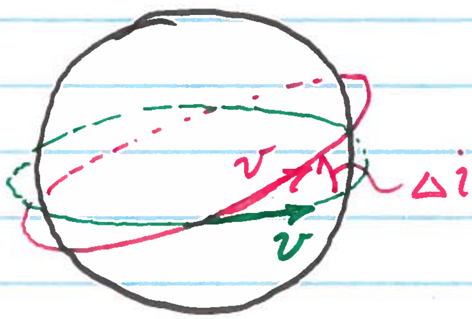
INCLINATION CHANGE Δi

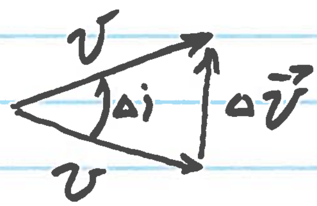
A) "SIMPLE" CHANGE

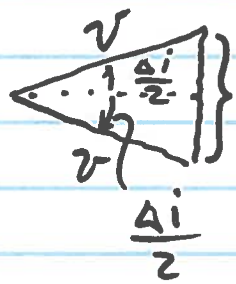
B) "INCLINATION CHANGE @ ∞"

C) COORDINATE BURN (change)

D) OPTIMAL INCLINATION CHANGE







$$\Delta v = 2 v \sin\left(\frac{\Delta i}{2}\right)$$

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B) Δi @ "∞"

ΔV_∞

$$\Delta V_{TOTAL} = \underset{circ \to Esc}{\Delta V_p} + \underset{\Delta i}{\Delta V_{A_\infty}} + \underset{Esc \to circ}{\Delta V_p}$$

$$\Delta V_{TOTAL} = \underbrace{\Delta V_A}_{\Delta i} \approx 0 + 2\Delta V_p$$

$$v_p = \sqrt{\frac{2\mu r_\infty}{r_p(r_\infty + \cancel{r_p})}} = \sqrt{\frac{2\mu}{r_p}}$$

$$\Delta V_\infty = 2\underbrace{v_\infty}_{0} \sin\left(\frac{\Delta i}{2}\right)$$

$$v_{Pcir} = \sqrt{\frac{\mu}{r_p}}$$

$$\Delta V_p = v_{Esc} - v_{cir} = (\sqrt{2}-1)\underbrace{\sqrt{\frac{\mu}{r_p}}}_{v_{circ}}$$

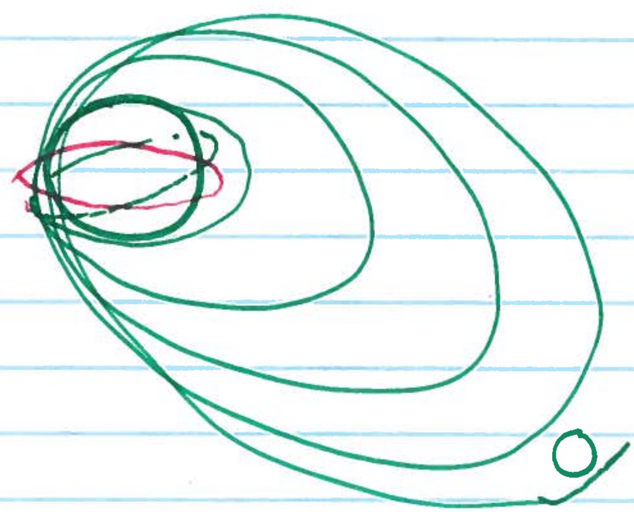
$$\underbrace{\Delta V_p}_{} = 2v \sin\left(\frac{\Delta i}{2}\right)$$

$$v_{circ} = \sqrt{\frac{\mu}{r_p}}$$

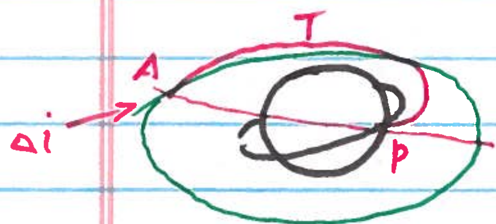
$$2\cancel{v_{circ}} \sin\left(\frac{\Delta i}{2}\right) = (\sqrt{2}-1)\cancel{v_{circ}}$$

$$\Delta i \doteq 48.9^\circ$$


Jupiter
~90°

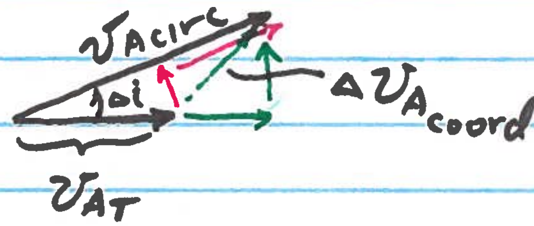


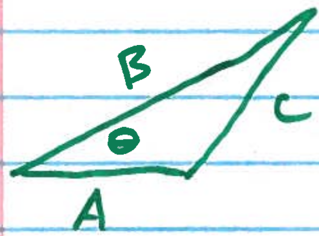
coordinated burn



$v_{P_{cir}}$, v_{P_T}

v_{A_T} , $v_{A_{cir}}$





Law of cosines

$$C^2 = A^2 + B^2 - 2AB\cos\theta$$

$$\Delta v_{A_{coord}} = \sqrt{v_{A_T}^2 + v_{A_{circ}}^2 - 2\, v_{A_T} v_{A_{cir}} \cos(\Delta i)}$$

Special problem #3 - Low Thrust Problem cancelled.

Example: Low Earth 200 Km Parking orbit @ 28° inclination
Moved to Geostationary orbit (0° inclination)

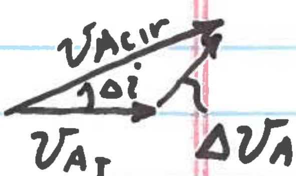
$$v_{P_{circ}} = \sqrt{\frac{\mu}{r_P}}$$

$$v_{P_T} = \sqrt{\frac{2\mu r_A}{r_P(r_P+r_A)}}$$

$$v_{A_T} = \sqrt{\frac{2\mu r_P}{r_A(r_P+r_A)}}$$

$$v_{A_{circ}} = \sqrt{\frac{\mu}{r_A}}$$

$$\Delta V_P = v_{P_T} - v_{P_{circ}} = \sqrt{\frac{\mu}{r_P}}\left\{\sqrt{\frac{2r_A}{r_P+r_A}} - 1\right\}$$



$$\Delta v_A = \sqrt{v_{A_T}^2 + v_{A_{circ}}^2 - 2 v_{A_T} v_{A_{circ}} \cos(28°)}$$

$$\Delta v_{TOTAL} = \Delta V_P + \Delta v_A$$

$$r_P = 6378 + 200 = 6578 \text{ Km}$$

r_A $\quad f_g = \frac{\mu}{r^2} = \underbrace{r\dot{\theta}^2}_{\text{centrepetal accel}}$

$$r = \left[\frac{\mu}{\dot{\theta}^2}\right]^{1/3}$$

$$\dot{\theta} = \frac{2\pi}{(23.9344\,hr)\left(\frac{3600\,sec}{hr}\right)} = 7.29\times10^{-5}\ \text{rad/s}$$

$$r_A = 42172 \text{ Km}$$

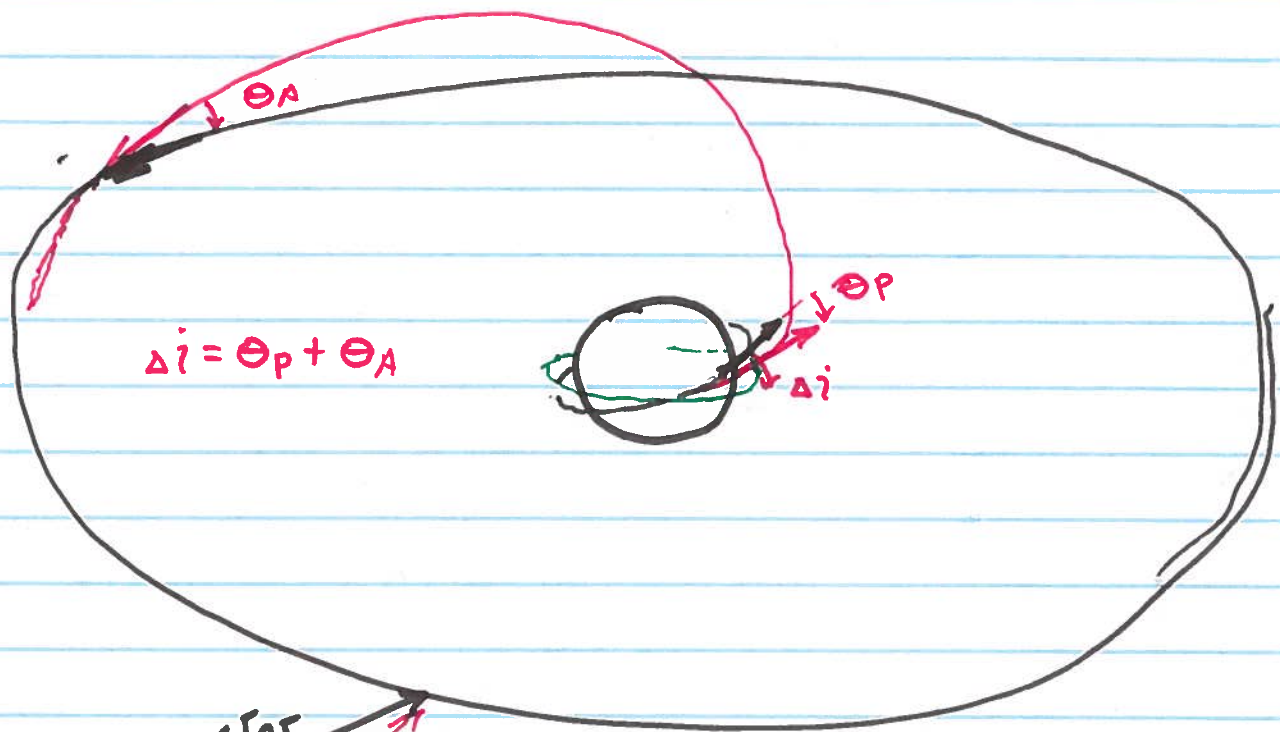
$$v_{P_{circ}} = \sqrt{\frac{\mu}{6578}} = \sim 7.784\ ^{km}/_{s}$$

$$\Delta v_P = 2.4548\ Km/s = \sqrt{\frac{\mu}{r_P}}\left\{\sqrt{\frac{2r_A}{r_P + r_A}} - 1\right\}$$

$$\Delta v_A \sim 1.825\ ^{Km}/_{s}$$

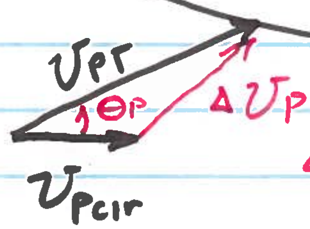
$$\Delta v_{TOTAL} = \Delta v_P + \Delta v_A$$

"Best" (optimal) inclination change



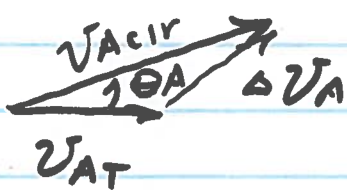
$$\Delta i = \theta_P + \theta_A$$

ΔV_P



$$\Delta V_P = \sqrt{\underbrace{V_{P_{cir}}^2 + V_{P_T}^2}_{A} - \underbrace{2 V_{P_{circ}} V_{P_T}}_{B} \cos\theta_P}$$

ΔV_A



$$\Delta V_A = \sqrt{\underbrace{V_{A_{circ}}^2 + V_{A_T}^2}_{C} - \underbrace{2 V_{A_{circ}} V_{A_T}}_{D} \cos\theta_A}$$

$$\theta_A = \Delta i - \theta_P$$

$$V_{P_{cir}} = \sqrt{\frac{\mu}{r_p}}$$

$$V_{P_T} = \sqrt{\frac{2\mu r_A}{r_p(r_p + r_A)}} \qquad V_{A_T} = \sqrt{\frac{2\mu r_p}{r_A(r_p + r_A)}}$$

$$V_{A_{circ}} = \sqrt{\frac{\mu}{r_A}}$$

$$\Delta V_{TOTAL} = \Delta V_P + \Delta V_A$$

$$= \sqrt{A + B\cos\theta_p} + \sqrt{C + D\cos(\Delta i - \theta_p)}$$

$$\frac{d\Delta V_{Total}}{d\theta_p} = 0 \quad \text{then solve for } \theta_p$$

$$= \frac{1}{2}\{A + B\cos\theta_p\}^{-1/2}\{-B\sin\theta_p\}$$

$$+ \frac{1}{2}\{C + D\cos(\Delta i - \theta_p)\}^{1/2}[D\sin(\Delta i - \theta_p)] = 0$$

alt = 200, 400, 800, 1600, 3200 . . . '

r = 42172