A) Two-Body Problem

B) $\Sigma \vec{F}^{Q} = m_Q \, {}^{N}\vec{a}^{Q}$

$\vec{F}_g^{Q}$

C) UNITS

D) Basis Transformation

(Direction Cosine Matrices)

DCM

$$\Sigma \vec{F}^{Q} = \vec{F}_{g_{Earth}}^{Q} + \vec{F}_{g_{sun}}^{Q} + \vec{F}_{g_{moon}}^{Q} + \vec{F}_{sp}^{Q}$$

Solar Pressure

$$+ \vec{F}_{mag.}^{Q} + \vec{F}_{Aero}^{Q}$$

$$F_g = \frac{G M_E m}{r^2} \Rightarrow \vec{F}_g^{Q} \triangleq \frac{-G M_E m_Q \vec{r}^{OQ}}{(r^{OQ})^3}$$

$$r^{OQ} = |\vec{r}^{OQ}|$$

$$\Rightarrow \frac{-G M_E \cancel{m_Q}}{(r^{OQ})^3} \vec{r}^{OQ} = \cancel{m_Q} \, {}^{N}\vec{a}^{Q}$$

$$\Rightarrow \mu \Rightarrow \boxed{\frac{-\mu}{(r^{OQ})^3} \vec{r}^{OQ} = {}^{N}\vec{a}^{Q}}$$

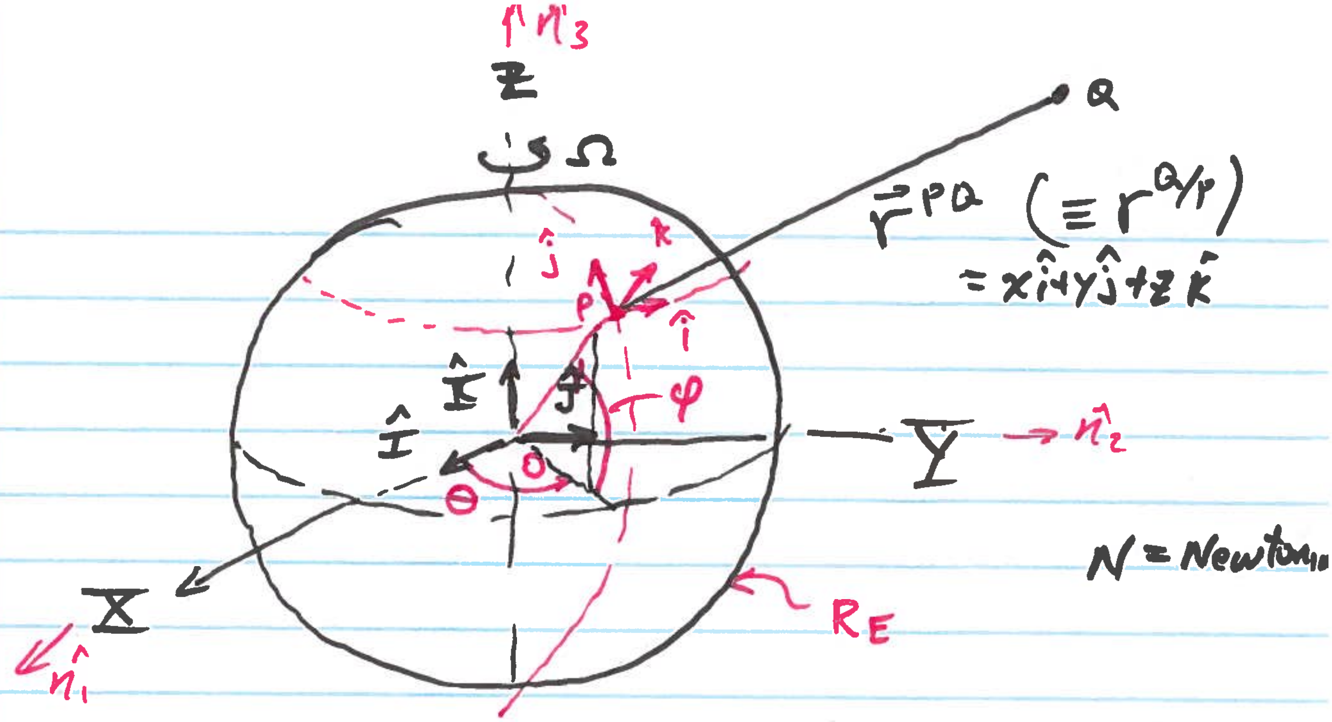
$$\boxed{\mu_E \cong G M_E} \qquad \mu = G(M_E + m_Q)$$

$$\boxed{\mu_E = 398600 \ \mathrm{km^3/s^2}}$$

SI: [Kg] [m] [s]
(Km)

English [lbf] [ft] [s]

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_n}{a_n} = m$$



$$\vec{F}_g^{\,Q} = m_Q \, {}^{N}\vec{a}^{\,Q}$$

$$\Rightarrow \; \frac{-\mu \vec{r}^{\,OQ}}{(r^{OQ})^3} = {}^{N}\vec{a}^{\,Q}$$

$A \equiv E$

$$= {}^{N}\vec{a}^{\,P} + {}^{A}\vec{a}^{\,Q/P} + {}^{N}\vec{\alpha}^{A} \times \vec{r}^{\,PQ} + {}^{N}\vec{\omega}^{A} \times ({}^{N}\vec{\omega}^{A} \times \vec{r}^{\,PQ}) + 2\,{}^{N}\vec{\omega}^{A} \times {}^{A}\vec{v}^{\,Q/P}$$

$R_E = 6378$ Km

$$\vec{r}^{\,OQ} = \vec{r}^{\,OP} + \vec{r}^{\,PQ} = R_E \hat{k} + (x\hat{i} + y\hat{j} + z\hat{k}) = x\hat{i} + y\hat{j} + (R_E + z)\hat{k}$$

(x, y, z) Measured

$${}^{E}\vec{v}^{\,Q/P} = \frac{{}^{E}d}{dt} r^{PQ} = \frac{{}^{E}d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} + \cancel{x \frac{{}^{E}d\hat{i}}{dt}}^{0} + \cancel{y \frac{{}^{E}d\hat{j}}{dt}}^{0} + \cancel{z \frac{{}^{E}d\hat{k}}{dt}}^{0} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$${}^{E}\vec{a}^{\,Q/P} = \frac{{}^{E}d}{dt} {}^{E}\vec{v}^{\,Q/P} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$${}^{N}\vec{\omega}^{E} = \Omega \hat{K} \qquad \Omega = \frac{2\pi}{1\,\text{day}[s]}$$

$$\boxed{\Omega \sim 7.292 \times 10^{-5}\ \text{rad}/s}$$

1day = 23hr 56mn 4 sec

~ 23.9344?

$${}^{N}\vec{a}^{Q} = {}^{N}\vec{a}^{P} + {}^{E}\vec{a}^{Q/P} + \cancel{{}^{N}\vec{\alpha}^{E}}^{\sim 0} \times \vec{r}^{PQ} + {}^{N}\vec{\omega}^{E} \times ({}^{N}\vec{\omega}^{E} \times r^{PQ}) + 2\,{}^{N}\vec{\omega}^{E} \times {}^{E}\vec{v}^{Q/P}$$

$${}^{N}\vec{a}^{P} = \cancel{{}^{N}\vec{a}^{O}}^{0} + \cancel{{}^{E}\vec{a}^{P/O}}^{0} + \cancel{{}^{N}\vec{\alpha}^{E}}^{0} \times \vec{r}^{OP} + {}^{N}\vec{\omega}^{E} \times ({}^{N}\vec{\omega}^{E} \times \vec{r}^{OP}) + \cancel{2\,{}^{N}\vec{\omega}^{E} \times {}^{E}\vec{v}^{P/O}}^{0}$$

$$= {}^{N}\vec{\omega}^{E} \times ({}^{N}\vec{\omega}^{E} \times \underbrace{\vec{r}^{OP}}_{R_E \hat{k}})$$

⇒ Must move from $\hat{i}, \hat{j}, \hat{k}$ to $\hat{I}, \hat{J}, \hat{K}$ and back easily

$$\vec{r}^{D} = A_1 \hat{a}_1 + A_2 \hat{a}_2 + A_3 \hat{a}_3 \Rightarrow \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_A$$

$$= X \hat{I} + Y \hat{J} + Z \hat{K} \Rightarrow \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{IJK}$$

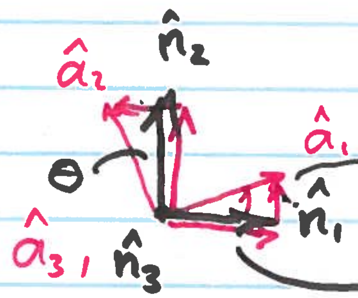
$$= B_1 \hat{b}_1 + B_2 \hat{b}_2 + B_3 \hat{b}_3 \rightarrow \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_B$$

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_A \neq \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_B \neq \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_N$$

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_B = \begin{bmatrix} {}^B C^A \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_A$$

WANT HAVE

Rot #1



$\hat{a}_1 = \cos\theta\,\hat{n}_1 + \sin\theta\,\hat{n}_2$

$\hat{a}_2 = -S_\theta \hat{n}_1 + C_\theta \hat{n}_2$

$\hat{a}_3 = \hat{n}_3$

$$\begin{array}{c|ccc} & \hat{n}_1 & \hat{n}_2 & \hat{n}_3 \\ \hline \hat{a}_1 & C_\theta & S_\theta & 0 \\ \hat{a}_2 & -S_\theta & C_\theta & 0 \\ \hat{a}_3 & 0 & 0 & 1 \end{array}$$

$\hat{n}_1 = C_\theta \hat{a}_1 - S_\theta a_2$

$\hat{n}_2 = S_\theta \hat{a}_1 + C_\theta a_1$

$\hat{n}_3 = \hat{a}_3$

$${}^A C^N = \begin{bmatrix} C_\theta & S_\theta & 0 \\ -S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$