UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 8th June 2017 9.00 to 12:00 noon

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also to be completed listing all questions attempted.

STATIONERY REQUIREMENTS
12-page Booklets and Treasury Tags
Rough Work Pad
Blue Cover Sheets
Yellow Cover Sheet

SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook (supplied)
Approved Calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 What is the lowest-frequency electromagnetic wave that can propagate along a waveguide of square cross-section with side 1 cm? [4]

A2 A block of copper has a heat capacity of 50 J K^{-1} and is initially at a temperature of 510 °C when it is dropped into a lake at 10 °C. Calculate the change in the entropy of the block and of the Universe. [4]

A3 A mass is constrained to move in a horizontal circle by means of a light rope fixed at one end. The mass is moving with speed v when the rope encounters a vertical pole of radius R, with the initial point of contact a distance L from the instantaneous position of the mass. The rope then winds itself around the pole. Assuming that gravity can be neglected, how long does it take after the time of impact of the rope on the pole until the mass hits the pole? [4]

A4 If M_{AB} is the magnetic flux linked by circuit A due a 1 A current in circuit B. Explain why $M_{AB} = M_{BA}$. [4]

A5 Sketch the velocity field created in a fluid by a large number of circular vortex loops each of radius r. The axes of the loops coincide and the centres of the loops are positioned with a constant spacing l ($\ll r$). [4]

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SECTION B

B6 Write brief notes on **three** of the following: [20]

(a) Maxwell's displacement current;

(b) the dielectric properties of plasmas;

(c) paramagnetism and diamagnetism;

(d) the Poynting vector.

B7 Write an essay on transmission lines. You should include in your answer an analysis of transmission lines in terms of circuit elements, diagrams of the electric and magnetic fields associated with two different transmission line geometries and discussions of characteristic impedance and the effect of different terminations of the transmission line. [20]

B8 Write brief notes on **three** of the following: [20]

(a) the Euler strut;

(b) gravitational slingshots;

(c) Euler's equations for the motion of a rigid body;

(d) Archimedes' Principle.

B9 Write an essay on fluid flow. Your answer should include a discussion of the convective derivative, Bernoulli's equation and its derivation, vorticity, potential flow and viscous fluid flow. [20]

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SECTION C

C10 Describe the Carnot cycle, and derive its efficiency. [4]

The operation of a particular heat engine is described by the Diesel cycle. The working substance is one mole of an ideal, diatomic gas, with $C_p/C_V = 1.4$. The gas has an initial volume V_1 and initial temperature T_1. It is first adiabatically compressed until it reaches a volume V_2. It then expands at constant pressure until it reaches a volume V_3 before undergoing an adiabatic expansion until it again reaches the volume V_1. It then gives out heat at constant volume until it returns to its initial state. Sketch the Diesel cycle in the $p-V$ plane. [2]

Calculate the temperatures at the ends of each part of the cycle in terms of V_1, V_2, V_3 and T_1. [4]

Write down the expressions for the heat flowing in or out of the gas for each part of the Diesel cycle and hence show that the efficiency of the cycle, η, is given by

$$\eta = 1 - \frac{1}{\gamma}\left[\left(\frac{V_3}{V_1}\right)^{\gamma} - \left(\frac{V_2}{V_1}\right)^{\gamma}\right] \times \left(\frac{V_3}{V_1} - \frac{V_2}{V_1}\right)^{-1}$$

where $\gamma = C_p/C_V$. [7]

Calculate the efficiency of the heat engine in the limit $V_3 \to V_2$ and compare this with the efficiency of a Carnot engine operating between the maximum and minimum temperatures in the cycle. [3]

C11 What is an equation of state in thermodynamics? Discuss how p, V, T equations of state can be used to identify whether a substance undergoes a phase transition. [3]

A mole of material is described by the equation of state

$$p(V - b) = RT,$$

and has a molar heat capacity C_V that is independent of temperature. Sketch the $p-V$ diagram of the system, and discuss the physical significance of b. [3]

Show that:

(a) the internal energy is a function of temperature only; [5]

(b) the ratio $\gamma = C_p/C_V$ is independent of temperature and pressure; [3]

(c) in an adiabatic change $p(V - b)^{\gamma}$ is constant. [4]

Does this system display a phase transition? [2]

[*You may use the Maxwell relation*

$$\left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial p}{\partial T}\right|_V .$$

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C12 The Maxwell–Boltzmann distribution for the particle speed v in a gas is

$$f(v) \propto v^2 \exp\left(-\frac{mv^2}{2k_\mathrm{B}T}\right).$$

Explain why it has this form, and show that the mean particle speed $\overline{v}$ and the mean of the square of the particle speed $\overline{v^2}$ are related by $\overline{v^2} = 3\pi/8(\overline{v})^2$. [4]

Consider a perfect gas with number density n. Show that the number of atoms striking unit area in unit time is $\frac{1}{4}n\bar{v}$, where $\bar{v}$ is the mean particle speed. [4]

Show that the mean kinetic energy of the atoms striking the wall of a container holding gas at temperature T is $2k_\mathrm{B}T$. [4]

Two identical large vessels containing argon initially at temperature T_0 and pressure p_0 are connected by a hole of diameter much less than the mean free path. One vessel is then heated to T_1 while the other remains at T_0. Under steady-state conditions, what pressure difference is then established between the two vessels? [4]

Now consider the same two containers. Initially one is filled with argon atoms with number density n_0, and the other with helium atoms with the same number density n_0. The temperatures of both containers are kept constant at T_0. Sketch how the number densities of argon and helium atoms in the two containers vary with time. Describe what would happen if the temperature of the two containers were different. [4]

[*If* $I_n = \int_0^\infty x^n \mathrm{e}^{-\alpha x^2}\,\mathrm{d}x$, *then*

$$I_n = \frac{n-1}{2\alpha} I_{n-2}, \quad I_1 = \frac{1}{2\alpha}, \quad I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}.$$

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C13 Write an essay on the transport properties of gases. You should include in your answer discussions of momentum (viscosity), heat and mass transport, and the dependence of the corresponding transport coefficients on the average velocities and temperature. You should also outline the derivation of the diffusion equation for heat transport and discuss a number of its solutions. [20]

END OF PAPER

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