UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 30th May 2019 9.00 am to 12.00 noon

PHYSICS A (2)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C, and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C, and D, each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains **9** *sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae, which you may quote without proof.*
Answers from **each** *Section should be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS
Booklets
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook
Approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof. Use a separate booklet for the whole of this section.*

1 Show that the group velocity of a wavepacket is given by $v_g = d\omega/dk$. [4]

2 Two light flexible strings are fixed together and held under tension. By what factor must their mass per unit length differ for half of the power carried by a transverse wave to be reflected at the boundary? [4]

3 Sketch a Michelson interferometer and indicate how it may be used to determine the wavelength of a spectral line. [4]

4 Draw a labelled sketch showing the carrier and charge density across a p-n junction [4]

5 Sodium, a monatomic free electron metal [density 0.97 g cm^{-3}, relative atomic mass 23] has an electron mobility of 5.2×10^{-3} m^2 V^{-1} s^{-1}, calculate the conductivity and the mean time between collisions for this material. [4]

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SECTION B

Attempt **two** *questions from this Section. Use a separate booklet for the whole of this section.*

B6 A Fabry-Perot etalon consists of two semi-transparent mirrors separated by an air gap of thickness d. Each mirror has an intensity reflection coefficient of R and an intensity transmission coefficient of T.

(a) Sketch a ray diagram showing multiple beam interference when an etalon is illuminated with collimated light at a small angle, θ. Thus, obtain an expression for the phase difference, δ, between successive beams of wavelength λ. [4]

(b) By summing the outgoing beams, the intensity transmission of the etalon can be shown to be [5]

$$|A|^2 = \frac{T^2}{(1-R)^2}\left(\frac{1}{1+\left[4R/(1-R)^2\right]\sin^2(\delta/2)}\right).$$

The finesse, $\mathcal{F}$, of an etalon is defined as the ratio of the separation of successive peaks as δ varies to the full-width at half-maximum of the peaks. Obtain an expression for $\delta_{1/2}$, the half-width at half-maximum of each transmitted peak and use it to show that

$$\mathcal{F} = \frac{\pi R^{1/2}}{1-R}.$$

(c) Two of the spectral lines for sodium are at approximately 589.0 and 589.6 nm. A measurement is to be performed at normal incidence, where the spectral lines are separated by at least 100 times their width. If spectral overlap is to be avoided, what are the constraints placed on R and d? Comment on whether such constraints are likely to be achievable in practise. Make a plot of the transmission versus wavelength that you expect from the etalon. [8]

[*You may assume the measured width of the spectral lines is determined only by the etalon properties.*]

(d) Outline the benefits and limitations of using such an etalon rather than a diffraction grating for spectroscopy. [3]

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B7 A damped driven simple harmonic oscillator has the equation of motion

$$m\ddot{x} + b\dot{x} + kx = \mathrm{Re}\left[F_0 e^{i\omega t}\right]$$

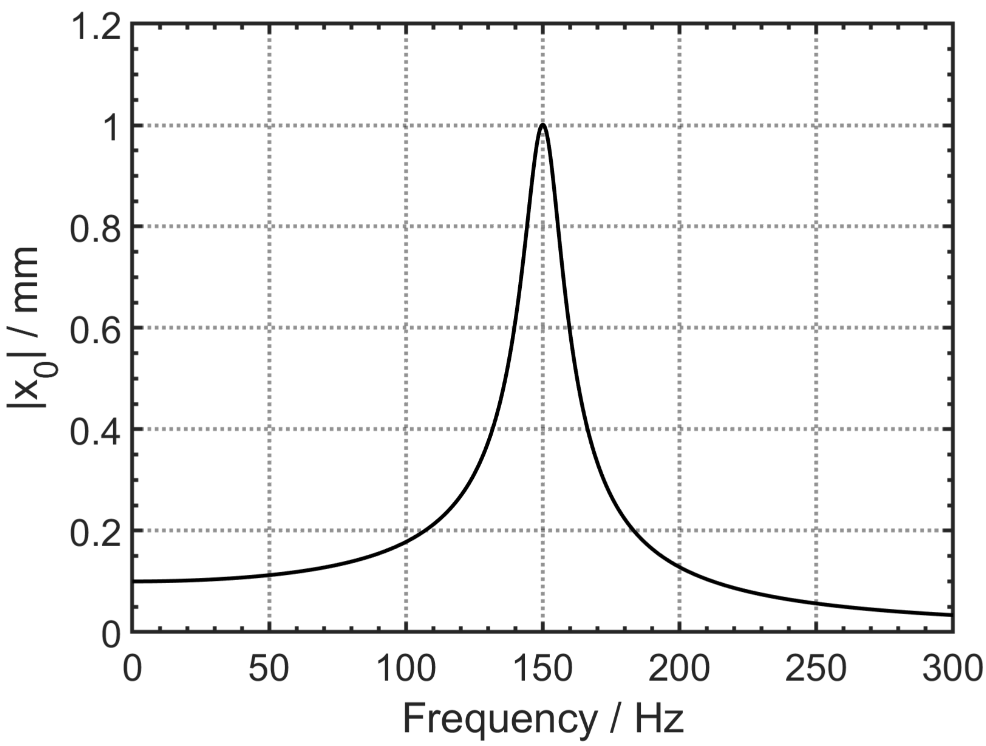
(a) Define all the symbols used and explain what each of the terms represents. [3]

(b) For the situation where the oscillator is undriven ($F_0 = 0$), by solving the equation of motion determine the relationship between m, b and k for the three different damping regimes. Describe the form of the resulting motion in each case. [4]

(c) Sketch the undriven amplitude response in each damping regime, when the oscillator is initially at $x = 0$, then abruptly given an initial velocity at $t = 0$. [3]

(d) Find an expression for the general solution, $x(t)$, in the lightly damped regime. [3]

(e) An electric toothbrush can be modelled as a damped, driven oscillator. When driven by an oscillatory driving force of $F_0 = 0.9\,\mathrm{N}$, the corresponding displacement, $|x_0|$, of a prototype is shown below, as a function of frequency. Explain the form of the curve and use it to estimate values for m, b, k and Q for the system. [7]



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B8 Light of a known wavelength diffracts as it passes through an aperture plane.

(a) Explain the difference between the Fresnel and Fraunhofer diffraction regimes and how Fourier transforms can be used to determine the diffraction patterns from multiple slit apertures in the Fraunhofer regime. [5]

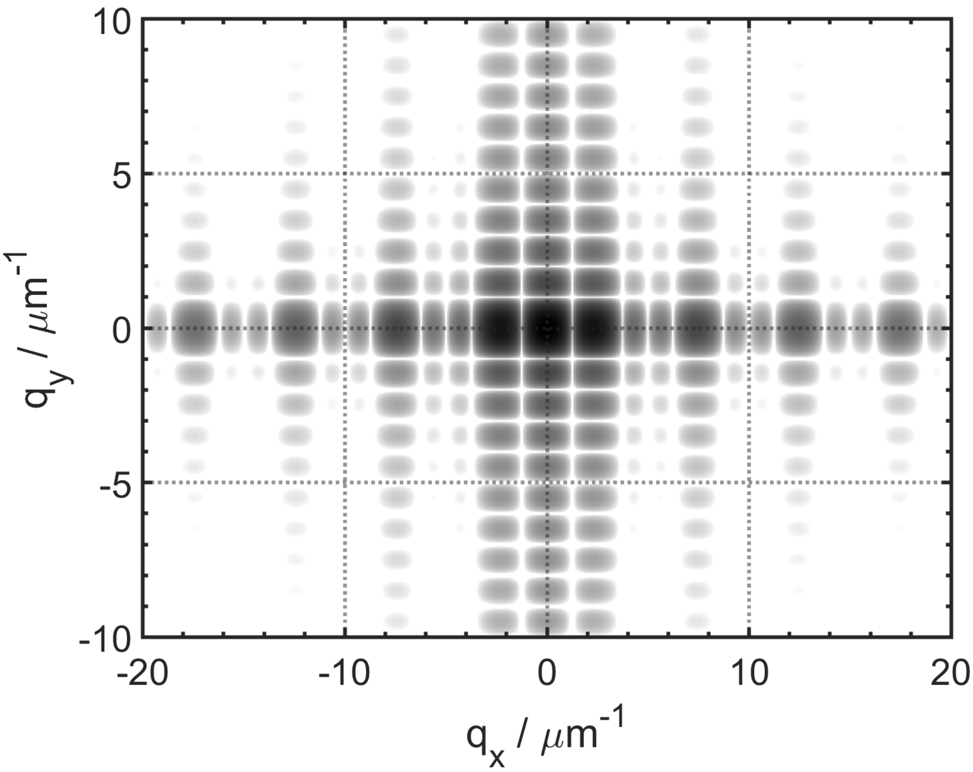
(b) Show that the diffracted amplitude from 4 long slits of width a, equally spaced where D is the distance between the centres of adjacent slits, is given by [6]

$$\psi_P \propto \left[\cos\left(\frac{qD}{2}\right) + \cos\left(\frac{3qD}{2}\right)\right]\mathrm{sinc}\left(\frac{qa}{2}\right)$$

where $q = 2\pi \sin\theta/\lambda$ and λ is the wavelength of the light.

(c) The 4 slits each have a width of $10\,\mu$m and their centres are separated by $20\,\mu$m. Draw a labelled diagram showing the diffraction pattern that would be observed on a 0.1 m wide screen placed on axis 0.5 m behind the slits, when they are illuminated with monochromatic light of wavelength 500 nm. [4]

(d) The figure below shows the 2D diffraction pattern formed when a pair of rectangular apertures in a plane mask, different to the mask in (c), are normally illuminated by monochromatic light of wavelength, λ. Darker print corresponds to higher intensity, on a logarithmic scale. The axes are scaled in units of $q = 2\pi \sin\theta/\lambda$. Draw a quantitatively labelled diagram showing the geometry of the apertures. [5]



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B9 Wave propagation can be modified by introducing additional boundary conditions to form a waveguide.

(a) Briefly discuss the benefits of using waveguides, compared to the unguided propagation of waves. Give two examples of the use of waveguides. [4]

(b) The z-displacement of a flexible membrane, which at rest lies in the $x-y$ plane, obeys the 2D wave equation [6]

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\sigma}{T}\frac{\partial^2 z}{\partial t^2}$$

where σ is the mass per unit area of the membrane and T is the tension per unit length, which is uniform in the $x-y$ plane. Show that

$$z = \sin(qy)\, e^{i(kx-\omega t)}$$

is a solution of the 2D wave equation and find the relationship between q, k and ω.

(c) If the membrane is fixed rigidly along the lines $y = 0$ and $y = a$, explain why only certain values of q are allowed and give an expression for q. [2]

(d) Show that waves will only propagate along the guide when [4]

$$\omega > \frac{n\pi}{a}\left(\frac{T}{\sigma}\right)^{1/2} \qquad n = 1, 2, 3, ...$$

(e) Sketch the dispersion relation for waves propagating along the guide in the three lowest frequency modes. [2]

(f) Explain, without detailed calculation, how and why the dispersion relationship would change if the flexible membrane was replaced with a membrane that resists bending. [2]

[*You may draw a useful analogy with a string that resists bending.*]

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SECTION C

Attempt **one** *question from this Section. Use a separate booklet for this section.*

C10 Condensed matter physics has a foundation in understanding the structure of materials.

(a) Explain the terms Bravais Lattice, unit cell and primitive unit cell. [3]

(b) If iron [atomic mass 56] can be modelled as a solid sphere of radius 0.124 nm, determine: the volume of the primitive unit cell and the density of iron in both the FCC and BCC forms. [6]

(c) Explain the concept of the reciprocal lattice and how it is calculated from the lattice vectors of a material. [3]

(d) A single crystal of iron is heated and undergoes a BCC - FCC phase transition. Describe how the reciprocal lattice of the material will evolve during this process. [3]

(e) Suggest an experimental technique that could be used to probe the structure of iron as it is heated. What problems might occur with making measurements as a result of the BCC - FCC phase transition? How might the new structure be probed? [5]

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C11 The free and nearly free electron model.

(a) Outline some of the 'failures' of the free electron model and the differences in approach taken in the nearly free electron model. [4]

(b) Given the form of the Bloch function, $\psi(x) = u(x)e^{i\phi x}$, show that for atoms spaced a distance a apart in a 1D loop of N atoms, [4]

$$\phi = \frac{2\pi n}{Na},$$

where x is a variable representing the distance around the loop and n is an integer.

(c) Using Bloch's Theorem, solve the Schrodinger Equation for the potential relating to the loop of atoms in (b), [8]

$$V(x) = \alpha \sum_{j=0}^{N-1} \delta(x - ja),$$

to show that

$$\cos(\phi a) = \frac{m\alpha}{\hbar^2 k}\sin(ka) + \cos(ka),$$

where $k^2 = 2mE/\hbar^2$.

(d) Taking $a = 1$ sketch how $\cos(\phi a)$ varies with k, for the case where $m\alpha/\hbar^2 = 10$ and comment on the implications for the electronic properties of the material. [4]

The derivative across the delta function satisfies the following:

$$\Delta\psi'(0) = \psi'_+(0) - \psi'_-(0) = \frac{2m\alpha}{\hbar^2}\psi(0).$$

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SECTION D

Attempt **one** *question from this Section. Use a separate booklet for this section.*

D12 Write an essay describing wave propagation in various media. [20]

D13 Write brief notes on **two** of the following:

(a) the Hall effect; [10]

(b) doping in semiconductors; [10]

(c) thermal conductivity of metals. [10]

END OF PAPER

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