UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 31st May 2012 1.30 to 4.30 pm

PHYSICS A (2)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C and D each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains 8 sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae which you may quote without proof.
Answers from **each** *Section must be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet. If several Booklets are used for a given Section join them together using a Treasury Tag.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Booklets and Treasury tags	Physics Mathematical formulae handbook (supplied)
Rough workpad	Approved calculator allowed
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

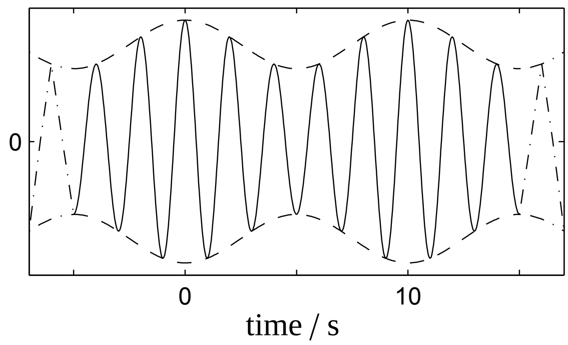
SECTION A

Answers should be concise and relevant formulae may be assumed without proof

1 Explain the phenomenon of Poisson's spot. [4]

2 Show how an elliptically polarised wave can be created by the superposition of a circularly polarised wave and a linearly polarised wave. [4]

3 What frequencies are present in the wavegroup shown by the solid line in the figure below? [4]



4 Draw annotated sketches of the atomic displacements for the acoustic and optical modes in a diatomic crystal. [4]

5 A monatomic free electron metal has a density of atoms of 2.5×10^{28} m^{-3} and a conductivity of 2.1×10^{7} Ω^{-1} m^{-1}. Evaluate the electronic mobility and mean time between collisions for this metal. [4]

SECTION B

B6 Define phase velocity u_p and group velocity u_g. [2]

Waves on deep water obey the dispersion relation for angular frequency ω

$$\omega^2 = gk + \frac{\sigma k^3}{\rho_1},$$

where g is the gravitational acceleration, σ and ρ_1 are respectively the surface tension and density of water, and k is the wavenumber. Verify that the group and phase velocities are equal at one particular wavelength, λ^*, and calculate the wave speed at λ^*, for $g = 9.8\ \mathrm{m\ s^{-2}}$, $\rho_1 = 10^3\ \mathrm{kg\ m^{-3}}$ and $\sigma = 72 \times 10^{-3}\ \mathrm{N\ m^{-1}}$. [4]

Find the ratio $u_\mathrm{g}/u_\mathrm{p}$ in the long and short wavelength limits. [2]

Consider a wavepacket which is compact at a given time. Describe or sketch the different time evolution when the wavepacket has wavelengths $\gg \lambda^*$ compared with when the wavelengths are $\ll \lambda^*$. [3]

If a thin elastic sheet, of thickness a and density ρ_2 covers the water surface, the dispersion relation is modified to

$$\omega^2\left(1 + \frac{\rho_2}{\rho_1}ak\right) = gk\left(1 + \frac{\sigma k^2}{\rho_1 g} + \frac{Bk^4}{\rho_1 g}\right),$$

where B is a modulus that quantifies the resistance of the sheet to bending. Take $\rho_2 = 0.8\rho_1$, $a = 10^{-4}$ m, $B = 10^{-10}$ J, and other values as above. Sketch, as a function of the wavenumber, the ratio of the frequencies observed with and without the covering sheet. Comment on the physical reasons for the behaviour of this ratio at small and large wavenumber. [5]

Assuming that this system is kept in a large container, and a square frame of edge $L = 6$ cm is lowered onto the surface pinning the boundaries, calculate by how much the lowest wave frequency allowed inside the frame is changed by the presence of the sheet. [4]

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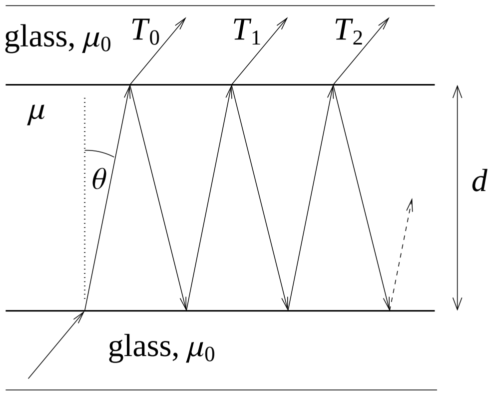
B7 A Fabry–Pérot interferometer consists of two flat glass plates separated by a distance d. The two inner surfaces are coated and have intensity reflection coefficient R; μ and μ_0 are the refractive indices of the medium between and outside the plates respectively. If an extended monochromatic source of light is observed through the interferometer, light is incident over a range of angles. The subsequent paths of one incident ray are shown in the figure.



Calculate the phase difference δ between the adjacent rays that exit the cavity as shown in the figure, and thus show that sharp bright rings are observed in the far-field when $2k_0\mu d\cos(\theta) = 2m\pi$, where m is an integer, k_0 is the wavenumber of the light in vacuum and θ is as labelled in the figure. [4]

Show that, for a fixed angle θ, the transmittance (the emitted fraction of the incident intensity) is given by [6]

$$T_e = \frac{(1-R)^2}{1+R^2-2R\cos(\delta)}.$$

The interferometer is now illuminated by a broadband source. Show that for a fixed angle θ, adjacent transmission peaks occur at a wavelength separation (called the free spectral range) given approximately by

$$\Delta\lambda \approx \frac{\lambda_0^2}{2\mu d\cos(\theta)},$$

where λ_0 is the central wavelength of the nearest transmission peak. [2]

A quantity, $\mathcal{F}$, known as the *finesse* of the interferometer is defined as $\mathcal{F} = \Delta\lambda/\delta\lambda$, where $\delta\lambda$ is the full-width half-maximum of the transmission for a particular order. It can be shown that the finesse is approximately

$$\mathcal{F} = \frac{\pi R^{\frac{1}{2}}}{1-R}.$$

A Fabry–Pérot interferometer is used to measure a spectrum that consists of two lines centred at $\lambda_1 = 670$ nm, with separation $\Delta\lambda_1 = 1$ nm, and another line at $\lambda_2 = 633$ nm. Derive appropriate values of d and R for measuring this spectrum, when observations are made varying θ. [5]

Discuss why a Fabry–Pérot interferometer can have a much higher resolving power than a Michelson interferometer of similar physical dimensions, and what the requirements on the flatness of the surfaces are, in order to achieve this higher theoretical resolving power. [3]

B8 State the conditions for Fraunhofer diffraction to apply, and how it can be achieved in practice. [2]

Explain why the complex amplitude ϕ is given by the Fraunhofer integral

$$\phi \propto \iint_{\Sigma} \psi_{\Sigma} \exp\left[-\mathrm{i}k\left(\frac{x_0 x + y_0 y}{R}\right)\right] \mathrm{d}x\,\mathrm{d}y$$

defining the quantities in this relation with a labelled sketch. [4]

Show, for the particular case of one-dimensional slits, that the aperture function can be recovered from a Fourier transform of the Fraunhofer diffracted field. [4]

An interferometric radio telescope is made of 10 dishes whose centres are separated by $D = 800$ m, aligned East–West on the equator, pointing directly overhead. Each dish has diameter 25 m. The interferometer operates at $\lambda = 6$ cm.

Consider first the dishes as synchronised emitters. Approximately how far away does an observer need to be in order to measure the Fraunhofer diffraction pattern? Show that in this case the intensity pattern observed as a function of angle is given approximately by:

$$I(\theta) \simeq I\left[\frac{\sin\left(10k_0\frac{D}{2}\theta\right)}{\sin\left(k_0\frac{D}{2}\theta\right)}\right]^2,$$

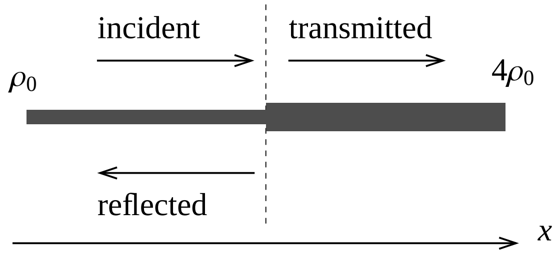
where θ is measured from the vertical, and $k_0 = 2\pi/\lambda$. State any assumptions made regarding the effect of the dish diameters. [4]

Sketch and label this intensity pattern. [3]

Now the same radio telescope is used in reception mode. Due to the Earth's rotation a bright source in the sky appears to move relative to the telescope. Through what angle must the source move to pass from the zero order maximum to the first minimum? [3]

B9 Consider a string of mass per unit length ρ held under a tension T. Explain what is meant by the characteristic impedance Z of the string, and how it depends on T and ρ. Evaluate the mean power transmitted by a travelling wave in terms of Z, ω and its amplitude A. [4]

Two long pieces of string of mass per unit length ρ_0 and $4\rho_0$ are connected together as shown in the figure, and held under a tension T, assumed to be constant.



Consider a sinusoidal transverse wave of the form

$$\psi = \mathrm{Re}\left(\psi_0 \exp\left[\mathrm{i}(kx - \omega t)\right]\right)$$

propagating in the lighter string in the x-direction, giving rise to reflected and transmitted waves. Obtain an expression for the disturbance which is set up in the light string, showing that it can be written as a sum of a standing wave and a travelling wave. [5]

Find the reflection and transmission coefficients, and the standing wave ratio (the ratio of maximum to minimum amplitude of oscillation) in the light string. [3]

An additional piece of string is inserted in between the two long pieces considered above. This new segment of string has different density and is of length a. Without detailed calculations, explain under what conditions, if any, it is possible to transmit fully the incident wave coming from the light string. [5]

A different piece of string is inserted between the two original strings. This new segment of string has density varying gradually from ρ_0 to $4\rho_0$, and has a length which is much greater than the incident wavelength. Under these conditions there is no reflection. What are the frequency, amplitude and wavelength of the wave in the heavier string, relative to the incident wave values? [3]

SECTION C

C10 Discuss, without mathematical detail, the origin of the band gap in semiconductors. [8]

In a particular direct gap semiconductor, the top of the valence band and the bottom of the conduction band are both located at the wavevector $\boldsymbol{k} = 0$. The band gap E_g is 0.18 eV, the effective mass of an electron in the conduction band is $m_e^* = 0.14\, m_e$, and the effective mass of a hole in the valence band is $m_h^* = 0.4 m_e$, where m_e is the free electron mass. A photon of energy 0.5 eV excites an electron from the valence to the conduction band. Sketch the energy bands and, on the assumption that a phonon is not involved and the wave vector of the photon is much less than the Fermi wavevector, show the electronic transition which occurs. [2]

Evaluate the initial and final energies of the electron, taking the zero of energy to be at the top of the valence band. [4]

If impurities are introduced into the semiconductor, new energy levels are introduced into the gap. Sketch their position for the introduction of *n*- and *p*-type impurities respectively, and briefly explain why they sit where they do. [4]

If the effective mass of the electron associated with a trivalent impurity is 0.1 m_e, and the dielectric constant of the material is 12, evaluate the ionisation potential of the singly-ionised impurity, commenting on this result. [2]

[*The energy levels of the hydrogen atom are given by*

$$E_n = -\frac{me^4}{2\hbar^2 n^2 (4\pi\epsilon_0)^2} = -\frac{1}{n^2} 13.6 \text{ eV}.$$
]

C11 Working within the free electron model, derive expressions for the Fermi energy E_F and the density of states $g(E_F)$. [6]

Hence evaluate the density of states at the Fermi energy for magnesium, which has a valence of two and an atomic concentration of 4.3×10^{28} m^{-3}. [4]

Using this value of $g(E_F)$, estimate the electronic contribution to the heat capacity of magnesium. [4]

Sketch the general form of the heat capacity of a metal as a function of temperature, briefly explaining the different regimes of behaviour (detailed mathematics is not required). [4]

Outline why the electronic contribution to the heat capacity of a metal is negligible at room temperature. [2]

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SECTION D

D12 Write an essay on the Q factor in different damping regimes, with physical examples. [20]

D13 Write short notes on **two** of the following:

(a) Umklapp processes and their relevance to thermal conductivity; [10]

(b) The Wiedemann–Franz law and electronic contributions to thermal and electronic conductivity; [10]

(c) Cyclotron resonance to measure effective mass of charged particles. [10]

END OF PAPER