UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 29th May 2014 1.30 pm to 4.30 pm

PHYSICS A (2)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C, and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C, and D, each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains **7** *sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae, which you may quote without proof.*
Answers from **each** *Section should be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS
Booklets and Treasury Tags
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook (supplied)
Students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof. Each Section should be answered in a separate booklet.*

1 A telescope in space has a circular mirror of diameter 1 m and images the sky at an energy of 1 eV. Estimate its angular resolution in radians. [4]

2 A beam of light travelling in air is incident normally on glass. The fraction of the incident beam power that is transmitted into the glass is 96%. Estimate the refractive index of the glass. [4]

3 A 440-Hz tuning fork has a Q-factor of order 10^4. The Fourier transform of the fork's sound is studied with a microphone connected to a computer, in order to measure the value of Q. Estimate the length of time that the sound waves must be sampled for. [4]

4 A two-dimensional Fermi gas at $T = 0$ has Fermi energy ε_F. Calculate the average energy per particle. [4]

5 X-rays of wavelength λ scatter elastically from a simple cubic crystalline lattice (of lattice spacing a). The initial and final X-ray wavevectors are $\boldsymbol{k}_{\rm i}$ and $\boldsymbol{k}_{\rm f}$, and the scattering vector is $\boldsymbol{G} = \boldsymbol{k}_{\rm f} - \boldsymbol{k}_{\rm i}$. Explain why the diffraction condition is that $\boldsymbol{G} = (2\pi/a)\{h, k, l\}$, where h, k, l are integers. [4]

SECTION B

Attempt **two** *questions from this Section. Each Section should be answered in a separate booklet.*

B6 A simple pendulum has no damping and oscillates freely at angular frequency ω_0. A small horizontal force $F_0 \sin(\omega t)$ is applied to the bob, which has mass m. Show that the horizontal displacement, x, of the bob, for small x, is given by

$$\ddot{x} + \omega_0^2 x = \frac{F_0}{m}\sin(\omega t).$$ [2]

Obtain the general solution of this equation. [4]

At $t = 0$, the bob is stationary at $x = 0$. Show that, at later times,

$$x = \frac{F_0}{m(\omega_0^2 - \omega^2)}\left(\sin(\omega t) - \frac{\omega}{\omega_0}\sin(\omega_0 t)\right).$$ [3]

Draw an annotated sketch of x versus t for the case $\omega = 2\omega_0$. [3]

Now consider damping, by adding the term $b\dot{x}/m$ to the first equation and rewriting the small horizontal force as $\mathrm{Im}\,[F_0 \exp(i\omega t)]$. By looking for solutions of the form $x = \mathrm{Im}[x_0 \exp(i\omega t)]$, show that the steady-state response function $R(\omega)$, defined by $x_0 = R(\omega)F_0$, can be written as

$$R(\omega) = \frac{1}{m(\omega_0^2 - \omega^2) + ib\omega}.$$ [4]

If the damping is small, derive approximate expressions for the response function at, and close to, ω_0. [4]

B7 A particular material has the property that transverse elastic waves travel in this material at a speed, v_m, that is independent of their frequency. A two-dimensional waveguide is made from this material. It is infinite in the x-direction, has width b in the y-direction, and is fixed along the lines $y = 0$ and $y = b$.

Consider a transverse elastic wave of angular frequency ω in this waveguide with a wavevector at a point (x, y) equal to (k_x, k_y). Explain why the resultant effect of the waveguide on wave propagation is a wave, ψ, travelling in the x-direction whose amplitude is modulated by a standing wave in the y-direction, that is,

$$\psi \propto \mathrm{Re}\left[\sin(k_y y)\exp(i\omega t - ik_x x)\right].$$ [5]

Explain why only some values of k_y are permitted, and derive the general relation between ω and k_x. [4]

Derive expressions for the phase and group velocities of guided waves travelling along the waveguide. [4]

If $b = 0.02$ m and $v_m = 400$ m s^{-1}, evaluate k_x for propagating transverse elastic waves at 15 kHz. [3]

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Explain what would propagate along the waveguide if you injected waves of frequency (a) 5 kHz, and (b) 50 kHz, into the waveguide. [4]

B8 Consider a slit of horizontal width a, and very long vertical extent, illuminated with light of wavelength λ. Use the fact that the amplitude of the Fraunhofer diffraction pattern is given by the Fourier transform of the aperture function (the aperture function is often written as $h(x)$).

Derive an expression for the intensity distribution of the Fraunhofer diffraction pattern of the slit as a function of $\sin\theta$, where θ is the angle (in the horizontal direction) at the centre of the slit that is subtended by the distance on the screen between the centre of the diffraction pattern and a position on the screen, and make an annotated sketch of the intensity distribution. [5]

Make an annotated sketch of how the above may be demonstrated experimentally, paying particular attention to light source and geometry. [2]

A diffraction grating of overall horizontal extent W is constructed from a large number of long, vertical slits, each of horizontal width a, with distance b between the centres of adjacent slits. Explain, with the help of annotated sketches, how the Fraunhofer diffraction pattern of this grating can be calculated using the convolution theorem, and make an annotated sketch of the grating's resultant Fraunhofer intensity distribution. [6]

State two separate effects of varying the ratio b/a. [2]

A diffraction grating, with a very small value of a, is used in a spectrograph. Show that the wavelength resolution, $\delta\lambda$, equals $\lambda b/mW$, where m denotes the order of diffraction. [3]

If the grating has 1600 slits mm^{-1} and the spectrograph is set up for $m = 2$, what is the minimum value of W needed to distinguish between two spectral lines at 372.6 nm and 372.8 nm? [2]

B9 Write down the form of the general Fresnel-Kirchhoff Diffraction Integral, explain the factors in it, and then explain the transitions from it to the Fresnel regime and to the Fraunhofer regime. [6]

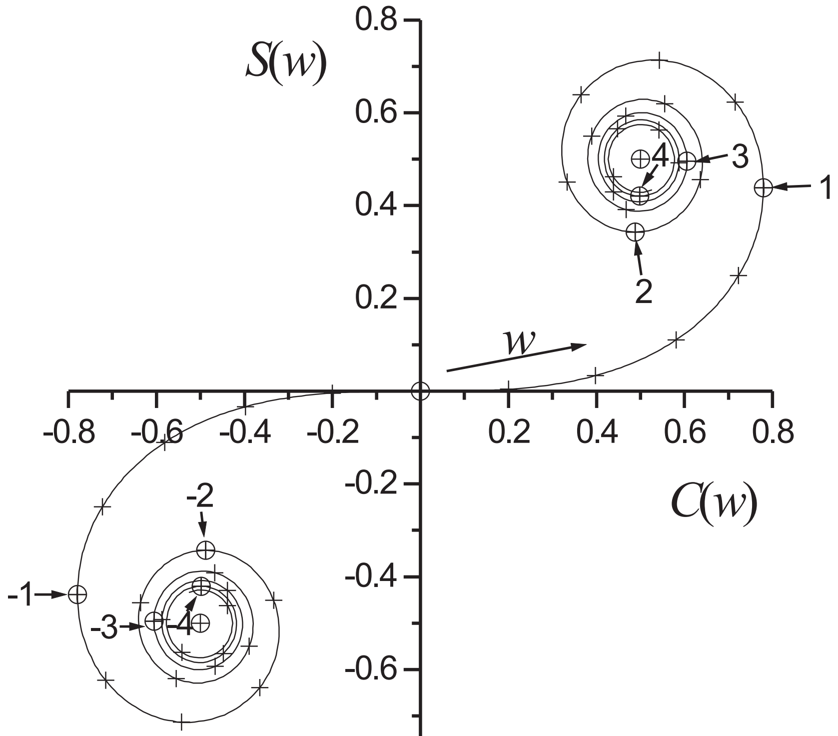
The use of the Cornu spiral for Fresnel diffraction requires geometric assumptions. State the assumption about the positions of the radiation source S, the aperture origin O, an aperture element at x, and a diffracted intensity point P, draw a diagram illustrating typical positions of S, O, x, and P, and state the geometric assumption about the aperture. [3]

Show that the optical path from S to P via the aperture element at x is $x^2/2R$ plus a constant , where $1/R = 1/a + 1/b$, a is the length SO, and b is the length OP. [3]

The Cornu spiral drawn below gives the values of $C(w)$ and $S(w)$ in the Fresnel Integral $C(w) + iS(w) = \int_0^w du\, \left[\cos\left(\frac{\pi u^2}{2}\right) + i\sin\left(\frac{\pi u^2}{2}\right)\right]$, where $u = x\sqrt{2/\lambda R}$.

A radio transmitter emits spherical waves of 3×10^9 Hz and a receiver is situated 50 m away from it; both transmitter and receiver are 5.0 m above the level ground. A long, opaque fence of height 5.5 m is placed between the transmitter and the receiver, perpendicularly to the line joining them, 5.0 m from the transmitter. Assuming that the waves are not reflected by the ground, use the Cornu spiral to estimate the ratio of the received intensity with the fence in place to the received intensity without the fence. [4]

The receiver is moved horizontally towards the fence. Draw an annotated sketch of the received intensity as a function of the receiver's distance from the fence. [4]



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SECTION C

Attempt **one** *question from this Section. Each Section should be answered in a separate booklet.*

C10 Explain why the molar heat capacity, C, of metals at low temperature, T, takes the form

$$C = \gamma T + \beta T^3 ,$$

where γ and β are material-dependent constants. [8]

What form do you expect the molar heat capacity to take in the high-temperature limit, and why? [4]

For the alkaline metal potassium, the Debye temperature $\theta_D \approx 90$ K. The body-centred cubic unit cell of potassium has a lattice constant of 0.52 nm. Derive an expression for the Debye frequency ω_D and hence estimate the speed of sound in potassium. [8]

C11 An effectively two-dimensional crystal has a square lattice. Draw the first Brillouin zone and the free-electron Fermi surface (i.e. a contour in 2D) in crystals formed from:

(i) monovalent, and

(ii) divalent atoms, in both insulating and conducting cases. [6]

Such a two-dimensional crystal is made of divalent atoms with a lattice parameter $a = 0.4$ nm. Within the free-electron model, calculate the Fermi energy. [5]

Now suppose that the same crystal has a direction-independent energy gap $E_g = 1$ eV at the Brillouin zone boundary. Within the nearly free-electron model, sketch the electron energy as a function of wavevector in the [10] and [11] directions and mark the zone boundary energies on your sketch. [4]

Explain whether this crystal is a metal or an insulator. [5]

SECTION D

Attempt **one** *question from this Section. Each Section should be answered in a separate booklet.*

D12 Write brief notes on **two** of the following:

(a) wave impedance and impedance matching in the context of acoustic or mechanical waves; [10]

(b) thin-film interference; [10]

(c) Michelson's interferometer. [10]

D13 Write an essay on the physics of the semiconductor n-p junction and its uses. Your discussion should include the effect of doping, the formation of a depletion layer, and the origin of the ideal diode I-V characteristic. [20]

END OF PAPER