Spaceflight Practice Problem - Attitude Dynamics

A 10cm x 10cm x 10cm cubesat is released into space by its parent vehicle with an angular velocity of

$$ {}^{N}\vec{\omega}^{C} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}_C $$ in the cubesat body frame, C.

The cubesat has inertia matrix $I^{C/C^*} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$

(a) Calculate the rotational kinetic energy

(b) Calculate the angular momentum

(c) If no external moment is applied, what will be the cubesat's angular velocity in 10 minutes?

(d) If the cubesat applies the following control moments to itself: $M_c = \begin{Bmatrix} 0.2 \\ -0.4 \\ +0.6 \end{Bmatrix}_c$

what is the cubesat's angular acceleration the instant it begins doing so?

Spaceflight Practice Problem Solution

(a) $KE_{rot} = \frac{1}{2} \overset{N\to C}{\omega} \cdot \overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N\to C}{\omega}$

(b) $\overset{N}{\vec{H}}^{C/O} = \underbrace{\overset{N\to C^*/O}{H}}_{\vec{r}^{\,OC^*} \times m_c \overset{N}{\vec{a}}^{C^*}} + \underbrace{\overset{N}{\vec{H}}^{C/C^*}}_{\overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N\to C}{\omega}}$

(c) $\overset{N}{\vec{\Sigma M}}^{C/C^*} = \frac{\overset{N}{d}}{dt} \overset{N\to C/C^*}{H} = 0 \rightarrow$ no change

(d) $\Sigma \vec{M}^{C/O} = \vec{r}^{\,OC^*} \times m_c \overset{N\to C^*}{a} + \overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N\to C}{\alpha} + \overset{N}{\vec{\omega}}^{C} \times \overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N}{\vec{\omega}}^{C}$

$\Sigma \vec{M}^{C/C^*} = \overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N\to C}{\alpha} + \overset{N}{\vec{\omega}}^{C} \times \overset{\Rightarrow}{H}^{C/C^*} \cdot \overset{N}{\vec{\omega}}^{C}$