UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 27th May 2021 (3 hours)

PHYSICS A (1)

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A will carry approximately 20% of the total marks. In Sections B, C, and D, each question carries approximately the same number of marks. The paper contains **5** *sides including this one.*

You may refer to the Mathematical Formulae Handbook supplied, which gives values of constants and contains mathematical formulae which you may quote without proof. You may also use an approved calculator.

If you are taking the exam in person, answers from **each** *Section should be written in separate Booklets. Write the letter of the Section and your candidate number, not your name, on the cover of* **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag. A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.*

If you are taking the exam online, **each** *Section should be scanned or photographed after the Examination and uploaded in a* **separate** *file according to the instructions provided. Before submitting your answers, ensure that all pages are of sufficient image quality to be readable.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 The normalised wave function for a particle in a one-dimensional box with infinite height walls at $x = -\frac{L}{2}$ and $x = +\frac{L}{2}$ is given by

$$\Psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right).$$

Calculate the probability of finding the particle within ±0.1 nm of the centre of the box if $L = 1$ nm. [4]

2 An electron in a plane-wave state of kinetic energy 8 eV is normally incident upon a rectangular potential barrier of height 10 eV and width 1 nm. Estimate the probability that the electron will tunnel through the barrier. [4]

3 Sketch a Stern-Gerlach apparatus and explain why, in general, it splits a beam of neutrons into two. [4]

4 An experiment is set up to detect dark matter particles. After 5 months of data taking, 8 events have been recorded. The fake signal rate is estimated to be 0.3 events per month. In order to claim a discovery, the probability of the observed signal being fake is required to be smaller than 10^{-6}. Explain whether this experiment can claim a discovery of dark matter.

[4]

5 A voltage signal with frequency spectrum covering the range 0-40 MHz is sampled with a digitiser that has quantization levels separated by 0.1 mV. What is the mimimum sampling rate that allows the signal to be faithfully reconstructed with at least 0.05 mV precision? [4]

SECTION B
Attempt **both** *questions from this Section.*

B6 An electron in a hydrogen atom experiences a magnetic field due to the presence of the proton, which appears as a current loop in the electron's rest frame. The magnetic moment of the electron interacts with this field producing an additional term in the Hamiltonian

$$\widehat{H}_1 = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\widehat{\boldsymbol{L}}\cdot\widehat{\boldsymbol{S}},$$

where m is the mass of the electron, r is the distance between the electron and the proton, V is the Coulomb potential energy, and $\widehat{\boldsymbol{L}}$ and $\widehat{\boldsymbol{S}}$ are the electron's orbital and spin angular momenta.

(a) Explain why the expectation value of $\widehat{H}_1$ can be written as

$$\langle\widehat{H}_1\rangle = \frac{\hbar^2}{4m^2c^2}[j(j+1) - l(l+1) - s(s+1)]\langle\frac{1}{r}\frac{dV}{dr}\rangle,$$

where j is the total angular momentum quantum number. [6]

(b) Calculate $\langle\widehat{H}_1\rangle$ in eV for an electron in the $l = 1$, $s = 1/2$, $j = 1/2$ state given that the radial part of the wavefunction is

$$\psi = \frac{1}{4\sqrt{6\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}\exp(-r/2a_0),$$

where $a_0 = 4\pi\epsilon_0\hbar^2/(me^2) = 5.29\times10^{-11}$ m is the Bohr radius and e is the charge of the electron. [8]

An electron is now bound in a three-dimensional harmonic oscillator potential.

(c) Sketch how the shifts in energy of the $l = 0$ and $l = 1$ states due to $\widehat{H}_1$ depend on the angular frequency of the oscillator. [6]

[*ψ is normalised such that $\int\psi\psi^*4\pi r^2dr = 1$. The integral $\int_0^\infty x^n\exp(-\alpha x) = n!/\alpha^{n+1}$ may be assumed.*]

B7 The potential energy of a one-dimensional harmonic oscillator of mass m and angular frequency ω can be written as

$$V(x) = \frac{m\omega^2x^2}{2}.$$

Using the raising and lowering operators,

$$\widehat{a}_\pm = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega\widehat{x} \mp i\widehat{p}),$$

show that for any eigenstate

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(a) the expectation values of the position and momentum are zero; [3]

(b) the expectation values of the potential and kinetic energies are each equal to $\frac{1}{2}\hbar\omega\left(n+\frac{1}{2}\right)$, where n is the quantum number of the state; and [5]

(c) the uncertainties Δx and Δp in position and momentum are related by

$$\Delta x \Delta p = \left(n+\frac{1}{2}\right)\hbar.$$

[4]

A superposition of the lowest two eigenstates in the one-dimensional harmonic oscillator at time $t = 0$ can be written as

$$|\Psi\rangle = \cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|1\rangle,$$

where θ and ϕ are constants.

(d) Show that the time-dependence of the position expectation value oscillates with angular frequency ω. What is the magnitude of the oscillation? [8]

[*You may assume that* $\widehat{a}_-|n\rangle = \sqrt{n}\,|n-1\rangle$ *and* $\widehat{a}_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$.]

SECTION C

Attempt the question from this Section.

C8 An experiment measures the rate of D^0 decays, R, as a function of the D^0 proper time, t. The following results are obtained,

t (ps)	R (counts/day)
0.25	1339
0.75	360
1.25	110
1.75	31
2.25	14
2.75	3

The fractional uncertainty in the rate is the constant value $\sigma = 0.15$ and the uncertainty in the measured proper time can be considered negligible. The following model is considered to describe this data,

$$R = R_0\, e^{-t/\tau}$$

where R_0 is a normalisation constant and τ is the lifetime of the D^0 particle.

(a) Explain how the data can be expressed so that, if the model is correct, the measurements would lie on a straight line of the form $y = mx + c$, where m and c are constants. [2]

(b) Perform such a transformation, and tabulate the values of x, y and the uncertainty on y, σ_y, corresponding to each measurement. [3]

(c) Using a least squares minimisation, determine estimates $\widehat{m}$ and $\widehat{c}$ of the constants m and c, and obtain the uncertainties in those estimates. How do these relate to the estimates and uncertainties of the model parameters, i.e. R_0 and τ ? [7]

(d) Given the fractional uncertainty in the count rates, σ, estimate the time required to acquire these data, assuming that the measurements for each proper time value are carried out independently. [4]

A different experiment determines the lifetime of the D^0 to be 0.478 ± 0.023 ps.

(e) Explain how the χ^2 test can be used to determine whether the data given above are compatible with this alternative measurement. What value of τ provides a better fit to the data? [4]

SECTION D

Attempt the question from this Section.

D9 Write an essay on the photoelectric effect, blackbody radiation and the Davisson and Germer experiment, and discuss how these represent an evidence of the failure of classical mechanics and how they are explained in quantum mechanics. [20]

END OF PAPER