UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

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Wednesday 6 June 2012 9.00 to 12.00 noon

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PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

*Each question in Sections B and C carries the same mark.*

*The **approximate** number of marks allocated to each part of a question in all sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

*Write the letter of the Section on the cover of each Booklet.*

*Write your candidate number, **not** your name, on the cover of each Booklet.*

*A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| Booklets and Treasury Tags | Physics Mathematical Formulae-Handbook (supplied) |
| Rough Work Pad | Approved Calculators allowed |
| Yellow Cover Sheet | |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

*Answers should be concise and relevant formulae may be assumed without proof.*

A1 Explain the physical basis of the terms $a/V^2$ and $b$ in the van der Waals equation of state

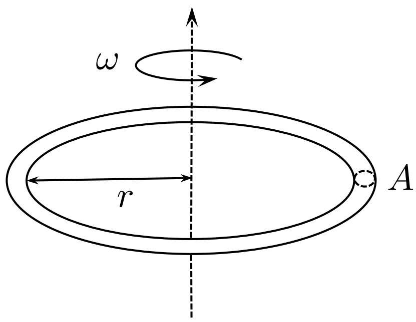
$$\left(p + \frac{a}{V^2}\right)(V - b) = RT\ .$$ [4]

A2 A lossless coaxial cable consists of an inner conductor of radius of 2.5 mm and an outer conductor of radius 5 mm. The space between the conductors is filled with non-magnetic dielectric material of relative permittivity $\varepsilon_r = 2$. Calculate the capacitance per unit length of the cable. [4]

A3 The orbit of Pluto has a period of 248 Earth years and an eccentricity $e = 0.25$. Given that the semi-major axis of the orbit of the Earth around the Sun is $1.5 \times 10^8$ km, calculate the furthest distance between Pluto and the Sun. [4]

A4 A long cylinder of dielectric material with relative permittivity $\varepsilon_r$ and radius $a$ is placed in a uniform electric field of magnitude $E_0$ perpendicular to its axis. Find an expression for the magnitude of the electric field strength inside the cylinder. [4]

A5 A hoop of radius $r$ and uniform circular cross section $A \ll \pi r^2$ is made from an elastic material of density $\rho$ and Young's modulus $E$. When the hoop is set in rotational motion about its axis, as shown in the diagram below, its radius is observed to increase slightly. Determine an approximate expression for the fractional change in the radius $r$ as a function of the angular speed $\omega$. [4]



## SECTION B

B6 Write brief notes on **three** of the following: [20]

(a) transmission lines;

(b) polarisation charge in dielectric materials;

(c) paramagnetism, diamagnetism and ferromagnetism;

(d) the dielectric properties of plasmas.

B7 Write an essay on Maxwell's equations. You should include a discussion of the physical basis of each of Maxwell's equations and the propagation of electromagnetic radiation in free space. [20]

B8 Write brief notes on **three** of the following: [20]

(a) fictitious forces in rotating systems;

(b) waves in elastic materials;

(c) Euler's equations for the rotation of a rigid body;

(d) energy methods in dynamical problems.

B9 Write an essay on fluid flow. Your answer should include a discussion of the convective derivative, the Bernoulli equation and its derivation, vorticity, potential flow and viscous fluid flow. [20]

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SECTION C

C10 Derive the Clausius-Clapeyron equation for the boundary between two phases,

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{T\Delta V},$$

defining the symbols involved. [5]

For the special case of steam in coexistence with water, in which steam can be assumed to behave as an ideal gas, show that the Clausius-Clapeyron equation can be written in the form

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{pL_{\mathrm{e}}}{RT^2},$$

where $R$ is the gas constant and $L_{\mathrm{e}}$ is the *molar* latent heat of evaporation of water, which can be assumed to be temperature independent. [2]

At atmospheric pressure $L_{\mathrm{e}} = 40.7\,\mathrm{kJ\,mol^{-1}}$. Estimate the vapour pressure of water at 273 K. [5]

Sketch the phase diagram of water in the $p - T$ plane, indicating the triple point. [2]

At the triple point of water ($T = 0.01°\mathrm{C}$ and $p = 612\,\mathrm{Pa}$) ice cubes float with 90 % of their volume submerged in water. Estimate the pressure at which ice and water are in equilibrium at −3°C, given that the latent heat of sublimation of ice at the triple point is $L_{\mathrm{s}} = 2.8 \times 10^6\,\mathrm{J\,kg^{-1}}$. [6]

[*The gas constant* $R = 8.31\,J\,K^{-1}mol^{-1}$ *and the density of water can be taken to be* $1000\,kg\,m^{-3}$.]

C11 A source delivers heat at a rate of $H$ per unit volume to a material with specific heat capacity $c$ and thermal conductivity $\kappa$. By considering the heat flux $\boldsymbol{J} = -\kappa\nabla T$ through an arbitrary closed surface, derive the equation for thermal diffusion in the presence of a heat source,

$$\frac{\partial T}{\partial t} = \frac{\kappa}{c}\nabla^2 T + \frac{H}{c}\,. \qquad [6]$$

A long straight cylindrical wire with electrical conductivity $\sigma$ and radius $a$ carries a uniform current $I$. The temperature of its surface is maintained at $T_0$ in a large water bath. Show that the temperature inside the wire at a radius $\rho$ is given by

$$T(\rho) = T_0 + \frac{I^2}{4\pi^2 a^4 \kappa\sigma}(a^2 - \rho^2)\,. \qquad [9]$$

The same wire is then placed in air at temperature $T_{\mathrm{air}}$. The wire loses heat from its surface according to Newton's law of cooling such that the heat flux per unit area from its surface is given by $\alpha\,[T(a) - T_{\mathrm{air}}]$, where $\alpha$ is a constant. Find an expression for $T(a)$ in the steady state. [5]

[*The Laplacian in cylindrical polar coordinates is*

$$\nabla^2 A = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 A}{\partial\phi^2} + \frac{\partial^2 A}{\partial z^2}\,.$$

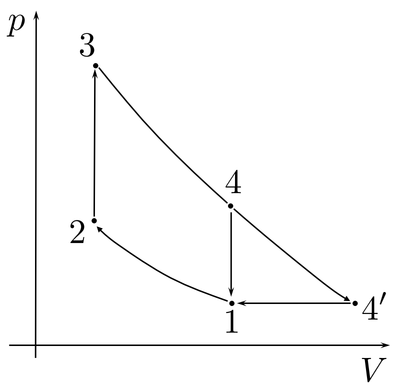
]

C12 Define the efficiency $\eta$ of a heat engine. [2]

The Otto cycle is shown below as the sequence $1 \to 2 \to 3 \to 4 \to 1$, where $1 \to 2$ and $3 \to 4$ are adiabats. The cycle is operated with an ideal gas. By expressing the heat flows in terms of the appropriate heat capacities, show that the efficiency of the Otto cycle is

$$\eta = 1 - r^{(1-\gamma)},$$

where $r = V_1/V_2$ is the ratio of the volumes at points 1 and 2, and $\gamma = C_p/C_V$ is the ratio of heat capacities. [6]




The Atkinson cycle, which is used in some modern hybrid vehicles, is shown as $1 \to 2 \to 3 \to 4' \to 1$ on the $p - V$ diagram, where $4 \to 4'$ is an extension of the adiabat $3 \to 4$. The efficiency of the Atkinson cycle depends on the ratio of pressures $r_p = p_3/p_2$. Find expressions for the temperatures at points 2, 3 and 4′ in the Atkinson cycle in terms of $T_1$, $r$, $r_p$ and $\gamma$. [7]

Hence, show that the efficiency of the Atkinson cycle is

$$\eta = 1 - \gamma\, r^{(1-\gamma)} \left[ \frac{r_p^{\frac{1}{\gamma}} - 1}{r_p - 1} \right].$$

[3]

By considering the $p - V$ diagram, explain why the Atkinson cycle is more efficient than the Otto cycle. [2]

C13 Write an essay on thermodynamic potentials, natural variables and Maxwell relations, including one example of an application of a Maxwell relation. [20]

END OF PAPER