UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Tuesday 31st May 2022 1.30 pm to 4.30 pm

PHYSICS B Paper 1

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A carries approximately 20% of the total marks. In Sections B and C, each question carries approximately the same number of marks.

The paper contains **6** *sides including this one.*

You may refer to the Mathematical Formulae Handbook supplied, which gives values of constants and contains mathematical formulae which you may quote without proof. You may also use an approved calculator.

If you are taking the examination in person, answers from ***each*** *Section should be written in separate Booklets. Write the letter of the Section and your candidate number, not your name, on the cover of* ***each*** *Booklet. If more than one Booklet is used for a given Section, join them together using a Treasury Tag. A separate master (yellow) cover should also be completed, listing all questions attempted in the paper.*

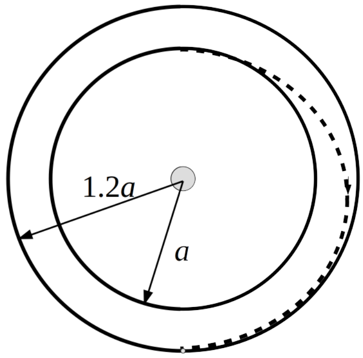
If you taking the examination online, ***each*** *Section should be scanned or photographed after the Examination and uploaded in a* ***separate*** *file, according to the instructions provided. Before submitting your answers, ensure that all pages are of sufficient image quality to be readable.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

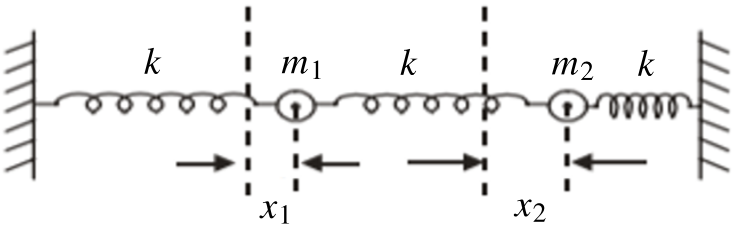
SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 A geostationary satellite is transferred into a circular orbit with 20% greater altitude, using an elliptical transfer orbit which is tangential to the two circular orbits as shown in the diagram below. How long does it take the satellite to reach its new orbit? [4]



A2 Two masses m_1 and m_2 are connected to each other and to two fixed points by three identical springs of force constant k as shown in the figure below. By considering the forces (or otherwise) on m_1 and m_2 when they are displaced by distances x_1 and x_2 respectively, find the angular frequencies of the normal modes for oscillations of small amplitude in the direction parallel to the springs. Gravity can be neglected. Describe the motions of the two bodies for each normal mode. [4]



A3 An electron is accelerated from rest through a potential difference of 150 V. What velocity does the electron acquire? [4]

A4 A 1 μF parallel plate capacitor filled with a dielectric medium of relative permittivity $\epsilon = 5$ is charged to 2500 V and then disconnected from the charging source. The dielectric is then removed; what is the resulting change in the stored energy? [4]

A5 A rectangular waveguide with cut-off frequency 10 GHz is used to transmit a signal which consists of pulses of 30 GHz radiation. Find the phase and group velocities at the signal frequency. [4]

SECTION B

B6 In an inertial frame S_0, the equation of motion for a rigid body which has an angular momentum $\boldsymbol{J}$ acted on by a couple $\boldsymbol{G}$ is

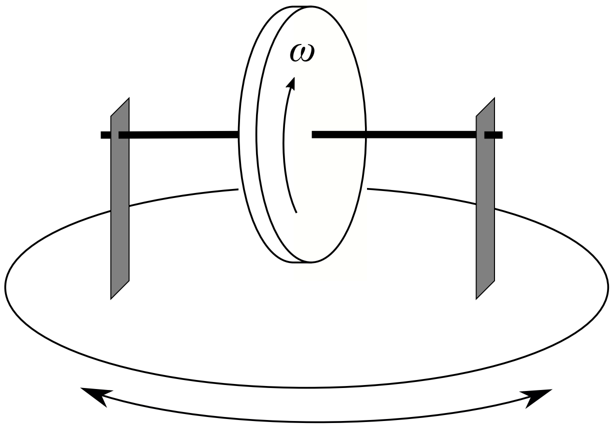
$$\boldsymbol{G} = \left.\frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t}\right|_{S_0}.$$

(a) Explain why, in a frame S which is rotating at an angular velocity $\boldsymbol{\Omega}$ relative to S_0, this equation can be rewritten as [4]

$$\boldsymbol{G} = \left.\frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t}\right|_{S} + \boldsymbol{\Omega} \times \boldsymbol{J}.$$

(b) Give an example of an application of this. [3]

A gyroscopic compass consists of a flywheel which rotates with a high angular velocity $\boldsymbol{\omega}$ about an axle through its centre. The axle is supported at each end such that it is constrained to lie in the local horizontal plane but free to rotate about a vertical axis, as shown below.



(c) If the compass is located at a latitude λ on the Earth's surface, show that, provided $\lambda \neq 90°$, the compass is only in equilibrium if its axle is aligned north–south. [4]

(d) Show that, for small angular deviations from the equilibrium orientation, the compass executes simple harmonic motion with an angular velocity of ω_{shm}, with [6]

$$\omega_{\mathrm{shm}}^2 \approx \frac{I_1}{I_2}\omega\Omega_{\mathrm{E}}\cos(\lambda),$$

where Ω_{E} is the angular velocity of the Earth's rotation, I_1 is the flywheel moment of inertia about its axle, and I_2 is its moment of inertia about any axis perpendicular to its axle through its centre of mass.

(e) If the moments of inertia are dominated by the flywheel, which is thin, and $\omega = 5000$ revolutions per minute, estimate ω_{shm} for the latitude of Cambridge ($\lambda = 52.2°$). [3]

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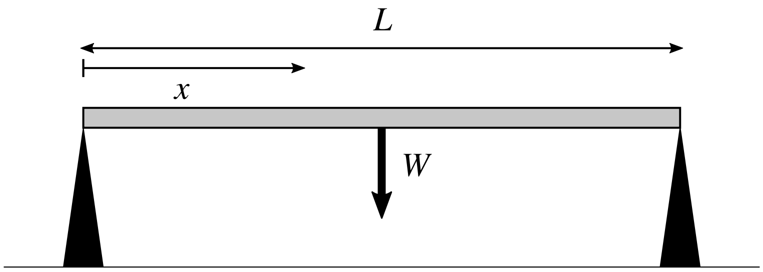
B7 (a) Explain what is meant by the term ‘bending moment’, B, and define the ‘moment of area’, I, applied to a beam under stress. [3]

(b) Show that, for small deflections, the radius of curvature, R, of a bent beam is [4]

$$R = \frac{EI}{B},$$

where E is Young’s modulus of the beam.

Consider a *light* uniform beam of length L, where the weight of the beam can be neglected. The beam rests on supports at each end of equal height, and has a downward load of W applied at its centre, as shown below.



(c) Show that the bending moment as a function of the distance along the beam, x, is given by [4]

$$B(x) = \begin{cases} -\frac{1}{2}xW & \text{for } 0 < x < \frac{1}{2}L, \\ -\frac{1}{2}W(L-x) & \text{for } \frac{1}{2}L < x < L. \end{cases}$$

Now consider a *heavy* uniform beam of length L, where the weight of the beam cannot be neglected. The beam has a rectangular cross section, with horizontal width a and vertical thickness b, density ρ, and again rests on supports at each end of equal height. There is no downward load on the beam apart from its own weight.

(d) Show that the bending moment as a function of x, due to the weight of the beam is [4]

$$-\tfrac{1}{2}\rho abgx(L-x).$$

(e) Assuming the deflections are small, find an expression for the vertical deflection from the horizontal for this heavy beam due to its own weight, as a function of x. [5]

SECTION C

C8 Consider a medium with relative permittivity and permeability $\epsilon = \mu = 1$ which is permeated by electric and magnetic fields.

(a) Explain the physical significance of the Poynting vector $\boldsymbol{N} = \boldsymbol{E}\times\boldsymbol{H}$, where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field strengths respectively. [2]

(b) Write down an expression for the electromagnetic energy density U in terms of $\boldsymbol{E}$ and $\boldsymbol{H}$. [3]

(c) Given that the rate of energy dissipation in a volume V is

$$\int_V \boldsymbol{E}\cdot\boldsymbol{J}\,\mathrm{d}V,$$

where $\boldsymbol{J}$ is the current density, prove Poynting's theorem [5]

$$\int_S \boldsymbol{N}\cdot\,\mathrm{d}\boldsymbol{S} = -\int_V \boldsymbol{E}\cdot\boldsymbol{J}\,\mathrm{d}V - \frac{\mathrm{d}}{\mathrm{d}t}\int_V U\,\mathrm{d}V,$$

where the surface S encloses the volume V.

A long straight cylindrical wire of radius a, length L (with $L \gg a$) and electrical conductivity σ carries a steady current I uniformly distributed across its cross-sectional area.

(d) Obtain an expression for the Poynting vector at a distance $r \leq a$ from the centre of the wire. [5]

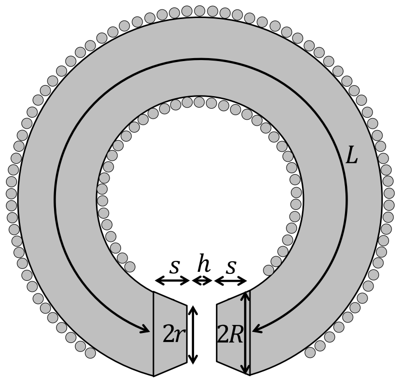
(e) Obtain an expression for the resistive (Ohmic) losses in the wire. [2]

(f) Derive the flux of the Poynting vector through the wire's surface, and explain the connection between this result and the result in part (e). [3]

C9 (a) Derive the boundary conditions for the magnetic flux density $\boldsymbol{B}$ and the magnetic field strength $\boldsymbol{H}$ at the interface between two magnetic materials with different relative magnetic permeabilities. [5]

Consider a toroidal electromagnet filled with a magnetic material of large relative permeability μ (i.e., $\mu \gg 1$). The torus contains a small air gap of length h. Over most of its length the torus has a circular cross section of radius R, but towards the gap the torus is tapered on both of its ends, such that its radius changes from R to r over a distance s towards the gap, with $s \gg (R - r)$.

The electromagnet has N windings through which a constant current of I is flowing. The length L of a circular arc which lies in the untapered part of the magnet's material is as shown in the figure below. You can assume $L \gg R$, and therefore L does not vary significantly within the material.



(b) Explain why the magnetic flux across the cross section of the torus is to a very good approximation conserved along the total length of the torus and within the gap. [3]

(c) Show that the magnetic field strength inside an untapered magnet (i.e. one with $s = 0$ and $r = R$) with no gap (i.e. $h = 0$) is [2]

$$H_{\text{in}} = \frac{NI}{L}.$$

(d) For an untapered magnet with a gap (i.e. $s = 0, r = R$ and $h \neq 0$), show that the magnetic field strength inside the gap is [3]

$$H_{\text{gap}} = \frac{\mu NI}{\mu h + L}.$$

(e) Show that the magnetic field strength inside the gap for the tapered magnet is [5]

$$H_{\text{gap}} = \frac{\mu NI}{\mu h + L\left(\frac{r^2}{R^2}\right) + 2s\left(\frac{r}{R}\right)}.$$

(f) By considering the ratio of the magnetic field strength inside the gap of an electromagnet with tapered ends to that of an otherwise identical electromagnet which is untapered, explain the benefit of the tapered ends in the limit $s/L \ll r/R$. [2]

END OF PAPER