UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 5 June 2014 13.30 to 16.30

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20-Page Booklets and Treasury Tags	Physics Mathematical Formulae Handbook (supplied)
Yellow Cover Sheet	Approved Calculators allowed
Rough Work Pad	

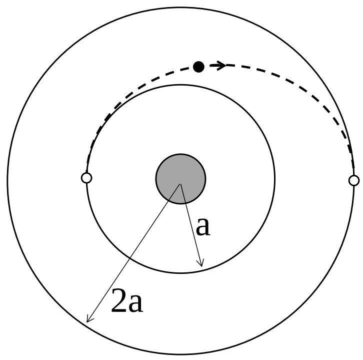
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

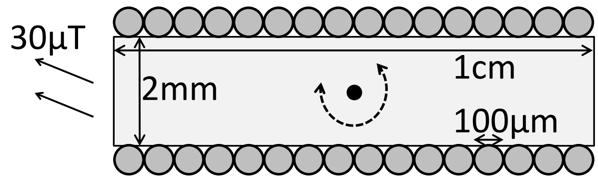
A1 The boiling point of liquid propane is 230.7 K at atmospheric pressure, and 233.4 K at 1.1 atm. What is the molar latent heat of vaporisation? [4]

A2 A geostationary satellite is transferred into an orbit with twice the radius, using the elliptical transfer orbit shown in the diagram below. How long does it take the satellite to reach its new orbit?



[4]

A3 A compass needle is constructed from a 1cm long electromagnet of Cu wire of radius 50 μm wound uniformly on a light rod with diameter 2 mm (as below), carrying a current of 100 mA. Calculate the period of small amplitude oscillations in the earth's magnetic field which has a horizontal component of 30 μT. (Cu density is 8960 kg m^{-3}).



[4]

A4 A spinning top consists of a disc of radius 2 cm with a light rod along its axis extending 1 cm below the disc. It spins at 20 revolutions per second about its axis, with the rod at 30° to the vertical and the lower end stationary. Find the period of precession. [4]

A5 A waveguide is used to transmit pulses at three times its cutoff frequency. Find the ratio between the group and phase velocities of this signal. [4]

SECTION B

B6 Write brief notes on **three** of the following:

(a) the Biot-Savart law;

(b) Ampère's law;

(c) magnetic scalar and magnetic vector potentials;

(d) guided waves. [20]

B7 Write an essay on electromagnetic induction. You should include a discussion of Faraday's Law, Lenz's rule, self and mutual inductance, and applications in transformers. [20]

B8 Write brief notes on **three** of the following:

(a) elastic properties of isotropic solids;

(b) the inertia tensor;

(c) the motion of two interacting bodies and the concept of reduced mass;

(d) viscosity and Reynolds number. [20]

B9 Write an essay on bound and unbound orbits in central potentials (not restricted to inverse-square laws), including discussion of Kepler's laws, elliptical orbits, gravitational slingshots, and Rutherford scattering. [20]

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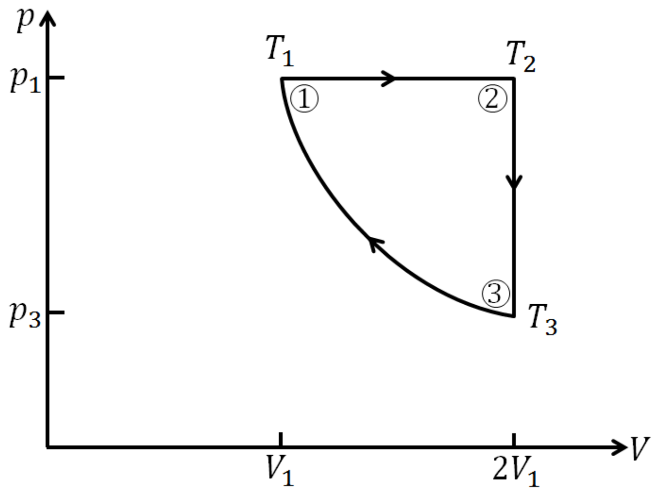
SECTION C

C10 Explain what is meant by a heat engine. Show (a) that no engine operating between two reservoirs at given temperatures can be more efficient than a Carnot engine, and (b) that operating a Carnot engine does not change the entropy of the universe. [5]



The diagram describes a three-stage gas engine using as its working substance a gas with molar heat capacity $c_V = \frac{3}{2}R$. In stage $1 \to 2$ the gas is maintained at constant pressure while temporarily placed in contact with a reservoir at temperature $2.5T_1$ until the volume is $2V_1$; in stage $2 \to 3$ it is placed in contact with a reservoir at $0.5T_1$ and allowed to cool at constant volume to a temperature T_3 such that in the final stage $3 \to 1$ it can be compressed adiabatically to its original state.

Show that $T_3 = 0.63T_1$. Show also that the work done on the gas in stage $3 \to 1$ is $W_{31} = 0.56p_1V_1$, and that the heat absorbed in stage $2 \to 3$ is $Q_{23} = -2.06p_1V_1$. Find the efficiency of the engine. [8]

Show that the entropy change of the *reservoir* in stage $1 \to 2$ is $-p_1V_1/T_1$. By how much does the entropy of the *universe* change during one full cycle of the engine? [3]

Compare this engine with a Carnot engine operating between the two reservoirs, in the light of the statements (a) and (b) above. [2]

If the final stage were *isothermal* at T_1, explain whether the efficiency of the engine would be greater or less? [2]

C11 The differential of the internal energy of a thin film of liquid with surface tension (force per unit length) γ and area A may be written $dU = TdS + \gamma dA$. Give the corresponding relation for the Helmholtz free energy, and derive the corresponding Maxwell relation. [3]

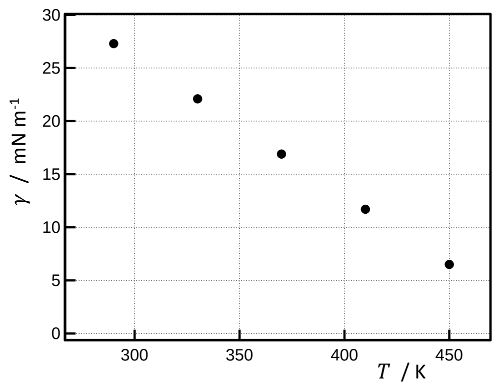
The surface tension of such a liquid is observed to fall with temperature, and to be independent of the area. Hence show

(a) that the entropy rises when the area is increased isothermally;

(b) that the temperature falls when the area is increased adiabatically; and

(c) that $dU = C_A dT + (\gamma - T\frac{d\gamma}{dT})dA$, where C_A is the heat capacity of the film at constant area. [7]

Further observation shows that the surface tension of the film is given by the graph below, and that C_A is almost constant at 70 mJ K^{-1} over the temperature range of interest.



Find the work done on the film, and the heat supplied to it, when the area is increased by 1 cm^2 reversibly and isothermally at temperature 330 K. [5]

Show that in reversible adiabatic changes, the relationship between temperature and area takes the form $C_A \ln T + \beta A = const$, and identify the quantity β. Find the (small) temperature change, and the internal energy change, if the area is increased by 1 cm^2 reversibly and adiabatically, starting at 330 K. [5]

C12 A system whose quantum states i have energy E_i is in thermal equilibrium with surroundings at temperature T. Explain what is meant by the partition function Z for the system. Derive an expression for the mean (expected) energy $\overline{E}$ in terms of Z and $\beta = 1/k_B T$, where k_B is the Boltzmann constant, and show that the heat capacity is given by

$$C = -\frac{\mathrm{d}\overline{E}}{\mathrm{d}\beta}\frac{1}{k_B T^2}.$$ [5]

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An atom with magnetic dipole moment μ in a field with flux density B has three energy levels: $-\mu B$, 0 and $+\mu B$. Show that its mean energy when in equilibrium with surroundings at temperature T is

$$\overline{E} = \frac{-2\mu B \sinh(\beta\mu B)}{1 + 2\cosh(\beta\mu B)}.$$

Sketch the mean energy against T and against B, and without further calculation sketch also the heat capacity against T. Give a physical explanation of the behaviour in each case. [6]

Find a simple approximate expression for $\overline{E}$ and C at temperature T in the high temperature limit ($T \gg \mu B/k_B$). The heat capacity of such a system in a field of 1.2 T is observed to be described closely by $C = a/T^2$ with $a = 9.33$ μeV K. What is the dipole moment? [5]

Show that the mean square of the energy can be expressed in general as

$$\overline{E^2} = \frac{1}{Z}\frac{\mathrm{d}^2 Z}{\mathrm{d}\beta^2},$$

and hence that its standard deviation is given by $\Delta E = \sqrt{k_B T^2 C}$. [4]

C13 Write an essay on the kinetic theory of gases, with reference to photons as well as molecules. Your answer should cover internal energy, pressure, surface flux, speed or frequency distributions, and equations of state. [20]

END OF PAPER