Rocket staging.

I_{sp} RP-1 + LOx $I_{sp} \sim 353$

LH_2 + LOx $I_{sp} \sim 455$

$\Delta V \sim 10$ km/s

$$\Delta V = V_E \ln\left(\frac{m_i}{m_f}\right)$$

$$10{,}000 = 353(9.81)\ln\left(\frac{m_i}{m_f}\right) \Rightarrow \left(\frac{m_i}{m_f}\right) \sim 18.0$$

$$= 455(9.81)\ln\left(\frac{m_i}{m_f}\right) \Rightarrow \left(\frac{m_i}{m_f}\right) \sim 9.4$$

Payload Ratio: $\dfrac{\text{mass Payload } (m_L)}{\text{mass Total } (m_{total}, m_i)} = \lambda, \eta, \pi$

ε = Structure ratio = $\dfrac{\text{mass Structure } (m_s)}{\text{Mass Structure + Mass Propellant}}$

m_L = PayLoad mass

m_s = Structural mass

m_p = Propellant mass

$$\Delta V = V_E \ln\left(\frac{m_i}{m_f}\right)$$

$$= -V_E \ln\left(\frac{m_f}{m_i}\right)$$

$$\frac{m_f}{m_i} = \frac{m_L + m_s}{m_L + m_s + m_p} = 1 - \frac{m_p}{m_i}$$

$$= 1 - \frac{m_p}{m_i}\left(\frac{m_s + m_p}{m_s + m_p}\right)$$

$$= 1 - \left(\frac{m_s + m_p}{m_i}\right)\left(\frac{m_p}{m_s + m_p}\right)$$

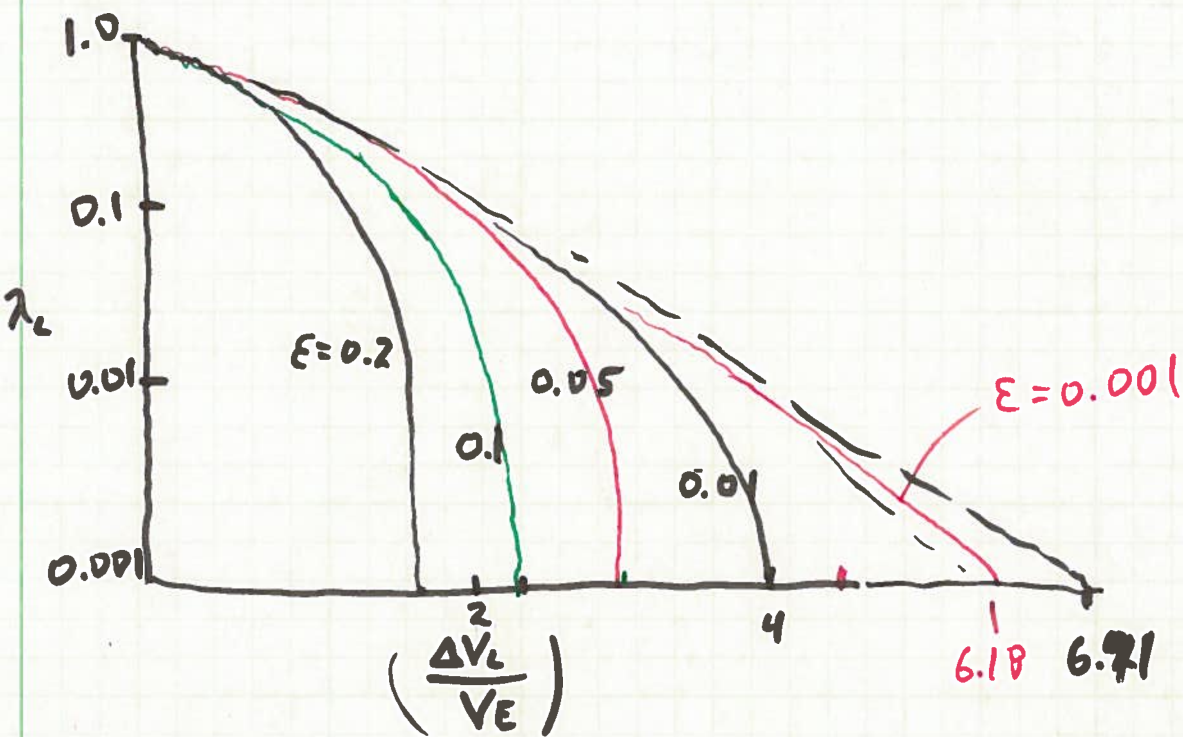
$$= 1 - \left[\underbrace{\left(\frac{m_s + m_p + m_L}{m_s + m_p + m_L}\right)}_{1} - \underbrace{\frac{m_L}{m_i}}_{\lambda}\right]\left[\underbrace{\left(\frac{m_s + m_p}{m_s + m_p}\right)}_{1} - \underbrace{\left(\frac{m_s}{m_s + m_p}\right)}_{\varepsilon}\right]$$

$$= 1 - (1 - \lambda)(1 - \varepsilon)$$

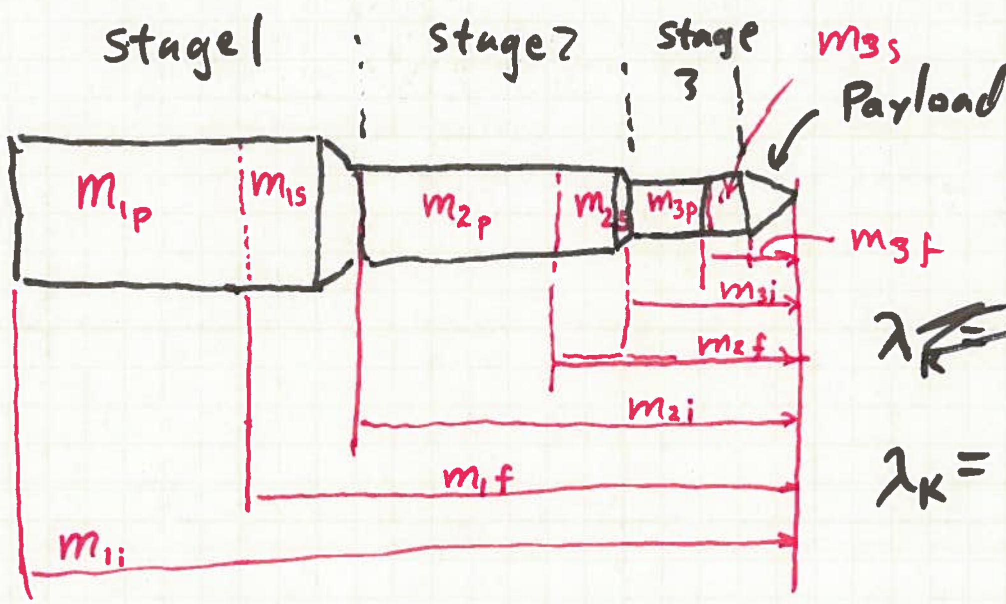
$$= \varepsilon + (1 - \varepsilon)\lambda$$

$$\Delta V = -V_E \ln\left[\varepsilon + (1 - \varepsilon)\lambda\right]$$

$$\boxed{\left(\frac{\Delta V_L}{V_E}\right) = -\ln\left[\varepsilon + (1 - \varepsilon)\lambda\right]}$$



3-stage rocket Example (Serie staging)



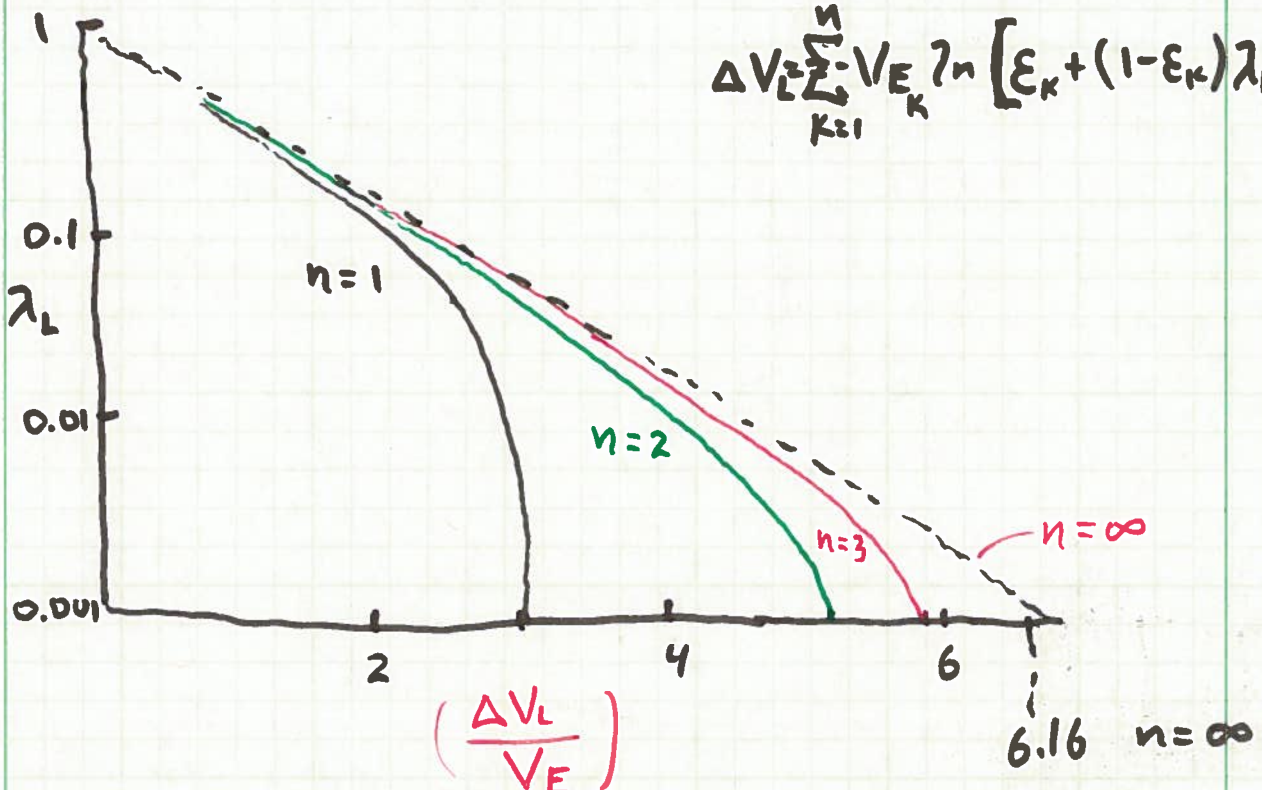
$$\lambda_K = \frac{m_{(K+1)i}}{m_{Ki}}$$

$$\lambda_L = \frac{m_L}{m_{1i}} = \left(\frac{m_{2i}}{m_{1i}}\right)\left(\frac{m_{3i}}{m_{2i}}\right)\cdots\left(\frac{m_{ni}}{m_{(n-1)i}}\right)\left(\frac{m_L}{m_{ni}}\right)$$

$$= \lambda_1 * \lambda_2 * \lambda_3 \times \cdots \lambda_n = \prod_{K=1}^{n} \lambda_K$$

$\varepsilon = 0.05$

$$\Delta V_L = \sum_{k=1}^{n} -V_{E_k} \ln\left[\varepsilon_k + (1-\varepsilon_k)\lambda_k\right]$$



$$\Delta V_{Total} = \Delta V_1 + \Delta V_2$$

$$= V_{E_1} \ln\left(\frac{m_{1i}}{m_{1f}}\right) + V_{E_2} \ln\left(\frac{m_{2i}}{m_{2f}}\right)$$

ΔV_{out}, ΔV_{Back}

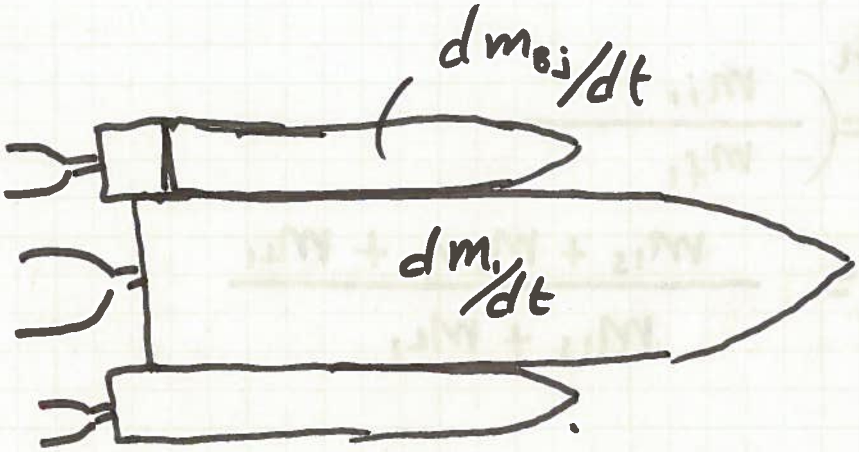
m_{P_B} Trip Back

Hohman

$$\Delta V_{BACK} = V_E \ln\left(\frac{m_L + m_s + \overset{?}{m_{PB}}}{m_L + m_s +}\right)$$

Required

$$\Delta V_{out} = V_E \ln\left(\frac{m_s + m_{PB} + \overset{?}{m}_{P_{out}}}{m_s + m_{PB}}\right)$$



$$\Delta V_{TOTAL} = \underbrace{\Delta V_0}_{\text{Primary + Boosters}} + \underbrace{\Delta V_1}_{\text{Primary w/ remaining } m_{1p}}$$

$$\Delta V_0 = -V_{E_0} \ln\left(\varepsilon_0 + (1-\varepsilon_0)\lambda_0\right)$$

$$V_{E_0} = \frac{V_{E_1}\left(\frac{dm_1}{dt}\right) + \sum V_{EBj}\left(dm_{Bj}/dt\right)}{\frac{dm_1}{dt} + \sum_{j=1}^{\nu} \frac{dm_{Bj}}{dt}}$$

$$\varepsilon_0 = \frac{m_{1s} + m_{Bsj}}{(m_{1s} + m_{Bsj}) + \sum m_{Bpj} + m_{p1/0}}$$

$m_{p1/0}$: mp primary burn in parallel w/ Booster

$$\lambda_0 = \frac{(m_{1s} + m_{p1/1} + m_{L1}}{m_{1i} + \sum m_{Bji} + m_L}$$

$$m_{p1} = m_{p1/0} + m_{p1/1}$$