UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 4 June 2015 13.30 to 16.30

PHYSICS B (Paper 2)

*Attempt **all** questions from Section A, **two** questions from Section B, and **two** questions from Section C.*

*Section A as a whole carries **approximately** one fifth of the total marks.*

Each question in Sections B and C carries the same mark.

*The **approximate** number of marks allocated to each part of a question in all Sections is indicated in the right margin.*

*Answers for each Section **must** be written in separate Booklets.*

Write the letter of the Section on the cover of each Booklet.

*Write your candidate number, **not** your name, on the cover of each Booklet.*

A single, separate master (yellow) cover sheet should also be completed, listing all questions attempted.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20-Page Booklets and Treasury Tags	Physics Mathematical Formulae Handbook (supplied)
Rough Work Pad	Approved Calculators allowed
Yellow Cover Sheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise and relevant formulae may be assumed without proof.

A1 A 1 kg meteorite travelling at $1\,\mathrm{km\,s^{-1}}$ and at a temperature of $1000\,^\circ\mathrm{C}$ crashes into a large lake. Estimate the change in entropy of the universe, assuming that all the water stays in the lake, given that the specific heat capacity of the meteorite is $700\,\mathrm{J\,kg^{-1}\,K^{-1}}$. [4]

A2 An infinitely long line charge carrying charge per unit length λ is placed parallel to and at a height h above an infinite horizontal perfectly conducting plane. Calculate the force per unit length on the line charge, and specify the direction of the force. [4]

A3 A particle at position $\mathbf{r}$ moves under a central force $\mathbf{F} = -A\mathbf{r}$ $(A > 0)$ in a path with maximum and minimum radius r_0 and $4r_0$. It is diverted into a circular orbit by a radial impulse. What is the radius of the circle? [4]

A4 The conductivities of stainless steel and gold are 1.5×10^6 and $4.1 \times 10^7\ \mathrm{S\,m^{-1}}$, respectively. If each of these materials is used to coat the inside of an identical waveguide, find the ratio of the resistances for high frequency currents in the plane of the coating film. Roughly how thick would the gold layer have to be in order to be useful at $10^{10}\,\mathrm{Hz}$? [4]

A5 A space probe of mass $m = 97\,\mathrm{kg}$ approaches the surface of a spherical comet moving at a speed of $1\,\mathrm{m\,s^{-1}}$. Just before impact it fires a harpoon of mass 1.4 kg into the surface to act as an anchor, at a speed of $70\,\mathrm{m\,s^{-1}}$ relative to the comet. During the subsequent motion a thruster pushes the probe continuously towards the comet with force 0.01 N. If the comet's radius is 2 km and the escape velocity is $0.44\,\mathrm{m\,s^{-1}}$, find the maximum height reached by the probe in this subsequent motion, assuming that the probe remains close to the surface. [4]

SECTION B

B6 Write brief notes on **three** of the following:

(a) the use of boundary conditions in solving problems that involve magnetic materials or electromagnets;

(b) inductors;

(c) the energy density and flux in static and oscillating electric and magnetic fields;

(d) reflection and transmission of electromagnetic waves at interfaces. [20]

B7 Write an essay about the need for, and uses of, the displacement current. You should include in your essay discussion of Maxwell's equations, electromagnetic waves in free space, metals and plasmas. [20]

B8 Write brief notes on **three** of the following:

(a) free precession of rigid bodies;

(b) conjugate momenta and conservation laws;

(c) moment of inertia tensors;

(d) Bernoulli's equation for incompressible flow, with two examples, including how to measure flow speed. [20]

B9 Write an essay on normal modes. You should include a discussion of the general principles, and several different examples. [20]

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SECTION C

C10 Explain what is meant by a *critical point* in the behaviour of a gas, illustrating your answer with the van der Waals equation for a gas of N particles

$$\left(p + \frac{aN^2}{V^2}\right)(V - bN) = Nk_\mathrm{B}T.$$

[5]

Consider instead the following approximate equation of state for a non-ideal gas, which takes the form of a truncated expansion in number density

$$p(n) = nk_\mathrm{B}T - \frac{b}{2}n^2 + \frac{c}{6}n^3,$$

where $n = N/V$ is the number density.

Sketch the isotherms $p(n)$ for different values of the temperature. Explain why the critical point corresponds to

$$\left.\frac{\mathrm{d}p}{\mathrm{d}n}\right|_T = \left.\frac{\mathrm{d}^2p}{\mathrm{d}n^2}\right|_T = 0,$$

indicating it on your sketch. [6]

Show that the critical density is $n_\mathrm{c} = b/c$, and find the critical temperature T_c and pressure p_c. [3]

Find the isothermal compressibility $\kappa_T = -\frac{1}{V}\left.\frac{\partial V}{\partial p}\right|_T$ at $n = n_\mathrm{c}$, and sketch its behaviour as a function of T. What does the equation predict for $T < T_\mathrm{c}$? [4]

Describe briefly what *actually* happens below the critical point. [2]

C11 State the Boltzmann distribution, and use it to find the average potential energy per particle of a gas of particles of mass m in thermal equilibrium at temperature T in a gravitational field of strength g. [5]

Explain what is meant by *equipartition of energy*. [4]

A large number of identical balls of mass m are dropped from the same height h inside a vertical tube. Assume that the balls are rough and collide elastically with each other and the walls and floor of the tube, and that the tube is sufficiently tall that no balls reach the top.

Assuming that the macroscopic motion of the balls may be described by statistical mechanics with an effective temperature T, show that

$$T = \frac{mgh}{4k_B}.$$ [6]

Find the fraction of balls that you would expect to find at a height greater than h at any instant. [5]

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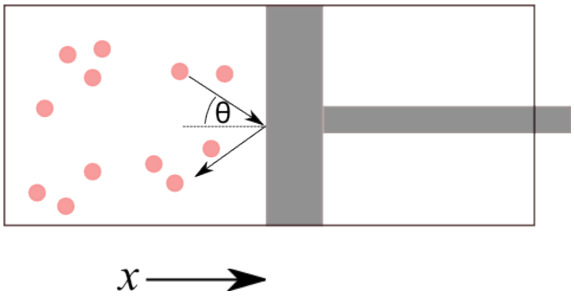
C12 The figure shows a cylinder containing a gas of particles.



Show that the flux on the piston (the number of molecules arriving per unit area per unit time) from particles moving at angles in the range $\theta \to \theta + \mathrm{d}\theta$ and speeds $v \to v + \mathrm{d}v$ is

$$\frac{1}{2} n f(v)\, v \cos\theta \sin\theta \,\mathrm{d}\theta \,\mathrm{d}v,$$

where n is the number density of the gas and $f(v)$ the distribution function of molecular speeds. [4]

Show that the pressure of a gas of particles of mass m and number density n is given by

$$p = \frac{1}{3} nm\langle v^2\rangle,$$

where $\langle v^2\rangle$ is the average squared velocity. [5]

Now suppose the piston moves inwards with velocity u, compressing the gas. No heat can flow through the walls of the container.

A particle with velocity (v_x, v_y, v_z) collides elastically with the moving piston. Find its velocity just after the collision. [*Hint: consider how things look in a frame moving with the piston.*] To first order in u, what is the kinetic energy of the particle *after* its collision with the piston? [6]

Using this result, find an expression for the rate at which the total energy of the gas is changing, and interpret the result in terms of the pressure. [5]

C13 Write an essay on the second law of thermodynamics, including a discussion of heat engines, Carnot's theorem, and entropy as a function of state. [20]

END OF PAPER