^2 + (y+h)^2 + z^2 + a^2 - 2a((x+d)\cos\theta + z\sin\theta) \right]^{1/2}$$

$$\text{Τέλος } \Phi_{3y}(x, y, z) = -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{d\theta}{R_3}$$

$$\text{όπου } R_3 = \left[ (x+d)^2 + (y-h)^2 + z^2 + a^2 - 2a((x+d)\cos\theta + z\sin\theta) \right]^{1/2}$$

Ηροδανωση για  $x < 0, y < 0, \phi = 0$

$$b) E_{\text{tot}} = E_1 + E_2 + E_3 + E_4$$

$$E_1 = -\frac{d\phi_1}{dx} - \frac{d\phi_1}{dy} - \frac{d\phi_1}{dz}$$

$$\rightarrow -\frac{d\phi_1}{dx} = -\frac{d}{dx} \left[ \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{d\theta}{R_1} \right] =$$

$$-\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \cdot \frac{d}{dx} \left( \frac{1}{R_1} \right) =$$

$$-\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \frac{1}{R_1^3} \left( -\frac{1}{2} \right) \cdot \frac{d}{dx} \left[ (x-d - a\cos\theta \cdot a)^2 \right]$$

$$= \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{x-d - a\cos\theta}{R_1^3} d\theta$$

$$\rightarrow -\frac{d\phi_1}{dy} = -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \frac{d}{dy} \left( \frac{1}{R_1} \right)$$

$$= -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \frac{1}{R_1^3} \left( -\frac{1}{2} \right) \cdot \frac{d}{dy} \left[ (y-h)^2 \right]$$

$$= \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{y-h}{R_1^3} d\theta$$

$$\rightarrow \frac{-d\phi_1}{dz} = -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \frac{d}{dz} \left( \frac{1}{R_1} \right)$$

$$= -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \left( -\frac{1}{2} \right) \cdot \frac{d}{dz} [(z - a\sin\theta)^2]$$

$$= \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} d\theta \frac{2 - a\sin\theta}{R_1^3}$$

$$\vec{E}_1 = \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{(x - d - a\cos\theta)\hat{i}_x + (y - h)\hat{i}_y + (z - a\sin\theta)\hat{i}_z}{R_1^3} d\theta$$

Obviously

$$\vec{E}_2 = \frac{-\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{(x - d - a\cos\theta)\hat{i}_x + (y + h)\hat{i}_y + (z - a\sin\theta)\hat{i}_z}{R_2^3} d\theta$$

$$\vec{E}_3 = \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{(x + d - a\cos\theta)\hat{i}_x + (y - h)\hat{i}_y + (z - a\sin\theta)\hat{i}_z}{R_3^3} d\theta$$

$$\vec{E}_4 = -\frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{(x + d - a\cos\theta)\hat{i}_x + (y + h)\hat{i}_y + (z - a\sin\theta)\hat{i}_z}{R_4^3} d\theta$$

$$\text{d}) \quad \vec{E}(x, z) = \epsilon_0 \hat{y} \left[ \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \right] = \epsilon_0 \hat{y} \left[ \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \right]$$

$$= \epsilon_0 \left[ \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{y - h}{R_1^3} d\theta - \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{y + h}{R_2^3} d\theta + \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{y + h}{R_3^3} d\theta \right]$$

$$\Rightarrow \frac{\lambda a}{4\pi\epsilon_0} \int_0^{2\pi} \frac{y - h}{R_4^3} d\theta \quad \stackrel{y=0}{\Rightarrow}$$

$$\vec{E}(x, z) = \frac{\lambda}{4\pi} \left[ \int_0^{2\pi} \left( -\frac{h}{R_1^3} - \frac{h}{R_2^3} + \frac{h}{R_3^3} + \frac{h}{R_4^3} \right) d\theta \right]$$

Προβληματικό  
 $\vec{E} = \vec{0}$  για  $x < 0, y < 0$

Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

```
askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisi6e.m +
```

```
1 function [X, Y] = askisi7Fol
2 % Define computational parameters
3 a=0.1;
4 d=1;
5 h=1;
6 white = [1 1 1];
7 black = [0 0 0];
8 red = [192 0 0]/255;
9 green = [119 172 48]/255;
10 darker_green = [ 26 108 28]/255;
11 purple = [125 46 143]/255;
12 magenta_light = [255 102 255]/255;
13 nfig=1;
14 Npoints=133;
15 xmin = 0;
16 xmax = 2;
17 ymin = 0;
18 ymax = 2;
19 xx = xmin:(xmax-xmin)/Npoints:xmax;
20 yy = ymin:(ymax-ymin)/Npoints:ymax;
21 [X, Y] = meshgrid(xx,yy);
22 % point by point calculation numerical integration
23 tic
24 for ix = 1:length(xx)
25     for iy = 1:length(yy)
26         x0 = X(ix,iy);
```

## Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

```
27 -     y0 = Y(ix,iy);
28 -     FA(ix,iy) = integral(@(thita)potential(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
29 -     end
30 -   end
31 -   disp('Point-by-point numerical integration time:')
32 -   toc
33 -   FB=0;
34 -   % point by point calculation numerical integration
35 -   tic
36 -   for ix = 1:length(xx)
37 -     for iy = 1:length(yy)
38 -       x0 = X(ix,iy);
39 -       y0 = Y(ix,iy);
40 -       ExA(ix,iy) = integral(@(thita)Ex_comp(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
41 -     end
42 -   end
43 -   disp('Point-by-point array_X numerical integration time:')
44 -   toc
45 -   tic
46 -   for ix = 1:length(xx)
47 -     for iy = 1:length(yy)
48 -       x0 = X(ix,iy);
49 -       y0 = Y(ix,iy);
50 -       EyA(ix,iy) = integral(@(thita)Ey_comp(thita,x0,y0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);
51 -     end
52 -   end
```

Command Window

Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

askisi7Fol.m    askisi77G.m    askisi6.m    ask6cd.m    askisi6e.m    +

```
53 -     disp('Point-by-point array_Y numerical integration time:')
54 -     toc
55 -     EyB=0;
56 -     ExB=0;
57 -     % Logical Variables
58 -     CircleA=(X>=0) & (Y>=0);
59 -     CircleB=(X<0) | (Y<0);
60 -     FC=FA.*CircleA;
61 -     FD=FB.*CircleB;
62 -     F=FC+FD;
63 -     ExC=ExA.*CircleA;
64 -     EyC=EyA.*CircleA;
65 -     ExD=ExB.*CircleB;
66 -     EyD=EyB.*CircleB;
67 -     Ex=ExC+ExD;
68 -     Ey=EyC+EyD;
69 -     % Plot Potential
70 -     xr(1) = 0.9;
71 -     yr(1) = 1;
72 -     xr(2) = 1.1;
73 -     yr(2) = 1;
74 -     Phi_max = max(max(F));
75 -     Phi_min = min(min(F));
76 -     Pm = 0.8*Phi_max;
77 -     figure(nfig);
78 -     hold off
```

Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

```
askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisibe.m +
```

```
79 - surface(X,Y,F), shading interp
80 - hold on
81 - pr = plot(xr,yr,'Linewidth',2);
82 - set(pr, 'Color', red);
83 - set(pr,'ZData',Phi_max+1+zeros(size(xr)))
84 - % Move the line charge to Z = Phi_max + 1
85 - set(gca,'FontSize',12,'FontWeight','bold')
86 - xlabel('x','FontSize',12,'FontWeight','bold')
87 - ylabel('y','FontSize',12,'FontWeight','bold')
88 - title('Normalized Potential','FontSize',10,'FontWeight','bold','Color','b')
89 - axis equal
90 - caxis([0 Pm])
91 - colorbar
92 -
93 - % Plot Lines
94 - %
95 - xr(1) = 0.9;
96 - yr(1) = 1;
97 - xr(2) = 1.1;
98 - yr(2) = 1;
99 - nfig = nfig + 1;
100 - figure(nfig);
101 - hold off
102 - cont = [ 0.01, 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7.5];
103 - [CS,H] = contour(X,Y,F,cont,'Linewidth',1,'Color','b');
104 - clabel(CS,H,cont);
```

Command Window

Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisibe.m +

```
105 - hold on
106 - pr = plot(xr,yr,'Linewidth',2);
107 - set(pr, 'Color', red);
108 - set(gca, 'FontSize',12,'FontWeight','bold')
109 - xlabel('x','FontSize',12,'FontWeight','bold')
110 - ylabel('y','FontSize',12,'FontWeight','bold')
111 - title('Normalized Lines', 'FontSize',10,'FontWeight','bold','Color','b')
112 - axis equal
113 - grid on
114 % Plot Electric Field (streamslice)
115 - xr(1) = 0.9;
116 - yr(1) = 1;
117 - xr(2) = 1.1;
118 - yr(2) = 1;
119 - nfig = nfig + 1;
120 - figure(nfig)
121 - hold off
122 - LL = sqrt(((Ex).^2 + (Ey).^2));
123 - quiver(X,Y,Ex./LL,Ey./LL,0.5)
124 - hold on
125 - hs = streamslice(X,Y,Ex,Ey,2);
126 - set(hs, 'Color','m','Linewidth',1.0);
127 - cont = [ 0.01, 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7.5];
128 - contour(X,Y,F,cont,'Linewidth',1,'Color','b');
129 - pr = plot(xr,yr,'Linewidth',2);
130 - set(pr, 'Color', red);
```

Command Window

Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

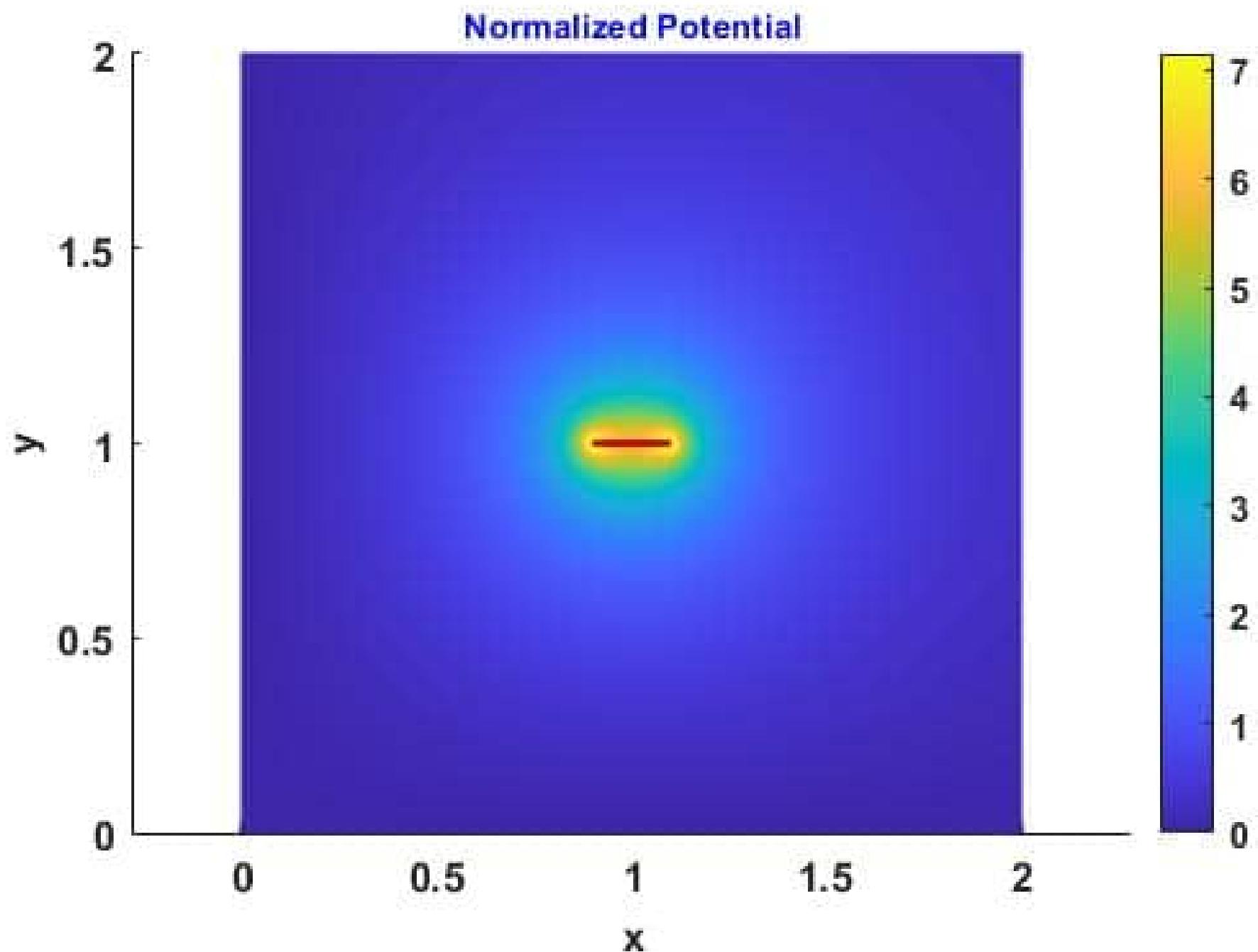
askisi7Fol.m askisi77G.m askisi6.m ask6cd.m askisibe.m +

```
131 - set(gca,'Fontsize',12,'Fontweight','bold')
132 - xlabel('x','Fontsize',12,'FontWeight','bold')
133 - ylabel('y','Fontsize',12,'FontWeight','bold')
134 - title('Normalized Electric Field','Fontsize',10,'FontWeight','bold','Color','b')
135 - axis equal
136 - grid on
137 % Functions
138 function Phi = potential(thita,x0,y0)
139 - a=0.1;
140 - d=1;
141 - h=1;
142 - PhiA= a./sqrt((x0-d).^2+(y0-h).^2+a.^2-2*a.* (x0-d).*cos(thita));
143 - PhiB=(-a)./sqrt((x0-d).^2+(y0+h).^2+a.^2-2*a.* (x0-d).*cos(thita));
144 - PhiC= a./sqrt((x0+d).^2+(y0+h).^2+a.^2-2*a.* (x0+d).*cos(thita));
145 - PhiD= (-a)./sqrt((x0+d).^2+(y0-h).^2+a.^2-2*a.* (x0+d).*cos(thita));
146 - Phi=PhiA+PhiB+PhiC+PhiD;
147 %
148 function dEx = Ex_comp (thita,x0,y0)
149 - a=0.1;
150 - d=1;
151 - h=1;
152 - R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
153 - R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
154 - R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
155 - R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
156 - dEx compA=(a*d).* (x0-d-a.*cos(thita))./R1.^3;
```

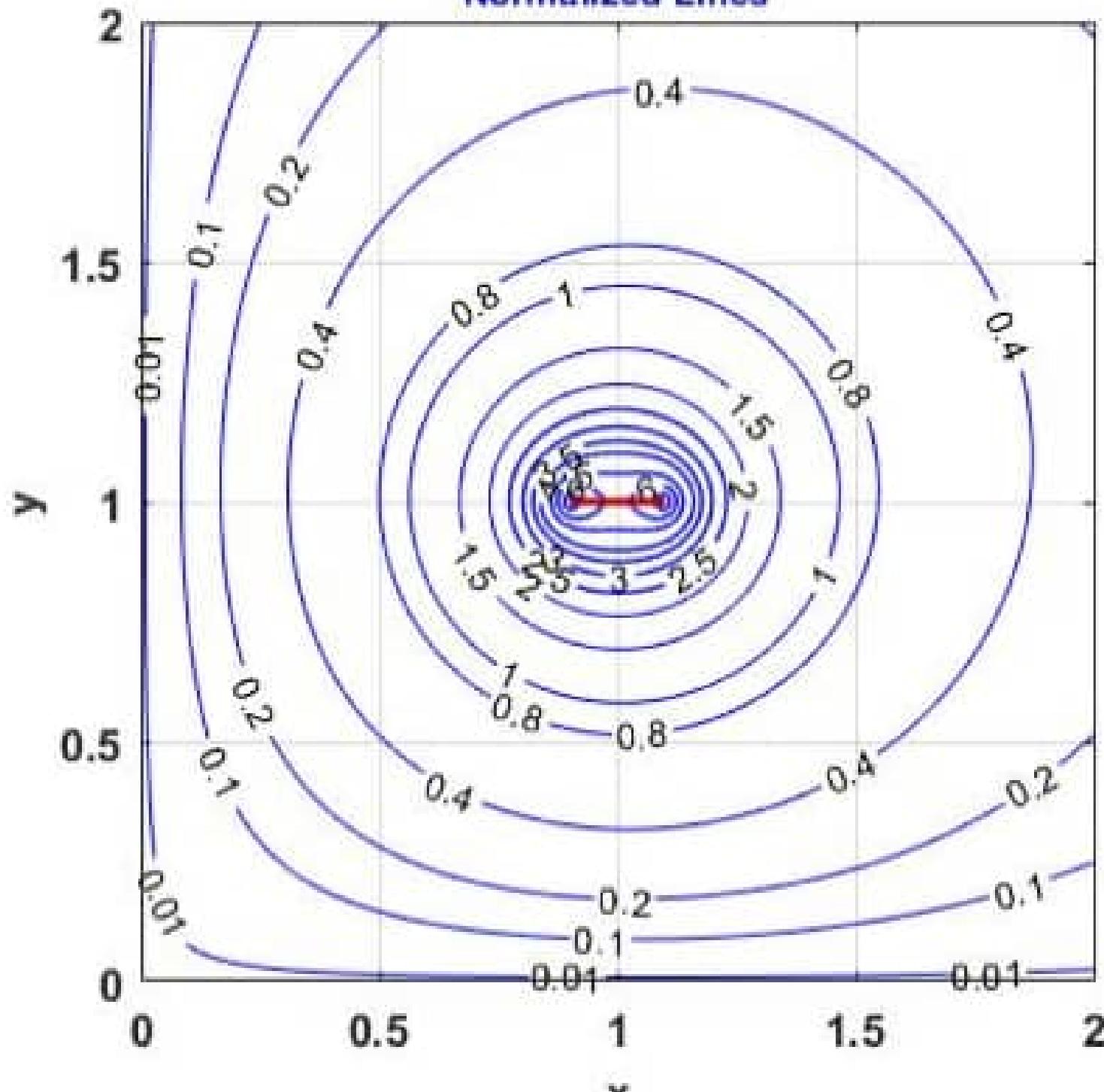
Editor - D:\πολυτεχνείο\matlab\askisi7Fol.m

```
askisi7Fol.m × askisi77G.m × askisi6.m × ask6cd.m × askisibe.m × +
```

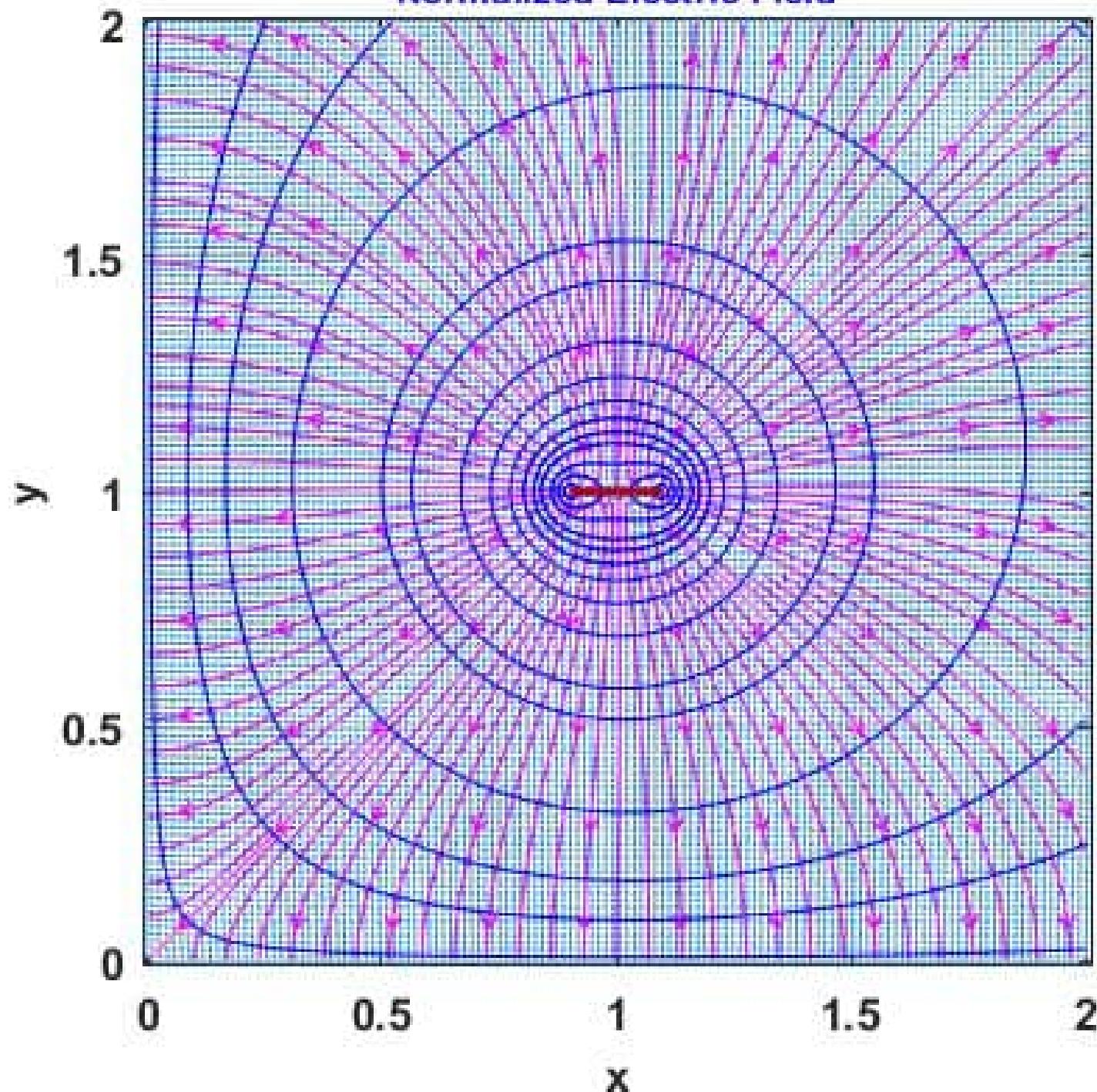
```
149 - a=0.1;
150 - d=1;
151 - h=1;
152 - R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
153 - R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
154 - R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
155 - R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
156 - dEx_compA=(a*d).* (x0-d-a.*cos(thita))./R1.^3;
157 - dEx_compB=(( -a)*d).* (x0-d-a.*cos(thita))./R2.^3;
158 - dEx_compC=(a*d).* (x0+d-a.*cos(thita))./R3.^3;
159 - dEx_compD=(( -a)*d).* (x0+d-a.*cos(thita))./R4.^3;
160 - dEx=dEx_compA+dEx_compB+dEx_compC+dEx_compD;
161 -
162 [- function dEy = Ey_comp (thita,x0,y0)
163 - a=0.1;
164 - d=1;
165 - h=1;
166 - R1=sqrt((x0-d).^2+(y0-h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
167 - R2=sqrt((x0-d).^2+(y0+h).^2+a.^2-(2*a).* (x0-d).*cos(thita));
168 - R3=sqrt((x0+d).^2+(y0+h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
169 - R4=sqrt((x0+d).^2+(y0-h).^2+a.^2-(2*a).* (x0+d).*cos(thita));
170 - dEy_compA=(a*d).* (y0-h)./R1.^3;
171 - dEy_compB=(( -a)*d).* (y0+h)./R2.^3;
172 - dEy_compC=(a*d).* (y0+h)./R3.^3;
173 - dEy_compD=(( -a)*d).* (y0-h)./R4.^3;
174 - dEy=dEy_compA+dEy_compB+dEy_compC+dEy_compD;
```



### Normalized Lines



### Normalized Electric Field



```
askisi7Fol.m X askisi77G.m X askisi6.m X ask6cd.m X askisi6e.m X +  
1 function [X, Z] = askisi7G  
2 % Define computational parameters  
3 nfig=1;  
4 Npoints=133;  
5 xmin = 0;  
6 xmax = +2;  
7 zmin = -2;  
8 zmax = 2;  
9 xx = xmin:(xmax-xmin)/Npoints:xmax;  
10 zz = zmin:(zmax-zmin)/Npoints:zmax;  
11 [X, Z] = meshgrid(xx,zz);  
12 % point by point calculation numerical integration  
13 tic  
14 for ix = 1:length(xx)  
15 for iz = 1:length(zz)  
16 x0 = X(ix,iz);  
17 z0 = Z(ix,iz);  
18 SA(ix,iz) = integral(@(thita) charge(thita,x0,z0),0,2*pi,'RelTol',1e-12,'AbsTol',1e-12);  
19 end  
20 end  
21 toc  
22 SB=0;  
23 % -----  
24 % Logical Variables  
25 % -----  
26 CircleA=(X>=0) & (Z>=-2) & (X<=2) & (Z<=2);
```

Editor - D:\πολυτεχνείο\matlab\askisi77G.m

```
askisi7Fol.m X askisi77G.m X askisi6.m X ask6cd.m X askisi6e.m X +
```

```
27 - CircleB=(X<0) | (Z<-2) | (X>2) | (Z>2);
28 - SC=SA.*CircleA;
29 - SD=SB.*CircleB;
30 - S=SC+SD;
31 * % Plot Lines Of Charge
32 - nfig = nfig + 1;
33 - figure(nfig);
34 - hold off
35 - [CS,H] = contour(X,Z,S,'Linewidth',1,'Color','b');
36 - clabel(CS,H)
37 - hold on
38 - set(gca,'FontSize',12,'FontWeight','bold')
39 - xlabel('x','FontSize',12,'FontWeight','bold')
40 - ylabel('z','FontSize',12,'FontWeight','bold')
41 - title('Normalized Charge','FontSize',10,'FontWeight','bold','Color','b')
42 - axis equal
43 - grid on
44 * % Functions
45 - function S = charge(theta,x0,z0)
46 - a=0.1;
47 - d=1;
48 - h=1;
49 - e0=8.85418782e-12;
50 - R1=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(theta)+z0.*sin(theta)));
51 - R2=sqrt((x0-d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(theta)+z0.*sin(theta)));
52 - R3=sqrt((x0+d).^2+ h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(theta)+z0.*sin(theta))));
```

Command Window

Editor - D:\πολυτεχνείο\matlab\askisi77G.m

```
askisi7Fol.m X askisi77G.m X askisi6.m X ask6cd.m X askisibe.m X +
```

```
33 - figure(nfig);
34 - hold off
35 - [CS,H] = contour(X,Z,S,'Linewidth',1,'Color','b');
36 - clabel(CS,H)
37 - hold on
38 - set(gca,'Fontsize',12,'Fontweight','bold')
39 - xlabel('x','Fontsize',12,'FontWeight','bold')
40 - ylabel('z','Fontsize',12,'FontWeight','bold')
41 - title('Normalized Charge','Fontsize',10,'FontWeight','bold','Color','b')
42 - axis equal
43 - grid on
44 % Functions
45 function S = charge(thita,x0,z0)
46 a=0.1;
47 d=1;
48 h=1;
49 e0=8.85418782e-12;
50 R1=sqrt((x0-d).^2+h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
51 R2=sqrt((x0-d).^2+h.^2+z0.^2+a.^2-(2*a).*((x0-d).*cos(thita)+z0.*sin(thita)));
52 R3=sqrt((x0+d).^2+h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(thita)+z0.*sin(thita)));
53 R4=sqrt((x0+d).^2+h.^2+z0.^2+a.^2-(2*a).*((x0+d).*cos(thita)+z0.*sin(thita)));
54 SA= ((-h))./R1.^3;
55 SB= ((-h))./R2.^3;
56 SC= (h))./R3.^3;
57 SD= (h))./R4.^3;
58 S=SA+SB+SC+SD
```

Command Window

