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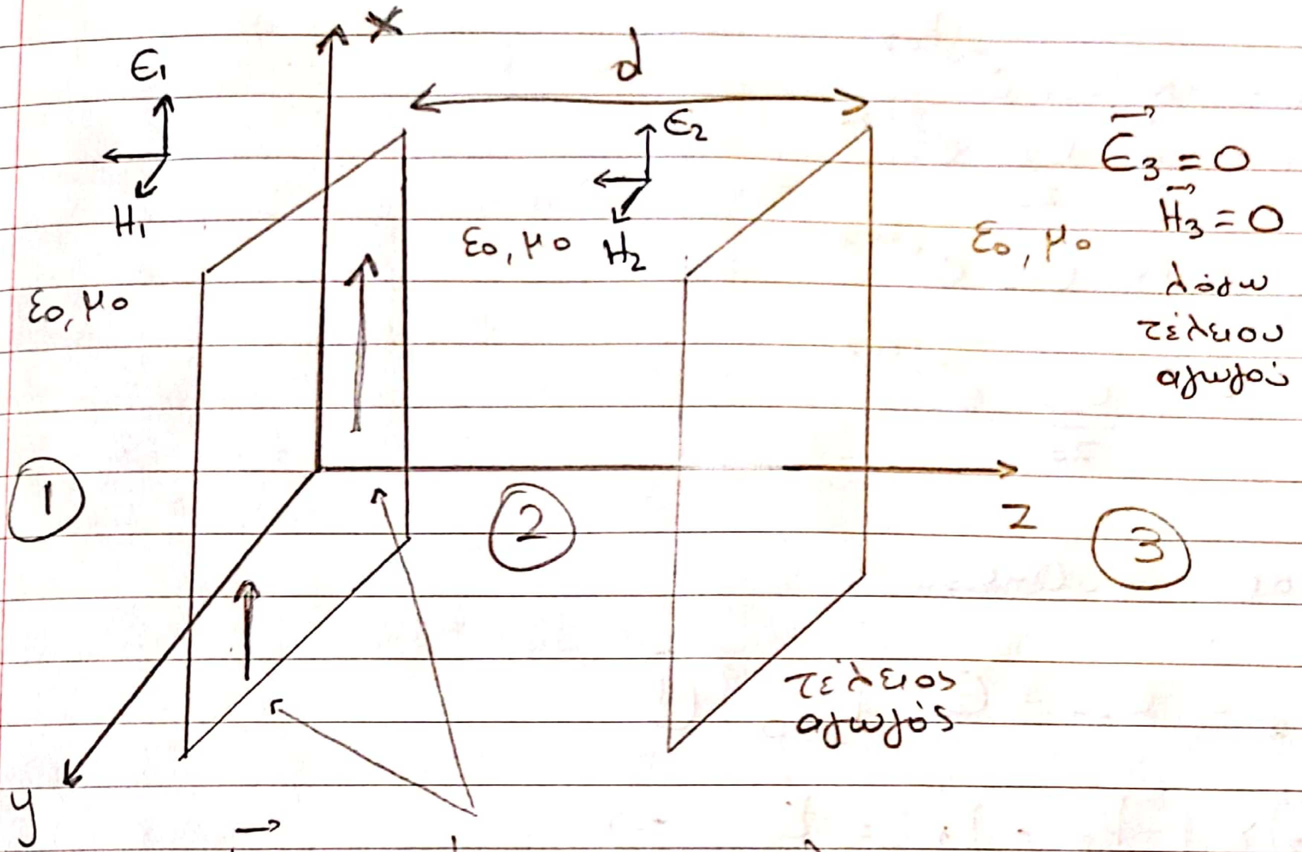
Άσκηση 8



$$\vec{K} = \hat{i}_x K_0 \cos(\omega t + \phi)$$

α) $$\vec{K} = \hat{i}_x K_0 e^{j\phi}$$

Για την περιοχή 1:

$$\vec{E}_1 = \hat{i}_x E_1 e^{jk_0 z}$$

$$\hat{K}_1 = -\hat{i}_z$$

Άρα

$$\vec{H}_1 = \frac{1}{Z_0}\left(\hat{K}_1 \times \vec{E}_1\right) = \frac{1}{Z_0}\begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ 0 & 0 & -1 \\ E_1 e^{k_0 j z} & 0 & 0 \end{vmatrix} =$$

$$\frac{1}{Z_0} E_1 e^{jk_0 z}\left(-\hat{i}_y\right) = -\hat{i}_y \frac{E_1}{Z_0} e^{jk_0 z}$$

Για την περιοχή 2:

$$\vec{E}_{2+} = \hat{i}_x E_{2+} e^{-jk_0 z}$$

$$\vec{H}_{2+} = \hat{i}_y \frac{E_{2+}}{Z_0} e^{jk_0 z}$$

$$\vec{E}_{2-} = \hat{i}_x E_{2-} e^{jk_0 z}$$

$$\vec{H}_{2-} = -\hat{i}_y \frac{E_{2-}}{Z_0} e^{jk_0 z}$$

Οριακές συνθήκες:

$$E_1 = E_{2+} + E_{2-} \Big|_{z=0} \quad (1)$$

$$(\hat{i}_n = \hat{i}_z) \times [\vec{H}_2 - \vec{H}_1] = \vec{K} \Rightarrow$$

$$-\hat{i}_x \left[\frac{E_{2+}}{Z_0} - \frac{E_{2-}}{Z_0} - \left(-\frac{E_1}{Z_0} \right) \right] = \hat{i}_x K_0 e^{j\phi} \Rightarrow$$

$$-E_1 - E_{2+} + E_{2-} = Z_0 K_0 e^{j\phi} \quad (2)$$

Για $x = d$:

$$E_{2+} e^{-jk_0 d} + E_{2-} e^{jk_0 d} = 0 \qquad \text{αφού } E_3 = 0$$

$$\Rightarrow \quad E_{2+} = -E_{2-} e^{2jk_0 d} \quad (3)$$

Απο (1), (2), (3) $\Rightarrow$

$$\begin{cases} E_{2-} = \frac{1}{2} z_0 K_0 e^{j\phi} e^{-j2k_0 d} \\ E_{2+} = -\frac{1}{2} z_0 K_0 e^{j\phi} \\ E_1 = -\frac{1}{2} z_0 K_0 e^{j\phi}\left(1 - e^{-j2k_0 d}\right) \end{cases}$$

Συνεπώς:

$$\vec{E_1} = E_1 \cdot \hat{i_x}\, e^{jk_0 z} \Rightarrow$$

$$\vec{E_1} = -\frac{1}{2} z_0 K_0 e^{j\phi}\left(1 - e^{-j2k_0 d}\right) e^{jk_0 z} \cdot \hat{i_x}$$

$$\vec{E_2} = \left[E_{2+} e^{-jk_0 z} + E_{2-} e^{jk_0 z}\right] \hat{i_x} \Rightarrow$$

$$\vec{E_2} = \frac{1}{2} z_0 K_0 e^{j\phi}\left[-e^{-jk_0 z} + e^{-j2k_0 d + k_0 z}\right] \hat{i_x}$$

Και

$$\vec{H_1} = -\frac{E_1}{z_0} e^{jk_0 z}\, \hat{i_y}$$ με E_1 το παραπάνω

$$\vec{H_2} = \left[\frac{E_{2+} - E_{2-}}{z_0}\right] e^{jk_0 z}\, \hat{i_y}$$ με E_{2+}, E_{2-} γνωστά

Περιοχή 3:

Λόγω τέλειου αγωγού $\vec{H_3} = 0$, $\vec{E_3} = 0$

β) $\vec{K} = (\hat{i_z} = \hat{i_n}) \times (\cancel{\vec{H_3}}^{0} - \vec{H_2})$

$\Rightarrow \vec{K} = \hat{i_z} \times (-\vec{H_2}) \Rightarrow$

$\vec{K} = \hat{i_x} \cdot \vec{H_2} = -\hat{i_x}\left[\left(-\frac{E_{2+} e^{-jk_od}}{Z_o}\right) + \frac{E_{2-} e^{jk_od}}{Z_o}\right]$

$= \hat{i_x} \frac{1}{Z_o}\left[-E_{2-} e^{jk_od} - E_{2-} e^{jk_od}\right]$

$= \frac{\hat{i_x}}{Z_o}\left(-2 E_{2-} e^{jk_od}\right)$

$= -\frac{\hat{i_x}}{\cancel{Z_o}} \cancel{2} \cdot \frac{1}{\cancel{2}} \cancel{Z_o} k_o e^{j\phi} e^{-j\cancel{2}k_od} \cancel{e^{jk_od}}$

$= -\hat{i_x} k_o e^{j\phi} e^{-jk_od}$

$= -\hat{i_x} k_o e^{j(\phi - k_od)}$

Άρα $\vec{K} = -\hat{i_x} k_o \cos(\omega t - k_od + \phi)$

γ) Στην περιοχή 1 το διάνυσμα Poynting είναι:

$\vec{P_1} = -\frac{1}{2Z_o}|\vec{E_1}|^2 \hat{i_z} = -\frac{1}{2\cancel{Z_o}} \frac{1}{\cancel{4}} Z_o^{\cancel{2}} k_o^2 \cancel{4} \sin^2(k_od) \hat{i_z}$

$= -\frac{1}{2} Z_o k_o^2 \sin^2(k_od) \hat{i_z}$

Για να είναι μέγιστο πρέπει $\sin^2(k_od) = max$

$\Rightarrow \sin(k_od) = 1$

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Άρα $k_o d = (2n+1)\frac{\pi}{2}$ για $n=0,1\ldots$

Άρα $d = (2n+1)\frac{\pi}{2k_o} = (2n+1)\frac{\lambda_o}{4}$

Αντίστοιχα ελάχιστα πρέπει $\sin^2(k_o d) = \min$

$\Rightarrow \sin(k_o d) = 0 \Rightarrow$

$k_o d = n\pi \rightarrow$

$d = n\frac{\pi}{k_o} \Rightarrow d = n\frac{\lambda_o}{2}$ για $n=0,1\ldots$

Στην περιοχή 2 το διάνυσμα Poynting είναι

$$\vec{P_2} = \frac{1}{2} R_e[\vec{E_2} \times \vec{H_2^*}] =$$

$$\frac{1}{2} Re\left[\hat{i_x} \frac{1}{2} z_o k_o e^{j\phi} e^{-jk_o d} 2j\sin(k_o(z-d)) \times \hat{i_y}\frac{1}{2}k_o e^{-j\phi} e^{jk_o d} 2\cos(k_o(z-d))\right]$$

$$= \frac{1}{2} \hat{i_z} \frac{1}{4} k_o^2 z_o 4 Re\left[j\sin(k_o(z-d))\cos(k_o(z-d))\right] = 0$$

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```
%aksisi8
K0=1;
e0=8.8541878128*(10^(-12));
m0=4*pi*(10.^(-7));
z0=sqrt(m0./e0);
%θεωρω K0*λ=2π
k0=2.*pi.*sqrt(e0.*m0);
fi=pi/4;
el=(3*10.^8);
d1=0.125;
d2=0.25;
d3=0.5;
%d1=0.125λ------------------------------------------------------------------------
%region1 Ex
z1=linspace(-2,0,100);
E11=-0.5.*z0.*K0.*exp(j*fi).*(1-exp((-j)*2*2*pi*d1)).*exp(j*2*pi*z1);
E11r=real(E11);
figure(1)
plot(z1,E11r,'b');
hold on
plot([0 0],[-300 300], 'g');
%region2
z2=linspace(0,d1,100);
E12= (-z0./2).*k0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z2) - exp(1i.*2.*pi.*z2).*exp(-2i.*2.*pi.*d1));
E12r=real(E12);
plot(z2,E12r,'b');
```

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```
hold on
plot([d1 d1],[-300 300], 'g');
%region3 Ex=0
plot([d1 0.5], [0 0], 'b');
xlabel('z/λ');
ylabel('E_(z.t=0) [V*m^(-1)]');
title('Electric current density for d=0.125λ');
hold off
%region1 Hy
H11=-(1./z0).*(z0./2).*K0.*exp(1i.*fi).*(exp(-2i.*2.*pi*d1)-1).*exp(1i.*2.*pi.*z1);
H11r=real(H11);
figure(2)
plot(z1, H11r, 'r');
hold on
plot([0 0], [-1.2 1.2], 'g');
%region2
H12 = (1./z0).*(-z0./2).*K0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z2) + exp(1i.*2.*pi.*z2).*exp(-2i.*2.*pi.*d1));
H12r = real(H12);
plot(z2, H12r, 'r');
hold on
plot([d1 d1], [-1.2 1.2], 'g');
%region3 Ex=0
plot([d1 0.5], [0 0], 'r');
xlabel('z/λ')
ylabel('H_y (z,t=0) [A*m^(-1)]');
title('Magnetic current density for d=0.125λ')
```

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```
hold off
%d2=0.25λ----------------------------------------------------------------------
%region1 Ex
z1=linspace(-2,0,100);
E21=(z0./2).*K0.*exp(1i.*fi).*(exp(-2i.*2.*pi*d2)-1).*exp(1i.*2.*pi.*z1);
E21r=real(E21);
figure(3)
plot(z1,E21r,'b');
hold on
plot([0 0],[-400 400], 'g');
%region2
z3=linspace(0,d2,100);
E22=(-z0./2).*K0.*exp(1i.*fi).*(exp(-i.*2.*pi.*z3) - exp(i.*2.*pi.*z3).*exp(-2i.*2.*pi.*d2));
E22r=real(E22);
plot(z3,E22r,'b');
hold on
plot([d2 d2],[-400 400], 'g');
%region3 Ex=0
plot([d2 0.5], [0 0], 'b');
xlabel('z/λ');
ylabel('E_(z.t=0) [V*m^(-1)]');
title('Electric current density for d=0.125λ');
hold off
%region1 Hy
H21 = -(1./z0).*(z0./2).*K0.*exp(1i.*fi).*(exp(-2i.*2.*pi*d2)-1).*exp(1i.*2.*pi.*z1);
H21r = real(H21);
```

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```
figure(4)
plot(z1, H21r, 'r');
hold on
plot([0 0], [-1.2 1.2], 'g');
%region2
H22 = (1./z0).*(-z0./2).*K0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z3) + exp(1i.*2.*pi.*z3).*exp(-2i.*2.*pi.*d2));
H22r = real(H22);
plot(z3, H22r, 'r');
hold on
plot([d2 d2], [-1.2 1.2], 'g');
%region3 Ex=0
plot([d2 0.5], [0 0], 'r');
xlabel('z/λ')
ylabel('H_y (z,t=0) [A*m^(-1)]');
title('Magnetic current density for d=0.25λ')
hold off
%d3=0.5λ-----------------------------------------------------------------------
%region1 Ex
Ex31=(z0./2).*K0.*exp(1i.*fi).*(exp(-2i.*2.*pi*d3)-1).*exp(1i.*2.*pi.*z1);
Ex31r=real(Ex31);
figure(5)
plot(z1, Ex31r, 'b');
hold on
plot([0 0], [-250 250], 'g');
%region2
z4=linspace(0,d3,100);
```

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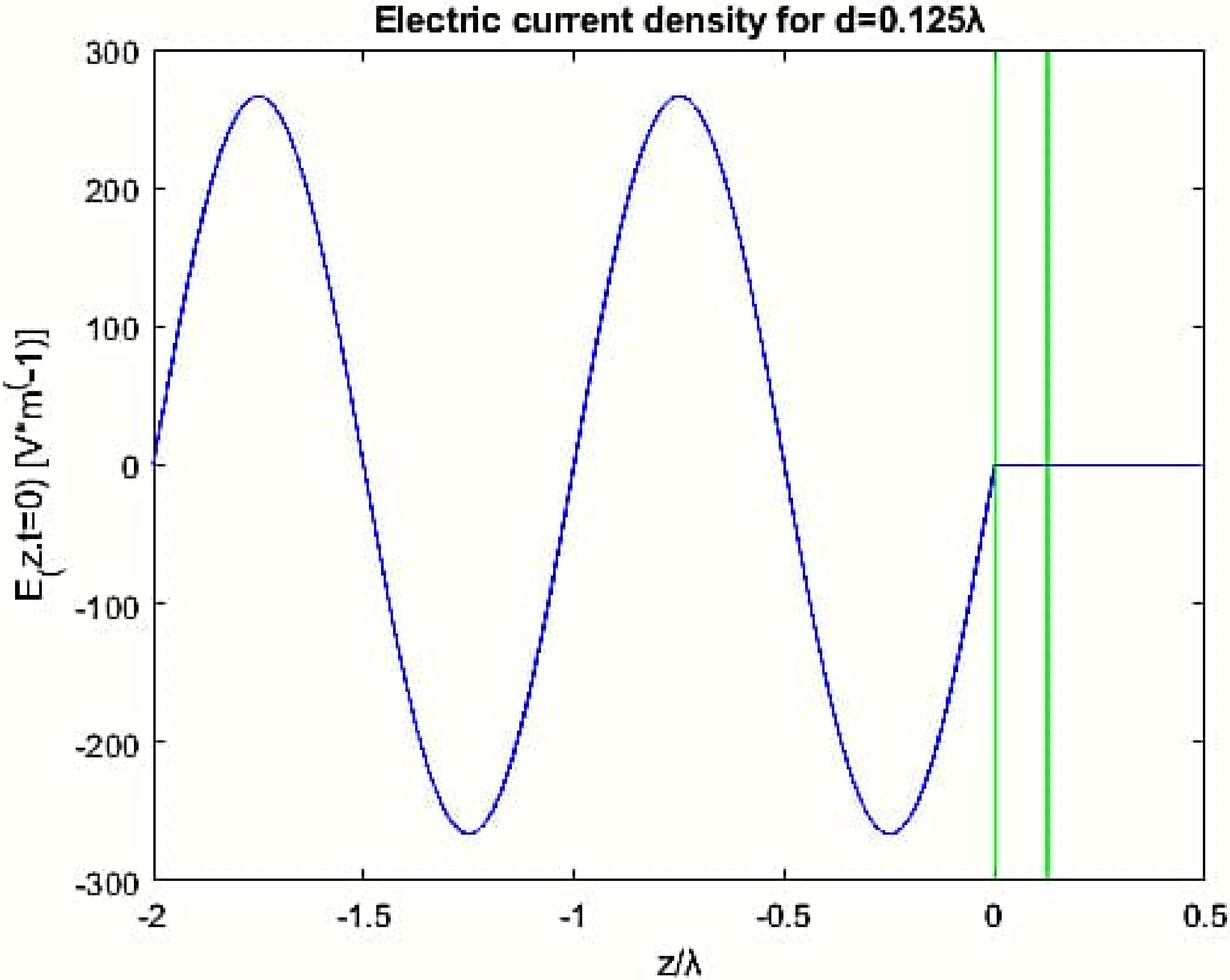
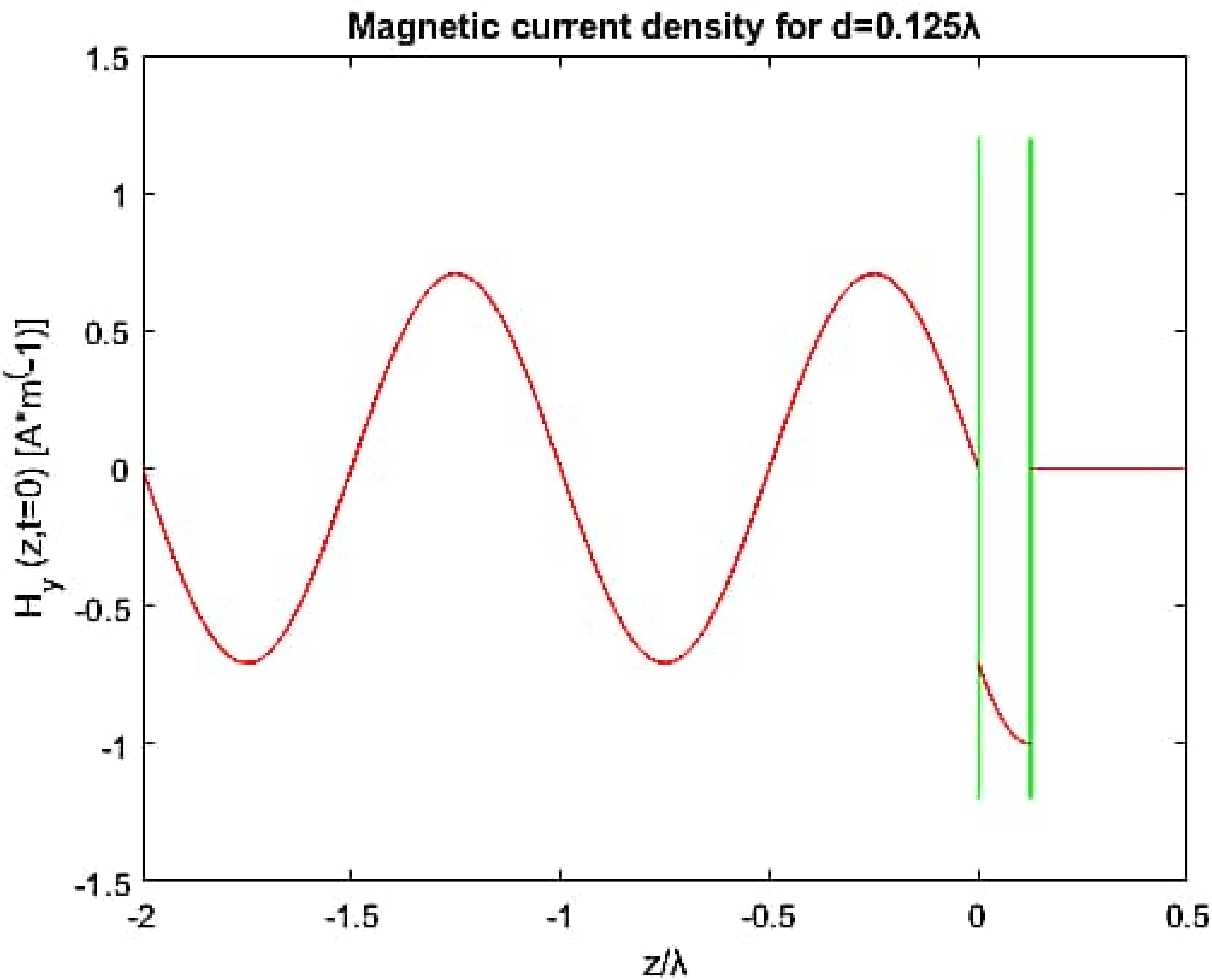
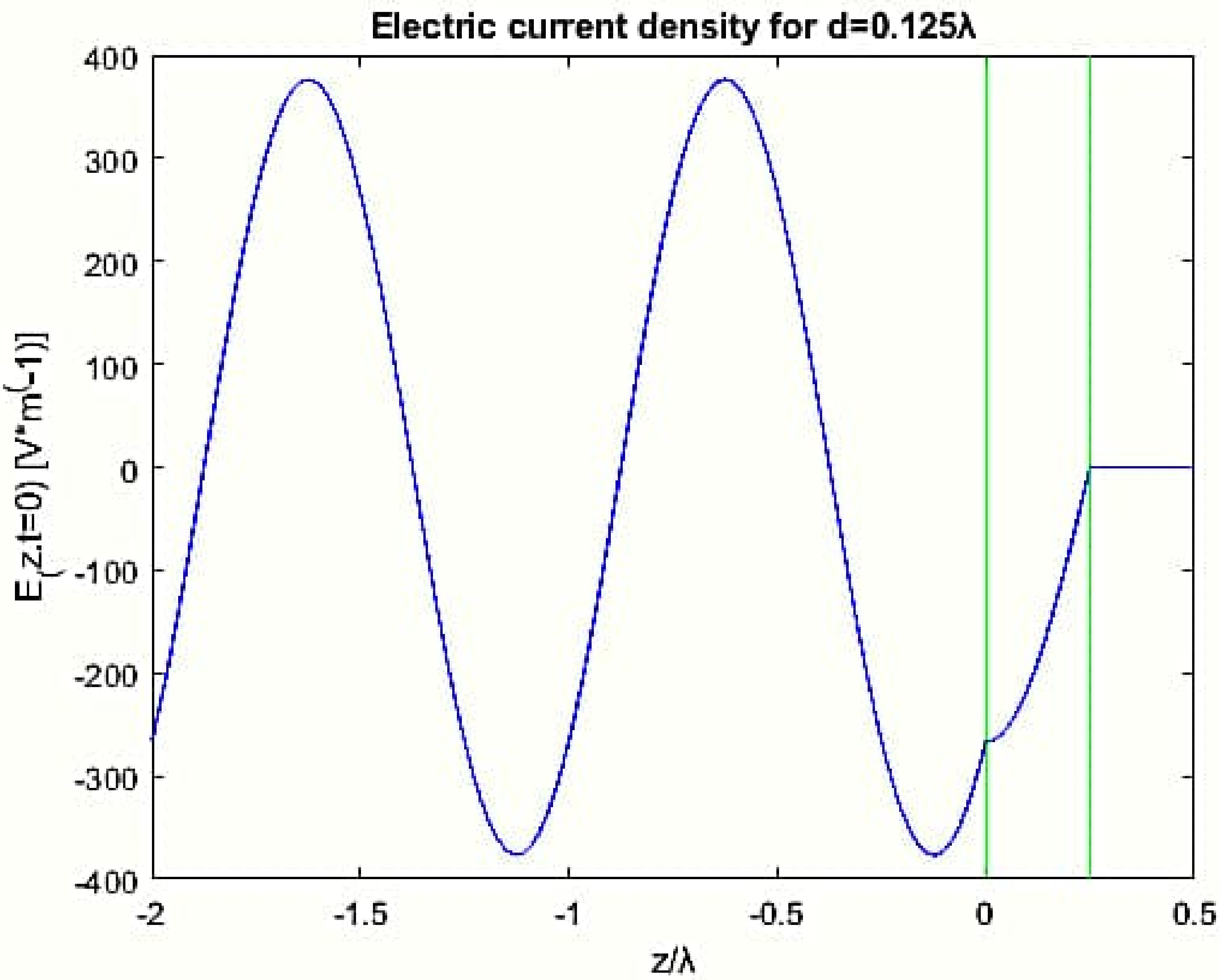
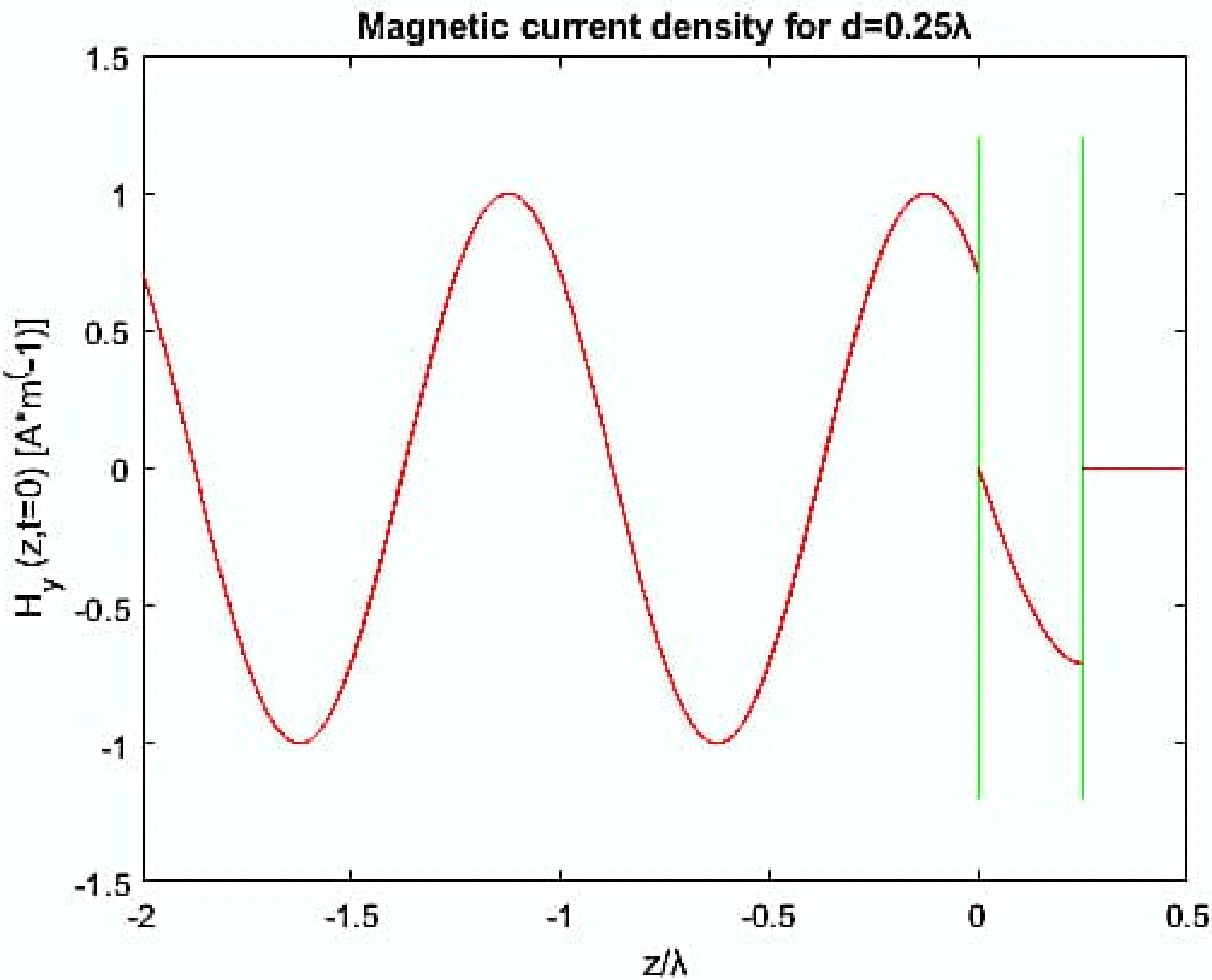
```
Ex32=(-z0./2).*K0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z4)-exp(1i.*2.*pi.*z4).*exp(-2i.*2.*pi.*d3));
Ex32r=real(Ex32);
plot(z4, Ex32r, 'b');
hold on
plot([d3 d3], [-250 250], 'g');
%regipon3 Ex = 0
plot([d3 1], [0 0], 'b');
xlabel('z/λ')
ylabel('E_x (z,t=0) [V*m^(-1)]');
title('Electric current density for d = 0.5λ')
hold off
%region1 Hy
H31=-(1./z0).*(z0./2).*K0.*exp(1i.*fi).*(exp(-2i.*2.*pi*d3)-1).*exp(1i.*2.*pi.*z1);
H31r=real(H31);
figure(6)
plot(z1, H31r, 'r');
hold on
plot([0 0], [-1 1], 'g');
%region2
H32=(1./z0).*(-z0./2).*K0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z4) + exp(1i.*2.*pi.*z4).*exp(-2i.*2.*pi.*d3));
H32r=real(H32);
plot(z4, H32r, 'r');
hold on
plot([d3 d3], [-1 1], 'g');
%region3 Ex=0
plot([d3 1], [0 0], 'r');
```

```
%region2
H32=(1./z0).*(-z0./2).*K0.*exp(1i.*fi).*(exp(-1i.*2.*pi.*z4) + exp(1i.*2.*pi.*z4).*exp(-2i.*2.*pi.*d3));
H32r=real(H32);
plot(z4, H32r, 'r');
hold on
plot([d3 d3], [-1 1], 'g');
%region3 Ex=0
plot([d3 1], [0 0], 'r');
xlabel('z/λ')
ylabel('H_y (z,t=0) [A*m^(-1)]');
title('Magnetic current density for d=0.5λ')
hold off
```

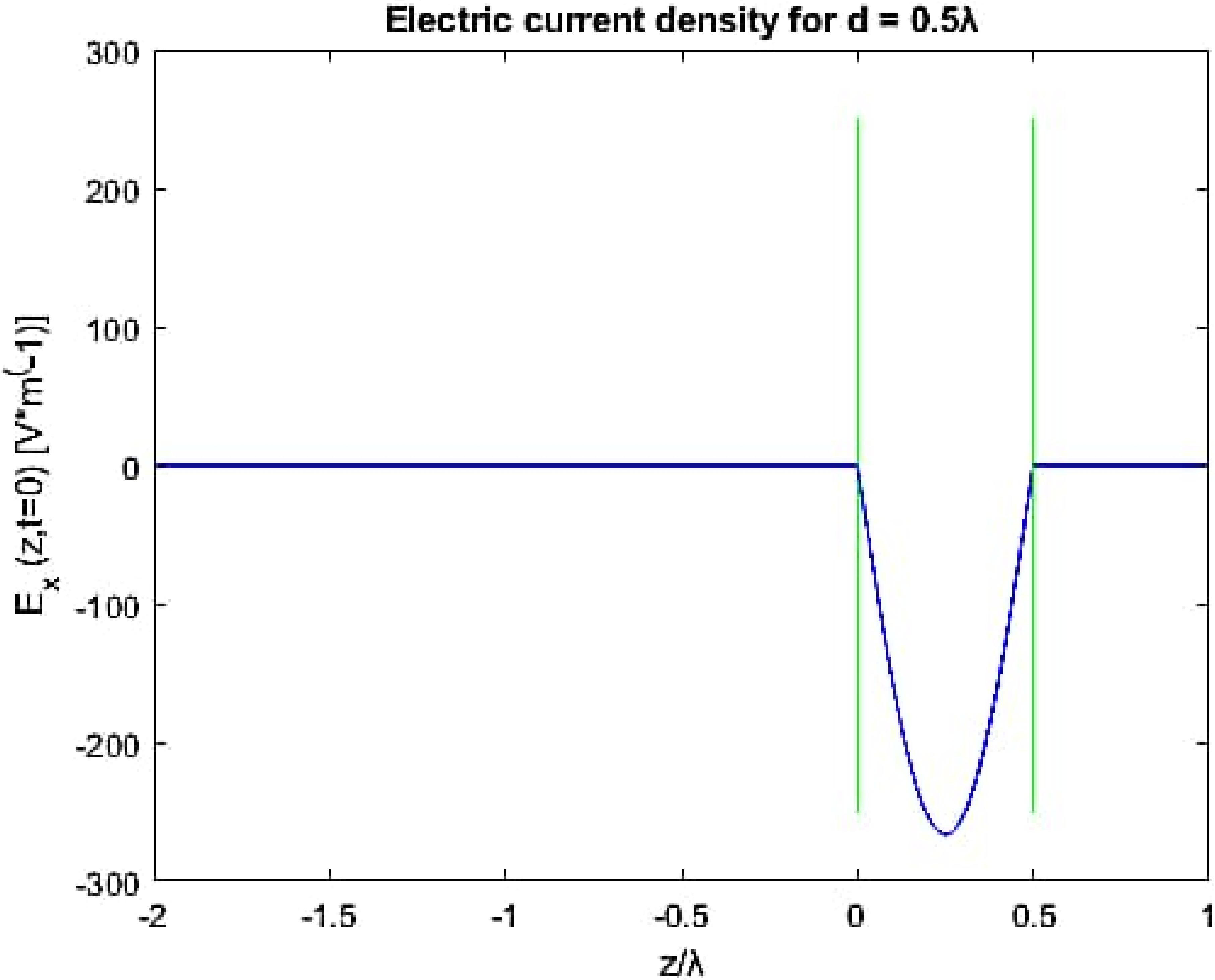

Electric current density for d=0.125λ
300
200
100
0
-100
-200
-300
E(z,t=0) [V*m^(-1)]
-2
-1.5
-1
-0.5
0
0.5
z/λ


Magnetic current density for d=0.125λ
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
z/λ
H_y (z,t=0) [A*m^(-1)]

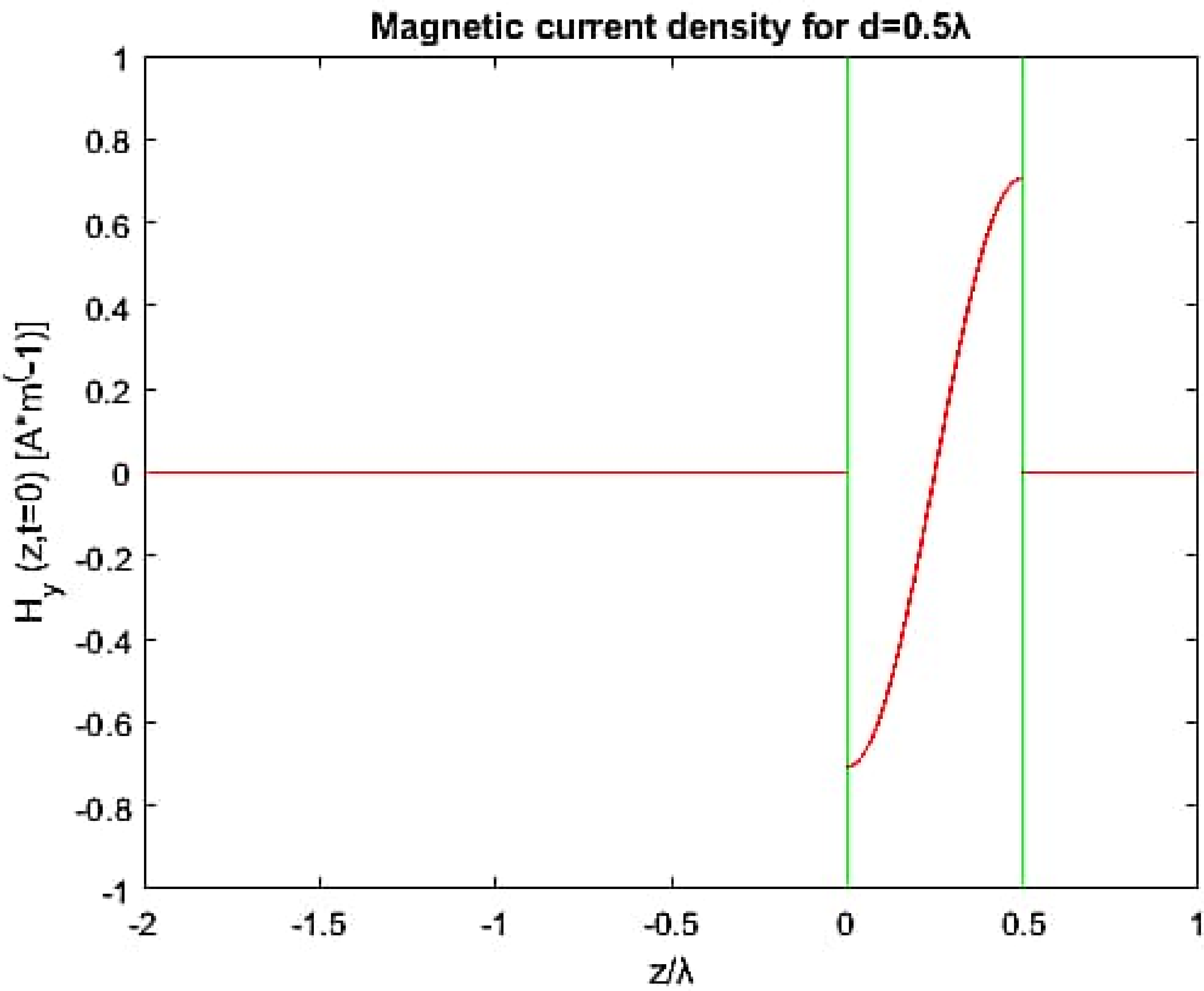

Electric current density for d=0.125λ
E(z,t=0) [V*m^(-1)]
z/λ
400
300
200
100
0
-100
-200
-300
-400
-2
-1.5
-1
-0.5
0
0.5



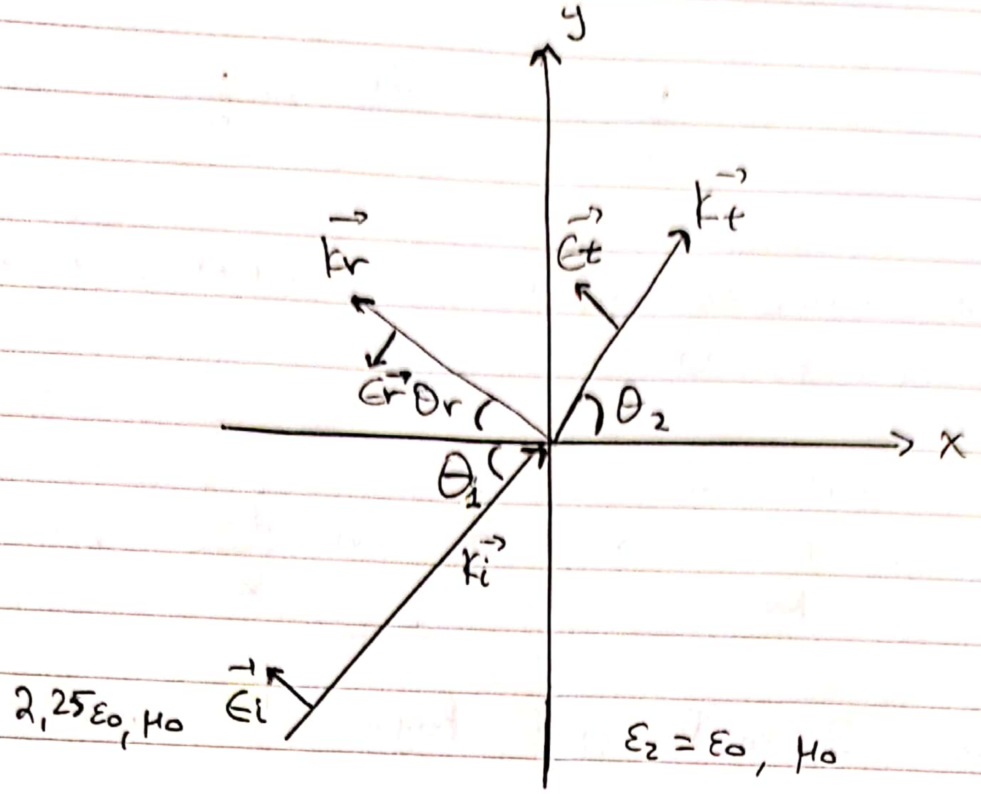
Magnetic current density for d=0.25λ
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
z/λ
H_y (z,t=0) [A*m^(-1)]



Electric current density for d = 0.5λ
300
200
100
0
-100
-200
-300
E_x (z,t=0) [V*m^(-1)]
-2
-1.5
-1
-0.5
0
0.5
1
z/λ


Magnetic current density for d=0.5λ
H_y (z,t=0) [A*m^(-1)]
z/λ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1

Άσκηση 9



a) $\hat{u}_{TM} = \cos\theta_1 \hat{i_y} - \sin\theta_1 \hat{i_x}$

$\vec{E_i} = E_0 \hat{u}_{TM} e^{-j\vec{k_i}\vec{r}} \Rightarrow \vec{E_i} = E_0[\cos\theta_i \hat{i_y} - \sin\theta_1 \hat{i_x}]e^{-j\vec{k_i}\vec{r}}$

$\theta_{cr} = \sin^{-1}\left(\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{\cancel{\varepsilon_0\mu_0}}{2{,}25\cancel{\varepsilon_0\mu_0}}}\right) = \sin^{-1}\left(\frac{2}{3}\right)$

$= 0{,}729$ ή $41{,}81^\circ$

Άρα η γωνία πρόσπτωσης είναι μεγαλύτερη απ' την γωνία ολικής ανάκλασης.

$\vec{k_i} = k_0 n_1 [\cos\theta_1 \hat{i_x} + \sin\theta_1 \hat{i_y}]$ με $n_1 = k_0 \omega\sqrt{\mu_1\varepsilon_1}$

Άρα

$\vec{E_i} = E_0\{\cos\theta_1 \hat{i_y} - \sin\theta_1 \hat{i_x}\} e^{-jk_0 n_1[x\cos\theta_1 + y\sin\theta_1]}$

Άρα

$\vec{\mathcal{E}_i} = Re[\vec{E_i} e^{j\omega t}] \Rightarrow$

$\vec{\mathcal{E}_i} = E_0 [\cos\theta_i \hat{i_y} - \sin\theta_1 \hat{i_x}] \cdot \cos(\omega t - k_0 n_1 [x\cos\theta_1 + y\sin\theta_1])$

Για $\theta_1 = \theta = 55^\circ$ έχω

$\cos\theta_1 = 0,5736$ και $\sin\theta_1 = 0,8192$

και $k_0 = \frac{2\pi}{\lambda_0} = 2\pi \cdot 10^6 \ m^{-1}$

Άρα τελικά $\vec{E_i} = E_0[0,5736 \hat{i_y} - 0,8192 \hat{i_x}] \cos[2\pi \cdot 3 \cdot 10^{14} t - 2\pi 10^6 \cdot 1,5(0,5736x + 0,8192y)]$

$$\vec{E_i} = E_0[0,5736 \hat{i_y} - 0,8192 \hat{i_x}] \cos[6\pi 10^{14} t - 3\pi 10^6 (0,5736x + 0,8192y)]$$

β) $$Z_i = \frac{Z_0}{\sqrt{\varepsilon_{ri}}} = \frac{Z_0}{n_i}$$

$$r_{TM} = \frac{Z_1 \cos\theta_1 - Z_2 \cos\theta_2}{Z_1 \cos\theta_1 + Z_2 \cos\theta_2} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - \frac{(n_1 \sin\theta_1)^2}{n_2^2}} = \frac{\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{j n_2}$$

$$n_1 \cos\theta_2 = -j \frac{n_1}{n_2} \sqrt{n_1^2 \sin^2\theta_1 - n_2^2}$$

$$= -1,5j \sqrt{2,25 \sin^2 55 - 1} = -j\, 1,070975$$

αφού $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Άρα

$$r_{TM} = \frac{0,5736 + j\, 1,070975}{0,5736 - j\, 1,070975} = 1 \angle 61,83^\circ \cdot 2 = e^{j 61,83 \cdot 2}$$

$$= e^{j 123,66}$$

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$$\vec{E_r} = E_o \cdot \hat{u}_{rm} \cdot e^{-j \vec{k_r} \vec{r}} \cdot r_{TM}$$

$$\hat{u}_{rm} = -\cos\theta_1 \hat{i_y} - \sin\theta_1 \hat{i_x}$$

$$\vec{E_r} = E_o [-\cos\theta_1 \hat{i_y} - \sin\theta_1 \hat{i_x}] e^{-jk_o n_1[-\cos\theta_1 x + \sin\theta_1 y] + 123,66^\circ}$$

Άρα $\vec{\mathcal{E}_r} = Re[\vec{E_r} e^{j\omega t}] \Rightarrow$

$$\vec{\mathcal{E}_r} = -E_o [\cos\theta_1 \hat{i_y} + \sin\theta_1 \hat{i_x}] \cos(\omega t - k_o n_1(-\cos\theta_1 x + \sin\theta_1 y) + 123,66^\circ)$$

$$\vec{\mathcal{E}_r} = -E_o [0,5736 \hat{i_y} + 0,8192 \hat{i_x}] \cos(6\pi 10^{14} t - 3\pi \cdot 10^6 (-0,5736 x + 0,8192 y) + 123,66^\circ)$$

γ) $t_{TM} = \dfrac{2 Z_2 \cos\theta_1}{Z_1 \cos\theta_1 + Z_2 \cos\theta_2} = \dfrac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \dfrac{2 \cdot 1,5 \cdot \cos 55}{1 \cdot \cos 55 - j 1,070975} =$

$$= 0,6687 + j 1,248 = 1,41639 e^{j 61,82^\circ}$$

$$\vec{E_t} = t_{TM} [\cos\theta_2 \hat{i_y} - \sin\theta_2 \hat{i_x}] E_o e^{-jk_o n_2 [x \cos\theta_2 + y \sin\theta_2]}$$

$$n_2 \cos\theta_2 = -j [n_1^2 \sin^2\theta_1 - n_2]^{1/2} = -j 0,71398$$

$$\cos\theta_2 = -j 0,71398 \qquad \sin\theta_2 = \frac{n_1 \sin\theta_2}{n_2} = 1,22873$$

Άρα

$$\vec{E_t} = 1,41659 e^{j 61,83} e^{-j\pi/2} [0,71398 \hat{i_y} - 1,22873 \hat{i_x}] \cdot E_o \cdot e^{-k_o 0,71398 x} \cdot e^{-jk_o 1,22873 y}$$

και

$$\vec{\mathcal{E}_t} = Re[\vec{E_t} \cdot e^{j\omega t}] \Rightarrow$$

$$\vec{E_t} = 1{,}41639 \cdot E_0 \cdot e^{-k_0 0{,}71398x}\left[0{,}71398\,\hat{i_y}\cos(\omega t - k_0 1{,}22873y - \frac{\pi}{2}\right.$$

$$\left. + 61{,}83^\circ) - 1{,}22873\,\hat{i_x}\cos(\omega t - k_0 0{,}71398 + 61{,}83^\circ)\right]$$

δ) $\vec{S_t} = \frac{1}{2}\vec{E_t} \times \vec{H_t^*}$

$$\nabla \times \vec{E_t} = -j\omega\mu_0 \vec{H_t} \Rightarrow$$

$$\vec{H_t} = \frac{j}{\omega\mu_0}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ \frac{d}{dx} & d/dy & 0 \\ E_{tx} & E_{ty} & 0 \end{vmatrix} \Rightarrow$$

$$\vec{H_t} = \hat{i_z}\left[\frac{d}{dx}E_{ty} - \frac{d}{dy}E_{tx}\right] \Rightarrow$$

$$\vec{H_t} = \hat{i_z}\frac{j}{\omega\mu_0}\left[-\cancel{j}k_0 n_2 E_0 t_{TM}\, e^{-jk_0 n_2(x\cos\theta_2 + y\sin\theta_2)}(\cos^2\theta_2 + \sin^2\theta_2)\right]$$

$$\vec{H_t} = \hat{i_z}\frac{k_0 n_2 E_0}{\omega\mu_0} t_{TM}\, e^{-jk_0 n_2(x\cos\theta_2 + y\sin\theta_2)}$$

Απο εξισωσεις Maxwell για ενα επιπεδο κυμα:

$$\vec{k_t} \times \vec{E_t} = \omega\mu_0 \vec{H_t} \Rightarrow$$

$$\vec{H_t} = \frac{1}{\omega\mu_0}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ k_{tx} & k_{ty} & 0 \\ E_{tx} & E_{ty} & 0 \end{vmatrix} = \frac{1}{\omega\mu_0}\hat{i_z}\left[k_{tx}\cdot E_{ty} - k_{ty}E_{tx}\right]$$

και $k_{tx} = k_0 n_2 \cos\theta_2$ και $k_{ty} = k_0 n_2 \sin\theta_2$

Αρα

$$\vec{H_t} = \hat{i_z}\frac{k_0 E_0}{\omega\mu_0} t_{TM}(n_2\cos^2\theta_2 + n_2\sin^2\theta_2)\, e^{-j\vec{k_t}\vec{r}}$$

$$\vec{H_t} = \hat{i_z} \frac{k_o \epsilon_o}{\omega \mu_o} t_{TM} n_2 e^{-j\vec{k_t}\vec{r}}$$

Απο $n_1 \sin\theta_1 = n_2 \sin\theta_2$ έχω

$$k_{tx} = k_o(-j)\sqrt{n_1^2 \sin\theta_1^2 - n_2^2} \quad \text{και}$$

$$k_{ty} = k_o n_1 \sin\theta_1$$

Αρα $$\vec{H_t} = \hat{i_z} \frac{k_o \epsilon_o}{\omega \mu_o} t_{TM} n_2 e^{-jk_o n_1 \sin\theta_1 y} e^{-k_o \sqrt{n_1^2 \sin\theta_1^2 - n_2^2}\, x}$$

Αρα $$\vec{S_t} = \frac{1}{2} \vec{E_t} \times \vec{H_t^*} = \begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ E_{xt} & E_{yt} & 0 \\ 0 & 0 & \frac{t_{TM}^* \epsilon_o^*}{Z_t} \end{vmatrix} \cdot \frac{e^{-jk_o n_2 (x\cos\theta_2 + y\sin\theta_2)}}{2}$$

$$= \frac{1}{2}\left[\hat{i_x} \frac{|E_o|^2 |t_{TM}|^2 \cos\theta_2}{Z_t} + \hat{i_y} \frac{|E_o|^2 |t_{TM}|^2 \sin\theta_2}{Z_t}\right] e^{-jk_o n_2 (x\cos\theta_2 - x\cos\theta_2^*)}$$

όπου $n_2 \cos\theta_2^* = j\sqrt{n_1^2 \sin\theta_1^2 - n_2^2}$

Αρα

$$\vec{S_t} = \left[\hat{i_x} \frac{|E_o|^2 |t_{TM}|^2 \cos\theta_2}{2Z_t} + \hat{i_y} \frac{|E_o|^2 |t_{TM}|^2 \sin\theta_2}{2Z_t}\right] e^{-2k_o \sqrt{n_1^2 \sin\theta_1^2 - n_2^2}\, x}$$

Αρα $\vec{P} = Re[\vec{S_t}]\big|_x =$

$$\vec{P} = \hat{i_y} \left(\frac{|E_o|^2 |t_{TM}|^2 \sin\theta_2}{2Z_t}\right) e^{-2k_o \sqrt{n_1^2 \sin\theta_1^2 - n_2^2}\, x}$$

$$P = 0$$

ε) $\frac{P_r}{P_i} = \frac{1}{2}\left(|r_{TE}|^2 + |r_{TM}|^2\right) \cdot 100\%$

$$r_{TE} = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

$$n_1\cos\theta_1 = 0,02212 \cdot 1,5 = 0,03319$$

$$n_2\cos\theta_2 = \frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{j\,n_2} = -j\,0,713983$$

$$r_{TE} = \frac{0,03319 + j\,0,713983}{0,03319 - j\,0,713983} = \frac{0,510873 - j\,0,04739}{-0,50867}$$

$$= -1,00433 + j\,0,093172$$

Αρα $\frac{P_r}{P_i} \approx \frac{1}{2}\left(|1|^2 + |1|^2\right) \cdot 100\% = 100\%$

και

$$\frac{P_t}{P_i} = \frac{1}{2}\left(|t_{TE}|^2 + |t_{TM}|^2\right) \cdot \frac{n_2\cos\theta_2}{n_1\cos\theta_1} \cdot 100\% \approx 0\%$$

Αρα έχουμε ολική ανάκλαση.

Editor - C:\Users\HOME\Documents\MATLAB\pedia7.m

pedia6.m | pedia7.m | pedia8.m | pedia9.m | +

```
%ασκηση9
e0=8.854*10.^(-12);
m0=4*pi*10.^(-7);
e1=2.25.*e0;
e2=e0;
m1=m0;
m2=m0;
n1=1.5;
n2=1;
z1=sqrt(m1./e1);
z2=sqrt(m2./e2);
f1=(0:pi/80:pi/2);
f2=acos(-j/n2*sqrt((n1.*sin(f1)).^2-n2.^2));
rTE=(z2.*cos(f1)-z1.*cos(f2))./(z2.*cos(f1)+z1.*cos(f2));
tTE=2.*z2.*cos(f1)./(z2.*cos(f1)+z1.*cos(f2));
rTM=(z1.*cos(f1)-z2.*cos(f2))./(z1.*cos(f1)+z2.*cos(f2));
tTM=2.*z2.*cos(f1)./(z1.*cos(f1)+z2.*cos(f2));
rTE1=abs(rTE);
rTE2=angle(rTE);
tTE1=abs(tTE);
tTE2=angle(tTE);
rTM1=abs(rTM);
rTM2=angle(rTM);
tTM1=abs(tTM);
tTM2=angle(tTM);
figure
```

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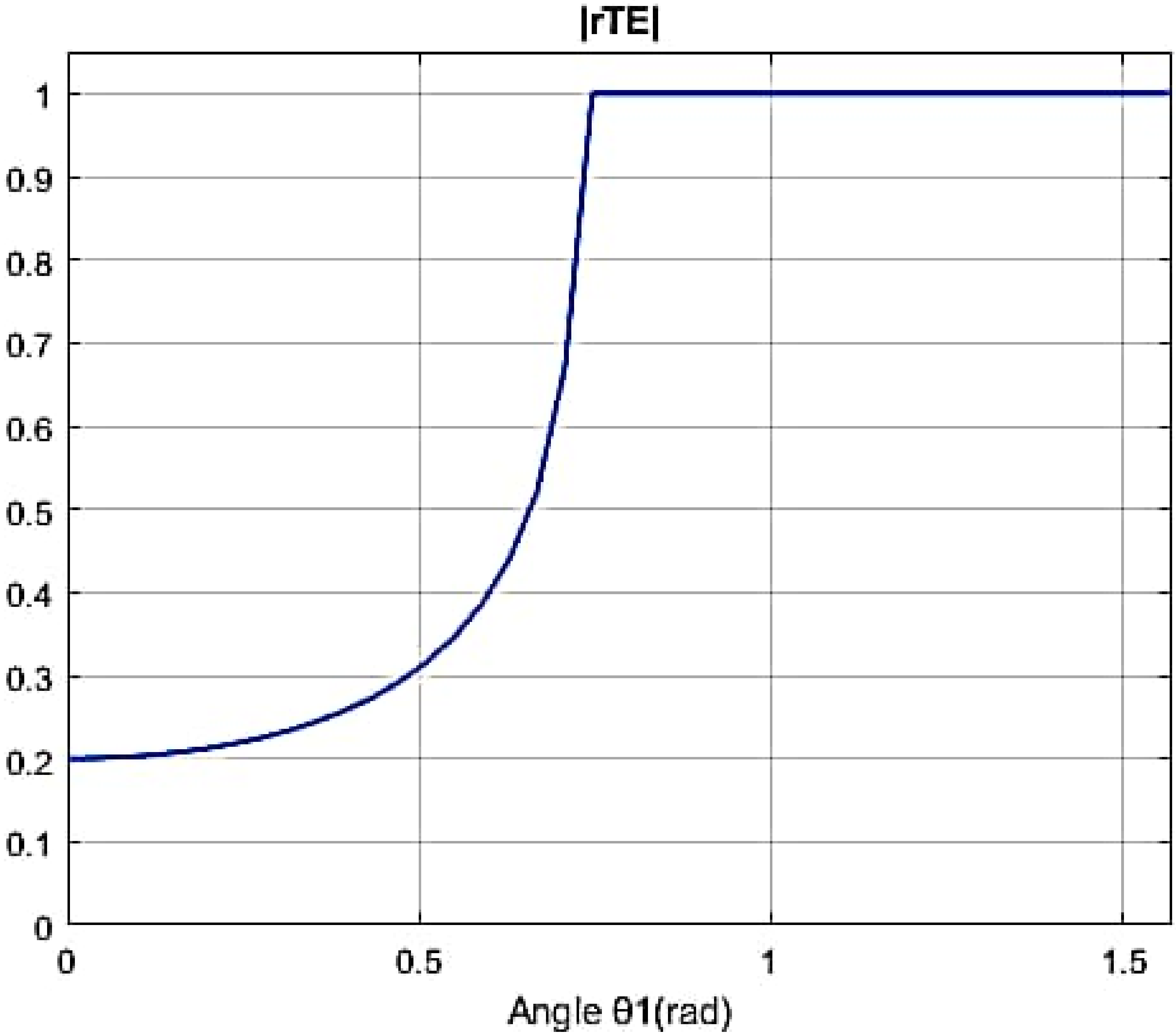
```
plot(f1,rTE1,'LineWidth',1.5);
title('|rTE|');
axis([0 1.57 0 1.05]);
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,rTE2,'LineWidth',1.5);
axis([0 1.58 -5 5]);
title('phase of rTE');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,tTE1,'LineWidth',1.5);
title('|tTE|');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,tTE2,'LineWidth',1.5);
axis([0 1.58 -5 5]);
title('phase of tTE');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,rTM1,'LineWidth',1.5);
title('|rTM|');
axis([0 1.57 0 1.05]);
```

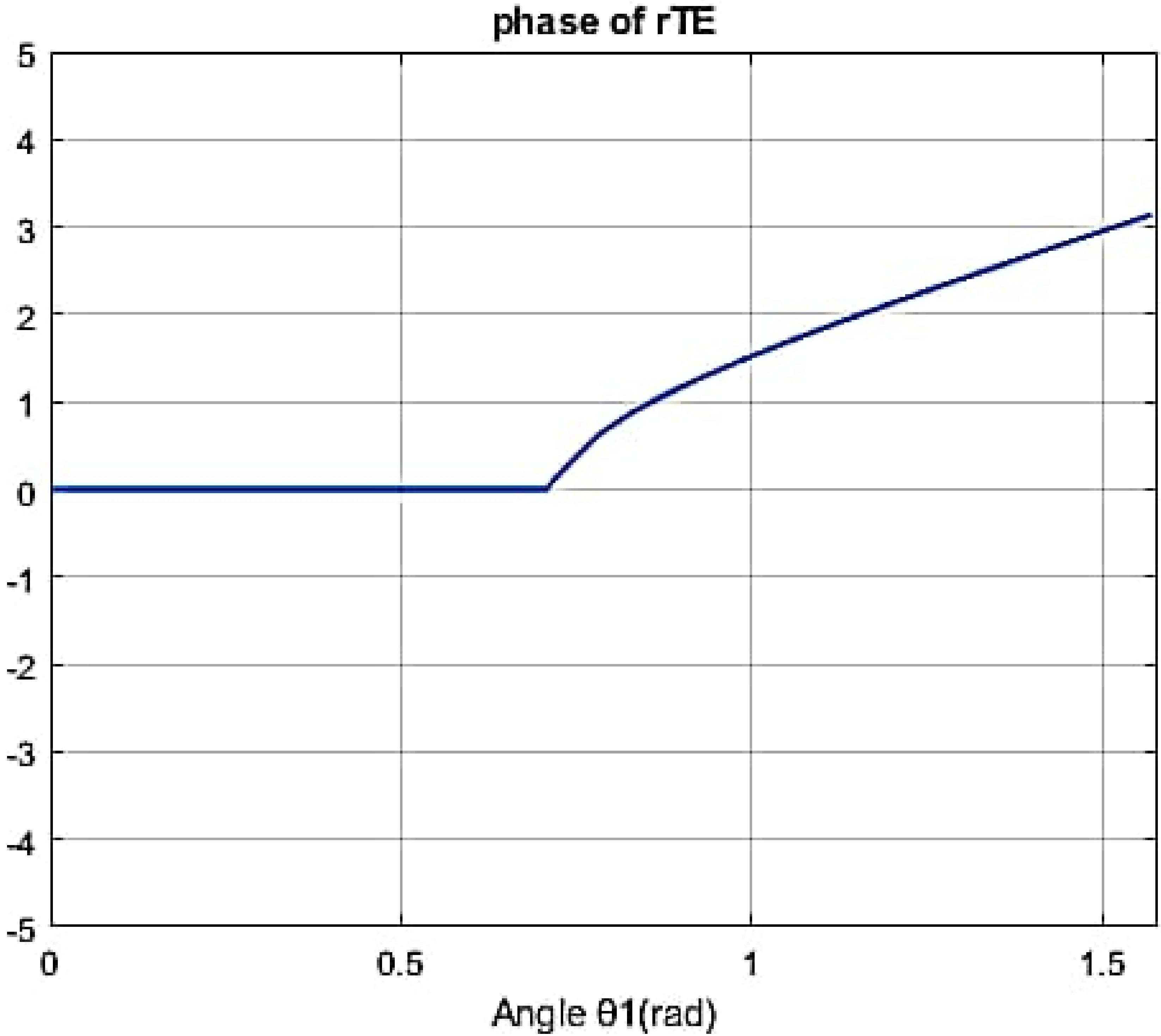
Editor - C:\Users\HOME\Documents\MATLAB\pedia7.m

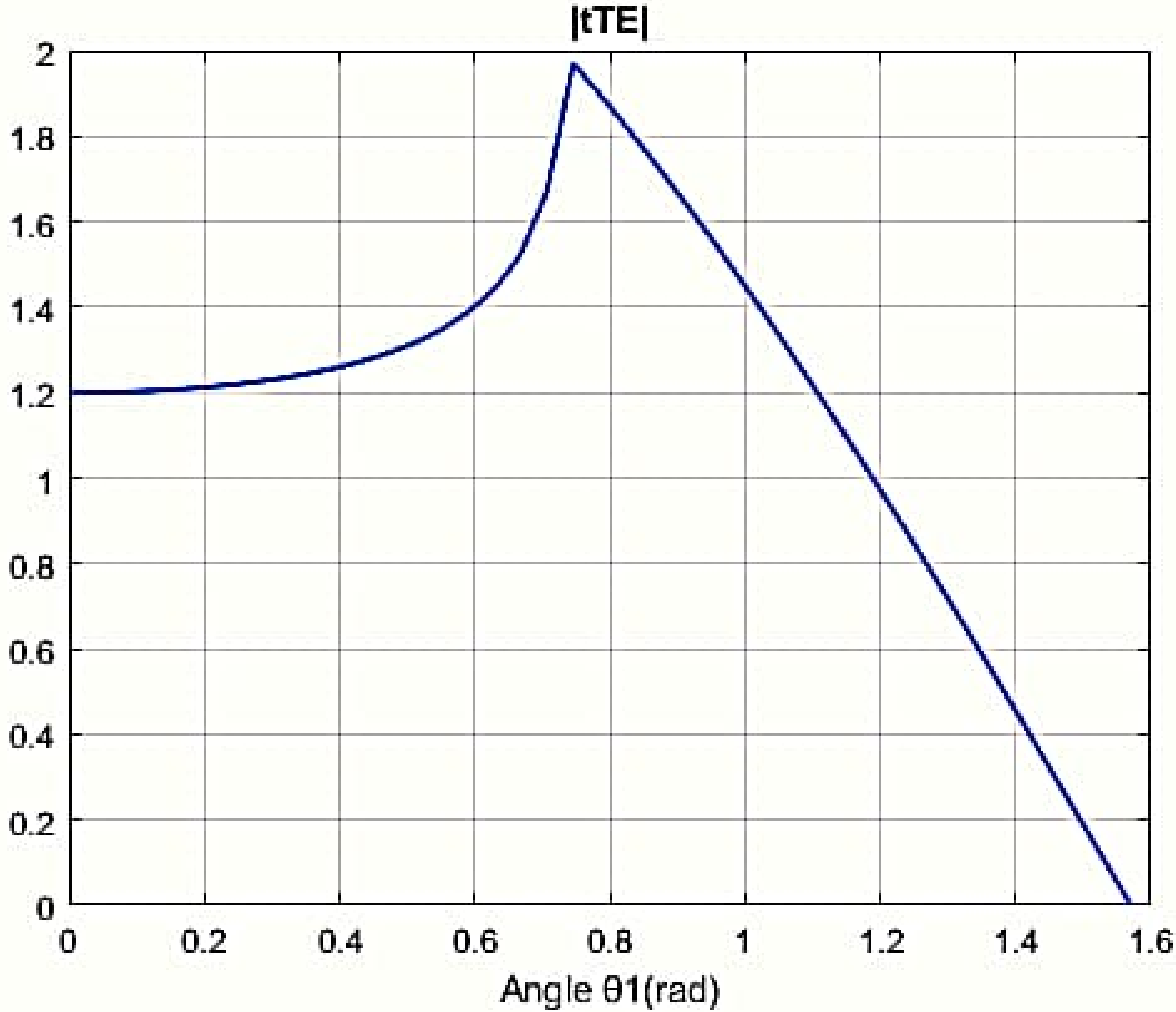
pedia6.m | pedia7.m | pedia8.m | pedia9.m | +

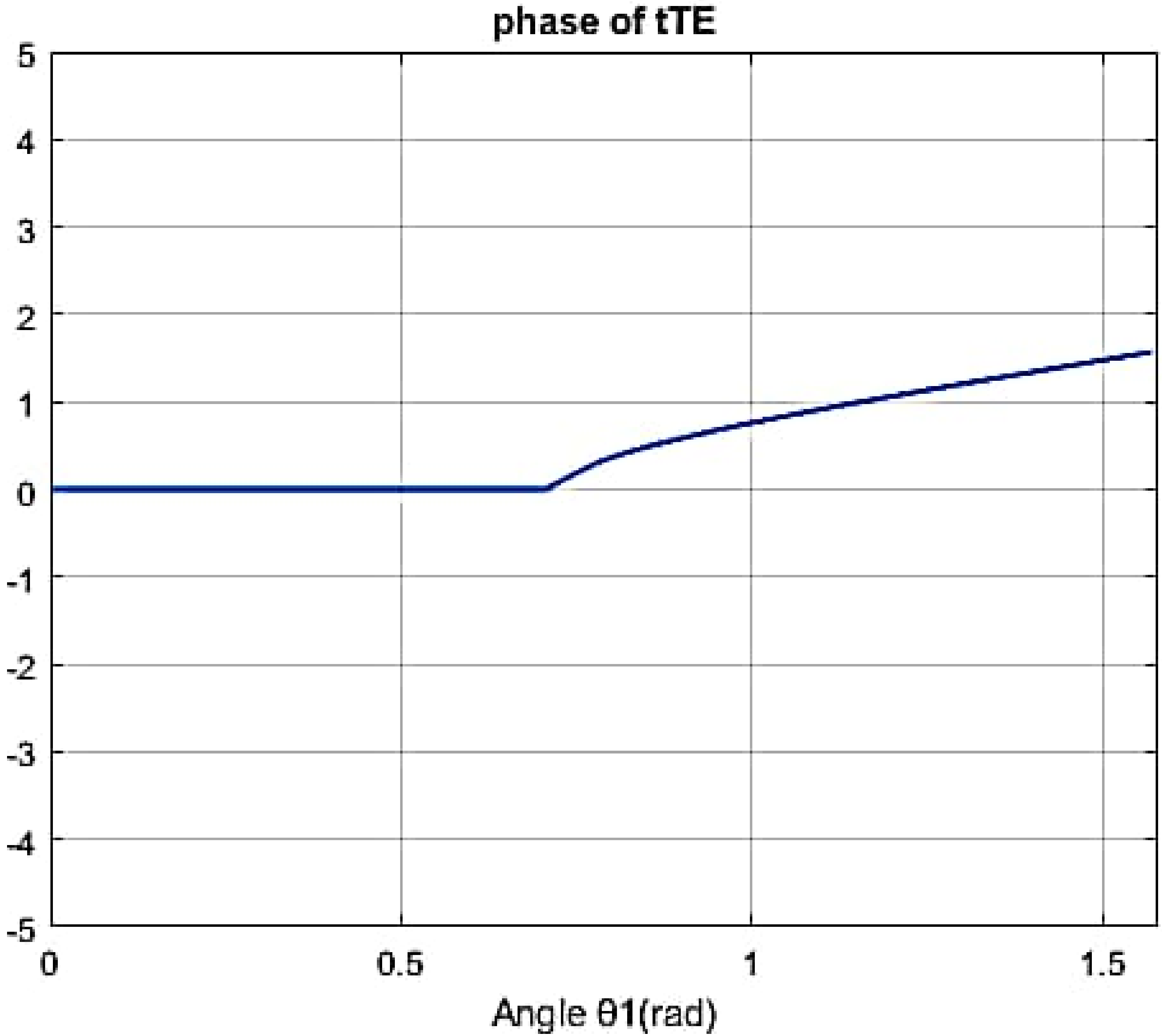
```
title('phase of tTE');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,rTM1,'LineWidth',1.5);
title('|rTM|');
axis([0 1.57 0 1.05]);
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,rTM2,'LineWidth',1.5);
axis([0 1.58 -4 4]);
title('phase of rTM');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,tTM1,'LineWidth',1.5);
title('|tTM|');
xlabel('Angle θ1(rad)');
grid on;
figure
plot(f1,tTM2,'LineWidth',1.5);
axis([0 1.58 -5 5]);
title('phase of tTM');
xlabel('Angle θ1(rad)');
grid on;
```

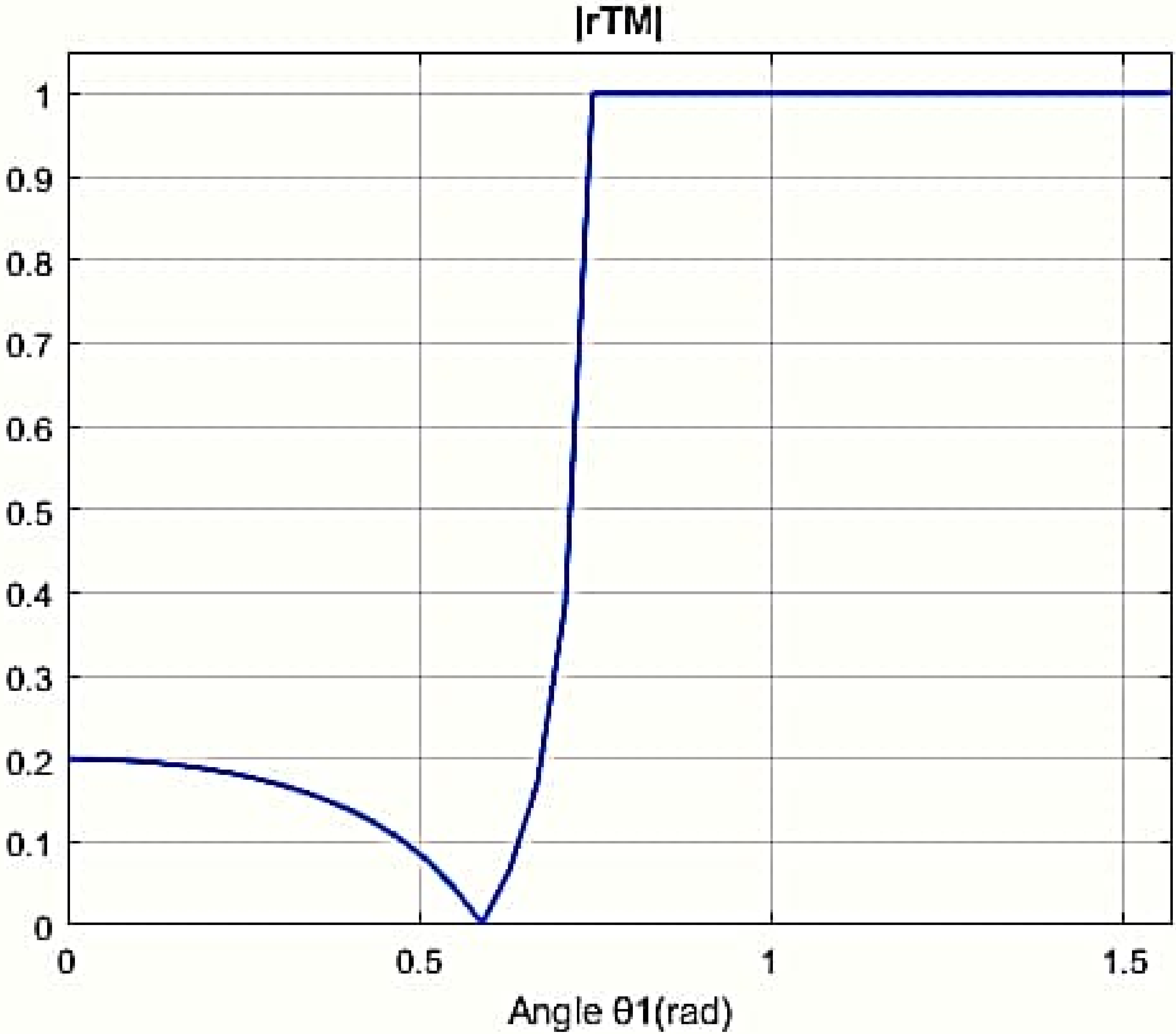


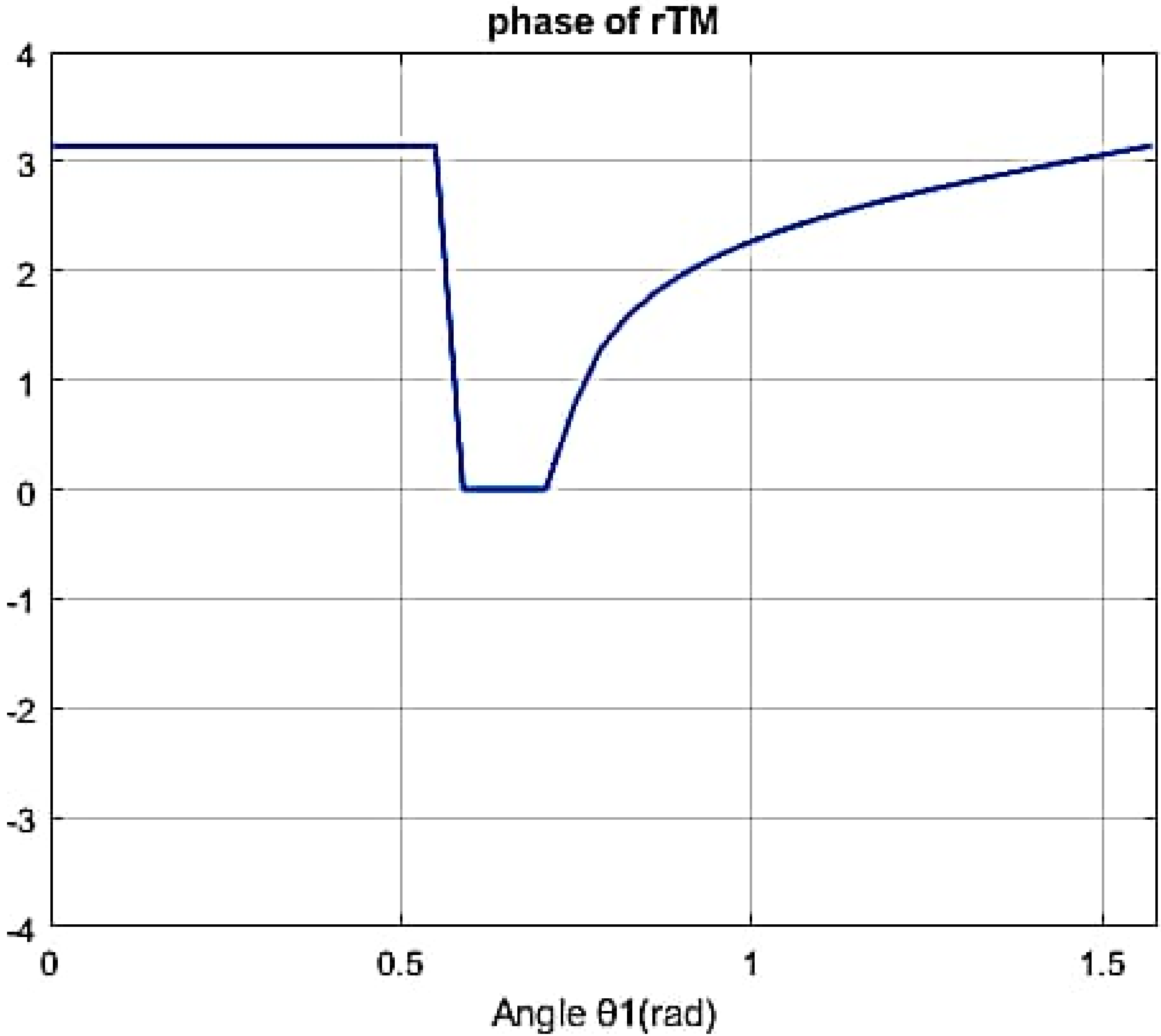
|rTE|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
Angle θ1(rad)

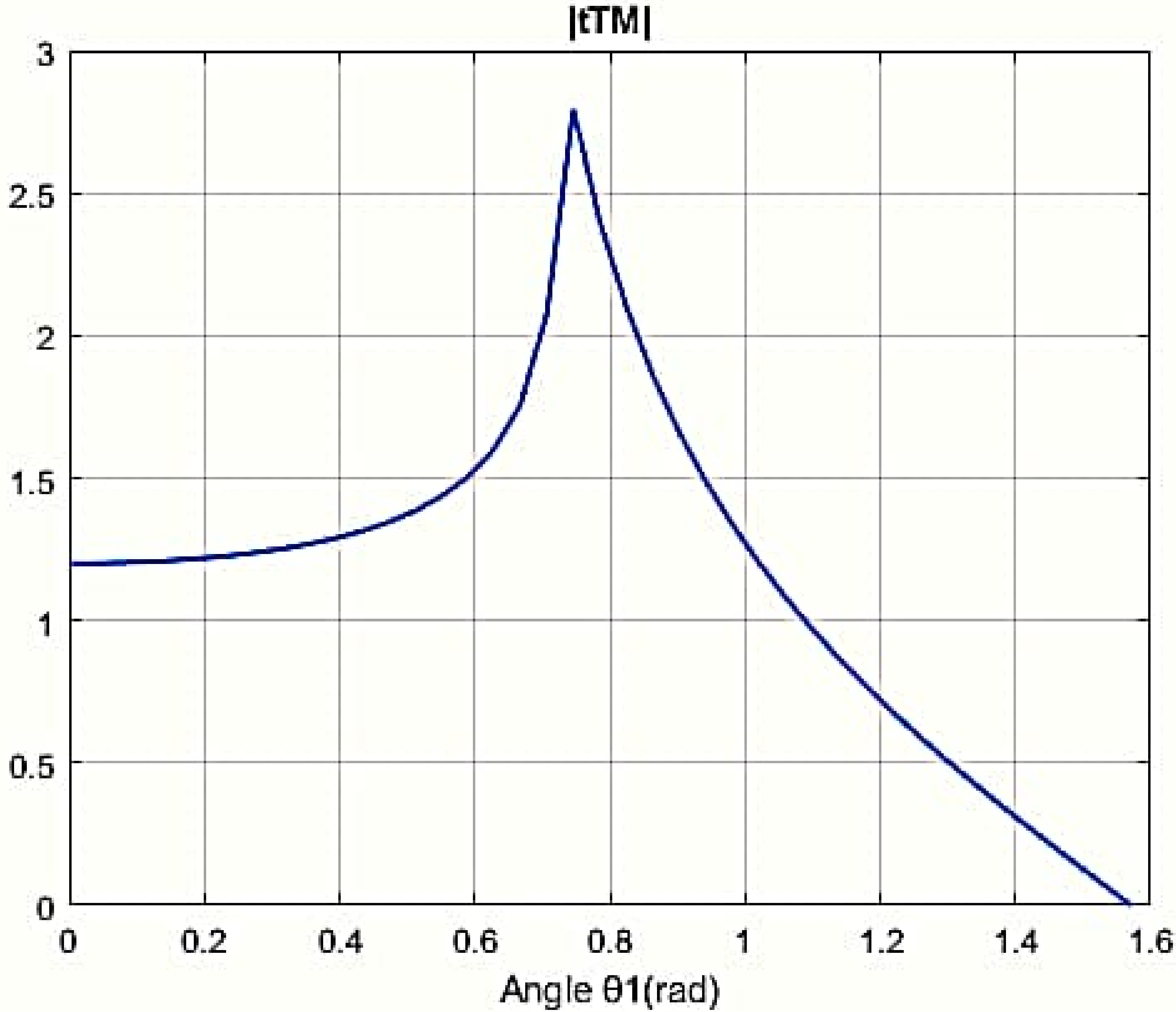

phase of rTE
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
Angle θ1(rad)

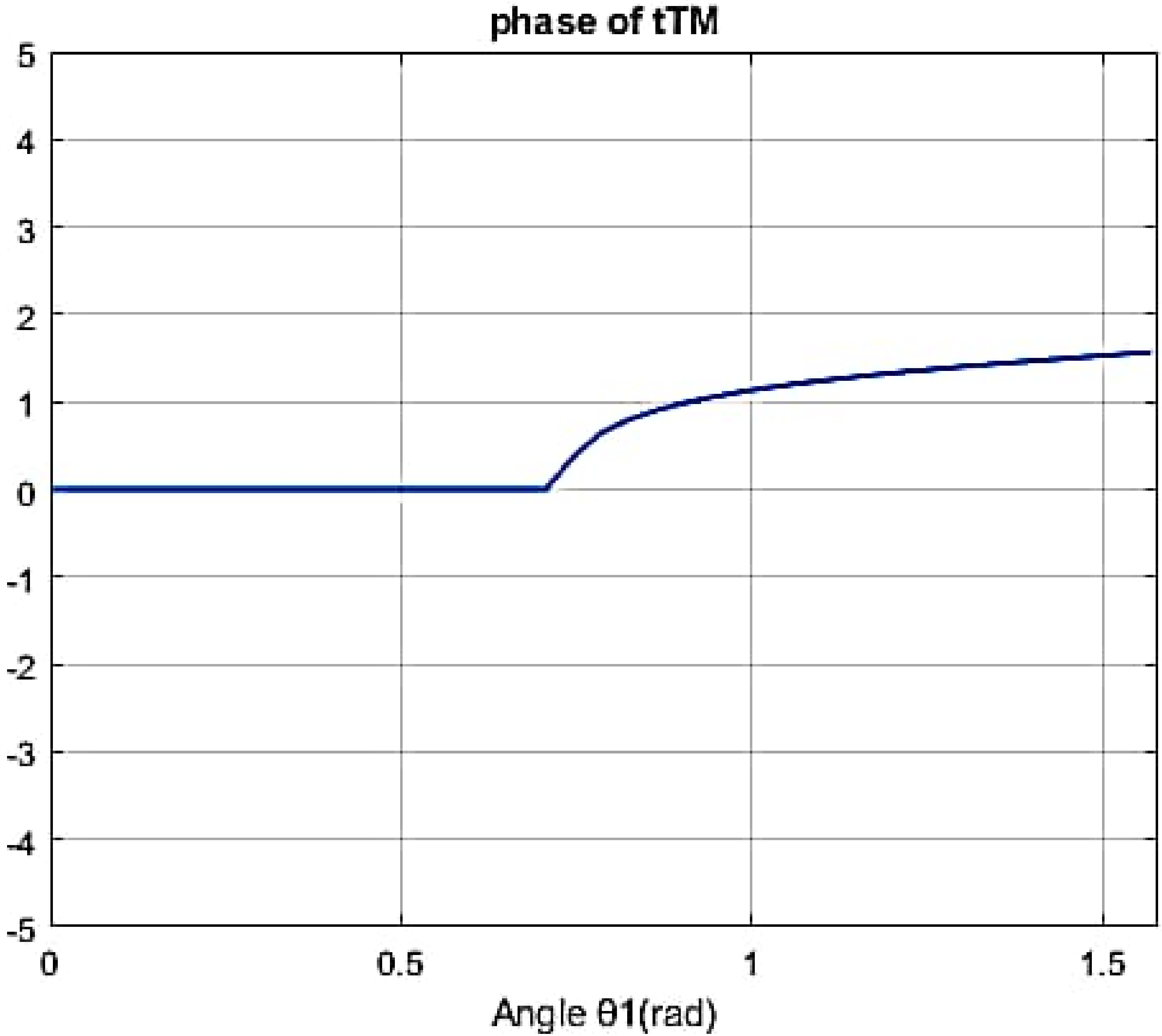

|tTE|
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Angle θ1(rad)


phase of tTE
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
Angle θ1(rad)


|rTM|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
Angle θ1(rad)

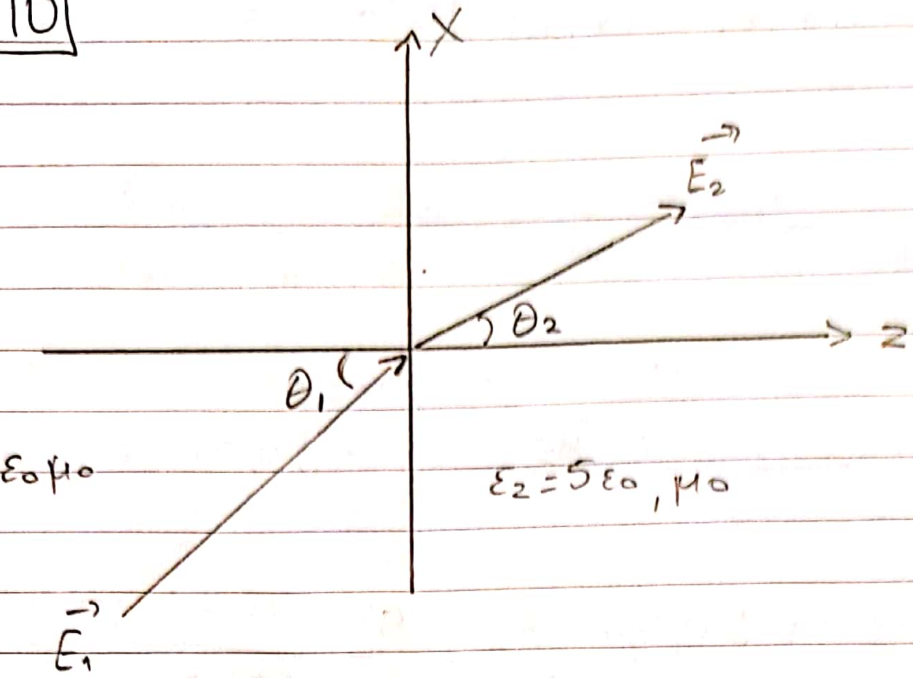

phase of rTM
4
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
Angle θ1(rad)


|tTM|
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Angle θ1(rad)



phase of tTM
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
Angle θ1(rad)

Ασκηση 10



a) $\theta_1 = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \tan^{-1}(\sqrt{5}) \simeq 65,91^\circ$

$\vec{k_i} = k_1[\sin\theta_1 \, \hat{i_x} + \cos\theta_1 \, \hat{i_z}] = \frac{\omega}{c}[0,912905 \, \hat{i_x} + 0,408171 \, \hat{i_z}]$

Επαλήθευση: Πρέπει $\vec{E} \cdot \vec{k_i} = 0$ αγνοώ την y συνιστώσα

$[2\cos\theta_1 \hat{i_x} - 2\sin\theta_1 \hat{i_z}][\sin\theta_1 \hat{i_x} + \cos\theta_1 \hat{i_z}] =$

$\cancel{2\cos\theta_1 \sin\theta_1} - \cancel{2\cos\theta \sin\theta_1} + \cancel{2\cos\theta_1 \sin\theta_1 \hat{i_y}} - \cancel{2\sin\theta_1\cos\theta_1 \hat{i_y}}$
$= 0$

Άρα $\vec{k_i} = 2\pi 10^6 [0,912192 \hat{i_x} + 0,408171 \hat{i_z}]$

b) $\vec{E_i} : [2\cos\theta_1 \hat{i_x} - j3\hat{i_y} - 2\sin\theta_i \hat{i_z}] e^{-j\vec{k}\vec{r}}$

$= [\underbrace{2}_{E_{TM}}\underbrace{(\cos\theta_1 \hat{i_x} - \sin\theta_1 \hat{i_z})}_{\hat{u}_{TM}} - \underbrace{j3}_{E_{TE}}\underbrace{\hat{i_y}}_{\hat{u}_{TE}}] e^{-j\vec{k}\vec{r}}$

$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\frac{(n_1\sin\theta_1)^2}{n_2^2}} = \frac{1}{n_2}\sqrt{n_2^2 - n_1^2\sin\theta_1^2}$

$= \frac{1}{n_2}\sqrt{5 - 0,912905} = \sqrt{\frac{4,08709}{5}} = \sqrt{0,8174914} \simeq 0,9$

$$r_{TE} = \frac{n_1 \cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{0{,}408171 - 2{,}02166}{0{,}408171 + 2{,}02166} =$$

$$-\frac{1{,}61348}{2{,}42983} \simeq -0{,}6667$$

$$t_{TE} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{2\cdot 0{,}408171}{2{,}42983} = \frac{0{,}816342}{2{,}42983}$$

$$\approx 0{,}335967$$

$$r_{TM} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{0{,}912698 - 0{,}9126..}{n_2\cos\theta_1 + n_1\cos\theta_2} \approx 0$$

$$t_{TM} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{2\cdot 0{,}408171}{\sqrt{5}\cdot 0{,}408171 + 0{,}9126} = \frac{0{,}816342}{0{,}9126 + 0{,}9126}$$

$$\approx \frac{0{,}816342}{1{,}8254} = 0{,}447214$$

$$\vec{E_r} = r_{TE}\, E_{TE}\, \hat{u}_{TE}\, e^{-j\vec{k_r}\vec{r}} = -0{,}6667\cdot(-j3)\cdot \hat{i_y}\, e^{-j\vec{k_r}\vec{r}}$$
$$= 2j\, e^{-j\vec{k_r}\vec{r}}\, \hat{i_y}$$

$$\vec{k_r} = k_0[\sin\theta_1\, \hat{i_x} - \cos\theta_1\, \hat{i_z}] = \frac{\omega}{c}[0{,}912905\,\hat{i_x} - 0{,}408171\,\hat{i_z}]$$

$$= 2\pi 10^6[0{,}912905\,\hat{i_x} - 0{,}408171\,\hat{i_z}]$$

Άρα $\vec{E_r} = 2j\, e^{-j2\pi\cdot 10^6[0{,}912905\,\hat{i_x} - 0{,}408171\,\hat{i_z}]}\cdot\hat{i_y}$

$$\vec{E_t} = \left[t_{TE}\, E_{TE}\, \hat{u}_{TE} + t_{TM}\, E_{TM}\, \hat{u}_{TM} \right] e^{-j\vec{k_t}\vec{r_t}}$$

$$= \left[\underbrace{0,335967(-j3)}_{1} \cdot \hat{i_y} + 0,447214 \cdot 2 \cdot (\cos\theta_1\, \hat{i_x} - \sin\theta_1\, \hat{i_z}) \right] e^{-j\vec{k_t}\vec{r_t}}$$

$$= \left[-j1,0079\, \hat{i_y} + 0,89442 (0,408171\, \hat{i_x} - 0,912905\, \hat{i_z}) \right] e^{-j\vec{k_t}\vec{r_t}}$$

$$\vec{k_t} = k_0\sqrt{5} \left[\sin\theta_2\, \hat{i_x} + \cos\theta_2\, \hat{i_z} \right] = 2\pi 10^6 \sqrt{5} \left[0,4082\, \hat{i_x} + 0,9129\, \hat{i_z} \right]$$

Άρα

$$\vec{E_t} = \left[-j\, \hat{i_y} + 0,89442 (0,408171\, \hat{i_x} - 0,912905\, \hat{i_z}) \right] e^{-2\pi 10^6 \sqrt{5} [0,4082x + 0,9129z]}$$

δ) $\vec{H_i} = \dfrac{k_0}{\omega\mu_0} (\vec{k_i} \times \vec{E_i}) = \dfrac{1}{Z_0} (\vec{k_i} \times \vec{E_i})$

$$= \frac{1}{Z_0} \begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ \sin\theta_1 & 0 & \cos\theta_1 \\ E_{ix} & E_{iy} & E_{iz} \end{vmatrix} e^{-j\vec{k_i}\vec{r_i}}$$

$$= \frac{1}{Z_0} \left[\hat{i_x}(-\cos\theta E_{iy}) + \hat{i_y}(\cos\theta E_{ix} - \sin\theta E_{iz}) + \hat{i_z}\sin\theta E_{iy} \right] \cdot e^{-j\vec{k_i}\vec{r_i}}$$

$$= \frac{1}{Z_0} \left[(-j3)(-\hat{i_x}\cos\theta + \hat{i_z}\sin\theta) + \hat{i_y} \cdot 2 \right] e^{-j\vec{k_i}\vec{r_i}}$$

$$\vec{H_r} = \frac{1}{Z_0} (\vec{k_r} \times \vec{E_r})$$

$$= \frac{1}{Z_0} \begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ \sin\theta & 0 & -\cos\theta \\ 0 & E_{ry} & 0 \end{vmatrix} e^{-j\vec{k_r}\vec{r_r}} = \frac{1}{Z_0} \left(\hat{i_z} \sin\theta E_{ry} + \hat{i_x} \cos\theta E_{ry} \right) e^{-j\vec{k_r}\vec{r_r}}$$

$$= \frac{1}{Z_0} \left(\hat{i_z}\, 2j \sin\theta + \hat{i_x}\, 2j \cos\theta \right) e^{-j\vec{k_r}\vec{r_r}}$$

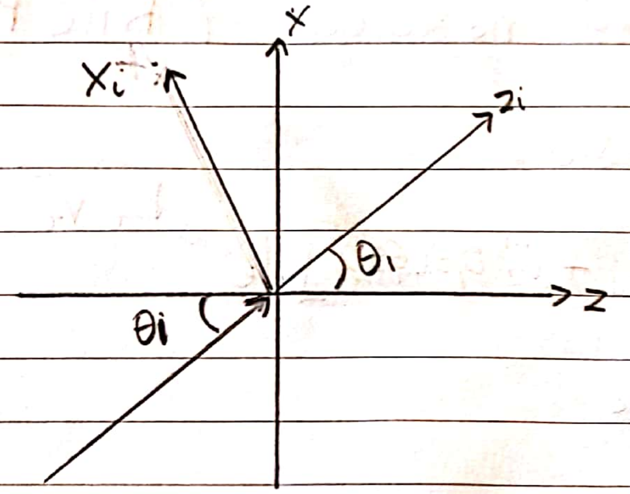
Άρα τελικά το συνολικό μαγνητικό πεδίο για z<0 με $Z_0 = 377$ είναι

$$\vec{H_1} = \vec{H_i} + \vec{H_r} \Rightarrow$$

$$\vec{H_1} = \frac{1}{Z_0}\left[(-j3)(-\hat{i_x}\cos\theta + \hat{i_z}\sin\theta) + 2\hat{i_y}\right] e^{-j\vec{k_i}\vec{r_i}} +$$

$$\frac{2j}{Z_0}\left[\hat{i_x}\cos\theta + \hat{i_z}\sin\theta\right] e^{-j\vec{k_r}\vec{r_r}}$$

δ)



$$\hat{i_{x_i}} = \cos\theta_i\,\hat{i_x} - \sin\theta_i\,\hat{i_z}$$
$$\hat{i_{y_i}} = \hat{i_y}$$
$$\hat{i_{z_i}} = \sin\theta_i\,\hat{i_x} + \cos\theta_i\,\hat{i_z}$$

$$\begin{bmatrix} E_{x_i} \\ E_{y_i} \\ E_{z_i} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 & -\sin\theta_i \\ 0 & 1 & 0 \\ \sin\theta_i & 0 & \cos\theta_i \end{bmatrix} \begin{bmatrix} 2\cos\theta_i \\ -j3 \\ -2\sin\theta_i \end{bmatrix} = \begin{bmatrix} 2 \\ -j3 \\ 0 \end{bmatrix} e^{-jk_0 z_i}$$

Προσπίπτον Ηλεκτρικό Πεδίο:

Scanned with CamScanner

Αρα $\vec{E_{xi}} = Re[\vec{E_{xi}} \cdot e^{j\omega t}] = 2\cos(\omega t - k_0 z_i)$

$$\vec{E_{yi}} = Re[\vec{E_{yi}}\, e^{j\omega t}] = 3\cos(\omega t - k_0 z_i - \frac{\pi}{2}) = 3\sin(\omega t - k_0 z_i)$$

Αρα $\frac{E_{xi}^2}{2^2} + \frac{E_{yi}^2}{3^2} = 1$

Άρα έχουμε ελλειπτική πόλωση
με $\tan\Theta_i = \frac{E_{yi}}{\vec{E_{xi}}} = \tan(\omega t - k_0 z_i)$

δεξιόστροφη ελλειπτική πόλωση (RHEP)

Διαδιδόμενο ηλεκτρικό πεδίο:

$$\vec{E_t} = [\hat{i_x}\, 0{,}8944\cos\theta_2 - \hat{i_y}\, j - \hat{i_z}\, 0{,}8944\sin\theta_2]\, e^{-j\vec{k_t}\vec{r_t}}$$

$$\hat{i}_{x_t} = \hat{i_x}\cos\theta_2 + \hat{i_z}\sin\theta_2$$
$$\hat{i}_{y_t} = \hat{i_y}$$
$$\hat{i}_{z_t} = -\hat{i_x}\sin\theta_2 + \hat{i_z}\cos\theta_2$$

$$\begin{bmatrix} E_{xt} \\ E_{yt} \\ E_{zt} \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \cdot \begin{bmatrix} 0{,}8944\cos\theta_2 \\ -j \\ -0{,}8944\sin\theta_2 \end{bmatrix} = \begin{bmatrix} 0{,}8944 \\ -j \\ 0 \end{bmatrix}$$

Αρα $E_{xt} = Re[E_{xt}\, e^{j\omega t}] = 0{,}8944\cos(\omega t - k_0\sqrt{\varepsilon_2}\, z_t)$
και

$$E_{yt} = Re[E_{yt}\, e^{j\omega t}] = \cos(\omega t - k_0\sqrt{\varepsilon_2}\, z_t - \frac{\pi}{2})$$
$$= \sin(\omega t - k_0\sqrt{\varepsilon_2}\, z_t)$$

$\left(\frac{E_{xt}}{0{,}8944}\right)^2 + \left(\frac{E_{yt}}{1}\right)^2 = 1$ Αρα ελλειπτική πόλωση με $\tan\theta_t = \dfrac{E_{yt}}{E_{xt}} = \tan(\omega t - k_0\sqrt{\varepsilon_2}\, z_t)$

Άρα έχουμε δεξιόστροφη ελλειπτική πόλωση (RHEP)

Και το ανακλώμενο κύμα έχει γραμμική πόλωση αφού είναι πολωμένο στην διεύθυνση y από Brewster

ε) $\dfrac{P_r}{P_i} = \dfrac{|r_{TE}|^2|E_{TE}|^2 + |r_{TM}|^2|E_{TM}|^2}{|E_{TE}|^2 + |E_{TM}|^2} = \dfrac{|0{,}6667|^2 \cdot 9}{9+4}$

$\dfrac{4{,}0004}{13} \simeq 0{,}307723$

$\dfrac{P_t}{P_i} = 1 - 0{,}307723 = 0{,}692277$ αφού μαζί με το $\dfrac{P_r}{P_i}$ αθροίζει στην μονάδα.

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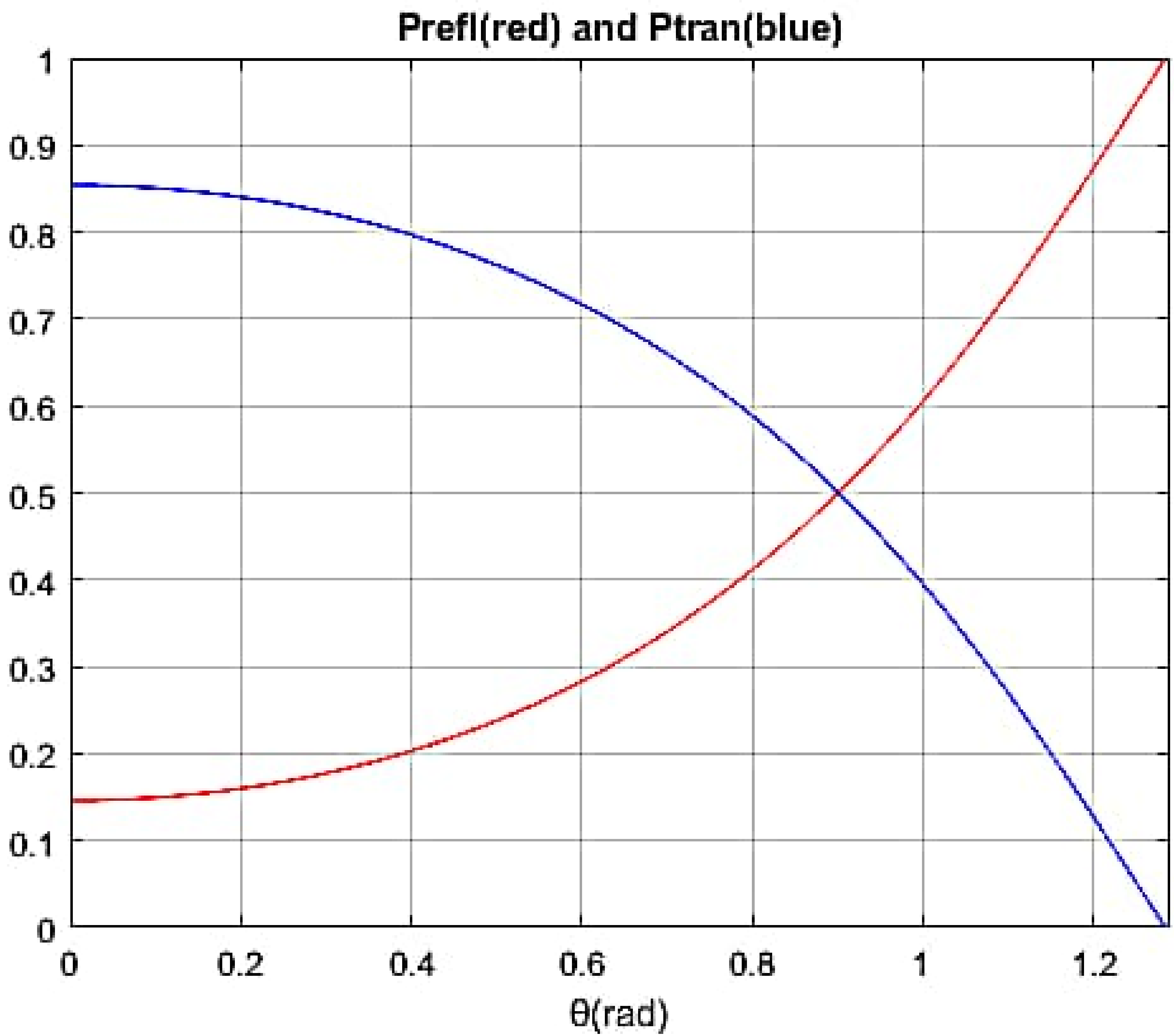
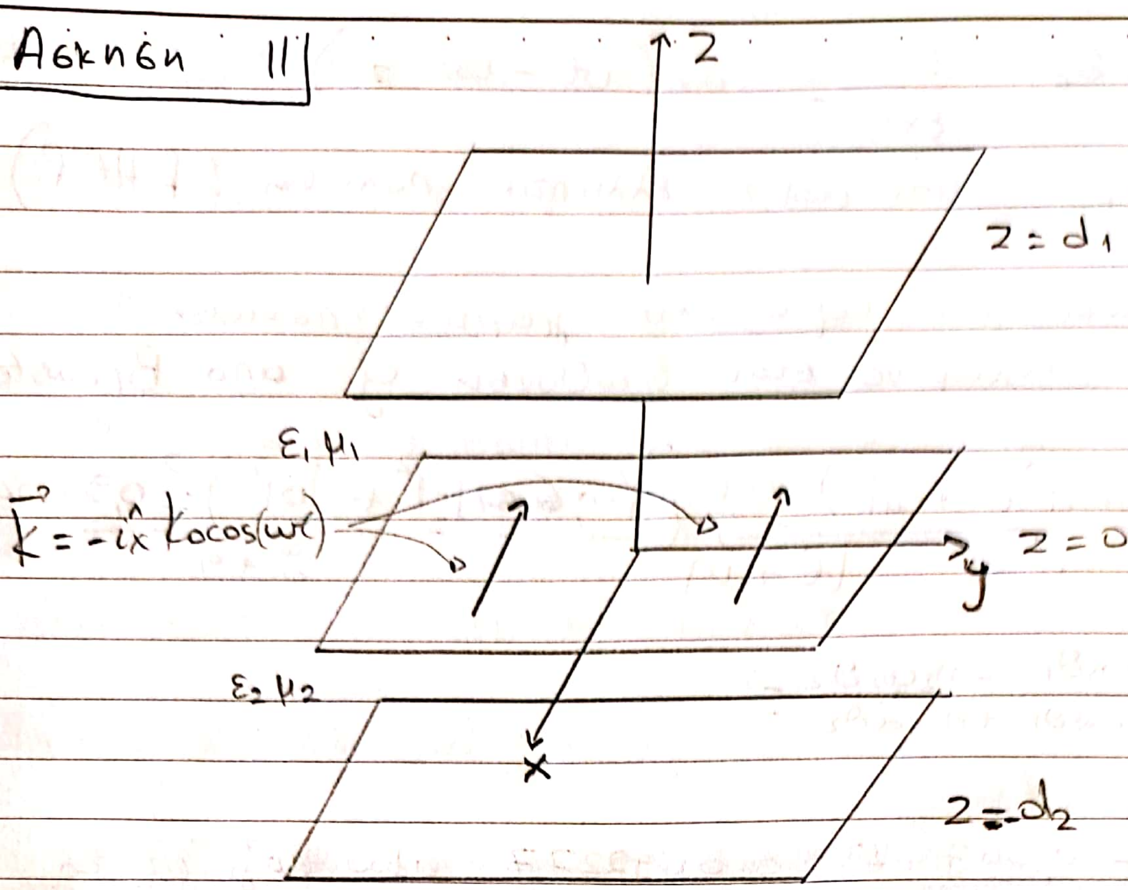
pedia6.m | pedia7.m | pedia8.m | pedia9.m

```
%ασκηση10
e0=8.854*10.^(-12);
m0=4*pi*10.^(-7);
e1=e0; e2=5.*e0;
m1=m0; m2=m0;
n1=1; n2=sqrt(5);
z1=sqrt(m1./e1);
z2=sqrt(m2./e2);
f1=(0:pi/80:pi/2);
f2=asin(n1.*sin(f1)./n2);
Ete=-3.*j;
Etm=2;
rTE=(n1.*cos(f1)-n2.*cos(f2))./(n1.*cos(f1)+n2.*cos(f2));
tTE=(2.*n1.*cos(f1))./(n1.*cos(f1)+n2.*cos(f2));
rTM=(n2.*cos(f1)-n1.*cos(f2))./(n2.*cos(f1)+n1.*cos(f2));
tTM=(2.*n1.*cos(f1))./(n2.*cos(f1)+n1.*cos(f2));
Pr=(rTE.^2.*Ete.^2+rTM.^2.*Etm.^2)./(Ete.^2+Etm.^2);
Pt=(n2./cos(f1)).*(tTE.^2.*Ete.^2.*real(cos(f2)./n1)+tTM.^2.*Etm.^2.*real(cos(f2)./n1))./(Ete.^2+Etm.^2);
figure
plot(f1,Pr,'r');
hold on
plot(f1,Pt,'b');
axis([0 1.29 0 1]);
grid on;
title('Prefl(red) and Ptran(blue)');
xlabel('θ(rad)');
```


Prefl(red) and Ptran(blue)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
θ(rad)

Άσκηση 11



α) Για την περιοχή 1:

$$\vec{E}_{1+} = +\hat{i_x} E_{1+} e^{-jk_1 z} \quad \text{και}$$

$$\vec{H}_{1+} = \frac{1}{Z_1}\left(\hat{k}_1 \times \vec{E}_{1+}\right) = \frac{1}{Z_1}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & 1 \\ E_{1+} & 0 & 0 \end{vmatrix} e^{-jk_1 z} = \frac{1}{Z_1}\hat{i_y} E_{1+} e^{-jk_1 z}$$

ομοίως:

$$\vec{E}_{1-} = +\hat{i_x} E_{1-} e^{jk_1 z} \quad \text{και}$$

$$\vec{H}_{1-} = \frac{1}{Z_1}\left(\hat{k}_1 \times \vec{E}_{1-}\right) = \frac{1}{Z_1}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & -1 \\ E_{1-} & 0 & 0 \end{vmatrix} e^{jk_1 z} = \frac{1}{Z_1}\left(-\hat{i_y}\right) E_{1-} e^{jk_1 z}$$

Άρα τελικά:

$$\vec{E_1} = \vec{E_{1+}} + \vec{E_{1-}} \Rightarrow$$

$$\vec{E_1} = \hat{i_x}\left[E_{1+} e^{-jk_1 z} + E_{1-} e^{jk_1 z} \right]$$

και

$$\vec{H_1} = \vec{H_{1+}} + \vec{H_{1-}} \Rightarrow$$

$$\vec{H_1} = \hat{i_y} \frac{1}{Z_1}\left[E_{1+} e^{-jk_1 z} - E_{1-} e^{jk_1 z} \right]$$

όπου $Z_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$ και $k_1 = \omega\sqrt{\varepsilon_1 \mu_1}$

Για την περιοχή 2:

$$\vec{E_{2+}} = \hat{i_x} E_{2+} e^{-jk_2 z} \quad \text{και}$$

$$\vec{H_{2+}} = \frac{1}{Z_2}\left(\hat{k_2} \times \vec{E_{2+}}\right) = \frac{1}{Z_2}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & 1 \\ E_{2+} & 0 & 0 \end{vmatrix} e^{-jk_2 z}$$

$$= \frac{1}{Z_2} \cdot \hat{i_y} E_{2+} e^{-jk_2 z}$$

ομοίως

$$\vec{E_{2-}} = \hat{i_x} E_{2-} e^{jk_2 z} \quad \text{και}$$

$$\vec{H_{2-}} = \frac{1}{Z_2}\left(\hat{k_2} \times \vec{E_{2-}}\right) = \frac{1}{Z_2}\begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & -1 \\ E_{2-} & 0 & 0 \end{vmatrix} e^{jk_2 z}$$

$$= \frac{1}{Z_2}\left(-\hat{i_y}\right) E_{2-} e^{jk_2 z}$$

και επειδή $\vec{E_2} = \vec{E_{2+}} + \vec{E_{2-}}$ $\quad \vec{H_2} = \vec{H_{2+}} + \vec{H_{2-}}$

$$\vec{E_2} = \hat{i_x}\left[E_{2+} e^{-jk_2 z} + E_{2-} e^{jk_2 z} \right]$$

και

$$\vec{H_2} = \hat{i_y} \frac{1}{Z_2}\left[E_{2+} e^{-jk_2 z} - E_{2-} e^{jk_2 z} \right]$$

όπου $Z_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$ και $k_2 = \omega\sqrt{\varepsilon_2 \mu_2}$

Στις περιοχές πάνω απ' το d_1 και κάτω απ' το d_2 έχω $\vec{E} = \vec{H} = \vec{0}$ λόγω τέλειων αγωγών.

Απο οριακές συνθήκες :

$$\hat{i_n} \times (E_{1+} - E_{1-}) = 0$$

$$\Rightarrow E_{1+} e^{-jk_1 d_1} + E_{1-} e^{+jk_1 d_1} = 0$$

$$E_{1+} = - E_{1-} e^{2jk_1 d_1} \quad (1)$$

Όμοια $\hat{i_n} \times (E_{2+} - E_{2-}) = 0$

$$E_{2+} = - E_{2-} e^{2jk_2 d_2} \quad (2)$$

Απο οριακές συνθήκες

$$H_1 - H_2 = K_0 \Rightarrow \frac{1}{Z_1}(E_{1+} - E_{1-}) - \frac{1}{Z_2}(E_{2+} - E_{2-}) = K_0 \quad (3)$$

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και

$$E_2 - E_1 = 0 \Rightarrow$$

$$E_{1+} + E_{1-} = E_{2+} + E_{2-} \quad (4)$$

β) $\vec{K_1} = \hat{i_z} \times [0 - \vec{H_1}] \Rightarrow$

$$\vec{K_1} = \begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & 1 \\ 0 & -H_1 & 0 \end{vmatrix} = \hat{i_x} \cdot H_1 = \hat{i_x} \frac{1}{Z_1}\left[E_{1+} e^{-jk_1 z} - E_{1-} e^{jk_1 z}\right]$$

$$= \hat{i_x} \frac{1}{Z_1}\left[E_{1+} e^{-jk_1 z} - E_{1-} e^{jk_1 z}\right]$$

Από (1)

$$\vec{K_1} = -\hat{i_x} \frac{1}{Z_1}\left[E_{1-} e^{jk_1 z} \cancel{-} (\cancel{+} E_{1-} e^{2jk_1 z}) \cancel{e^{-jk_1 z}}\right]$$

Άρα για $z = d_1$

$$\vec{K_1} = -\hat{i_x} \frac{2 E_{1-} e^{jk_1 d_1}}{Z_1} \quad \text{ή αλλιώς από (1)}$$

$$\vec{K_1} = \hat{i_x} \frac{2 E_{2+} e^{-jk_1 d_1}}{Z_1}$$

ομοίως

$$\vec{K_2} = \hat{i_z} \times [H_2 - 0] \Rightarrow \vec{K_2} = \begin{vmatrix} \hat{i_x} & \hat{i_y} & \hat{i_z} \\ 0 & 0 & 1 \\ 0 & H_2 & 0 \end{vmatrix} =$$

$$-\hat{i_x} H_2 = -\hat{i_x} \frac{1}{Z_2}\left[E_{2+} e^{-jk_2 z} - E_{2-} e^{jk_2 z}\right]$$

και από (2) για $z = d_2$

$$\Rightarrow \vec{K_2} = -\hat{i_x} \frac{2 E_{2+} e^{jk_2 d_2}}{Z_2}$$

γ) $$\underbrace{\begin{bmatrix} e^{-jk_1d_1} & e^{+jk_1d_1} & 0 & 0 \\ 0 & 0 & e^{-jk_2d_2} & e^{+jk_2d_2} \\ 1 & 1 & -1 & -1 \\ 1/Z_1 & -1/Z_1 & -1/Z_2 & 1/Z_2 \end{bmatrix}}_{A} \begin{bmatrix} E_{1+} \\ E_{1-} \\ E_{2+} \\ E_{2-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_0 \end{bmatrix}$$

$k_1 = k_2 = k$

$$\det A = \frac{2}{Z_0} \cdot \left(e^{-2jk} - e^{2jk}\right)$$

$$A_{E_{2+}} = \begin{bmatrix} 0 & 0 & e^{-jk} \\ e^{-jk} & e^{+jk} & 0 \\ 1 & 1 & -1 \end{bmatrix} \cdot (-k_0)$$

$$\det A_{E_{2+}} = -k_0\left(e^{-2jk} - 1\right)$$

Άρα $$E_{2+} = \frac{\det A_{E_{2+}}}{\det A}$$

δ) $$A_{E_{1+}} = -k_0 \begin{bmatrix} 0 & e^{jk} & e^{-jk} \\ e^{jk} & 0 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

$$\det A_{E_{1+}} = k_0 \, e^{jk}\left(e^{-jk} - e^{jk}\right) = k_0\left(1 - e^{2jk}\right)$$

Άρα $$E_{1+} = \frac{\det A_{E_{1+}}}{\det A}$$

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$$A_{E_1-} = k_0 \begin{bmatrix} 0 & e^{jk} & e^{-jk} \\ e^{-jk} & 0 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

$$\det A_{E_1-} = k_0\left(1 - e^{-2jk}\right)$$

Άρα $E_{1-} = \dfrac{\det A_{E_1-}}{\det A}$

και

$$A_{E_2-} = k_0 \begin{bmatrix} 0 & 0 & e^{jk} \\ e^{-jk} & e^{jk} & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\det A_{E_2-} = k_0\left(1 - e^{2jk}\right)$$

Άρα $E_{2-} = \dfrac{\det A_{E_2-}}{\det A}$

Με την μέθοδο Cramer βρίσκω τα E_{1+}, E_{1-}, E_{2+} και E_{2-} και τα τοποθετώ στον κώδικα στο ματλαμπ. Οι συχνότητες που μηδενίζεται η $\det A$ είναι όταν $e^{-2jk} = e^{2jk}$ κάθε 0,075 GHz

Για E_{2+} βλέπω πως όταν μηδενίζεται η $\det A$ απειρίζεται το πλάτος. Ωστόσο αυτό δεν συμβαίνει για κάθε μηδενισμό της ορίζουσας του A αλλά μόνο για περιττές τιμές του $f = 0{,}075\,n$ GHz. Συνεπώς μόνο όταν το sin πηγαίνει απ' τα αρνητικά στα θετικά

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pedia6.m | pedia7.m | pedia8.m | pedia9.m

```matlab
%ασκηση11
%γ
K0=1;
d1=1;
d2=1;
e0=8.8541878128*10^(-12);
m0=4*pi*10.^(-7);
e1=e0;
e2=e0;
m1=m0;
m2=m0;
c=3e8;
f=10^9:1.25*10^9/1000000:1.5*10^9;
k=2*pi/c.*f; %k1=k2=k
z0=sqrt(m0./e0); %z0=z1=z2
%A=[exp(-i*x1(ifr)) exp(+i*x1(ifr)) 0 0;0 0 exp(-i*x2(ifr)) exp(+i*x2(ifr));1 1 -1 -1; 1/z1 -1/z1 -1/z2 1/z2];
detA=2/z0.*(exp(-2i.*k)-exp(2i*k));
subplot(2,1,1)
plot(f,abs(detA),'Linewidth',2);
title('Determinant of A');
xlabel('frequency (Hz)');
E2p=-K0*(exp(-2i.*k)-1)./detA;
subplot(2,1,2)
plot(f,abs(E2p),'Linewidth',2);
title('|E2+|');
xlabel('frequency (Hz)');
```

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```
%δ
Freq=1.15*10.^9;
kk=2*pi/c.*Freq;
z2=-d2:0.00001:0;
z1=0:0.00001:d1;
detAA=2/z0.*(exp(-2i.*kk)-exp(2i*kk));
E1p=K0*(1-exp(2i*kk))./detAA;
E1n=K0*(1-exp(-2i*kk))./detAA;
E2pp=-K0*(exp(-2i.*kk)-1)./detAA;
E2n=K0*(1-exp(2i*kk))./detAA;
E1=E1p.*exp((-i*kk).*z1)+E1n.*exp((i*kk).*z1);
H1=E1p/z0.*exp((-i.*kk).*z1)-E1n/z0.*exp((i.*kk).*z1);
E1r=real(E1);
E1i=imag(E1);
H1r=real(H1);
H1i=imag(H1);
E2=E2pp.*exp((-i*kk).*z2)+E2n.*exp((i*kk).*z2);
H2=E2pp/z0.*exp((-i.*kk).*z2)-E2n/z0.*exp((i.*kk).*z2);
E2r=real(E2);
E2i=imag(E2);
H2r=real(H2);
H2i=imag(H2);
figure
subplot(4,1,1);
plot(z2,E2r, 'b',z1,E1r, 'b')
ylim([-2*10^(-13),2*10^(-13)]);
```

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pedia6.m | pedia7.m | pedia8.m | pedia9.m | +

```
ylabel('real E [V/m]');xlabel('z[m]');title('Real part of Electric Field Phasor');
subplot(4,1,2);
plot(z2,E2i, 'r',z1,E1i, 'r')
ylabel('imaginary E [V/m]');xlabel('z[m]');title('Imaginary part of Electric Field Phasor');
subplot(4,1,3);
plot(z2,H2r, 'black',z1,H1r, 'black')
ylabel('real H [V/m]');xlabel('z[m]');title('Real part of Magnetic Field Phasor');
subplot(4,1,4);
plot(z2,H2i, 'm',z1,H1i, 'm')
ylim([-5*10^(-16),5*10^(-16)]);
ylabel('imaginary H [V/m]');xlabel('z[m]');title('Imaginary part of Magnetic Field Phasor');
%Με πανομοιοτυπη διαδικασια για f=1.5GHz που ειναι ο 1ος μηδενισμος της
%οοριζουσας Α εχουμε
F=1.05*10.^9;
K=2*pi/c.*F;
z2=-d2:0.00001:0;
z1=0:0.00001:d1;
detAA=2/z0.*(exp(-2i.*K)-exp(2i*K));
E11p=K0*(1-exp(2i*K))./detAA;
E11n=K0*(1-exp(-2i*K))./detAA;
E22pp=-K0*(exp(-2i.*K)-1)./detAA;
E22n=K0*(1-exp(2i*K))./detAA;
E11=E11p.*exp((-i*K).*z1)+E11n.*exp((i*K).*z1);
H11=E11p/z0.*exp((-i.*K).*z1)-E11n/z0.*exp((i.*K).*z1);
E11r=real(E11);
E11i=imag(E11);
```

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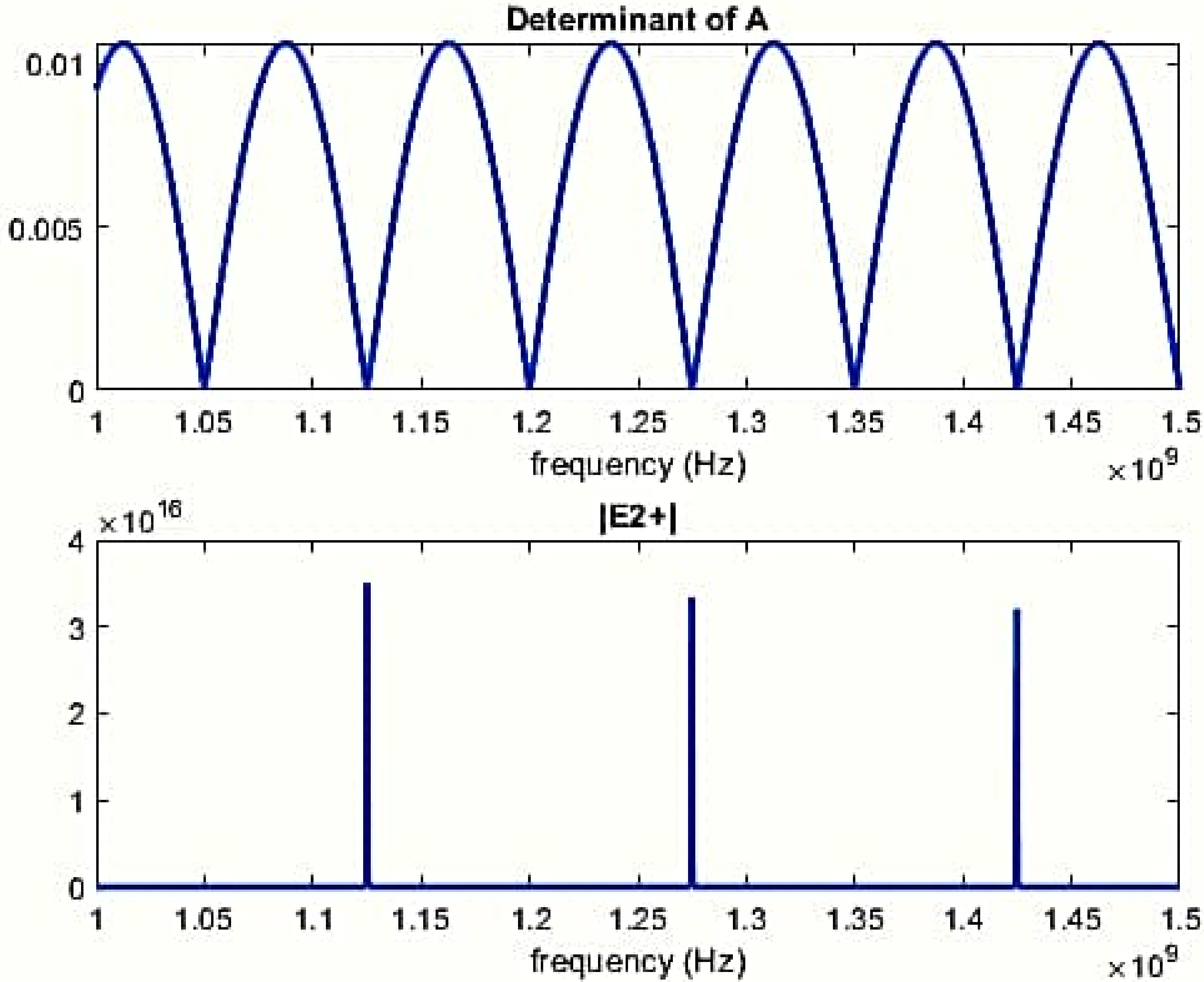
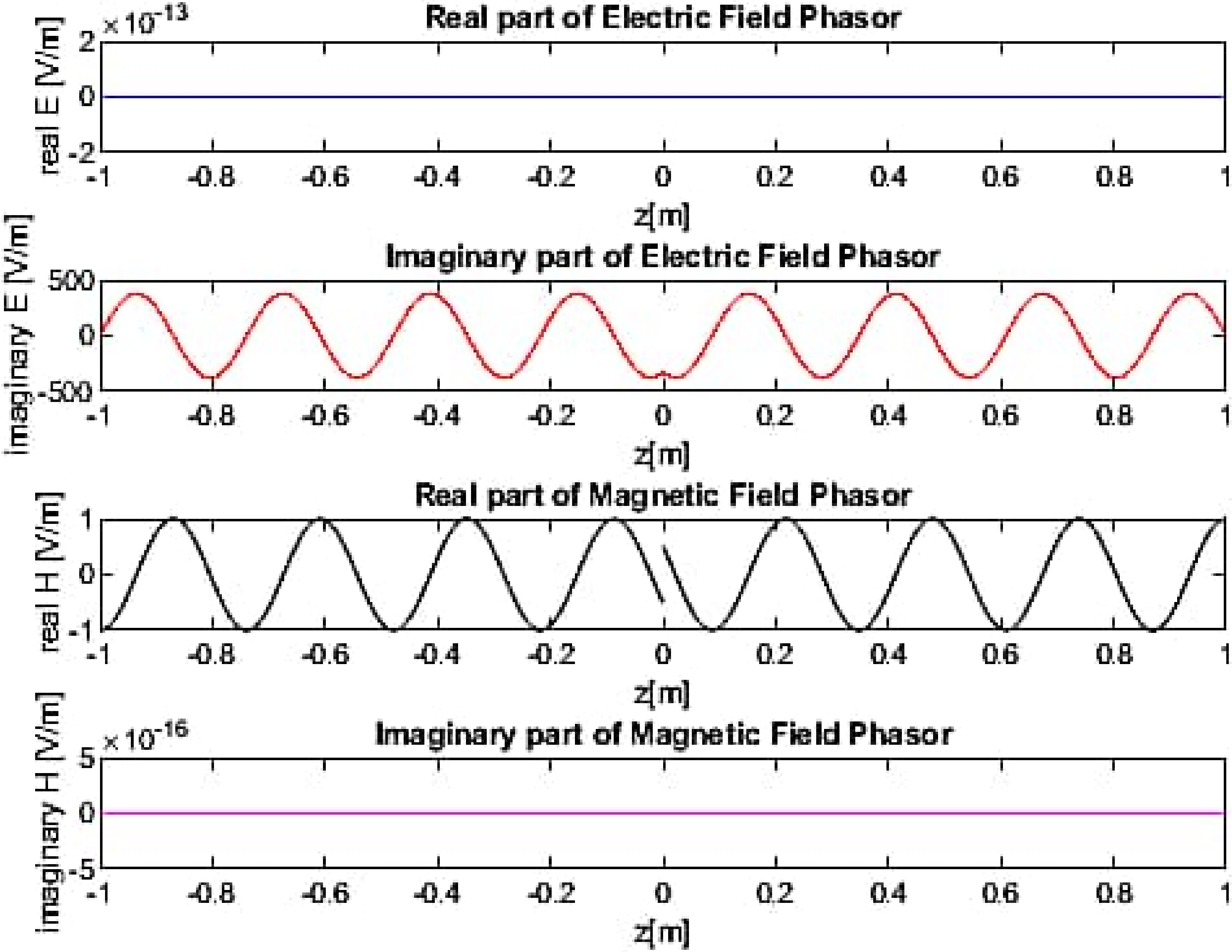
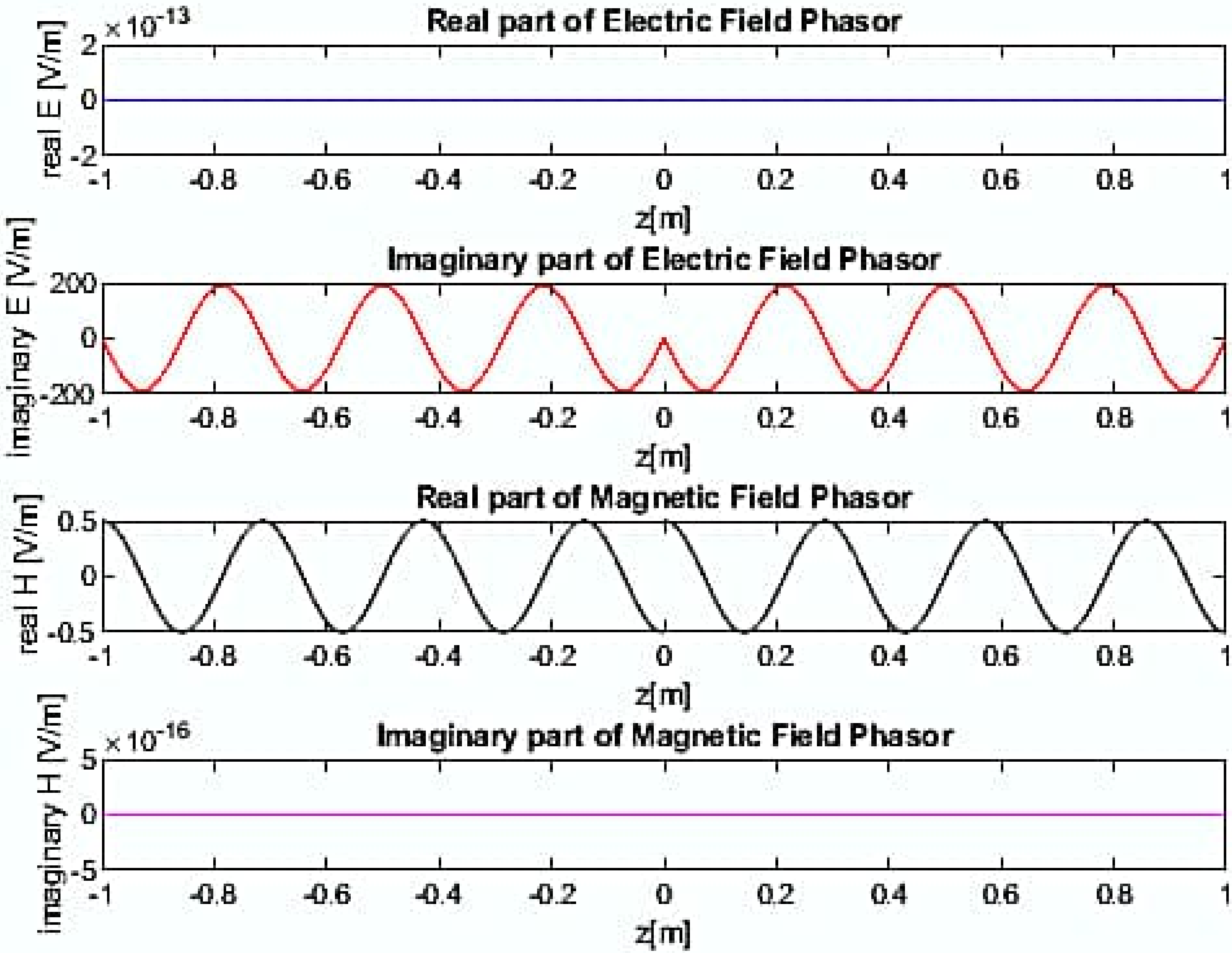
pedia6.m | pedia7.m | pedia8.m | pedia9.m

```
H11=E11p/z0.*exp((-i.*K).*z1)-E11n/z0.*exp((i.*K).*z1);
E11r=real(E11);
E11i=imag(E11);
H11r=real(H11);
H11i=imag(H11);
E22=E22pp.*exp((-i*K).*z2)+E22n.*exp((i*K).*z2);
H22=E22pp/z0.*exp((-i.*K).*z2)-E22n/z0.*exp((i.*K).*z2);
E22r=real(E22);
E22i=imag(E22);
H22r=real(H22);
H22i=imag(H22);
figure
subplot(4,1,1);
plot(z2,E22r, 'b',z1,E11r, 'b')
ylim([-2*10^(-13),2*10^(-13)]);
ylabel('real E [V/m]');xlabel('z[m]');title('Real part of Electric Field Phasor');
subplot(4,1,2);
plot(z2,E22i, 'r',z1,E11i, 'r')
ylabel('imaginary E [V/m]');xlabel('z[m]');title('Imaginary part of Electric Field Phasor');
subplot(4,1,3);
plot(z2,H22r, 'black',z1,H11r, 'black')
ylabel('real H [V/m]');xlabel('z[m]');title('Real part of Magnetic Field Phasor');
subplot(4,1,4);
plot(z2,H22i, 'm',z1,H11i, 'm')
ylim([-5*10^(-16),5*10^(-16)]);
ylabel('imaginary H [V/m]');xlabel('z[m]');title('Imaginary part of Magnetic Field Phasor');
```


Determinant of A
0.01
0.005
0
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
frequency (Hz)
×10^9
×10^16
|E2+|
4
3
2
1
0
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
frequency (Hz)
×10^9


Real part of Electric Field Phasor
×10^-13
real E [V/m]
2
0
-2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
z[m]
Imaginary part of Electric Field Phasor
imaginary E [V/m]
500
0
-500
z[m]
Real part of Magnetic Field Phasor
real H [V/m]
1
0
-1
z[m]
Imaginary part of Magnetic Field Phasor
×10^-16
imaginary H [V/m]
5
0
-5
z[m]


Real part of Electric Field Phasor
×10^-13
real E [V/m]
2
0
-2
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
z[m]
Imaginary part of Electric Field Phasor
imaginary E [V/m]
200
0
-200
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
z[m]
Real part of Magnetic Field Phasor
real H [V/m]
0.5
0
-0.5
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
z[m]
Imaginary part of Magnetic Field Phasor
×10^-16
imaginary H [V/m]
5
0
-5
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
z[m]