UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Wednesday 16th June 2021 (3 hours)

PHYSICS A (2)

Attempt **all** *questions.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. Section A will carry approximately 20% of the total marks. In Sections B, C, and D, each question carries approximately the same number of marks. The paper contains **5** *sides including this one.*

You may refer to the Mathematical Formulae Handbook supplied, which gives values of constants and contains mathematical formulae which you may quote without proof. You may also use an approved calculator.

If you are taking the exam in person, answers from **each** *Section should be written in separate Booklets. Write the letter of the Section and your candidate number, not your name, on the cover of* **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag. A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.*

If you are taking the exam online, **each** *Section should be scanned or photographed after the Examination and uploaded in a* **separate** *file according to the instructions provided. Before submitting your answers, ensure that all pages are of sufficient image quality to be readable.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof.*

1 Gravity waves on deep water have a dispersion relation $\omega = \sqrt{gk}$, where g is the acceleration due to gravity. An observer on an oil platform sees a wavepacket approach which contains 5 crests at any one time. How many crests strike the platform as the wavepacket passes? [4]

2 Explain, with the aid of a diagram, why soap bubbles exhibit complex, multi-coloured patterns rather than, say, pure red, green, and blue. [4]

3 A pianist in a concert hall plays a short note at 440 Hz that is heard to die away after roughly 5 seconds. Estimate the Q-value of the concert hall, giving reasons. [4]

4 In Cartesian coordinates, the lattice vectors of the primitive unit cell of a crystal are $a(0, \frac{1}{2}, \frac{1}{2})$, $a(\frac{1}{2}, 0, \frac{1}{2})$ and $a(\frac{1}{2}, \frac{1}{2}, 0)$.

Name the lattice type and calculate the reciprocal lattice vectors. [4]

5 A one-dimensional crystal has total length L and the unit cell is of length a. By applying periodic boundary conditions to the whole crystal, show that the first Brillouin zone contains one allowed wavevector for each unit cell in the crystal. [4]

SECTION B
Attempt **all** *questions from this Section.*

B6 Transverse waves travel along a long string of mass per unit length ρ_L stretched to tension T.

(a) Define the mechanical impedance Z_L of the string and derive an expression for it in terms of T and ρ_L. [3]

A second long string of mass per unit length ρ_R and impedance Z_R is connected to the right of the first one and a sinusoidal wave is incident on the boundary from the left.

(b) Give expressions for the amplitudes of the waves that are transmitted and reflected at the boundary, in terms of Z_L and Z_R, relative to the amplitude of the incident wave. [4]

(c) Show that the ratio of mean transmitted power to the mean incident power is independent of the wavelength and show that energy is conserved at the boundary. [4]

A third string, of length l and impedance Z_M, is inserted between the first two strings.

(d) Show that the boundary conditions may be written as

$$\begin{aligned} 1 + r &= a + b \\ Z_L(1 - r) &= Z_M(a - b) \\ \gamma a + \gamma^{-1} b &= t \\ Z_M(\gamma a - \gamma^{-1} b) &= Z_R t \end{aligned}$$

where a, b, r, and t are complex numbers and γ is a complex number of unit modulus, all of which you should define. [3]

(e) By solving for γ, or otherwise, find the relations between Z_L, Z_M, and Z_R for which no reflection results and describe the corresponding wavelengths at which this occurs. [6]

B7 In the Fresnel approximation for diffraction of light, the complex amplitude at point P is given by

$$\psi_P \propto \int \int_{\Sigma} h(x, y) \exp \frac{ik(x^2 + y^2)}{2R} dxdy.$$

(a) Define the quantities appearing in this expression and explain the approximations involved in its derivation. [7]

An optical experiment is performed in which a square aperture of size d is illuminated by a monochromatic point source placed on an axis normal to the square and passing through its centre. Observations are made at a point on the axis on the opposite side to the source.

(b) Give an expression for the complex amplitude at such a point in terms of the functions $C(w) = \int_0^w du \ \cos\left(\frac{\pi u^2}{2}\right)$ and $S(w) = \int_0^w du \ \sin\left(\frac{\pi u^2}{2}\right)$. [5]

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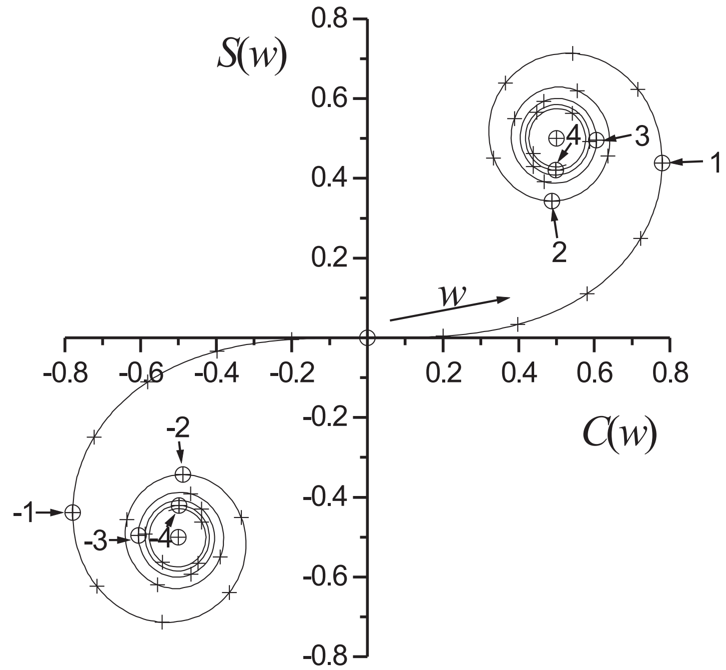
(c) Suppose that d =1mm and that the source and observation point are each 1m away from the aperture. Using the Cornu spiral shown, estimate the factor by which the intensity at the observation point is changed, compared to the unobscured intensity, when the light has wavelength of 1000nm. [4]

Observations are now made at points off the axis.

(d) Describe how the complex amplitude can be found using the Cornu spiral. [4]



SECTION C

Attempt the question from this Section.

C8 The Hamiltonian for the electronic states in a one-dimensional crystal is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\,,$$

where m is the mass of the electron and $V(x)$ is the lattice potential.

The wavefunctions of the electronic states in a one-dimensional crystal with lattice constant a can be written in the following form

$$\psi_k(x) = \sum_n C_{k,n}\,\exp(i[k + nG_1]x)\,,$$

where $G_1 = \frac{2\pi}{a}$.

(a) Explain the meanings of all the terms in this expression and briefly justify its form. [4]

(b) By writing the wavefunction as a combination of the two plane waves $\exp(ikx)$ and $\exp(i[k - G_1]x)$, show that, when the wavefunction is written in this form, the energies of the electronic states are given by

$$\frac{1}{2}\left(\frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 [k - G_1]^2}{2m}\right) \pm \frac{1}{2}\sqrt{\left(\frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 [k - G_1]^2}{2m}\right)^2 + 4|V(G_1)|^2} \ ,$$

where $|V(G_1)|$ is the magnitude of the lattice potential at wavevector G_1 which you can assume to be $\ll \frac{\hbar^2 G_1^2}{2m}$. [7]

(c) Calculate the energies of the states at wavevector $k = [\frac{G_1}{2} + \Delta k]$ to second order in Δk. [5]

(d) The effective mass m^* is given by $\hbar^2 \left(\frac{\partial^2 E}{\partial k^2}\right)^{-1}$, where E is the energy of the electronic state. Calculate the effective masses of the two bands at wavevector $\frac{G_1}{2}$ and comment on the physical implications for electrons in each band with wavevectors close to this value. [4]

SECTION D

Attempt the question from this Section.

D9 Write brief notes on the following:

(a) electrical conduction in metals; [10]

(b) the Debye model. [10]

END OF PAPER