0) Problem 4.26 Type problems

1) Transfer trajectories

A) Impulsive (high Thrust - Short duration)

B) Low Thrust (Long Duration)

$$\vec{r}_0 = r_{01}\hat{n}_1 + r_{02}\hat{n}_2 + r_{03}\hat{n}_3$$

$$\vec{V}_0 = V_{01}\hat{n}_1 + V_{02}\hat{n}_2 + V_{03}\hat{n}_3$$

$$\vec{h} = \vec{h}_0 = \vec{r}_0 \times \vec{V}_0$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} \qquad \varepsilon = \frac{\vec{v} \cdot \vec{v}}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$$

Keppler $\rightarrow$ θ_1 if given Δt

$$r(\theta) = \frac{h^2}{\mu(1 + e\cos\theta)}$$

$$r(\theta) = r_{11}\hat{n}_1 + r_{12}\hat{n}_2 + r_{13}\hat{n}_3$$

$$\hat{p} \triangleq \frac{\vec{e}}{e}, \quad \hat{w} \triangleq \frac{\vec{h}}{h}, \quad \hat{q} \triangleq \hat{w} \times \hat{p}$$

$$\left[{}^{PF}C_0^{N}\right] = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ w_1 & w_2 & w_3 \end{bmatrix} = \text{Body } \overset{ZXZ}{313}$$

$$\Omega_0, \; i, \; \omega_0$$

$$\Omega(t_1) = \Omega_0 + \dot{\Omega} t_1$$

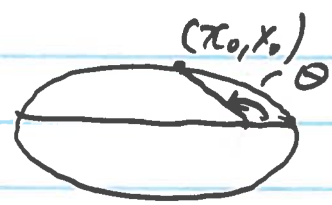
$$\Omega_0 = ATAN2(\underset{Y}{\overset{w_1}{C_{31}}}, \underset{X}{\overset{-w_2}{-C_{32}}}), \quad i = \cos^{-1}(C_{33})$$

$$\omega_0 = ATAN2(\overset{p_3}{C_{13}}, \overset{q_3}{C_{23}}) \qquad \omega = \omega_0 + \dot{\omega} t_1$$

$$\dot{\Omega} = -\left[\frac{3\sqrt{\mu} R^2 J_2}{2(1-e^2)^2 a^{7/2}}\right] \cos i$$

$$\dot{\omega} = -\left[\frac{3\sqrt{\mu} R^2 J_2}{2(1-e^2)^2 a^{7/2}}\right] \left(\frac{5}{2}\sin^2 i - 2\right)$$

$$\begin{Bmatrix} x_0 \\ y_0 \\ \cancel{z_0} \to 0 \end{Bmatrix}_{PF} = \left[{}^{PF}C_0^{N}\right] \begin{Bmatrix} r_{01} \\ r_{02} \\ r_{03} \end{Bmatrix}_N$$



$$\theta_0 = ATAN2(y_0, x_0)$$

Θ_0

Kepler

$$\sqrt{\frac{\mu}{a^3}}\,(\overbrace{t - T_0}^{\tau_0}) = E_0 - e\sin E_0$$ ← Elliptic traj.

$$\tan\left(\frac{\Theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{E}{2}\right)$$

→ solve for E_0

Solve Keplers for time ~~at~~ τ_0

~~$\ast$~~ ~~at~~ $\tau_1 = \tau_0 + 96(3600)$

$$\sqrt{\frac{\mu}{a^3}}\,(\tau_1) = E_1 - e\sin E_1$$

↑ solve Numerically for E_1

E_1 $$\tan\left(\frac{\Theta_1}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{E_1}{2}\right)$$

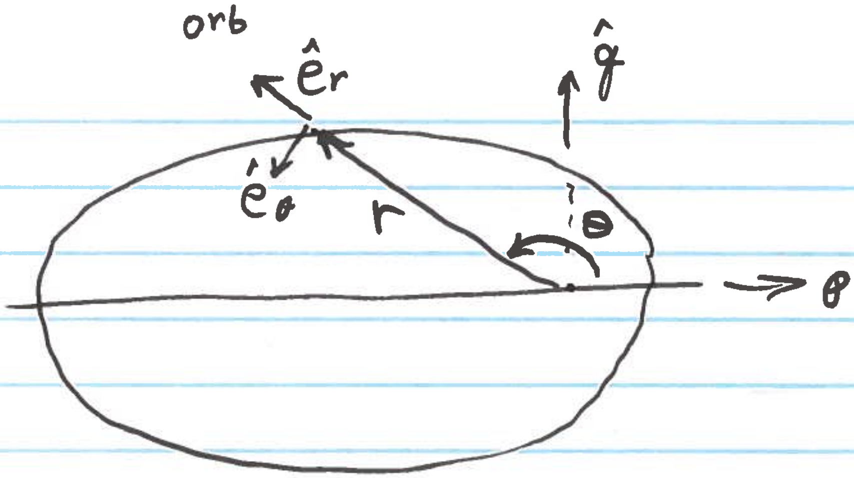
solve for Θ_1

$$r_1 = r(\Theta_1) = \frac{h^2}{\mu(1+e\cos\Theta_1)}$$

$$\begin{Bmatrix} r_{11} \\ r_{12} \\ r_{13} \end{Bmatrix}_N = \left[{}^{N}C^{orb}\right]\begin{Bmatrix} r_1 \\ 0 \\ 0 \end{Bmatrix}_{orb}$$

↑ want

$$\left[{}^{N}C^{orb}\right] = \left[{}^{N}C^{PF}\right]\left[{}^{PF}C^{orb}\right]$$



$$\left[{}^{orb}C^{PF}\right] = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{PF}C^{N}\right] = \left[{}^{PF}C^{B}\right]\left[{}^{B}C^{A}\right]\left[{}^{A}C^{N}\right]$$

$$\left[{}^{A}C^{N}\right] = \begin{bmatrix} C\Omega & S\Omega & 0 \\ -S\Omega & C\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \Omega = \Omega_0 + \dot{\Omega} J_1$$

$$\left[{}^{B}C^{A}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_i & S_i \\ 0 & -S_i & C_i \end{bmatrix}$$

$$\left[{}^{PF}C^{B}\right] = \begin{bmatrix} C\omega & S\omega & 0 \\ -S\omega & C\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \omega = \omega_0 + \dot{\omega} J_1$$

Low Thrust Transfer

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$$

$$\varepsilon = \frac{-\mu}{2a}$$

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}\left(-\frac{\mu}{2a}\right) = \frac{\mu}{2a^2}\frac{da}{dt}$$

$$\frac{d\varepsilon}{dt} = P = \vec{F}\cdot\vec{v} = \vec{F}\cdot(v\hat{t})$$
$$= \tilde{a}^{p}\cdot v\hat{t}$$
$$= \left(\dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}\right)\cdot v\hat{t}$$

$$= \underbrace{\dot{v}}_{A_t} v$$

$$\frac{v^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a} \quad \Rightarrow \quad v = \sqrt{\mu}\left[\frac{2}{r} - \frac{1}{a}\right]^{1/2}$$

$$\frac{d\varepsilon}{dt} = \frac{\mu}{2a^2}\frac{da}{dt} = A_t\left[\sqrt{\mu}\left(\frac{2}{r} - \frac{1}{a}\right)^{1/2}\right]$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

$$\frac{\sqrt{\mu}}{2a^2}\frac{da}{dt} = A_t\left[\frac{1+2e\cos\theta+e^2}{a(1-e^2)}\right]^{1/2} \qquad e \sim 0$$

$$\frac{\sqrt{\mu}}{2a^2}\frac{da}{dt} = (A_t)\left[\frac{1}{a}\right]^{1/2}$$

const?

$$\frac{da}{a^{3/2}} = \frac{2}{\sqrt{\mu}}\,dt(A_t)$$

$A_t = \text{const}$
$\text{mass} = \text{const}$

$$\Rightarrow \int_{a_0}^{a_f} \frac{da}{a^{3/2}} = \frac{2A_t}{\sqrt{\mu}}(t - t_0)$$

$$A_t = \frac{F_{thrust}}{m}$$

$a_0 \sim r_0$
$a \sim r$

$$m = m_0 - \dot{m}t$$

$$\int_{a_0}^{a} \frac{da}{a^{3/2}} = \frac{2}{\sqrt{\mu}} \int_{t_0}^{t} \frac{\text{Thrust}}{(m_0 - \dot{m}t)} dt$$

$$\Rightarrow \boxed{\frac{1}{\sqrt{r_0}} - \frac{1}{\sqrt{r}} = \left(\frac{\text{Thrust}}{\sqrt{\mu}\,\dot{m}}\right) \ln\left[\frac{m_0}{m_0 - \dot{m}t}\right]}$$