UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Saturday 23th May 2015 9.00 am to 12.00 noon

PHYSICS A (1)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C, and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C, and D, each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains **6** *sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae, which you may quote without proof.*
Answers from **each** *Section should be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Booklets and treasury tags	Mathematical Formulae Handbook
Rough workpad	Linear graph paper
Yellow master coversheet	Approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof. Use a separate booklet for the whole of this section.*

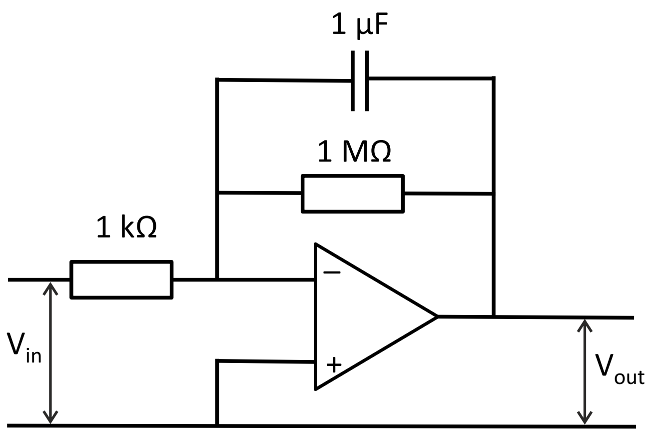
1 A particle is described by a wavefunction $\psi(x) = a + ix$ in the region $0 < x < a$ and $\psi = 0$ elsewhere. Calculate the probability of finding the particle in the region $a/2 < x < a$. [4]

2 Sketch the wavefunctions and corresponding energies of the eigenstates of a quantum harmonic oscillator, labelling your diagram to explain the key physical features. [4]

3 Show that, in spherical polar coordinates, the operator for the z-component of the orbital angular momentum is given, in spherical coordinates, by $-i\hbar(\partial/\partial\phi)$. [4]

4 Stellar magnitudes, m, are determined by the equation $m = -2.5\ \log_{10}(f/f_0)$, with f the stellar flux, and f_0 a normalisation constant. Given a fractional error of 10% in the stellar flux measurement, find the corresponding fractional error in m. [4]

5 For the following system containing an ideal op-amp, draw an annotated sketch of the gain $|V_{\text{out}}/V_{\text{in}}|$ as a function of frequency from 0.01 Hz to 100 Hz for a sinusoidal input. [4]



SECTION B

Attempt **two** *questions from this Section. Use a separate booklet for the whole of this section.*

B6 The particle flux associated with wavefunction ψ is given by

$$J = \Re\left[\psi^* \frac{\hbar}{i\,m}\nabla\psi\right],$$

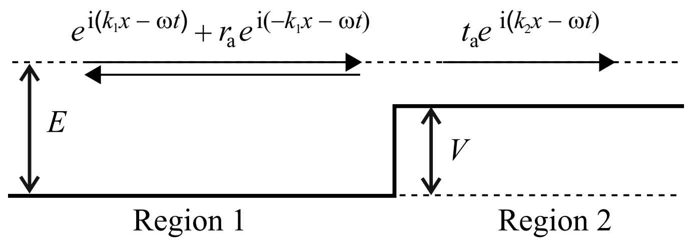
where m is the mass of the particle. Find the particle flux associated with a plane wave $Ae^{i(kx-\omega t)}$. [3]

A region of space contains two superimposed plane waves propagating in opposite directions:

$$\psi = Ae^{i(kx-\omega t)} + Be^{i(-kx-\omega t)},$$

where A and B are complex constants. By explicitly evaluating the expression for particle flux above, show that the net particle flux in this region is equal to the difference in fluxes of the forward- and backward-propagating waves in isolation. [6]

A particle of kinetic energy E and mass m is incident on a potential step of height V, as shown below. Calculate the amplitude reflection coefficient, r_a, and the amplitude transmission coefficient, t_a, as a function of E and V. [5]



Show that the net particle flux in Region 1 is equal to the transmitted particle flux in Region 2. [3]

Evaluate the transmission probability for electrons, when $E = 3$ eV and $V = 2$ eV. [3]

B7 An infinitely deep quantum well extends from $x = 0$ to $x = a$, and contains a particle of mass m. Find the energies and normalised wavefunctions of the eigenstates of the system, as a function of the quantum number $n = 1, 2, 3 \ldots$. [6]

A particle is in the ground state of a quantum well of width a. The quantum well suddenly expands to a width $2a$, leaving the wavefunction unchanged. The energy of the system is then immediately measured. Calculate the probability that the system will be found in the ground state of the expanded well. [6]

At time $t = 0$, the system is in an eigenstate of the Hamiltonian, $|\phi_n\rangle$, with energy E_n. Using the time-dependent Schrödinger equation, find the wave function at time t. [4]

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For the quantum well system above, describe in general terms how you would calculate the probability of finding the system in the new ground state if a time t elapses between the expansion and the measurement. [4]

B8 Show that the Hamiltonian for a particle of mass m in a one-dimensional potential $\frac{1}{2}m\omega^2x^2$ can be written in the form

$$\hat{H} = \left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\hbar\omega\ ,$$

where $\hat{a} = \sqrt{m\omega/2\hbar}\,(\hat{x} + i\hat{p}/m\omega)$ and $\hat{a}^\dagger = \sqrt{m\omega/2\hbar}\,(\hat{x} - i\hat{p}/m\omega)$. [5]

Evaluate the commutators $\left[\hat{a}, \hat{a}^\dagger\right]$, $\left[\hat{H}, \hat{a}\right]$ and $\left[\hat{H}, \hat{a}^\dagger\right]$. [5]

The state $|\phi\rangle$ is an eigenstate of $\hat{H}$ with energy E. Show that $\hat{a}^\dagger|\phi\rangle$ is also an eigenstate of $\hat{H}$, and find its energy. [4]

Given that $\hat{a}|0\rangle = 0$, find (unnormalised) wavefunctions for the ground state, $|0\rangle$, and for the first excited state, and verify that these are orthogonal. [6]

B9 For a system of two identical particles, explain what constraint is placed on the overall wavefunction by considering the operation of exchanging the two particles. [3]

For a system of two fermions whose wavefunction can be written as the product of a spatial part and a spin part, explain what are the possible combinations of exchange symmetries for the spatial and spin parts of the wavefunction. [3]

Consider the hydrogen molecule H_2. Explain what is meant by *para*-hydrogen and *ortho*-hydrogen. [2]

The total angular momentum of a hydrogen molecule is described by the quantum number ℓ. Show that ℓ can only take even values for *para*-hydrogen and odd values for *ortho*-hydrogen. [*The spherical harmonic functions $Y_{l,m}(\theta, \phi)$ have the property $Y_{l,m}(\pi - \theta, \phi + \pi) = (-1)^l Y_{l,m}(\theta, \phi)$.*] [4]

When liquid hydrogen is prepared, it consists of 25% *para*-hydrogen with $\ell = 0$ and 75% *ortho*-hydrogen with $\ell = 1$. Over a period of days, the *ortho*-hydrogen converts to the lower-energy *para*-hydrogen state. Calculate the energy released when this occurs in 1 kg of liquid hydrogen. [*The bond length in the hydrogen molecule is* 74 pm.] [6]

Comment on the consequences for storage of liquid hydrogen. [2]

SECTION C

Attempt **one** *question from this Section. Use a separate booklet for this section.*

C10 Write an essay on the use of the density operator in quantum mechanics. [20]

C11 Write brief notes on **two** of the following:

(a) double-slit experiments and their implications for quantum mechanics; [10]

(b) orbital angular momentum in quantum mechanics; [10]

(c) the Schrödinger equation. [10]

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SECTION D

Attempt **one** *question from this Section. Use a separate booklet for this section.*

D12 In 1929, Edwin Hubble measured the speeds V of nine galaxies that are moving away from us. These measurements, as a function of the distances d to these galaxies (which Hubble assumed to be known accurately), are given in the Table below.

distance d (kpc)	speed V (km s^{-1})
50	40
650	170
800	380
900	500
1100	750
1400	780
1600	730
1650	1000
2000	1100

The parsec (pc) is a unit of distance used in astronomy.

Plot these measurements on a suitable graph of V against d. [6]

Hubble decided that his measurements showed that $V = Hd$, where H is a constant of proportionality. Find the value of H in units of km s^{-1} Mpc^{-1} from the above data by explicitly minimising χ^2; in this context, χ^2 is given by [8]

$$\chi^2 = \sum_i \frac{(V_i - Hd_i)^2}{\sigma^2} ,$$

where i refers to each measurement and σ is the measurement error in speed V_i, assumed to be constant.

Estimate the error in H from the above data. [6]

D13 Write brief notes on **two** of the following:

(a) methods for reducing systematic errors; [10]

(b) Bayes' theorem, with an example of its use; [10]

(c) shot noise. [10]

END OF PAPER