UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 31st May 2018 9.00 am to 12.00 noon

PHYSICS A (2)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C, and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C, and D, each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains **8** *sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae, which you may quote without proof.*
Answers from **each** *Section should be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Booklets	Mathematical Formulae Handbook
Rough workpad	Approved calculator
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof. Use a separate booklet for the whole of this section.*

A1 The Hubble space telescope has a primary mirror of 2.4 m diameter. Estimate the angular resolution in radians at a photon energy of 2.5 eV. What is the resolution in metres when observing a source at a distance of 2×10^{17} m? [4]

A2 A diffraction pattern is observed at 0.5 m from an aperture. What is the outer radius of the second Fresnel half-period zone at the aperture, if the aperture is positioned at 0.5 m from a monochromatic point source of wavelength 400 nm? [4]

A3 To perform ultrasound imaging, an acoustic wave is transmitted into the body and reflections are detected at the surface of the body. By calculating the energy transmitted at an air-body boundary and a water-body boundary, explain why water-based gel is applied to the body surface during ultrasound imaging. [4]
[*You may assume acoustic impedances of:* $Z_{air} = 4 \times 10^2$ $kg\ m^{-2}\ s^{-1}$, $Z_{water} = 1.48 \times 10^6$ $kg\ m^{-2}\ s^{-1}$ *and* $Z_{body} = 1.63 \times 10^6$ $kg\ m^{-2}\ s^{-1}$.]

A4 Draw labelled diagrams showing the phonon dispersion curves for a solid modelled as a) a monatomic chain and b) a diatomic chain where the atoms are i) of the same mass and ii) of different masses. Indicate the position of the first Brillouin zone boundary in each case. [4]

A5 Given a Hall coefficient of 2.4×10^{-10} $m^3\ C^{-1}$, calculate the number of carriers per atom for Beryllium of density 1.85 g cm^{-3} and relative atomic mass 9. Suggest an interpretation of your result. [4]

SECTION B

Attempt **two** *questions from this Section. Use a separate booklet for the whole of this section.*

B6 A damped simple-harmonic oscillator has an equation of motion

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0,$$

where x is the displacement from equilibrium, m the mass, $\gamma = b/m$ the friction coefficient and $\omega_0 = \sqrt{k/m}$ the resonant angular frequency.

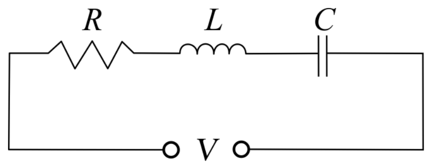
(a) By substituting a solution of the form Ae^{pt} for $t > 0$, identify the conditions for heavy, critical and light damping in terms of γ/m and ω_0. [4]

(b) The oscillator is now driven by $\mathrm{Re}\left[\frac{F_0}{m}e^{i\omega t}\right]$, where F_0 is the driving force and m the mass. By substituting the trial solution $x = \mathrm{Re}\left[Ae^{i\omega t}\right]$ into the equation of motion for a damped, driven, simple-harmonic oscillator, show that $A = R(\omega)F_0$, where $R(\omega)$ is the response function [6]

$$R(\omega) = \frac{1}{m\left[(\omega_0^2 - \omega^2) + i\gamma\omega\right]}.$$

Draw an annotated sketch of the amplitude and phase of the response function as a function of frequency ω. How would the amplitude sketch be modified for an oscillator with (i) a large Q factor or (ii) a very small Q factor?

(c) For the circuit shown below, [3]



in which $V(t) = \mathrm{Re}[V_0 e^{i\omega t}]$, show that the equation for the charge, q, on the capacitor is

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{q}{LC} = \frac{V}{L}.$$

(d) Show that the angular frequency at which the maximum amplitude occurs is [5]

$$\omega_{res} = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

(e) Why is the bandwidth of this resonance independent of capacitance? How can the performance of the circuit as a narrow linewidth band-pass filter be optimised? [2]

(TURN OVER)

B7

(a) With the aid of a diagram, state the condition for Fraunhofer diffraction and detail the ways in which this condition can be achieved in practice. [4]

(b) The Fraunhofer integral is given by: [4]

$$\psi_P \propto \iint_\Sigma \psi_\Sigma h(x,y) \exp\left[-ik\left(\frac{x_0 x + y_0 y}{R}\right)\right] dxdy.$$

Define the variables in this equation and, using your diagram, outline how it is derived.

(c) Two narrow slits are separated by distance D and illuminated by light at a wavelength of 400 nm. Sketch the intensity of the diffraction pattern of these slits as a function of angle viewed on a distant screen. What is the angular separation of the fringes if $D = 0.1$ mm? [3]

(d) One slit is now covered by a piece of glass that shifts the phase of the incident light by π. Sketch the new intensity diffraction pattern and annotate with the new angular separation. [3]

(e) The pair of narrow slits is replaced with two long rectangular apertures of width a separated by D. The same piece of glass is placed so as to cover the left hand aperture. Show that the resulting intensity diffraction pattern has an angular dependence proportional to [6]

$$\sin^2\left(\frac{qD}{2}\right)\mathrm{sinc}^2\left(\frac{qa}{2}\right),$$

where $q = 2\pi/\lambda \sin\theta$. How does this relate to the result for two narrow slits? Sketch the resulting intensity diffraction pattern, annotating the positions of the angular minima for $D = 0.1$ mm and $a = 0.02$ mm.

B8

(a) Explain what is meant by the terms ‘division of amplitude’ and ‘division of wavefront’ in the context of interferometry. [2]

(b) Draw a labelled diagram of a Michelson interferometer. [3]

(c) For monochromatic light entering the Michelson interferometer, assuming an equal intensity of the interfering beams of $I_0/2$, show that the intensity of the resulting fringe pattern varies as [5]

$$I(x) = I_0\left(1 + \mathrm{Re}\left[e^{ikx}\right]\right),$$

where k is the wavenumber and x is the difference in paths travelled by the two beams. Draw a graph of the intensity variation of the fringe pattern, annotating the positions of the minima.

(d) The light entering the Michelson interferometer is now replaced with the light from a sodium lamp. Sketch how the output intensity of the fringe pattern changes as a function of mirror displacement for the sodium D line, using the fact that the D line is actually a doublet centered at 589.3 nm with $\Delta\lambda$=0.6 nm. Annotate your sketch with appropriate values for the periodicities. [6]

(e) A broadband light source is now used. Explain how this configuration can be used to perform Fourier transform spectroscopy. For a wavelength of 500 nm, what value of mirror displacement is needed to achieve a spectral resolution of 0.01 nm? [4]

(TURN OVER)

B9 A light string, of mass ρ per unit length, is stretched under tension T along the x axis.

(a) Show that the motion arising from small transverse displacements $\Psi(x,t)$ can be described by the wave equation [5]

$$\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 \Psi}{\partial x^2}.$$

(b) Give a general definition of the term *wave impedance* and derive the wave impedance of the string. [2]

(c) The mean power required to drive an oscillator can be expressed by [3]

$$\langle P \rangle = \frac{1}{2}\,\mathrm{Re}\,[\mathbf{F}\mathbf{u}^*]\,,$$

where **F** is the force applied and **u** is the velocity, both described in complex notation. What is the mean power carried along the string when it is driven by force **F** at velocity **u**?

(d) State the conditions that must be satisfied by Ψ and $\partial\Psi/\partial x$ at any point on the string. [2]

(e) The wave encounters a discontinuity in mass density on the string. Explain the following observations by considering the mass densities on either side of the discontinuity ρ_1 and ρ_2: [8]

(i) A phase change is observed in the reflected wave at the boundary.
(ii) The wave amplitude and energy is entirely reflected at the boundary.
(iii) No reflection is observed at the boundary.

SECTION C

Attempt **one** *question from this Section. Use a separate booklet for this section.*

C10

(a) State the main assumptions of the free electron model. [3]

(b) Derive an expression for the density of states and the Fermi energy for a 3D electron gas within the free electron model. [6]

(c) Sketch the form of the Fermi Dirac distribution as the temperature is gradually raised from a starting point of 0K. Explain any assumptions you have made. [3]

(d) Explain how these results can be used to argue that the electronic heat capacity can be approximated as $\frac{9}{2}Nk_B\frac{T}{T_F}$. [4]

(e) Calculate the electronic heat capacity of sodium with density 0.97 g cm^{-3} and relative atomic mass 23 at 300K given that the Fermi energy can be taken to be 3.24 eV. Comment on the discrepancy between your calculated value and the measured heat capacity of sodium at 300K, which is approximately 28 J Mol^{-1} K^{-1}. [4]

C11

(a) Explain why the concept of effective mass is useful. Show that the effective mass of an electron can be written as: [6]

$$m^* = \hbar^2 \left(\frac{d^2\varepsilon}{dk^2}\right)^{-1}.$$

(b) By considering the motion of an electron in an electric field, show that the electrical conductivity, σ, can be written in terms of the number of electrons per volume n, their charge e and the electron mobility μ: [4]

$$\sigma = ne\mu.$$

(c) Calculate the drift velocity for copper with density 8.94 g cm^{-3} and relative atomic mass 63.5 in a 3 mm wire at a current of 13 A. Find an approximate value for the distance travelled by an electron during a 50 Hz mains cycle. Assume that copper has one electron per atom available for conduction. [5]

(d) Sketch a diagram showing the temperature dependence of electrical resistivity, explaining the different regimes present. Referring to the diagram, suggest a reason why pure iron generally has a lower electrical resistivity than mild steel which is approximately 99.75% iron and 0.25% carbon. [5]

(TURN OVER)

SECTION D

Attempt **one** *question from this Section. Use a separate booklet for this section.*

D12 Write an essay on doping in semiconductors including a brief discussion of how this gives rise to the concept of a P-N junction. [20]

D13 Write brief notes on **two** of the following:

(a) convolution and the convolution theorem as applied to the interpretation of Fraunhofer diffraction patterns; [10]

(b) thin film interference; [10]

(c) dispersion, group velocity and phase velocity. [10]

END OF PAPER