UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 7th June 2018 9.00 to 12:00 noon

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also to be completed listing all questions attempted.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
12-page Booklets and Treasury Tags	Physics Mathematical Formulae Handbook (supplied)
Rough Work Pad	Approved Calculators allowed
Yellow Cover Sheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 Explain why a waveguide cannot support TM_{n0} or TM_{0m} modes. [4]

A2 A domestic heat pump extracts heat from the ground at a temperature 5 °C and delivers heat to an underfloor heating system at temperature 25 °C. If the heat pump consumes 1 kW of electricity, how much heat is delivered to the heating system if the heat pump is perfect? [4]

A3 The position x of a non-relativistic particle at time t is given by the equation $x = At^n$ during a period of time. Find the potential energy of the particle as a function of x during this period of time. [4]

A4 Estimate the mean free path of the molecules in air at a temperature of 25 °C and a pressure of 100 kPa. [4]

A5 A defect in a material produces an electronic state which has energy ϵ. The state may be unoccupied, occupied by a spin up electron, occupied by a spin down electron or occupied by two electrons. The interaction energy of two electrons occupying the state is U, which can be positive or negative. Sketch the occupancy of the level as a function of the electronic chemical potential. [4]

[*The chemical potential can be assumed to be the same for spin up and spin down electrons and thermal effects can be ignored.*]

SECTION B

B6 Write brief notes on **three** of the following: [20]

(a) forces between currents;

(b) the continuity equation for electric charges;

(c) transformers;

(d) the magnetic vector potential.

B7 Write an essay on electrostatic fields in dielectric media. You should include in your answer discussions of polarisation, electric field and electric displacement, boundary conditions at interfaces and electrostatic energy. You should also discuss dielectric slabs, rods and spheres in uniform electric fields. [20]

B8 Write brief notes on **three** of the following: [20]

(a) the Hohmann transfer orbit;

(b) the Coriolis force;

(c) the Lagrangian formulation of dynamics;

(d) bending of beams.

B9 Write an essay on normal modes. Your answer should include the definition of a normal mode and discussions of methods for calculating normal modes, symmetry and the extent to which real systems exhibit normal modes. You should also discuss a variety of examples of normal modes. [20]

(TURN OVER

SECTION C

C10 The differential of the internal energy of a thin film of liquid with surface tension (force per unit length) γ and area A is $dU = TdS + \gamma dA$, where T is temperature and S is entropy. Given the corresponding relation for the Helmholtz free energy $F = U - TS$, derive the corresponding Maxwell relation [3]

$$\left.\frac{\partial \gamma}{\partial T}\right|_A = -\left.\frac{\partial S}{\partial A}\right|_T .$$

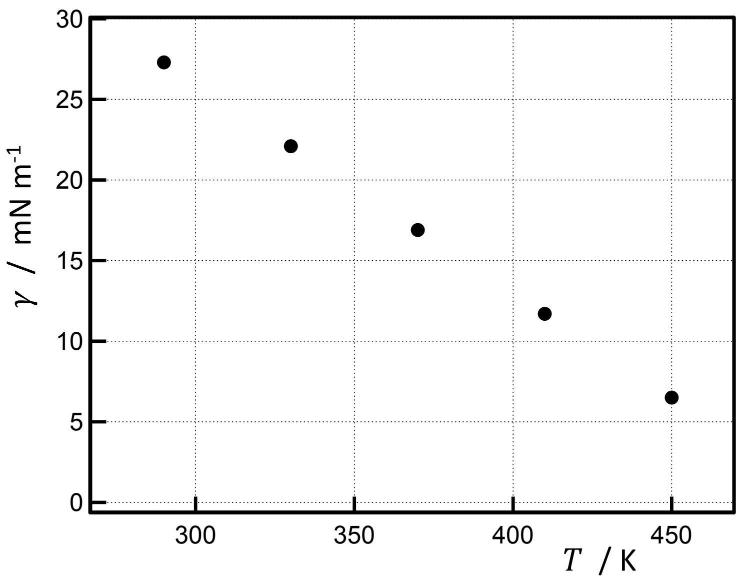
The surface tension of such a liquid is observed to fall with temperature, and to be independent of the area. Hence show that:

(a) the entropy rises when the area is increased isothermally; [2]

(b) the temperature falls when the area is increased adiabatically; [2]

(c) $dU = C_A dT + \left(\gamma - T\frac{d\gamma}{dT}\right) dA$, where C_A is the heat capacity of the film at constant area. [3]

The surface tension of the film is given by the graph below, and C_A is almost constant with a value of 70 mJ K^{-1} over the temperature range of interest.



Calculate the work done on the film, and the heat supplied to it, when the area is increased by 1 cm^2 reversibly and isothermally at temperature 330 K. [4]

Show that in reversible adiabatic changes, the relationship between temperature and area takes the form $C_A \ln T + \beta A =$ constant, and determine the quantity β. [3]

Calculate the temperature change, if the area of the film is increased by 1 cm^2 reversibly and adiabatically starting at 330 K, and comment on the change of internal energy in this process. [3]

C11 Discuss what is meant by *black body radiation*? [2]

For a cavity with rectangular cross-sections of sides $L_x \times L_y \times L_z$, show that the possible frequencies for radiation within it are [3]

$$\nu = \frac{c}{2\pi}\left[\left(\frac{\pi l}{L_x}\right)^2 + \left(\frac{\pi m}{L_y}\right)^2 + \left(\frac{\pi n}{L_z}\right)^2\right]^{1/2},$$

where c is the speed of light and l, m and n are positive integers.

Hence show that the number of modes in the range $\nu \rightarrow \nu + \mathrm{d}\nu$ is [3]

$$\mathrm{d}N = L_x L_y L_z \frac{8\pi\nu^2}{c^3}\mathrm{d}\nu.$$

Show that the average number of quanta in an oscillator of energy $h\nu$ is [2]

$$\frac{1}{\mathrm{e}^{h\nu/k_\mathrm{B}T} - 1}.$$

Hence show that the thermal energy of such a cavity is [4]

$$U = L_x L_y L_z \int_0^\infty \frac{8\pi\nu^2}{c^3}\,\frac{h\nu\,\mathrm{d}\nu}{\mathrm{e}^{h\nu/k_\mathrm{B}T} - 1},$$

and thus depends on the cavity size and temperature as

$$U \propto L_x L_y L_z T^4.$$

Sketch the spectral energy distribution of a black body for three different temperatures (low, intermediate and high). [2]

The black body energy of a narrow cavity with sides $L_x \gg L_z$ and $L_y \gg L_z$ can be modelled by considering the spectrum of radiation of a two-dimensional cavity. Show that at low temperatures the black body energy varies as [4]

$$U \propto L_x L_y T^3.$$

(TURN OVER

C12 Explain why the transport of heat through a material obeys the diffusion equation. [2]

A source delivers heat at a rate of H per unit volume to a material with a heat capacity c per unit volume and thermal conductivity κ. By considering the heat flux $J = -\kappa\nabla T$ through an arbitrary closed surface, derive the equation for thermal diffusion in the presence of a heat source, [4]

$$\frac{\partial T}{\partial t} = \frac{\kappa}{c}\nabla^2 T + \frac{H}{c}.$$

A long straight cylindrical wire with electrical conductivity σ and radius a carries a uniform current I. The temperature of its surface is maintained at T_0. Show that

(a) the rate of heating H is [3]

$$H = \frac{I^2}{\sigma\pi^2 a^4};$$

(b) the temperature inside the wire at a radius ρ is given by [6]

$$T(\rho) = T_0 + \frac{I^2}{4\pi^2 a^4 \kappa\sigma}(a^2 - \rho^2).$$

The same wire is then placed in air at temperature T_{air}. The wire loses heat from its surface according to Newton's law of cooling such that the heat flux per unit area from its surface is given by $\gamma(T(a) - T_{\mathrm{air}})$, where γ is a constant. Find an expression for $T(a)$ in the steady state. [5]

[*The Laplacian in cylindrical polar coordinates is:*

$$\nabla^2 A = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 A}{\partial\phi^2} + \frac{\partial^2 A}{\partial z^2}.$$
]

C13 Write an essay on the 3rd Law of Thermodynamics including the relation between Gibbs free energy and entropy as temperature tends to 0 K. You should also discuss the behaviour of ^{3}He and ^{4}He close to 0 K, and the liquefaction of gases. [20]

END OF PAPER