UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Wednesday 5 June 2013 9.00 to 12.00 noon

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.

STATIONERY REQUIREMENTS
20-Page Booklets and Treasury Tags
Metric Graph Paper
Yellow Cover Sheet
Rough Work Pad

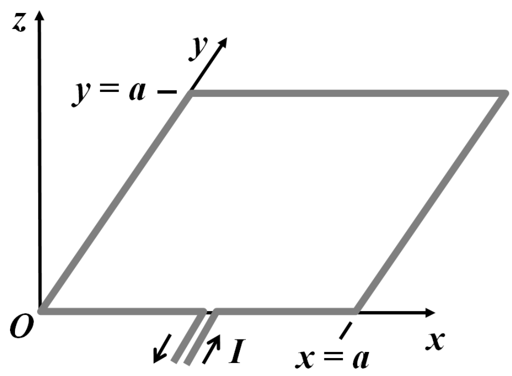
SPECIAL REQUIREMENTS
Physics Mathematical Formulae Handbook (supplied)
Approved Calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

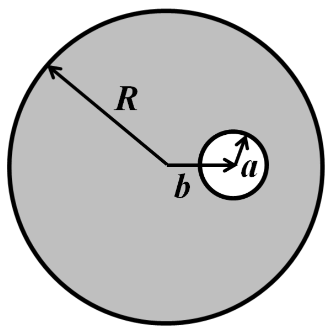
Answers should be concise and relevant formulae may be assumed without proof.

A1 A square loop of wire of side a = 1m lies in the xy-plane as shown below and carries a current I = 5 A. An inhomogeneous magnetic flux density B is applied along the z-axis. The field varies in the xy-plane as $B_z(x, y) = \alpha xy$ with $\alpha = 1\ \mathrm{T\ m^{-2}}$. Calculate the magnitude and direction of the total force on the loop. [4]



A2 Given that the Moon orbits the Earth in the same sense as the Earth's rotation and has an orbital period of 27.3 days, calculate the time difference between high tides on Earth. [4]

A3 A long straight cylindrical conductor of radius R has a cylindrical hole of radius a that runs parallel to its axis at distance b as shown. The conductor carries a current I uniformly distributed over its cross-section. Calculate the magnetic flux density B inside the hole and show that it is uniform across the hole. [4]



A4 Two isotopes of atomic masses m_1 and m_2 are to be separated using a gas centrifuge which consists of a hollow cylinder of radius a rotating about its axis with angular speed ω. The gas mixture is injected at radius a and is extracted near the axis. By assuming the gas inside the cylinder to be in thermodynamic equilibrium at temperature T and subject to a centrifugal potential, calculate the isotope concentration ratio of the extracted gas in terms of the injected ratio. [4]

A5 Consider a system of N non-interacting particles each of which can either be in a state with energy ϵ or $-\epsilon$. Write down expressions for the total energy and the entropy of the system as a function of the difference in occupation numbers of the two states. [4]

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SECTION B

B6 Write brief notes on **three** of the following:

(a) Displacement current and its role in Maxwell's equations;

(b) Dielectrics;

(c) Mutual inductance;

(d) Boundary conditions and uniqueness in electrostatic problems. [20]

B7 Write an essay on magnetic materials. You should include a discussion of the different types of magnetic materials, magnetisation, permeability and their application in electromagnets. [20]

B8 Write brief notes on **three** of the following:

(a) Lagrangian dynamics;

(b) Viscosity;

(c) Hyperbolic orbits;

(d) Elastic waves. [20]

B9 Write an essay on free and forced precession. You should include a discussion of Poinsot's treatment, the major axis theorem and the motion of gyroscopes. [20]

SECTION C

C10 For an ideal gas with n molecules per unit volume, show that the number of molecules striking a unit area of its enclosure per unit time is $\frac{1}{4}n\bar{v}$ where $\bar{v}$ is their mean speed. [5]

Hence, given that the Maxwell-Boltzmann distribution of speeds v for molecules of mass m at temperature T is

$$f(v) \sim v^2 \exp\left(-\frac{mv^2}{2k_B T}\right),$$

show that the mean kinetic energy for molecules striking the enclosure is $2k_B T$. [5]

Two identical vessels containing an ideal gas at temperature T_0 and pressure p_0 are connected by a hole of diameter d much less than the mean free path. One vessel is then heated to and maintained at temperature T_1 while the other is maintained at T_0. Under these steady state conditions, what is the pressure difference between the two vessels ? [8]

Under these conditions, what is the total rate of heat transfer between the two vessels? [2]

C11 Explain what is meant by a *thermodynamic potential* and its *natural variables* in thermodynamics. In thermodynamic equilibrium, which potential is minimized at constant temperature and volume, and which at constant temperature and pressure? [4]

A fluid flows into and out of a thermally isolated vessel through a number of tubes. Explaining any assumptions made, show that under steady state conditions the total enthalpy of the fluid entering is the same as that leaving. [6]

Helium enters such a closed system as a flow of compressed gas at a temperature of 14 K and at pressure p. A fraction α emerges as liquid and the rest as gas at 14 K, both at atmospheric pressure p_0. Find an expression for α in terms of the enthalpy $H(p)$ of He gas and the enthalpy H_ℓ of the liquid. [6]

From the data below, find the maximum value of α and the initial pressure p for which this occurs. [4]

Pressure	0	10	20	30	40		atm
Enthalpy $H(p)$ of He gas at 14 K	8.74	7.85	7.31	7.18	7.26	$\times 10^4$	J kg^{-1}

[*The enthalpy H_ℓ of liquid He at p_0 is* 1.01×10^4 *J kg*$^{-1}$.]

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C12 Explain why a heat pump is a more efficient way to heat a building than direct electrical heating. [2]

The source for a heat pump consists of a large horizontal array of tubes buried at a uniform depth z below ground level. A fluid pumped through the tubes extracts heat from the ground which is used to heat a house, maintaining the inside temperature at T_{H}. The annual variation of the outside air temperature can be expressed as $T_{\mathrm{air}} = T_0 - T_{\mathrm{A}} \cos(\omega t)$, with $t = 0$ corresponding to 1 January. The heat extracted from the ground by the pump is negligible compared with other heat flows, and the heat capacity of the source itself can be ignored. If the thermal conductivity and heat capacity per unit volume of the soil are κ and C respectively, show that the temperature T_{S} of the source as a function of time is

$$T_{\mathrm{S}} = T_0 - T_{\mathrm{A}} \cos(\omega t - z/\delta) \exp(-z/\delta)$$

where $\delta = \sqrt{2\kappa/\omega C}$. [8]

If $T_{\mathrm{H}} = 20$°C, $T_0 = 15$°C, and $T_{\mathrm{A}} = 20$°C, and the heating is turned off when $T_{\mathrm{air}} > T_{\mathrm{H}}$, compare the energy costs of heating the house on the coldest day of the year using: (i) direct electrical heating; (ii) a heat pump with the outside air as the source; (iii) a heat pump with a deep ground source. [4]

Draw a labelled graph showing the variations of T_{air} and T_{S} for $z = (\pi\delta)/2$ and $\pi\delta$ and discuss the optimum depth for the source. [6]

[*Assume that the heat loss from the house is proportional to the difference between the inside and outside air temperatures, and ignore the daily temperature variations of the outside air.*]

C13 Write an essay on phase transitions. You should include a discussion of liquefaction of a van der Waals gas, phase diagrams, the Clausius-Clapeyron equation, and phase equilibrium. [20]

END OF PAPER