UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Friday 18th June 2021 1.00 pm to 4.00 pm

PHYSICS B Paper 2

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A carries 20% of the total marks. In Sections B and C, each question carries the same number of marks.

The paper contains **5** *sides including this one.*

SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook allowed
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 Describe what the Van der Waals equation below represents and explain the meanings of the terms a and b; [4]

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT.$$

A2 State the third law of thermodynamics and explain using diagrams/plots the unattainability of absolute zero. [4]

A3 What is the lowest frequency of electromagnetic radiation which can propagate in a rectangular waveguide with cross-section with sides of 40 and 20 mm? [4]

A4 A magnetic dipole, $\boldsymbol{m}$, is rotated in a uniform magnetic flux density, $\boldsymbol{B}$. The angle between $\boldsymbol{m}$ and $\boldsymbol{B}$ is θ. Draw a plot of the potential energy of the dipole as a function of θ. [4]

A5 Two isolated spherical conductors of radii 50 and 100 mm are charged to 2 and 4 kV, respectively. They are then connected by a fine resistive wire. How much heat is generated in the wire in total, if its resistance is sufficient to overdamp the system? [4]

SECTION B

B6 Write brief notes on **three** of the following: [20]

(a) the free precession of a symmetric top;

(b) Lagrangian dynamics;

(c) the elastic bending of beams;

(d) the Magnus effect.

B7 Write an essay on waves in conducting media. You should include in your answer the distinction between plasmas and metals, as well as discussions on refractive index and the skin effect. [20]

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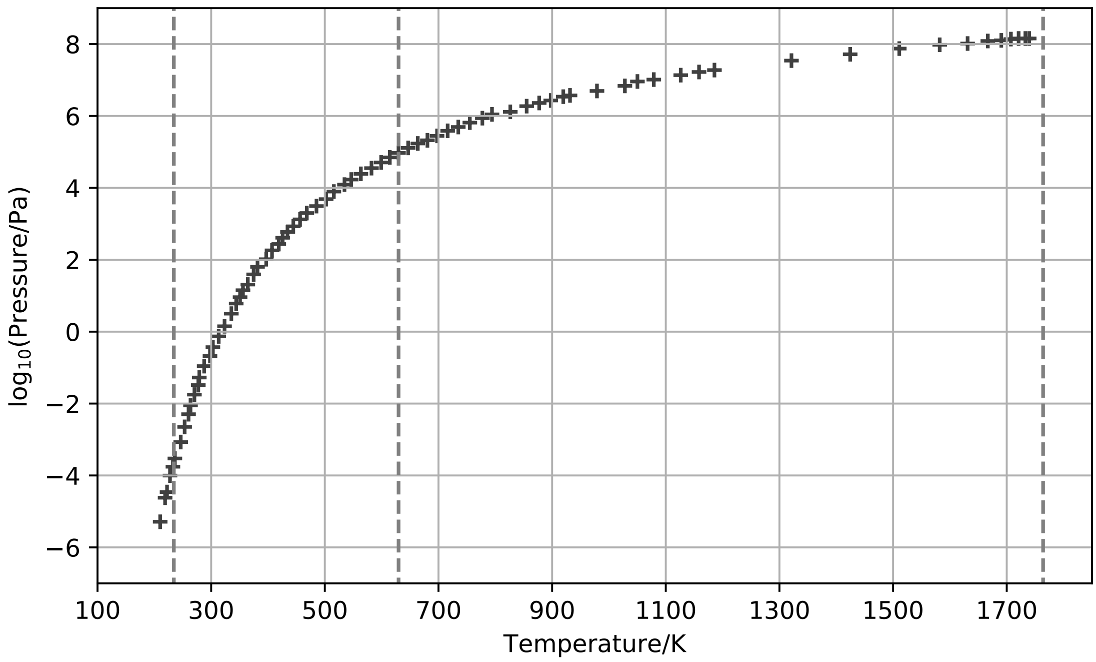
SECTION C

C8 (a) Define all the terms in the Clausius–Clapeyron equation for the boundary between two phases: [3]

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{T\Delta V}.$$

(b) Outline a derivation of the Clausius–Clapeyron equation, explaining the conditions under which it is valid. [4]

The plot below shows a set of measurements of the vapour pressure of Mercury. The vertical dashed lines indicate, from left to right, the temperatures of the triple point, the boiling point and the critical point.



(c) Show that Clausius–Clapeyron equation predicts that the vapour pressure above a liquid obeys the approximate equation [6]

$$p = p_0 e^{-L/RT}.$$

State the assumptions required, and explain why these are physically reasonable for liquid Mercury in equilibrium with its vapour between 300 and 500 K.

(d) Estimate the latent heat of vapourisation of Mercury at around 400 K. [3]

(e) Sketch a labelled phase diagram for Mercury in the p–T plane which includes a solid, a liquid and a gas phase (you may assume that Mercury contracts when it freezes). [4]

[*The gas constant* $R = 8.31\ \mathrm{J\,K^{-1}mol^{-1}}$.]

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C9 The partition function Z can be written

$$Z = \sum_j g_j \exp\left(\frac{-\epsilon_j}{k_\mathrm{B}T}\right).$$

(a) Describe the thermodynamical model to which this applies, and define all the terms in this expression. [5]

(b) Show that the internal energy of a system U_sys can be written as [3]

$$U_\mathrm{sys} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta},$$

where $\beta = 1/(k_\mathrm{B}T)$.

A quantum mechanical rotor can be used to describe the rotational states of a freely spinning molecule. The rotor has energy levels

$$\frac{\hbar^2 J(J+1)}{2I},$$

where I is the moment of inertia of the rotor and J is an integer ≥ 0. These energy levels have degeneracy $2J+1$. Thus the 3 lowest energy levels have energies of 0, $\hbar^2/I$ and $3\hbar^2/I$ respectively and degeneracies of 1, 3, and 5 respectively.

(c) Write down an expression for the partition function for the rotor. [2]

(d) Show that the partition function for the rotor has the form [6]

$$Z \approx 1 + A\mathrm{e}^{-B/T}$$

at low temperatures and

$$Z \approx CT$$

at high temperatures, and find expressions for the constants A, B and C.

(e) Find the energy and heat capacity of the rotor at high temperatures and comment on the value of the heat capacity. [4]

END OF PAPER