UNIVERSITY OF
CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 6th June 2019 9.00 to 12:00 noon

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also to be completed listing all questions attempted.

STATIONERY REQUIREMENTS
12-page Booklets
Rough Work Pad
Yellow Cover Sheet

SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook (supplied)
Approved Calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 A lead block has mass 0.5 kg and heat capacity 130 J/Kkg. Calculate the change in entropy of the Universe when: [4]

a) the block is at temperature 100 °C and placed in a lake at 10 °C;

b) the block is at 10 °C and dropped from a height of 200 m into a lake at 10 °C.

A2 Draw the $p - T$ phase diagram of water including the triple point and describe the key features. [4]

A3 A waveguide with cut-off frequency 20 GHz is used to transmit a signal which consists of pulses of 40 GHz radiation. Find the phase and group velocities of the signal. [4]

A4 A lossless coaxial cable consists of an inner conductor of radius 3 mm and an outer conductor of radius 6 mm. The space between the conductors is filled with non-magnetic dielectric material of relative permeability $\epsilon_r = 2$. Calculate the capacitance per unit length and the inductance per unit length of the cable. [4]

A5 An audio-frequency transformer is used to transform the 4 Ω resistance of a secondary load into a resistance that equals the 80 Ω output resistance of the voltage generator attached to the primary coil. The primary coil has a self inductance of 0.75 H. The magnetic core of the transformer is a toroid of average radius 10 cm, cross-sectional area 30 cm^2 and relative permeability $\mu_r = 500$.

If the impedance of the primary coil is ≥ 200 Ω, calculate the output impedance and the minimum number of primary and secondary turns that must be wound onto the core so as to achieve impedance matching between the voltage generator and the secondary load. [4]

[*You may assume an ideal transformer with perfect coupling and zero coil resistance.*]

SECTION B

B6 Write brief notes on **three** of the following: [20]

(a) electromagnetic waves from Maxwell's equations;

(b) dielectrics;

(c) skin depth in conducting materials;

(d) transmission lines.

B7 Write an essay on magnetic materials. You should include a discussion of the different types of magnetic materials, magnetisation, permeability and their application in electromagnets. [20]

B8 Write brief notes on **three** of the following: [20]

(a) rotating frames and fictitious forces;

(b) elasticity of beams and Euler's strut instability;

(c) potential flow past a sphere;

(d) tidal forces and tides.

B9 Write an essay on rigid body dynamics. Your narrative should include an introduction to the tensor of inertia, Euler's equations, and applications to describing the motion of a lamina and a symmetric top. [20]

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SECTION C

C10 A van der Waals equation of state of 1 mole of an interacting gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

where P is the pressure, V the total available volume, a and b are empirical constants, T is the temperature and R the gas constant. Rearrange the van der Waals equation such that we have an expression for the pressure as function of all other parameters. Discuss the meaning of the resulting expression. [3]

Now consider the expression for the pressure in the limit $a \approx 0$. Use a Maxwell relation to obtain the volume dependence of the entropy of the system obeying this equation of state. [2]

In the limit $V \to \infty$, the entropy of the van der Waals gas $S^{\text{vdW.gas}}(V,T)$ approaches the ideal gas entropy $S^{\text{id.gas}}(V,T)$. Use this information to derive an expression for the excess entropy [4]

$$\Delta S^{\text{ex.}} \equiv S^{\text{vdW.gas}}(V,T) - S^{\text{id.gas}}(V,T)$$

at volume V and temperature T for a van der Waals gas obeying the equation of state $P = RT/(V - b)$.

Discuss the effect of the parameter a in the van der Waals equation of state on $\Delta S^{\text{ex.}}$. [2]

The van der Waals gas has a liquid-to-vapour transition at low temperature. Sketch a low-temperature isotherm for which a liquid and vapour phase coexist and annotate the main features. [3]

Derive the volume of the denser phase (the liquid) at this low temperature. [6]

C11 State the Clausius and Kelvin formulations of the 2nd Law of Thermodynamics and sketch the corresponding heat engine representation. [2]

During a momentary fluctuation, heat can flow from cold to hot. Does this observation indicate that there is a limit to the applicability of the 2nd Law of Thermodynamics? Explain your answer. [2]

A heat pump (assumed to be ideal) can deliver up to 0.5 kW of power. The heat pump is used to heat a small house. If the temperature inside the house has to be maintained at T_{high} = 293 K and the temperature outside is T_{low} = 273 K, what is the maximum amount of heat that the ideal heat pump can deliver per unit time? [4]

Now consider the house, maintained at a temperature $T_{\text{high}} = 293K$, is losing heat to the environment (temperature T_{low}) at a rate [10]

$$\dot{q} = \lambda(T_{\text{high}} - T_{\text{low}}) ,$$

with λ = 0.1 kW/K. What is the lowest outside temperature for which the heat pump can maintain the inside temperature at 293 K?

Calculate the power that a conventional gas heater would have to deliver to maintain the same inside temperature. [2]

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C12 Consider a one-dimensional model polymer chain consisting of a large number N of links, each of length a, such that the fully stretched chain is $L_0 = Na$. The joints between the links are completely flexible and have no excluded volume. Such a chain can be modelled as a random walk with each step having equal probability to step 'left' or 'right' along the linear polymer axis.

Calculate the number Ω of possible arrangements of the links of the chain, if the ends of the chain are separated by a distance $L < L_0$. [3]

Approximate the entropy S of the chain as a function of L using Stirling's formula. [4]

Show that the tension in the chain, $\mathcal{F}$, can be written as [6]

$$\mathcal{F} = -T\left(\frac{\partial S}{\partial L}\right)_T = \frac{k_B T}{Na^2}L\,.$$

Now consider the case where the number of links stepping right N_R is much larger than the number of links stepping left (N_L). Show that [3]

$$\Omega \approx \frac{N^{N_L}}{N_L!}\,.$$

Calculate the resulting tension in the chain as function of N_L in this limit. [3]

What happens in the limit $N_L \to 0$? [1]

[*Stirling's formula is:* $\ln N! \approx N\ln N - N$]

C13 Write an essay comparing the definitions of temperature in classical thermodynamics and statistical mechanics. Explain how Boltzmann's constant can be obtained. [20]

END OF PAPER