UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Saturday 1 June 2013 9.00 to 12.00 noon

PHYSICS B (Paper 1)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

*Each question in Sections B and C carries the same mark.*

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

*Write the letter of the Section on the cover of each Booklet.*

*Write your candidate number, **not** your name, on the cover of each Booklet.*

*A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| 20-Page Booklets and Treasury Tags | Physics Mathematical Formulae |
| Rough Work Pad | Handbook (supplied) |
| Yellow Cover Sheet | Approved Calculators allowed |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

*Answers should be concise and relevant formulae may be assumed without proof.*

A1 Calculate the change in the volume of a 1 $m^3$ cube of concrete when subjected to a hydrostatic pressure of 1 atm. The Young's modulus and Poisson's ratio of concrete are 10 GPa and 0.2 respectively. [4]

A2 Two long cylindrical conductors with radii $a_1$ and $a_2$ lie parallel to each other, separated by a distance $d$ that is large compared to both $a_1$ and $a_2$. Determine an expression for the capacitance per unit length between the two conductors. [4]

A3 By how much does the entropy of the universe increase when taking a hot bath? Assume that 0.1 $m^3$ of cold water at a temperature of 280 K is mixed with 0.1 $m^3$ of hot water at 340 K and assume the specific heat capacity of water, $c = 4.2$ J $K^{-1}$ $g^{-1}$, to be temperature independent. Neglect the heat capacity of the bath itself. [4]

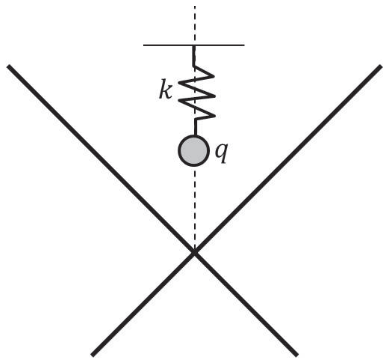
A4 If the Earth had no atmosphere its surface temperature would be $T_0 = 280$ K due to the incoming radiation from the Sun. In a simplified model for estimating the effect of the atmosphere on the Earth's surface temperature, the atmosphere is assumed to behave like a perfect greenhouse; it transmits all incoming, predominantly higher energy, radiation from the Sun and acts as a perfect blackbody for absorption and emission of lower energy, infrared radiation. By considering the radiation balance outside the atmosphere and on the surface of the Earth derive an expression for the temperature of the surface of the Earth in terms of $T_0$. [4]

A5 A hollow sphere of density $\rho$ has outer radius $R_2$ and inner radius $R_1$. Determine its moment of inertia for rotation about an axis tangential to its outer surface.
[*The moment of inertia of a solid sphere about an axis through its centre is* $I = \frac{2}{5}MR^2$.] [4]

## SECTION B

B6 Explain the method of image charges in electrostatics, and provide two examples of its use. [6]

A small uncharged sphere of mass $m$ is suspended by a spring of spring constant $k$ at a height $y_0$ above the intersection of two perfectly conducting planar sheets angled at $\pm 45$ degrees to the vertical, as shown below.




A charge $q$ is then placed onto the sphere slightly changing its equilibrium height. Show that the electrostatic force is approximately

$$F_{\mathrm{el}} \approx \frac{\beta}{y_0^2}$$

and find $\beta$. [6]

For small vertical displacements, calculate the fractional change in the oscillation frequency of the sphere relative to the uncharged case. [6]

Considering also lateral displacements, explain whether this system is in unstable or stable equilibrium. [2]

B7 A transmission line has capacitance $C$ and inductance $L$ per unit length. Show that the voltage and current along the transmission line obey wave equations, and derive the wave speed and characteristic impedance. [6]

Two long strips of perfect conductor of width $a$ and separated by a distance $d$ ($\ll a$) form a coplanar transmission line filled by a medium of dielectric constant $\varepsilon_r = 2.3$. Write down an expression for $C$ and, using Ampère's Law, show that $L = \mu_0 d/a$ (edge effects should be ignored). Hence calculate the wave speed and the characteristic impedance. [4]

At a point at a distance $l_1$ along the transmission line the separation of the strips changes to $\alpha d$ (as shown). By calculating the voltage and current on either side of this point, derive expressions for the voltage and energy transmission coefficients at this point. [5]

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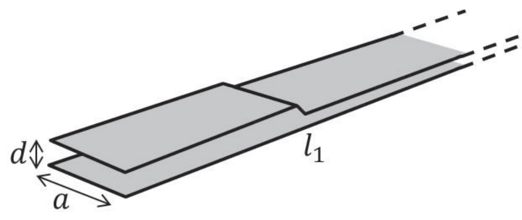
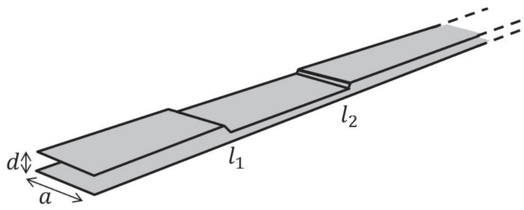


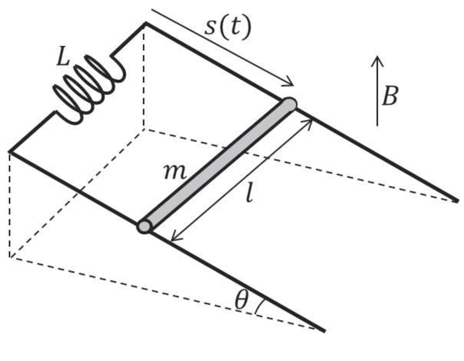

Compare the voltage reflection coefficients for the cases where $\alpha = \alpha_0$ and $\alpha = \alpha_0^{-1}$. [2]




The separation reverts to its original value at a distance $l_2$. Describe qualitatively how the transmission through the whole line (which is terminated to avoid end reflections) depends on the frequency of the voltage applied. [3]

B8 State Faraday's Law and Lenz's Law, defining all the terms used. [4]

Two parallel, rigid and perfectly-conducting rods separated by a distance $l$ are inclined at an angle $\theta$ to the horizontal and connected at their top ends by an inductance $L$ (much larger than the self-inductance of the arrangement of rods). A horizontal perfectly-conducting rigid rod of mass $m$ connects the two rails and can slide smoothly along them (without rolling). A vertical magnetic flux density $B$ is applied.




Find the forces that the moving rod experiences, and sketch their directions. [5]

If the rod is initially at rest at $s = 0$, show that the equation of motion is

$$m\ddot{s} = mg\sin\theta - \kappa B^2 s$$

and find an expression for $\kappa$. [4]

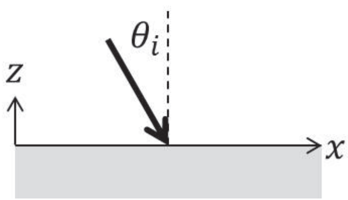
Hence find the distance along the rails where the rod comes to rest, and describe the subsequent motion. [5]

Describe briefly how the dynamics change if the resistivity of the rod is not negligible. [2]

B9 Show that the dielectric constant at angular frequency $\omega$ of a plasma with electron number density $n$ is

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

and find an expression for $\omega_p$. [5]




A plasma is confined to $z < 0$ and an electromagnetic wave is incident from the vacuum above ($z > 0$) at an angle $\theta_i$ to the vertical. The electric field is linearly polarised and lies in the $xz$ plane. What are the ratios between the electric and magnetic field amplitudes in the vacuum and in the plasma? [2]

If radio waves of 10 GHz are used and the plasma has an electron density of $10^{12}$ cm$^{-3}$, calculate the refractive index of the plasma. [3]

State the continuity conditions for the electric and magnetic field components at the plasma boundary. [2]

The field just inside the plasma at $z = 0$ is measured to be $(E_x, E_y, E_z) = (1, 0, 3)\, e^{-i\omega t}$ V m$^{-1}$. Calculate the angle of incidence and find the mean power per unit area of the incident wave. [6]

Discuss qualitatively what would happen if the radio wave frequency were reduced to 1 GHz? [2]

SECTION C

C10 Explain what is meant by a fictitious force. [3]

Show that the acceleration $\boldsymbol{a}$ of a particle with mass $m$ and velocity $\boldsymbol{v}$ as measured in a frame that is rotating with constant angular velocity $\boldsymbol{\omega}$ is given by

$$m\,\boldsymbol{a} = \boldsymbol{F} - 2m\,(\boldsymbol{\omega}\times\boldsymbol{v}) - m\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}),$$

where $\boldsymbol{F}$ is the force experienced in the inertial frame. [4]

With the aid of a diagram, explain why the Earth is approximately a flattened sphere with polar radius slightly smaller than its equatorial radius. [2]

In the Earth's frame, at a latitude $\lambda$, state the direction of the force acting on a stationary mass relative to the local vertical and find an approximate expression for the magnitude of the vertical component of the centrifugal force relative to the gravitational force. [2]

[*Take the radius of the Earth to be 6,400 km and* $g = 9.81\,ms^{-2}$.]

At a latitude $\lambda$ in the Northern hemisphere a projectile is fired vertically upwards from a point $O$ with an initial speed $u$. It rises to a height $h_0$ and subsequently falls under gravity, landing at point $X$. Neglecting air resistance, what are the displacements of $X$ from $O$ in the East-West and North-South directions. [6]

[*When considering the Coriolis force, take the Earth to be a sphere and neglect any horizontal velocity components.*]

The projectile is now dropped from a height $h_0$ directly above $O$ and subsequently hits the ground at point $X'$. Compare the displacement of $X'$ from $O$ with the case where the projectile was initially fired upwards. [3]

C11 Most of the visible matter in a particular galaxy is concentrated in the small galactic core of mass $M$. If in addition there is a spherically symmetric uniform distribution of dark matter of density $\rho$, centred on the galactic core and extending out to large radii, show that the force on a star of mass $m$ at radius $r$ from the galactic core is given by

$$\boldsymbol{F}(\boldsymbol{r}) = -\frac{GMm}{r^2}\,\hat{\boldsymbol{r}} - \alpha G m r\,\hat{\boldsymbol{r}},$$

and find an expression for $\alpha$ in terms of $\rho$. [4]

By writing the kinetic energy of the star in terms of $\dot{r}$ and $\dot{\phi}$, write down the expression for the total energy for an orbit in this potential. [2]

Explain why the angular momentum $\boldsymbol{J}$ of the star is a constant of the motion. [2]

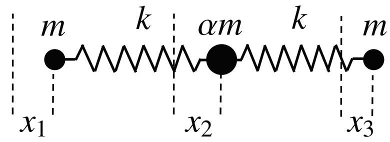
By expressing $\dot{\phi}$ in terms of $|\boldsymbol{J}|$, find the radial equation of motion and determine an expression for the angular speed $\Omega$ for a circular orbit of radius $r_0$ (writing your answer in terms of $G$, $M$, $r_0$ and $\alpha$). [5]

Find the angular frequency $\omega$ of small oscillations in the radial direction about this circular orbit. [4]

Hence show that such a nearly circular orbit can be approximated by a precessing ellipse and, assuming that $\alpha r_0^3/M \ll 1$, determine the precession frequency. [3]

C12 What is meant by a normal mode of a dynamical system? [3]

Three masses, $m$, $\alpha m$ and $m$, are connected by two identical light springs of spring constant $k$ and are constrained to move only in the $x$-direction, as shown below. Calculate the frequencies and eigenvectors of the normal modes of the system. [7]




Explain the dependence of each of these eigenvalues and eigenvectors on $\alpha$. [3]

For the case where all three masses are equal ($\alpha = 1$), write down the general expression for the positions of the masses in terms of the normal modes. [2]

The masses are initially at rest when the mass on the left is given a sharp tap with a hammer which imparts an impulse to that mass, such that its instantaneous speed is $v$. For the case where $\alpha = 1$, calculate the relative fractions of the total energy of the system associated with each of the three normal modes. [5]

C13 For irrotational flow of an incompressible fluid, explain why it is possible to define a velocity potential such that $\boldsymbol{v} = \nabla\Phi$ and show that $\Phi$ satisfies Laplace's equation. [3]

Stating clearly any assumptions made, derive Bernoulli's equation,

$$p + \tfrac{1}{2}\rho v^2 + \rho\phi_g = \text{constant}\,,$$

where $p$ is the pressure, $\rho$ is the fluid density and $\phi_g$ is the gravitational potential. [4]

A fixed vertical cylindrical pole of radius $a$ is immersed in a stream which is flowing horizontally with speed $v$. The fluid flow is irrotational and can be described by a velocity potential. Using cylindrical polar coordinates where $r = 0$ defines the axis of the cylinder, write down the boundary conditions for $\boldsymbol{v}$ at $r = a$ and $r \to \infty$ . [2]

Taking the general solution of Laplace's equation in cylindrical coordinates to be

$$\Phi = A\theta + B\ln\frac{r}{a} + \sum_{n=1}^{\infty}(C_n r^n + D_n r^{-n})\cos(n\theta)\,,$$

explain why only the $C_1$ and $D_1$ terms can be present for the potential describing the fluid flow around the pole and hence show that

$$\Phi = v\left(r + \frac{a^2}{r}\right)\cos\theta\,.$$

[4]

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Using Bernoulli's equation, find an expression for the pressure at the surface of the cylinder as a function of $\theta$. [3]

Hence determine the total force that the fluid exerts on the cylinder and explain your result. [4]

END OF PAPER