$$\pi_1 = \frac{m_1}{m_1+m_2} \qquad \pi_2 = \frac{m_2}{m_1+m_2}$$

$$X_1 = -\pi_2 r_{12} \qquad X_2 = \pi_1 r_{12}$$

$$\mu_1 = \pi_1 \mu \quad , \quad \mu_2 = \pi_2 \mu \qquad \Omega = \sqrt{\frac{\mu}{r_{12}^3}}$$

$$\pi_2 = 1 - \pi_1$$

$$\hat{i}: \cancelto{0}{\ddot{x}} - \Omega^2 x - 2\Omega \cancelto{0}{\dot{y}} = -\frac{\mu_1}{r_1^3}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \pi_1 r_{12})$$

$$\hat{j}: \cancelto{0}{\ddot{y}} - \Omega^2 y + 2\Omega \cancelto{0}{\dot{x}} = -\left\{\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right\} y$$

$$\hat{k}: \cancelto{0}{\ddot{z}} = -\left\{\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right\} z$$

Equilibrium where $\dot{x} = \dot{y} = \dot{z} = \ddot{x} = \ddot{y} = \ddot{z} = 0$

$$x = x_e + \delta x \qquad \delta x \lll 1$$
$$y = y_e + \delta y \qquad \delta y \lll 1$$
$$z = z_e + \delta z \qquad \delta z \lll 1$$

$$-\Omega^2 x = -\left\{\frac{\mu_1}{r_1^3}(x+\pi_2 r_{12}) + \frac{\mu_2}{r_2^3}(x-\pi_1 r_{12})\right\}$$

$$-\Omega^2 y = -\left\{\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right\} y$$

$$0 = -\left\{\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right\} z \Rightarrow z_e = 0$$

$$\Omega = \sqrt{\frac{\mu}{r_{12}^3}}$$

$$\mu_1 = \pi_1 \mu = (1-\pi_2)\mu \quad , \quad \mu_2 = \pi_2 \mu$$

$\hat{i}$: $$\Rightarrow (1-\pi_2)(x+\pi_2 r_{12})\left(\frac{1}{r_1^3}\right) + \frac{\pi_2}{r_2^3}(x+\pi_2 r_{12} - r_{12}) = \frac{x}{r_{12}^3}$$

$\hat{j}$: $$(1-\pi_2)\left(\frac{1}{r_1^3}\right) + \pi_2\left(\frac{1}{r_2^3}\right) = \frac{1}{r_{12}^3}$$

$$a = \left(\frac{1}{r_1^3}\right), \quad b = \left(\frac{1}{r_2^3}\right)$$

$$c = \left(\frac{1}{r_{12}^3}\right)$$

$$\begin{bmatrix} (1-\pi_2)(x+\pi_2 r_{12}) & \pi_2(x+\pi_2 r_{12} - r_{12}) \\ (1-\pi_2) & \pi_2 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} x \\ 1 \end{Bmatrix} c$$

$$\Rightarrow a = b = c$$

$$r_1 = r_2 = r_{12}$$

$$\delta \underset{\sim}{r} = \begin{Bmatrix} \delta x \\ \delta y \\ \cancel{\delta z} \end{Bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \delta \underset{\sim}{\ddot{r}} + \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \delta \dot{r}$$

$$+ \begin{bmatrix} -\frac{3}{4} & -\frac{3\sqrt{3}}{2}(\pi_2 - 1) \\ \frac{-3\sqrt{3}}{2}\left(\pi_2 - \frac{1}{2}\right) & \frac{-9}{4} \end{bmatrix} \{\delta r\} = \{0\}$$

$$\delta r = \delta r_0 e^{i\lambda t}$$

$$\delta \dot{r} \Rightarrow \delta r_0 i \lambda e^{i\lambda t}$$

$$\delta \ddot{r} = -\lambda^2 e^{i\lambda t} \delta r_0$$

$$= \begin{bmatrix} \lambda^2 - 3/4 & -2\lambda - \frac{3\sqrt{3}}{2}\left(\pi_2 - \frac{1}{2}\right) \\ 2\lambda - \frac{3\sqrt{3}}{2}\left(\pi_2 - \frac{1}{2}\right) & \lambda^2 - \frac{9}{4} \end{bmatrix} \begin{Bmatrix} A \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix}$$

$$\Rightarrow \lambda^4 + \lambda^2 - \frac{27}{4} \pi_2 (\pi_2 - 1) = 0$$

$$\Rightarrow \lambda^2 = \frac{1}{2}\left[-1 \pm \sqrt{1 - 27\pi_2(1 - \pi_2)}\right]$$

$e^{i\lambda t}$ $\qquad e^{(x+iy)} = e^{-x}(A\sin yt + B\cos yt)$

$$e^{\overbrace{(x+iy)}^{\lambda^2}}$$

$$re^{i\theta} = r[\cos\theta + i\sin\theta]$$

$$\sqrt{r}\, e^{i\frac{\theta}{2}}$$

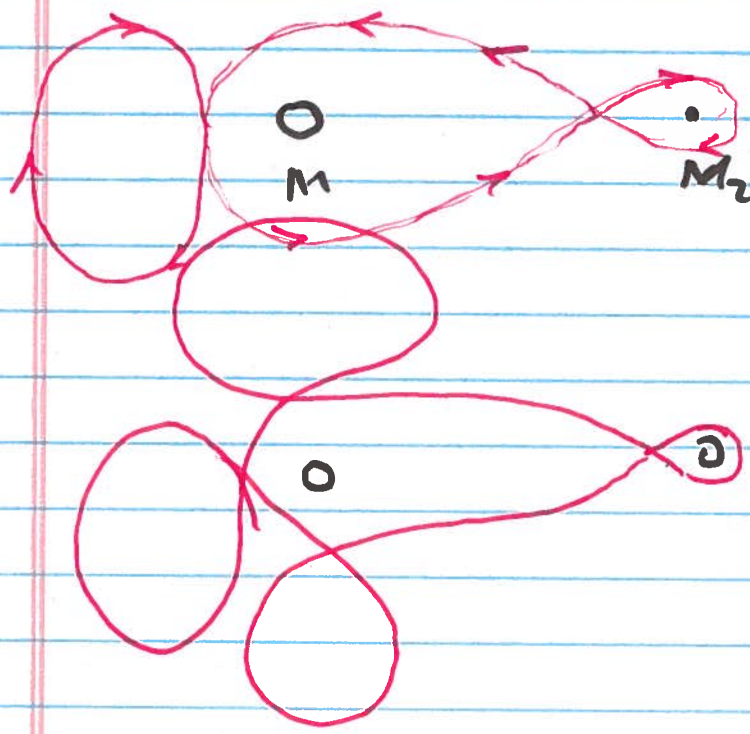
$$-1 \pm \sqrt{1 - 27\pi_2(\pi_2 - 1)} = 0$$

$\lambda \overset{\pi_2}{\Rightarrow}$ STABLE solution iff

$\pi_2 < 0.03852$ or

$\pi_2 > 0.96148$

$$\pi_2 = \frac{m_2}{m_1 + m_2}$$



M1
M2