UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Saturday 30 May 2015 9.00 to 12.00 noon

PHYSICS B (Paper 1)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.

STATIONERY REQUIREMENTS
20-Page Booklets and Treasury Tags
Rough Work Pad
Yellow Cover Sheet

SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook (supplied)
Approved Calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 Find the maximum electrostatic energy that can be stored on an isolated metal sphere of radius 1 mm, if the breakdown strength of air is $3 \times 10^6\ \mathrm{V\,m^{-1}}$. [4]

A2 Two people stand diametrically opposite each other on the edges of a horizontal circular platform of radius a rotating about its axis at angular speed ω. One throws a ball directly at the other at speed $v \gg a\omega$. By what horizontal distance will the ball miss the second person? [4]

A3 Explain how the dynamic viscosity of a gas depends on pressure and temperature, giving a brief justification for each. [4]

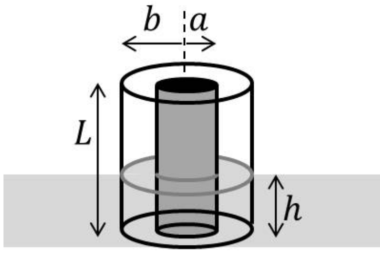
A4 A conducting ring can float above the end of a vertical solenoid in which an alternating current is flowing. Explain *qualitatively* why, giving relevant formulae. [4]

A5 A photon in the visible region is absorbed by a thermally isolated, macroscopic body at room temperature. Write down an expression for the change in entropy, and find the large factor by which the number of accessible microstates of the body increases. [4]

SECTION B

B6 State the equations governing the electrostatic fields D and E in a dielectric medium. Derive the boundary conditions satisfied by D and E at the interface between two dielectric media. [6]

A long solid metal cylindrical rod of radius a is surrounded concentrically by a thin metal cylinder of radius b, both of length L. These are placed vertically onto the flat bottom of a wide glass container into which a non-conducting liquid of density ρ and dielectric constant ε is poured, up to a height $h < L$.



Show that the capacitance of this arrangement is

$$C = \frac{2\pi\varepsilon_0}{\ln(b/a)}(L - h + \varepsilon h).$$ [6]

If a constant potential difference V is now applied between the rod and cylinder, find the change in the equilibrium height of the liquid enclosed. You may ignore capillary forces. [5]

Estimating the sensitivity in μm V^{-1}, discuss briefly how this arrangement might perform when used as a voltmeter. [3]

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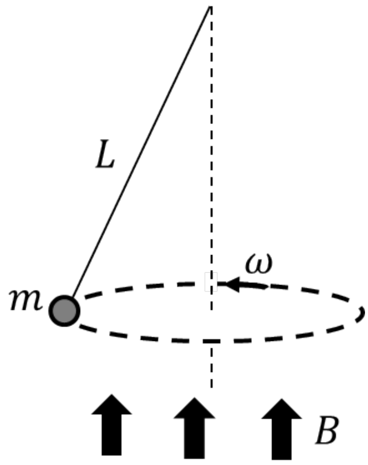
B7 Show the relationship between Faraday's law, Lenz's law and one of Maxwell's equations, defining all the terms used. [6]

A conducting wire of length L supporting a metal sphere of mass m and capacitance C is hung from a grounded rigid pivot and made to rotate as a conical pendulum at angular frequency ω. A uniform magnetic field B oriented vertically is then applied.



Assuming the sphere is heavy ($m \gg CB^2L^2$), show that the charge on the sphere

$$Q = \frac{1}{2}BC(\omega L^2 - g^2/\omega^3).$$ [7]

If a small electric field E aligned parallel to the magnetic field is now turned on, find the fractional change in the radius of the circular motion (to lowest order in E). [5]

Discuss briefly how the charge induced on the sphere might be measured in this system. [2]

B8 Explain what is meant by the impedance of a transmission line, and state how it depends on the inductance and capacitance per unit length. Outline why impedance is a useful concept. [4]

A transmission line of impedance Z_1 is connected to a second line of impedance Z_2. Using incident forward-travelling waves

$$\begin{aligned} V_i &= V_1 \exp\{\mathrm{i}(kz - \omega t)\} \\ I_i &= I_1 \exp\{\mathrm{i}(kz - \omega t)\} \end{aligned}$$

with corresponding backward-travelling waves, show that the voltage reflection coefficient is

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}.$$ [4]

A coaxial transmission line of outer radius b and inner radius a has impedance

$$Z = \frac{Z_0}{2\pi n} \ln(b/a)$$

when filled with a material of refractive index n. Two coaxial cables with identical inner radii and slightly different outer radii b and $b + \Delta b$ are connected together. Show that the fractional reflected power depends quadratically on Δb. Estimate the fractional tolerance in b needed to keep power reflections below 0.01% when manufacturing $50\,\Omega$ cables with $n = 2.3$. [5]

A long $50\,\Omega$ coaxial cable is pierced at regular intervals L by thin conducting needles which locally connect the inner to outer conductors with resistance $R_n \gg 50\,\Omega$. Find the voltage reflection coefficient r at each needle. Show that summing the reflected waves gives a total power reflection coefficient for microwaves of frequency ν

$$\left| \frac{r}{1 - (1 + r)^2 \mathrm{e}^{\mathrm{i}\phi}} \right|^2$$

carefully defining $\phi(\nu)$. [5]

How does this reflected power coefficient change with frequency? [2]

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B9 An electromagnetic wave of frequency f propagates down a rectangular waveguide of dimensions $a \times b$ where $a > b$, filled with a medium of dielectric constant ε (which can be varied). Find the electric field distribution of the lowest-order mode, assuming that the E-field is parallel to the guide wall of width b, and show that it satisfies the boundary conditions at all walls. [6]

Show that waves of frequency $f < c/(2a\sqrt{\varepsilon})$ have an evanescent form, with the electric field decaying by 1/e in a distance ℓ given by

$$\frac{1}{\ell^2} = \pi^2 \left\{ \frac{1}{a^2} - \frac{4\varepsilon f^2}{c^2} \right\}$$ [5]

For a fixed frequency $f = c/4a$, sketch a graph of $l(\varepsilon)$ showing how it changes as the dielectric constant of the medium is steadily increased, and describe what happens in each region of this graph. [3]

Find the corresponding magnetic field distribution of the lowest-order mode when $f < c/(2a\sqrt{\varepsilon})$, and hence the time-averaged power flow. Compare and contrast briefly how this situation compares to that of a wave experiencing the 'skin effect' at a bulk metal surface. [6]

SECTION C

C10 A single particle moves in an elliptical orbit under an attractive central force A/r^2. By considering the energy at the ends of the orbit, or otherwise, show that the semi-major axis a is related to the energy E via $a = -A/2E$. [4]

Explain how the position of two interacting bodies can be found from that of a single body with reduced mass moving about a fixed centre of force. [4]

Two stars A and B with equal mass m move under their mutual gravitational attraction. Initially, they are separated by a distance a_1, with A at rest and B moving with speed u at 90° to the line between them. Show that the maximum u for which the stars subsequently remain bound together is $u_E = 2\sqrt{Gm/a_1}$. [4]

In a particular case, u is determined to be $\frac{1}{2}u_E$. Show that the minimum subsequent separation is $a_2 = \frac{1}{3}a_1$, and find the velocity of B where this occurs. [4]

Find the period of the relative motion. [2]

Make a simple sketch of the path followed by each star. [2]

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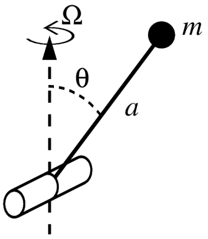
C11 Outline the Lagrangian formulation of dynamics, and compare it with the direct use of Newton's Laws. [5]

A small mass m is mounted on the end of a light rigid rod of length a, the other end of which is fixed to the midpoint of a light elastic horizontal rod rotating with a fixed vertical angular velocity Ω as in the diagram. The elastic rod applies a restoring torque $\mu\theta$ to the rigid rod when the latter is displaced by an angle θ from the upward vertical.



Show that the Lagrangian for the system is

$$L = \frac{1}{2}ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\Omega^2\sin^2\theta - \frac{1}{2}\mu\theta^2 - mga\cos\theta,$$

and find an expression for the acceleration $\ddot{\theta}$. [3]

In one such apparatus, the torsion constant μ is $2mga$. By approximating $\ddot{\theta}$ near $\theta = 0$, show that for $\Omega^2 < g/a$ there is an equilibrium at $\theta = 0$ around which the mass oscillates with angular frequency given by $\omega_1^2 = g/a - \Omega^2$. [3]

Sketch $\ddot{\theta}$ against θ over the range $0 < \theta < \pi/2$ for $\Omega^2 > g/a$, and show that in these circumstances the stable equilibrium moves to an angle θ_0 in this range. [3]

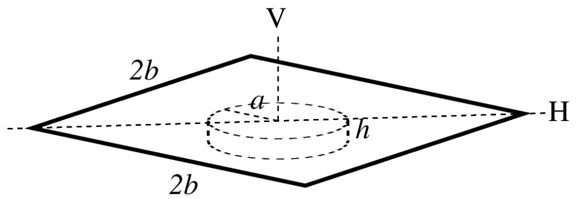
Find the value of Ω for which θ_0 is $\pi/4$, and the angular frequency ω_2 with which the mass oscillates around this angle. [4]

How does the system behave if $\Omega^2 = g/a$? [2]

C12 The relations between the normal components of stress and strain in an elastic material may be written in the form $Ee_1 = \tau_1 - \sigma\tau_2 - \sigma\tau_3$, with similar expressions for e_2 and e_3. Explain the quantities appearing in the equation with the aid of a diagram. [4]

Find, in terms of E and σ, the modulus defined by $M = \tau_1/e_1$ when the material is constrained so that $e_2 = e_3 = 0$. [3]

A rigid thin square plate with sides of length $2b$ and mass m is attached horizontally to a fixed table via a short light rubber cylinder with radius $a\,(< b)$ and height $h\,(\ll a)$ as in the diagram, by glue covering the full area of both ends.



Show that when the plate is twisted through a small angle ϕ about a vertical axis V through its centre, the restoring torque is $T_{\mathrm{V}} = \frac{\pi}{2}Ga^4\phi/h$, where G is the shear modulus. [3]

Show also that when the plate is instead tilted through a small angle θ about an axis H through its opposite corners, the restoring torque is $T_{\mathrm{H}} = \frac{\pi}{4}Ma^4\theta/h$. [4]

Find the angular frequencies ω_{H} and ω_{V} of small oscillations about these axes, neglecting any motion of the centre of mass. [4]

In a particular case, the ratio $\omega_{\mathrm{H}}/\omega_{\mathrm{V}}$ is found to be 7.9. Deduce the Poisson ratio for the rubber. [2]

[*Note: you may assume without proof that $G = E/2(1+\sigma)$.*]

C13 The shear stress necessary to sustain a velocity gradient in a viscous fluid may be written in the form

$$\tau = \eta \frac{\partial u}{\partial z}.$$

Explain the quantities in this relation with the aid of a diagram. [3]

An incompressible fluid with density ρ moves in the x-direction only, with velocity $u(z, t)$. Show that where gravitational forces are negligible, u satisfies the differential equation

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \eta \frac{\partial^2 u}{\partial z^2}, \qquad (\star)$$

where $p(x)$ is the pressure. [5]

Find the velocity $u(z)$ for *steady-state* flow driven by a uniform pressure gradient $p' = \partial p / \partial x$ between two wide plates of which one, at $z = 0$, is stationary and the other, at $z = a$, is moving with speed u_0 in the x-direction. [4]

Find also the total volume flow rate between the plates, per unit width. [2]

Show that, in the absence of any pressure gradient, a *time-dependent* velocity of the form

$$u(z, t) = \mathrm{Re}\left\{A e^{\mathrm{i}(kz - \omega t)}\right\}$$

also satisfies equation ($\star$) for an appropriate *complex* value of k, which you should find. Explain the motion this describes. [4]

A single wide plate immersed in a large tank of water oscillates in its own plane with amplitude 10 cm at frequency 1 Hz. At what distance from the plate does the fluid move with amplitude 2 mm? [*The viscosity of water is* $\eta = 1.00 \times 10^{-3}$ Pa s.] [2]

END OF PAPER