UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part IB

Thursday 1st June 2017 1.30 to 4.30 pm

PHYSICS A (2)

Attempt **all** *questions from Section A,* **two** *questions from Section B,* **one** *question from Section C, and* **one** *question from Section D.*
Section A will carry approximately 20% of the total marks.
In Sections B, C, and D, each question carries approximately the same number of marks.
The approximate number of marks allocated to each question or part of a question is indicated in the right margin.
The paper contains **8** *sides including this one and is accompanied by a Mathematical Formulae Handbook giving values of constants and containing mathematical formulae, which you may quote without proof.*
Answers from **each** *Section should be written in separate Booklets.*
Write the letter of the Section and your candidate number, not your name, on the cover of **each** *Booklet. If several Booklets are used for a given Section, join them together using a Treasury Tag.*
A separate master (yellow) cover sheet should also be completed, listing all questions attempted in the paper.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Booklets and treasury tags	Mathematical Formulae Handbook
Rough workpad	Approved calculator
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

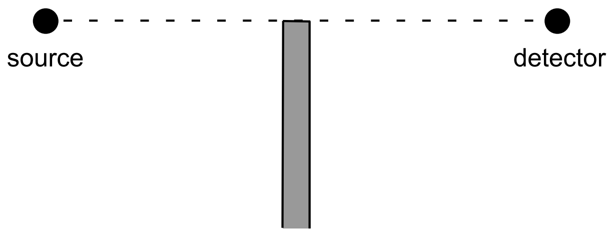
SECTION A

Attempt **all** *questions from this Section. Answers should be concise and relevant formulae may be assumed without proof. Use a separate booklet for the whole of this section.*

1 A wave is travelling at speed c along a string with mass per unit length ρ. The wave strikes the end of the string which is attached to a damper that applies a transverse force $-\alpha v$, where v is the transverse velocity of the end of the string, and $\alpha > 0$. Find the value of α such that half the incident power is reflected at the end of the string. [4]

2 A microscope using light of wavelength 600 nm has an objective lens of diameter 0.5 cm and focal length 2 cm. Estimate the spatial resolution that the microscope can achieve. [4]

3 A semi-infinite obstruction is placed between a point source of light and a detector, as shown below. Sketch the Cornu spiral, and use your sketch to find the intensity measured at the detector as a fraction of the intensity that would be measured if the obstruction were not present. [4]



4 Calculate the ratio of the volumes of the non-primitive face-centred cubic and body-centred cubic unit cells assuming the constituent atoms are identical hard spheres of radius r which are in contact with their nearest neighbours. [4]

5 Potassium [density 0.862 g cm^{-3}, relative atomic mass 39] is a monovalent material which is a good approximation to a free-electron metal. In such a material it can be shown that N electrons each of mass m_e in a volume V impart a short-range repulsive pressure

$$P = \frac{\hbar^2}{5m_e}(3\pi^2)^{2/3}\left(\frac{N}{V}\right)^{5/3} .$$

Estimate the isothermal bulk modulus $K_T = -V\,(\partial P/\partial V)_T$ of potassium assuming this pressure is the only contribution to it. [4]

SECTION B

Attempt **two** *questions from this Section. Use a separate booklet for the whole of this section.*

B6 A damped harmonic oscillator has the equation of motion

$$m\ddot{x} + b\dot{x} + kx = F.$$

Using the complex notation where F and x respectively represent a sinusoidally oscillating force and the steady-state displacement response at a frequency ω, find an expression for the frequency-dependent complex response function, $R(\omega) = x(\omega)/F(\omega)$, and sketch its amplitude and phase as a function of frequency in the case of light damping. [5]

Find an expression for the amplitude resonance frequency, ω_a, and find an approximate expression for $|R(\omega)|$ in the limit when $\omega \gg \omega_0 = \sqrt{k/m}$. [4]

A sensitive optical experiment is mounted on a large metal table of mass m. The table is supported by four legs, each with spring constant $1 \times 10^4\,\mathrm{N\,m^{-1}}$, and can move up and down in the vertical direction. A damper with damping constant $b = 500\,\mathrm{N\,s\,m^{-1}}$ is attached to the table. Vibrations in the room provide a sinusoidal driving force of amplitude $F_0 = 5\,\mathrm{N}$ at a frequency of 10 Hz on the table. Assuming that the damping is light and that the resonance frequency is much less than 10 Hz, estimate what minimum mass of table is required to ensure that the amplitude of vibration of the table, $|x_0|$, is less than 1 µm. For this value of mass, calculate the actual value of $|x_0|$, and verify that this value is indeed close to 1 µm. [5]

A piece of equipment of mass 20 kg is suddenly placed on the table. Draw an annotated sketch of the displacement of the table a function of time. [3]

What value of b would be preferable in practice? [3]

B7 Explain what is meant by group velocity, and show that it is given by $v_g = d\omega/dk$. [6]

Show that for a dispersive medium where the phase velocity v_p depends on wavelength, λ,

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}.$$ [4]

When light is refracted at an interface between two media, the angle of incidence, θ_1, and the angle of propagation of the refracted light, θ_2, are related by

$$v_2 \sin\theta_1 = v_1 \sin\theta_2,$$

where v_1 and v_2 represent the phase velocities of the light in the first and second media, respectively. Light of wavelength λ is incident from a vacuum onto an interface with a medium where the speed of light is $v_p(\lambda)$. The angle of incidence is 45.00°. For $\lambda = 500$ nm, $\theta_2 = 17.80°$. When the wavelength is changed to 498 nm, θ_2 decreases by 0.010°, and when the wavelength is changed to 502 nm, θ_2 increases by 0.010°. Find the group velocity in the medium at a wavelength of 500 nm. [6]

A short pulse of light contains wavelengths centred around 500 nm. Find how long it would take for the pulse to propagate through 1 cm of the medium, and describe qualitatively but in detail what would happen to the pulse as it propagates. [4]

B8 State the convolution theorem for Fourier transforms, and show with the aid of diagrams how it can be used to predict the Fraunhofer diffraction pattern of a grating with a finite number of very narrow slits. [6]

A diffraction grating with 500 slits per mm is illuminated at normal incidence with parallel light of wavelength 400 nm. Find the angle of diffraction of the second-order diffraction peak. [2]

Working with the second-order diffraction peaks, how wide must the grating be in order to resolve two spectral lines close to 400 nm but separated in wavelength by 0.2 nm? [3]

The diffracted light is collected by a lens of focal length 0.25 m. An array detector with a pixel width t is placed at the focus and used to measure the diffraction pattern. What value of t is required to resolve the two spectral lines? [4]

The width of each slit in the diffraction grating used in the experiment above is 1 μm. Explain why this width of slits causes a problem in the experiment. [5]

B9 A thin, uniform, transparent film of refractive index n and thickness d is deposited on a flat substrate of refractive index $n_2 > n$. A laser beam of wavelength λ is incident from the surrounding air onto the film, at an angle θ_i from the normal. Show that the phase difference between light reflected at the air-film interface and light passing into the film and being reflected at the film-substrate interface is given by

$$\delta = \frac{4\pi n d}{\lambda} \cos\theta,$$

where θ is the angle of propagation inside the film, given by $n \sin\theta = \sin\theta_i$. [5]

Light of wavelength λ and amplitude A is incident on the top surface of the film. A reflection of amplitude $0.15A$ occurs at the top interface (air-film). The wave propagating into the film is partially reflected at the bottom interface (film-substrate) and emerges from the top surface of the film with amplitude $0.03A$. Neglecting any dependence of the reflection and transmission coefficients on wavelength or angle, and assuming that there is no reflection of light leaving the film at the film-air interface, find an expression for the reflected intensity as a function of δ, and sketch it. [5]

In a real experiment, the film refractive index is 1.2 and the wavelength is 500 nm. For angles θ_{i} close to $10°$, bright fringes in the reflected intensity occur at a spacing of $0.4°$. Find the film thickness, d. [6]

Describe qualitatively and explain the behaviour seen when monochromatic light is reflected at normal incidence from an oil film of non-uniform thickness on the surface of a puddle. [4]

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SECTION C

Attempt **one** *question from this Section. Use a separate booklet for this section.*

C10 Give a brief physical description of a phonon. [2]

Consider a one-dimensional chain of particles where the particles alternate between type A and type B, which have different masses. If the spring constant joining the masses is α, show that the angular frequency ω of small amplitude oscillations along the length of the chain is given by

$$\omega^2 = \frac{\alpha}{m_A m_B}\left\{(m_A + m_B) \pm \left[(m_A + m_B)^2 - 4m_A m_B \sin^2 qa\right]^{\frac{1}{2}}\right\}, \qquad (*)$$

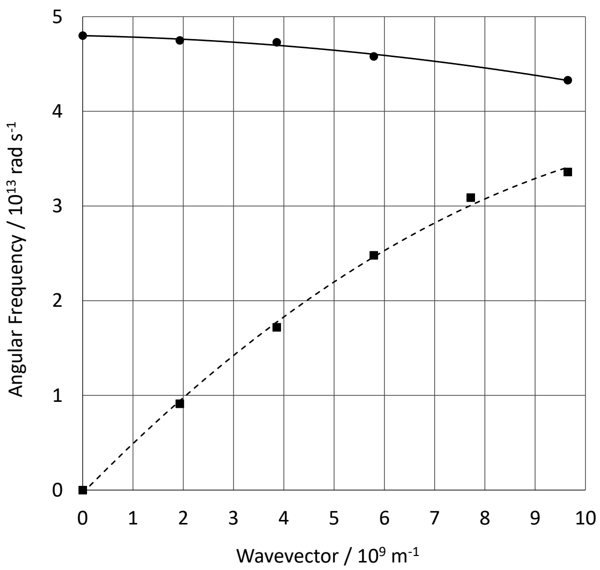
where m_A and m_B are the masses of the two types of particles, q is the wavevector and a is the spacing between adjacent particles in the chain. [6]

Comment briefly on the physical meaning of the two solutions in equation $(*)$. [2]

In the limit where the wavevector is small, show that the angular frequencies in equation $(*)$ can be described by

$$\omega \approx \sqrt{\frac{2\alpha(m_A + m_B)}{m_A m_B}} \qquad \text{and} \qquad \omega \approx \sqrt{\frac{2\alpha a^2 q^2}{m_A + m_B}}.$$ [4]

The figure below shows the dispersion relationship for phonons travelling in the [111] direction in NaCl, which approximates well to the one-dimensional model considered above as it has alternating planes of Na and Cl atoms.



Using the small-wavevector limit and the figure, estimate the value for α in this system. [*The relative atomic masses of sodium and chlorine can be taken as 23 and 35.5 respectively.*] [3]

Using the data in the figure, find a value for the speed of sound in the [111] direction in NaCl and then use this to find the spacing between a plane of Na atoms and the neighbouring plane of Cl atoms. [3]

C11 Explain, without mathematical details, what is meant by a semiconductor and what differentiates it from a metal. Explain the difference between direct and indirect band gaps and hence describe why it might be easier to make a light-emitting diode from gallium arsenide rather than silicon. [5]

Again without mathematical details explain the concept of doping in semiconductors, including the difference between p and n type. Sketch the energy level structure of a p-n junction. [5]

If the electrons in germanium have an effective mass $m_{\mathrm{e}}^* = 0.1 m_{\mathrm{e}}$ and a dielectric constant of $\epsilon = 16$, then by modifying the Bohr model for hydrogen, determine the energy required to ionise donor states in the material, and the effective radius of the donor states. At 300 K, explain whether it is likely that a significant proportion of the donor atoms will be ionised. [3]

A germanium [density 5.3 g cm^{-3}, relative atomic mass 73] sample is doped by replacing one in every million atoms with single-electron donors. Assuming all of the donors are ionised calculate the ratio of conductivity of the doped sample to the undoped case at 300 K where there are $\approx 3 \times 10^{19}$ conduction electrons per m^3. [2]

Describe how the carrier concentration could be measured. [3]

Estimate the doping concentration at which the orbits of the donor states start to overlap. [2]

[*The Bohr equations for the energies and radii of electron states in a hydrogen atom are*

$$E_n = \frac{m_{\mathrm{e}} e^4}{2\hbar^2 n^2 (4\pi\epsilon_0)^2}, \qquad r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{m_{\mathrm{e}} e^2}.$$

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SECTION D

Attempt **one** *question from this Section. Use a separate booklet for this section.*

D12 Write an essay on diffraction patterns of simple circular holes and obstructions, in both the Fraunhofer and Fresnel regimes. [20]

D13 Write brief notes on **two** of the following:

(a) the Debye theory of heat capacity in solids; [10]

(b) the concept of effective mass in condensed matter physics and its relation to electrical transport properties; [10]

(c) the reciprocal lattice and Brillouin zones. [10]

END OF PAPER