UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 2nd June 2022 1.30 pm to 4.30 pm

PHYSICS B Paper 2

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A carries approximately 20% of the total marks. In Sections B and C, each question carries approximately the same number of marks.

The paper contains **5** *sides including this one.*

You may refer to the Mathematical Formulae Handbook supplied, which gives values of constants and contains mathematical formulae which you may quote without proof. You may also use an approved calculator.

If you are taking the examination in person, answers from ***each*** *Section should be written in separate Booklets. Write the letter of the Section and your candidate number, not your name, on the cover of* ***each*** *Booklet. If more than one Booklet is used for a given Section, join them together using a Treasury Tag. A separate master (yellow) cover should also be completed, listing all questions attempted in the paper.*

If you taking the examination online, ***each*** *Section should be scanned or photographed after the Examination and uploaded in a* ***separate*** *file, according to the instructions provided. Before submitting your answers, ensure that all pages are of sufficient image quality to be readable.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 Calculate the change in volume of a 1 m^3 cube of basalt (a type of rock) when subjected to a constant pressure of 1 atm. The Young's modulus and Poisson's ratio of basalt are 60 GPa and 0.25 respectively. [4]

A2 Acetone of viscosity 3.16×10^{-4} N s m^{-2} and density 789 kg m^{-3} flows through a cylindrical pipe with a mean velocity of 0.5 m s^{-1}. Estimate the minimum diameter of the pipe if the flow is to be turbulent. [4]

A3 A lead block has mass 0.5 kg and specific heat capacity 710 J $K^{-1}kg^{-1}$. Calculate the approximate change in entropy of the Universe when: (a) the block, initially at a temperature of 200 °C, is placed in a lake at a temperature of 15 °C; (b) the block is at 15 °C and is dropped from a height of 200 m into a lake at a temperature of 15 °C. [4]

A4 Starting from the equation $\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V$, derive the Maxwell relations [4]

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \qquad \text{and} \qquad \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p .$$

A5 A gas consists of atoms with two energy levels separated by 27×10^{-6} eV. What is the population ratio between the two levels at temperatures of: (a) 500 K, and (b) 2 mK? [4]

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SECTION B

B6 Write an essay on fictitious forces in dynamics, i.e. centrifugal and Coriolis forces, including examples. [20]

B7 Write brief notes on **three** of the following: [20]

(a) the method of images in electrostatics;

(b) the polarisation of insulators in an electric field;

(c) the magnetic vector potential;

(d) the reflection and transmission of electromagnetic waves at the interface between two media.

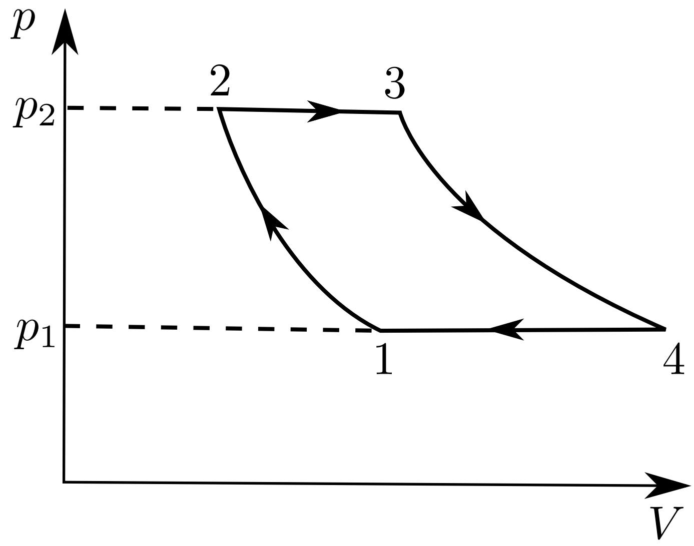
SECTION C

C8 (a) Describe the operation of a Carnot heat engine, including as part of your description an annotated sketch of a p, V diagram of the heat engine cycle. [3]

(b) Derive an expression for the efficiency η of an ideal Carnot heat engine operating between heat reservoirs at temperatures T_1 and T_2 $(T_2 > T_1)$. [4]

(c) Describe the conditions necessary for this efficiency to be achieved and explain why the efficiency of practical engines is always less than this. [2]

An idealised cycle (representing the operation of a simple gas engine in terms of the gas pressure p and volume V) is shown below; the paths $1 \to 2$ and $3 \to 4$ are adiabatic, paths $2 \to 3$ and $4 \to 1$ are at constant pressure, the gas can be assumed to be ideal and mechanical losses can be neglected.



(d) Sketch the cycle in a T, V diagram. [2]

(e) Show that the efficiency of the engine is given by [5]

$$\eta = 1 - \left(\frac{p_2}{p_1}\right)^{(1-\gamma)/\gamma}$$

where $\gamma = C_p/C_V$ is the ratio of the gas specific heats at constant pressure and at constant volume.

(f) One such engine uses an ideal gas with diatomic molecules as the working fluid. The engine operates with a value of p_2/p_1 of 10 and the ratio of the gas volumes at points 3 and 2 in the cycle is $V_3/V_2 = 2$. Find its efficiency and compare this result with the efficiency of an ideal Carnot engine operating between the same maximum and minimum temperatures. [4]

C9 (a) Show that the number of radiation states with energies of $\epsilon \rightarrow \epsilon + \mathrm{d}\epsilon$ inside a cubic reflecting cavity of dimension L is [3]

$$\mathrm{d}N = \frac{L^3\epsilon^2}{\hbar^3 c^3 \pi^2}\mathrm{d}\epsilon.$$

The mean energy of a quantum harmonic oscillator of angular frequency ω at temperature T is given by

$$\overline{U} = \frac{\hbar\omega}{\exp(\hbar\omega/k_\mathrm{B}T) - 1},$$

where k_B is the Boltzmann constant.

(b) Explain why the equilibrium radiation energy per unit volume inside the cavity at angular frequencies $\omega \rightarrow \omega + \mathrm{d}\omega$ is given by [4]

$$u(T,\omega)\,\mathrm{d}\omega = \frac{\omega^2\,\mathrm{d}\omega}{\pi^2 c^3}\frac{\hbar\omega}{\exp(\hbar\omega/k_\mathrm{B}T) - 1}$$

when the temperature of the cavity is T.

(c) Treating thermal radiation as a ‘gas’ of photons and adding together the contributions of photons travelling in all possible directions, show that the power radiated per unit area and per unit angular frequency by a black body of temperature T is given by [5]

$$e(T,\omega) = \frac{u(T,\omega)c}{4},$$

and hence that the total emitted power per unit area is proportional to T^4.

(d) A spacecraft contains no internal heat sources but is heated by radiation from the Sun. Show that the equilibrium surface temperature of the spacecraft is given by [4]

$$T \propto T_\mathrm{S}\sqrt{\frac{r}{R}},$$

where T_S is the surface temperature of the Sun, r is the radius of the Sun and R is the distance from the spacecraft to the centre of the Sun. You may assume that $R \gg r$ and that the spacecraft and the Sun are both spherical black bodies.

The surface temperature of the Sun is approximately 6000 K and the radiation spectrum of the Sun peaks at wavelengths of around 0.5 µm. It has been determined that the spacecraft will cease operation if its surface temperature exceeds 600 K.

(e) Considering the radiation absorption and emission at the spacecraft surface, discuss whether the spacecraft could be operated closer to the Sun if it is painted with (i) paint which is perfectly absorbing (‘black’) at all wavelengths, or (ii) paint which is perfectly absorbing at wavelengths shorter than 2 µm and less absorbing (‘grey’) at wavelengths longer than 2 µm. [4]

END OF PAPER