UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Saturday 28th May 2022 (3 hours)

PHYSICS A (1)

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A will carry approximately 20% of the total marks. In Sections B, C, and D, each question carries approximately the same number of marks. The paper contains **5** *sides including this one.*

You may refer to the Mathematical Formulae Handbook supplied, which gives values of constants and contains mathematical formulae which you may quote without proof. You may also use an approved calculator.

If you are taking the exam in person, answers from **each** *Section should be written in separate Booklets. Write the letter of the Section and your candidate number, not your name, on the cover of* **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag. A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.*

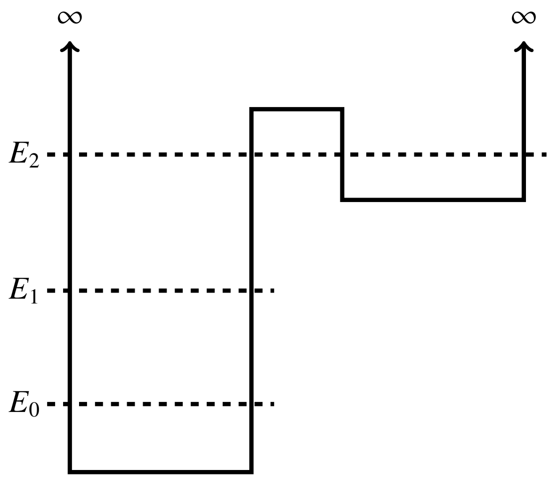
If you are taking the exam online, **each** *Section should be scanned or photographed after the Examination and uploaded in a* **separate** *file according to the instructions provided. Before submitting your answers, ensure that all pages are of sufficient image quality to be readable.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 A particle moves in the potential shown below. Given that the lowest three energy eigenstates have energies E_0, E_1, E_2 as marked, sketch the corresponding wavefunctions.



[4]

2 Estimate the order of magnitude of the distance scale in metres that can be probed at the Large Hadron Collider using a beam of protons with an energy of 6.5 TeV. [4]

3 A particle is described by a wavefunction $\psi(x) \propto \sin x + i(1 - \cos x)$ in the region $0 < x < 2\pi$ and $\psi(x) = 0$ elsewhere. Calculate the probability of finding the particle in the region $0 < x < \pi/2$. [4]

4 A muon detector gives a signal if traversed by a muon with a probability of 90%, and gives a false signal if traversed by any other type of particle with a probability of 20%. The detector is used to select muons from a beam of particles that contains 95% of muons. What is the fraction of misidentified particles in the selected sample? [4]

5 A smartphone tuning app is able to tune the fifth string of a guitar to 110 Hz with a precision of 0.07 Hz. Estimate the minimum sampling frequency and sampling time needed for this task. [4]

SECTION B
Attempt **both** *questions from this Section.*

B6 An isotropic simple harmonic oscillator has hamiltonian

$$\widehat{H} = \frac{\widehat{p}_x^2 + \widehat{p}_y^2}{2m} + \frac{m\omega^2}{2}(\widehat{x}^2 + \widehat{y}^2).$$

(a) Deduce real-positive values $\alpha_x, \alpha_y, \beta_x$, and β_y such that the operators

$$\begin{aligned}\widehat{a}_x &= \alpha_x \widehat{x} + i\beta_x \widehat{p}_x,\\ \widehat{a}_x^\dagger &= \alpha_x \widehat{x} - i\beta_x \widehat{p}_x,\\ \widehat{a}_y &= \alpha_y \widehat{y} + i\beta_y \widehat{p}_y,\\ \widehat{a}_y^\dagger &= \alpha_y \widehat{y} - i\beta_y \widehat{p}_y\end{aligned}$$

satisfy the commutation relations

$$[\widehat{a}_x, \widehat{a}_x^\dagger] = [\widehat{a}_y, \widehat{a}_y^\dagger] = 1,$$

with all other commutators vanishing, and such that the hamiltonian is proportional to

$$(\widehat{a}_x^\dagger \widehat{a}_x + \widehat{a}_y^\dagger \widehat{a}_y + 1).$$

[3]

(b) Show that the eigenvalues of $\widehat{a}_x^\dagger \widehat{a}_x$ are given by the non-negative integers and find the energy spectrum of the hamiltonian. [4]

(c) Denoting the eigenstate of $\widehat{a}_x^\dagger \widehat{a}_x$ with eigenvalue n by $|n\rangle$, show that one may choose $\widehat{a}_x^\dagger |n\rangle = \sqrt{n+1}|n+1\rangle$ and $\widehat{a}_x|n\rangle = \sqrt{n}|n-1\rangle$. [4]

(d) Show that the operator

$$\widehat{Q} := -i\hbar(\widehat{a}_x^\dagger \widehat{a}_y - \widehat{a}_y^\dagger \widehat{a}_x)$$

is hermitian and, by expressing it in terms of $\widehat{x}, \widehat{y}, \widehat{p}_x$, and $\widehat{p}_y$, give a physical interpretation of it. [3]

(e) Find the possible eigenvalues of $\widehat{Q}$ for energy eigenstates with energy $3\hbar\omega$. [6]

B7 A particle of mass m is confined to move on a circular ring of radius R.

(a) Explain why the hamiltonian may be written as

$$\widehat{H} = -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\phi^2},$$

where ϕ is the angular co-ordinate. [3]

(TURN OVER

(b) Show that the energy eigenvalues are given by

$$\frac{\hbar^2 n^2}{2mR^2},$$

where n is an integer, and find the normalized energy eigenstates. [4]

Suppose now that a second particle is added to the ring, with the same mass as the first particle. The two particles do not interact with one another.

(c) Assuming the particles are spinless and distinguishable, what are the degeneracies of the ground state, the first excited state, the fourth excited state, and the state with energy $\frac{25\hbar^2}{2mR^2}$? [7]

(d) Assuming instead that the particles are spin-half and indistinguishable, and denoting the two possible spin states for particle i by $|\uparrow\rangle_i$ and $|\downarrow\rangle_i$, with $i \in \{1, 2\}$, write down the possible two-particle wavefunctions for the ground state and first excited state. Specify clearly the degeneracies in each case. [6]

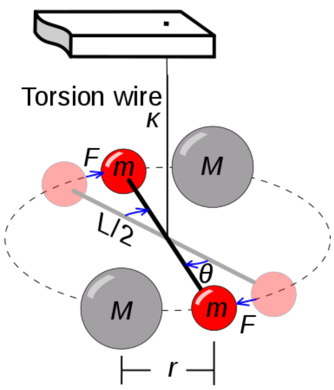
SECTION C

Attempt the question from this Section.

C8 Henry Cavendish first measured the gravitational force between masses in the laboratory using a torsion balance like that shown in the figure. The gravitational pull between the big and small metal spheres, of mass M and m respectively, makes the hanging rod of length L rotate against the force exerted by the wire torsion. At equilibrium, the force balance of the system is such that

$$\frac{\kappa}{L}\theta = G\frac{mM}{r^2}$$

where θ is the rotation angle, r is the distance between the centres of the big and small spheres, and κ and G are constants. A group of Part IB Physics students tried to measure the gravitational constant, G, using a similar apparatus with $m = 38$ g, $M = 1.5$ kg and $L = 10$ cm, obtaining the following measurements:



r (mm)	θ (mrad)
16	66
18	55
22	44
32	25

The uncertainty in the rotation angle measurement is 2 mrad and the uncertainty in r can be considered to be negligible.

(a) The torsion constant of the wire, κ, can be obtained from a measurement of its oscillation period, T, through

$$T = 2\pi\sqrt{\frac{mL^2}{2\kappa}}.$$

The period is measured to be 498.2 ± 6.0 seconds. Calculate the value and uncertainty of κ. [4]

(b) Explain how the data can be expressed so that, if the model is correct, the measurements would lie on a straight line of the form $y = cx$, where c is a constant with dimensions of area. [2]

(c) Using a least squares minimisation, show that the best estimate of the constant c in the previous model, $\widehat{c}$, is given by

$$\widehat{c} = \frac{\overline{xy}}{\overline{xx}}.$$

What is the best fit value of G according to this model? Obtain an estimate of the error in $\widehat{c}$ using the error quadrature formula. [6]

(d) Consider an alternative model of the type $y = c'x + d$. Using a χ^2 test determine which model provides a better description of the data. What can be inferred regarding systematic errors in the measurement from the estimate of d? [8]

SECTION D

Attempt the question from this Section.

D9 Write an essay on time dependence in quantum mechanics. You should include discussions of state propagation, stationary states, expectation values, Ehrenfest's theorem and the time evolution operator. [20]

END OF PAPER