UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Friday 28th May 2021 1.00 pm to 4.00 pm

PHYSICS B Paper 1

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A carries 20% of the total marks. In Sections B and C, each question carries the same number of marks.

The paper contains **6** *sides including this one.*

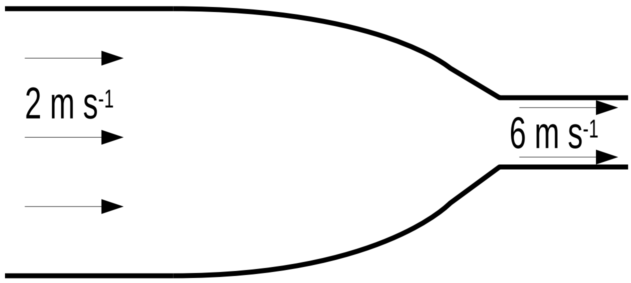
SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook allowed
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 Water enters a Venturi tube with velocity 2 m s^{-1} and exits with velocity 6 m s^{-1} as shown in the diagram below.



Assuming the flow of water is laminar, calculate the change in pressure. [4]

A2 The moment of inertia tensor for a rigid body is:

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ kg m}^2.$$

Find the principal moments of inertia of the body. [4]

A3 A parcel is dropped from a stationary helicopter 200 m above the ground at the equator. The pilot intends the parcel to land at the point vertically below the helicopter, however, they forgot to account for the Coriolis force. By how much does the parcel miss its mark and in which direction? [4]

A4 What is the relationship between Young's modulus E, the bulk modulus B and the Poisson ratio σ for an isotropic material under pressure? What restriction does this place on the value of σ? [4]

A5 An ideal monatomic gas at temperature 400 K is adiabatically expanded so that the final volume is eight times the original volume. What is the final temperature of the gas? [4]

SECTION B

B6 (a) State Kepler's laws of planetary motion. [3]

A spacecraft of mass m orbits a mass $M \gg m$ at a distance that varies between a minimum of r_p and maximum of r_a. The speed of the spacecraft at these two extreme distances is v_p and v_a respectively.

(b) Give the energy E and the angular momentum J of spacecraft at the two extreme distances. [4]

(c) Hence show that [4]

$$J^2 = \frac{2GMm^2}{\left(\dfrac{1}{r_\mathrm{p}} + \dfrac{1}{r_\mathrm{a}}\right)}.$$

A spacecraft orbits the Earth in a circle of radius r_0 at a speed v_0.

(d) Explain, including a diagram, how it is possible to transfer the spacecraft to a larger circular orbit using two short burns of the spacecraft rocket engine that increase its speed, but do not change its direction of motion. [3]

The spacecraft rocket engine contains enough fuel to change its *linear* momentum by mv_0, either in a single short burn, or in a series of short burns. Any change in mass of the spacecraft due to a burn is negligible, and the change of linear momentum is proportional to the amount of fuel used.

(e) Show that in first burn of the orbital transfer, about a quarter of the available fuel is used when the larger circular orbit has a radius of $3r_0$. [3]

(f) Find how much fuel is used in the second burn to place the satellite in the circular orbit with radius $3r_0$. [3]

B7 (a) Explain what is meant by a *normal mode* of a mechanical system. [2]

A sphere is suspended from a thin vertical fibre of length ℓ_1. If one end of the fibre is twisted by an angle ϕ, it produces a restoring torque that is proportional to ϕ/ℓ_1. It is found that the sphere undergoes torsional oscillations at an angular frequency ω_0. An identical sphere is now suspended vertically below the first sphere using another length ℓ_2 of the same fibre.

(b) If the angular displacements of the first and second spheres are ϕ_1 and ϕ_2, show that the equations of motion are [4]

$$\ddot{\phi}_1 = -\omega_0^2 \left[\phi_1 - \frac{\ell_1}{\ell_2}(\phi_2 - \phi_1) \right]$$

and

$$\ddot{\phi}_2 = -\omega_0^2 \frac{\ell_1}{\ell_2}(\phi_2 - \phi_1).$$

(c) Hence show that the frequencies ω of the torsional normal modes of the system are given by [4]

$$\omega^2 = \omega_0^2 \left(\frac{1}{2} + \frac{\ell_1}{\ell_2} \pm \sqrt{\frac{1}{4} + \left(\frac{\ell_1}{\ell_2}\right)^2} \right).$$

(d) Sketch how ω^2 varies as a function of the ratio $R = \ell_2/\ell_1$, and give the values for ω^2 when R tends to zero, and R tends to infinity. [6]

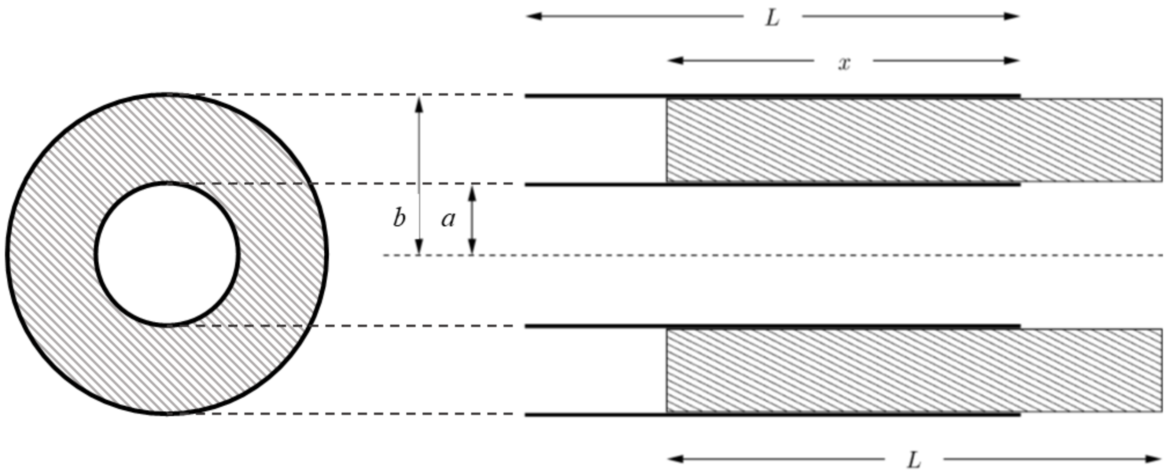
(e) Interpret in terms of simpler physical systems the lower frequency mode when R tends to zero, and the higher frequency mode when R tends to infinity. [4]

SECTION C

C8 (a) Given a free charge density ρ in a dielectric medium with relative premittivity ϵ, write down the expressions for the electrostatic fields $\boldsymbol{D}$ and $\boldsymbol{E}$. [2]

(b) Derive the boundary conditions satisfied by $\boldsymbol{D}$ and $\boldsymbol{E}$ at an interface between two dielectric media. [2]

The diagram shows the cross section and longitudinal section of a cyclindrical capacitor, with its axis horizontal. It is length L and with inner radius a and outer radius b. A uniform, solid dielectric material of relative permittivity ϵ and length L fills a length x of the capacitor in the region between the two cylinders, where $x \gg b$.



Cross section *Longitudinal section*

(c) Show that the capacitance of the system is [6]

$$C = \frac{2\pi\epsilon_0}{\ln(b/a)}[L - x + \epsilon x]$$

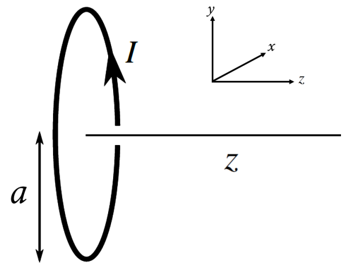
(d) By considering energy, or otherwise, find an expression for the force, both magnitude and direction, experienced by the dielectric, when the capacitor is under a constant potential difference V. [4]

(e) If the dielectric is allowed to fill the capacitor fully under a constant potential difference V, find the change of total charge on the capacitor from that of the initial configuration, as illustrated above. [3]

(f) By sketching the $\boldsymbol{E}$ field lines, explain qualitatively why a force on the dielectric arises when the capacitor is under a constant potential difference V. [3]

C9 (a) State the Biot–Savart law, and give an example of its application in calculating magnetic flux density $\boldsymbol{B}$. [4]

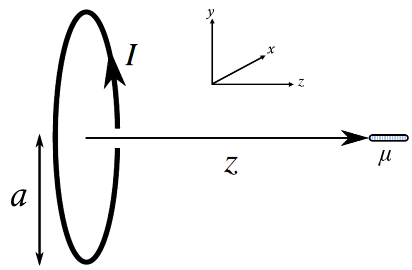
A loop of radius a lying in the x–y plane carries a static current I.



(b) Show that the magnetic flux density $\boldsymbol{B}$ at the point $(0, 0, z)$ is given by $(0, 0, B_z)$, where [4]

$$B_z = \frac{\mu_0 a^2 I}{2(a^2 + z^2)^{3/2}}.$$

A thin rod of magnetic material of relative permeability μ and volume V lies along the z-axis with its centre at $(0, 0, z)$.



(c) Assuming that the length of the rod is small compared with its distance to the loop along z-axis, show that the magnetisation is [3]

$$M = \frac{(\mu - 1)a^2 I}{2\mu(a^2 + z^2)^{3/2}}.$$

(d) Find the force $\boldsymbol{F}$ that the magnetic rod experiences. [5]

(e) What happens to the force if the direction of current in the loop is reversed. [2]

(f) Explain qualitatively how the strength of this force would change if the rod were replaced by a body of the same volume and material but a different shape. [2]

END OF PAPER