1) Kepler's 2nd Law $\frac{dA}{dt} = \frac{h}{2}$

2) Eccentricity $\vec{e}$

3) Actual Anomaly θ

4) Position & Velocity

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

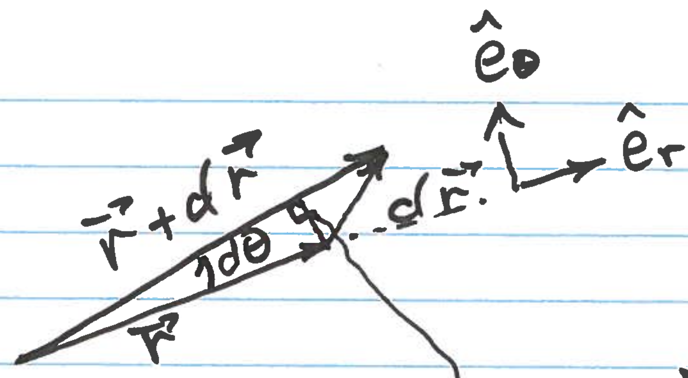
$$v_\theta(\theta) = \frac{\mu}{h}(1+e\cos\theta)$$

$$v_r(\theta) = \frac{\mu}{h} e \sin\theta$$

4) circular orbits $e=0$

5) Parabolic trajectory $-1<e<1$

6) Hyperbolic trajectories $e>1$



$$\vec{r} = r\hat{e}_r$$
$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{h} = \vec{r} \times \vec{v}$$
$$r\hat{e}_r \times (\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$
$$\underbrace{r^2\dot{\theta}}_{h}\,\hat{k}$$

$$dA = \frac{1}{2}(r+dr)(r\,d\theta)$$
$$= \frac{1}{2}(r^2 d\theta + \cancel{r\,dr\,d\theta}^{\,0})$$
$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}\underbrace{r^2\dot{\theta}}_{h}$$

Eccentricity $\vec{e}$

$$\vec{F}_g^{\,Q} = m_Q \, {}^N\vec{a}^{\,Q}$$

$$\vec{f}_g = \vec{a}$$

$$-\frac{\mu \vec{r}}{r^3} = \ddot{\vec{r}}$$

$$-\frac{\mu \vec{r}}{r^3} \times \vec{h} = \underbrace{\ddot{\vec{r}} \times \vec{h}}$$

$$\frac{{}^N d}{dt}\left(\dot{\vec{r}} \times \vec{h}\right) = \ddot{\vec{r}} \times \vec{h} + \dot{\vec{r}} \times \cancel{\frac{{}^N d}{dt} \vec{h}}$$

$$\vec{r} \times \vec{h} = \vec{r} \times \left(\vec{r} \times \dot{\vec{r}}\right) = \vec{r}\left(\underbrace{\vec{r} \cdot \dot{\vec{r}}}_{r\dot{r}}\right) - \dot{\vec{r}}\left(\underbrace{\vec{r} \cdot \vec{r}}_{r^2}\right)$$

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right)$$

$$\vec{r} \cdot \dot{\vec{r}} = r\hat{e}_r \cdot \left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right) = r\dot{r}$$

$$\Rightarrow \frac{\vec{r} \times \vec{h}}{r^3} = \frac{\vec{r}\dot{r} - \dot{\vec{r}} r}{r^2} = -\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\Rightarrow \mu \frac{{}^N d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{{}^N d}{dt}\left(\dot{\vec{r}} \times h\right)$$

$$\frac{{}^N d}{dt}\Big[\underbrace{\dot{\vec{r}} \times \vec{h} - \mu\left(\frac{\vec{r}}{r}\right)}_{= C}\Big] = 0$$

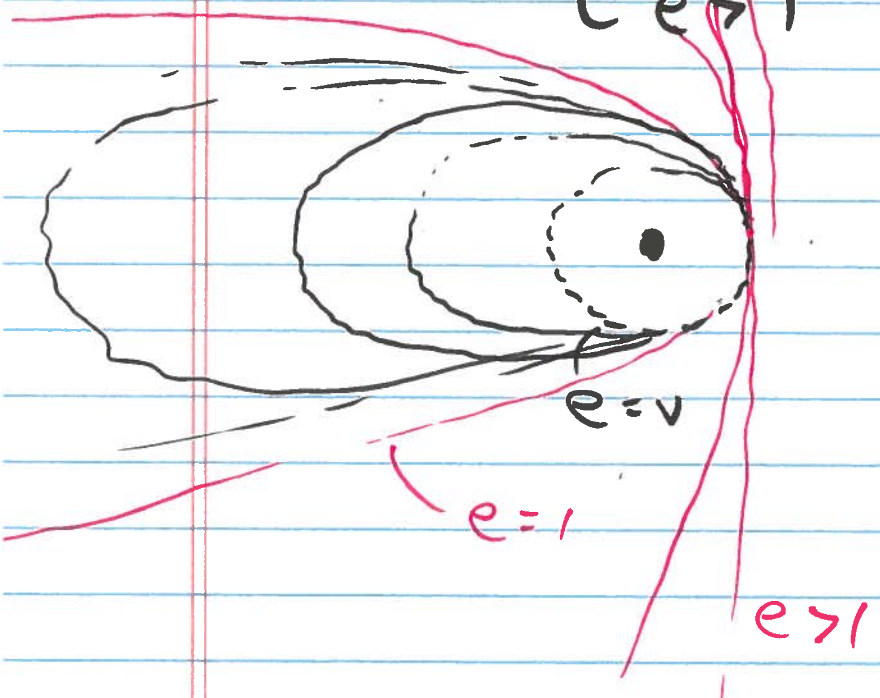
$$\dot{\vec{r}} \times \vec{h} - \mu\left(\frac{\vec{r}}{r}\right) = \vec{C}$$

$$\Rightarrow \frac{\vec{v} \times \vec{h}}{\mu} - \left(\frac{\vec{r}}{r}\right) = \frac{\vec{C}}{\mu} = \vec{e}$$

$$\boxed{\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}} \qquad e = |\vec{e}|$$

$$\vec{h} = \vec{r} \times \vec{v}$$

$$e \begin{cases} -1 < e < 1 & \text{Ellipse} \qquad e = 0 \text{ circular} \\ e = 1 & \text{parabola} \\ e > 1 & \text{hyperbola} \end{cases}$$



produce predictive relation

$$\frac{\vec{v}\times\vec{h}}{\mu} - \frac{\vec{r}}{r} = \vec{e}$$

$$\frac{\vec{r}\cdot(\vec{v}\times\vec{h})}{\mu} - \underbrace{\frac{\vec{r}\cdot\vec{r}}{r}}_{r} = \underbrace{\vec{r}\cdot\vec{e}}_{re\cos\theta}$$

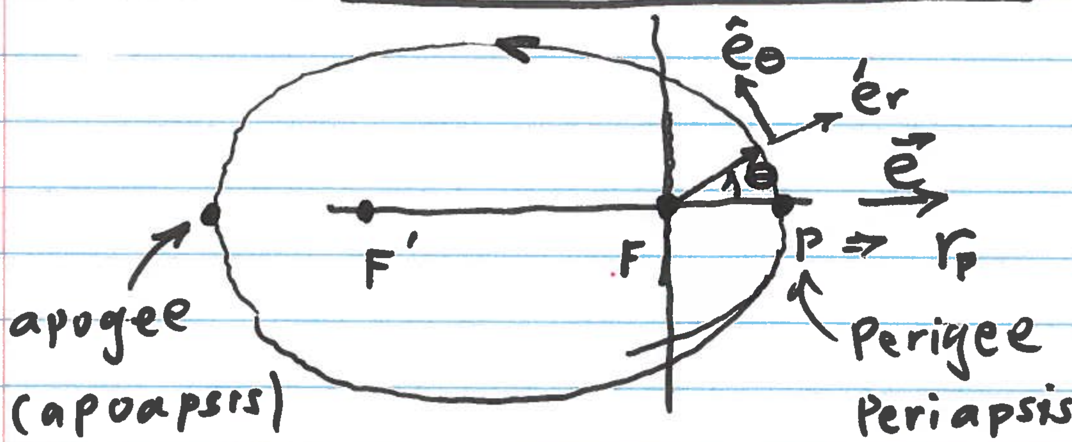
Actual

$$\underbrace{\vec{r}\cdot(\vec{v}\times\vec{h})}_{h^2} = \vec{v}\cdot(\vec{h}\times\vec{r}) = \vec{h}\cdot(\underbrace{\vec{r}\times\vec{v}}_{\vec{h}}) = h^2$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\Rightarrow \quad \frac{h^2}{\mu} - r = re\cos\theta$$

$$\Rightarrow \boxed{r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}}$$



$v_r = \dot{r}$

$r > 0$

outbound $\dot{r} > 0$

$\vec{v}\cdot\vec{r} > 0 \Rightarrow$ outbound

inbound $\dot{r} < 0$

$\Rightarrow \vec{v}\cdot\vec{r} < 0 \Rightarrow$ Acos wrong velocity

$$\vec{r} = r\,\hat{e}_r \qquad r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

$$\vec{v} = \underbrace{\dot{r}}_{v_r}\hat{e}_r + \underbrace{r\dot{\theta}}_{v_\theta\ (v_\perp)}\hat{e}_\theta$$

$$v_\theta = r\dot{\theta}$$

from before $h = r^2\dot{\theta}$

$$\dot{\theta} = \frac{h}{r^2}$$

$$v_\theta = r\left(\frac{h}{r^2}\right) = \frac{h}{r}$$

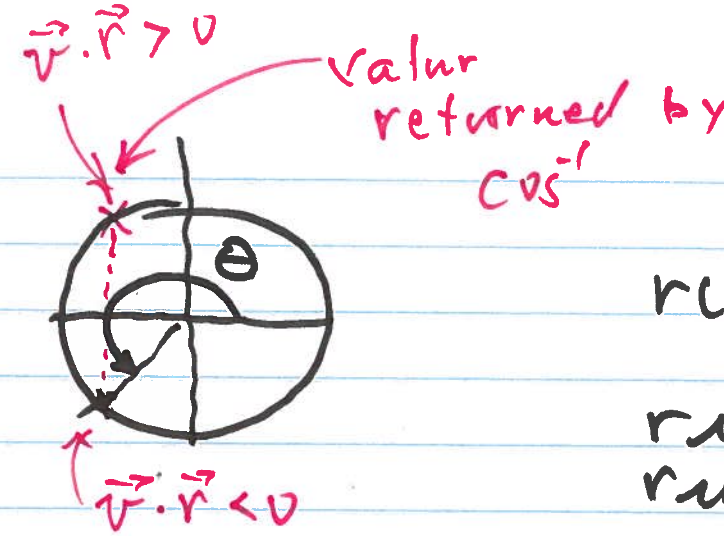
$$\frac{h^2}{\mu(1+e\cos\theta)}$$

$$\boxed{v_\theta = \frac{\mu}{h}(1+e\cos\theta)}$$

$$v_r = \dot{r} = \frac{d}{dt}\left\{\frac{h^2}{\mu(1+e\cos\theta)}\right\}$$

$$\boxed{v_r = \frac{\mu}{h}e\sin\theta}$$

$$\vec{v}\cdot\vec{r} = (v_r\hat{e}_r + v_\theta\hat{e}_\theta)\cdot r\hat{e}_r = v_r r$$



$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

$$r\mu(1+e\cos\theta) = h^2$$

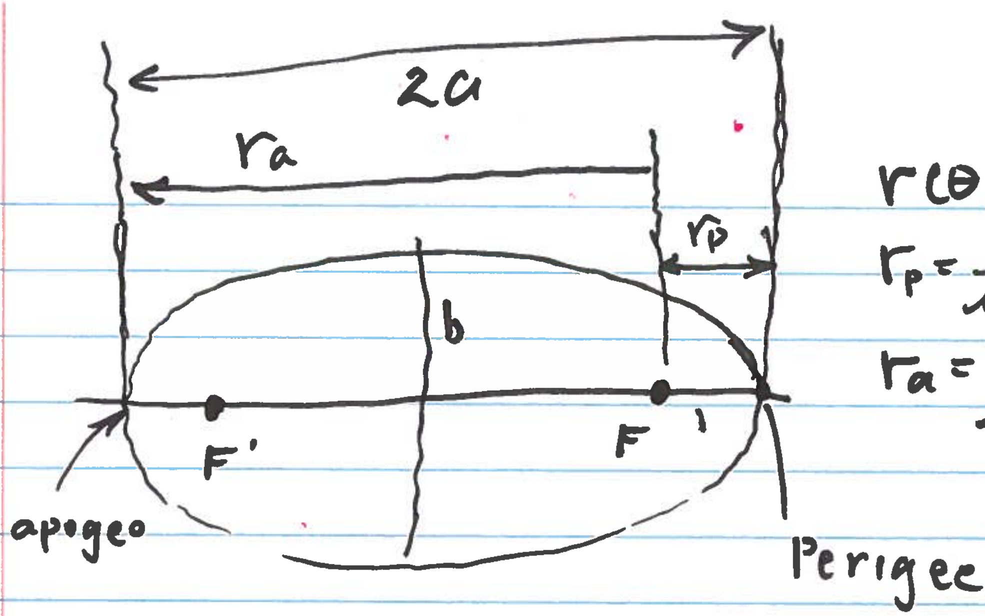
$$r\mu + r\mu e\cos\theta = h^2$$

$$e = \cos^{-1}\left[\frac{h^2 - r\mu}{r\mu}\right]$$

$0<\theta<\pi$ (180)

wrong 50% of time.

$\theta_{correct} = 360 - \theta_{ACOS}$ when on return leg.



$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

$$r_p = \frac{h^2}{\mu(1+e)}$$

$$r_a = \frac{h^2}{\mu(1-e)}$$

$$a = \frac{r_p + r_a}{2} \quad \Rightarrow \quad r_p = a(1-e), \quad r_a = a(1+e)$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$\Rightarrow h^2 = a\mu(1-e^2)$$

$$r(\theta) = a\frac{1-e^2}{1+e\cos\theta} = \frac{r_p(1+e)}{1+e\cos\theta}$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

Evalute @ r_p $(\theta = 0$

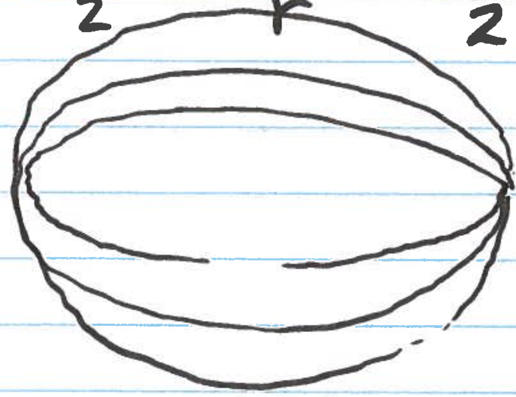
$$v^2 = v_\theta^2 = \left[\frac{\mu}{h}(1+e\cos\theta)\right]^2$$

$$r(\theta) = r_p = \frac{h^2}{\mu(1+e)}$$

& use $h^2 = a\mu(1-e^2)$

B

$$\Rightarrow \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a} = \begin{cases} <0 & \text{Elliptical} \\ 0 & \text{Parabolic} \\ >0 & \text{hyperbolic} \end{cases}$$



$$\Rightarrow \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \left\{ \begin{array}{l} \text{Ellipse} \\ \text{or} \\ \text{Parabola} \end{array} \right.$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{\mu}{2a} \left\{ \text{Hyperbola} \right.$$

circular orbits $e=0$

$$\vec{f}_g = \vec{a}$$

$$\vec{f}_g = \frac{\mu r \vec{e}_r}{r^3} = -\frac{\mu \hat{e}_r}{r^2}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

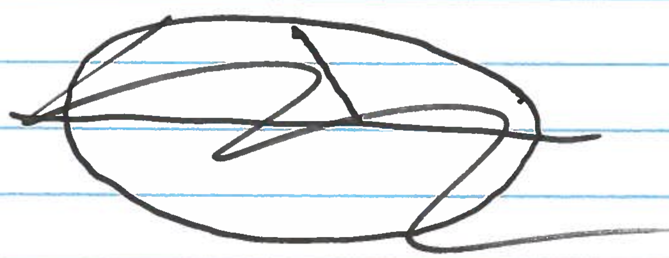
$$(\cancel{\ddot{r}}^{\,0} - r\dot{\theta}^2)\hat{e}_r + (\underbrace{r\ddot{\theta} + 2\cancel{\dot{r}}^{\,0}\dot{\theta}}_{r\ddot{\theta} = 0})\hat{e}_\theta = -\frac{\mu}{r^2}\hat{e}_r$$

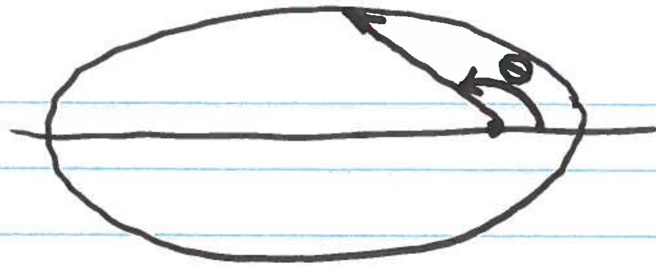
$$\ddot{\theta} \to 0 \Rightarrow \dot{\theta} = \text{constant}$$

$$\dot{\theta}^2 r = \frac{\mu}{r^2} \qquad \dot{\theta} = \sqrt{\frac{\mu}{r^3}}$$

Orbit period $T = \frac{2\pi}{\dot{\theta}}$

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{\mu}}}$$





Elliptic

Area of Ellipse $= \pi a b$

$$\frac{dA}{dt} = \frac{h}{2}$$

$$dA = \frac{h}{2}\,dt$$

$$A = \oint dA = \frac{h}{2}\int_0^T dt$$

$$A = \pi a b = \frac{h}{2} T$$

$$T_{\text{Ellipse}} = \frac{2\pi a b}{h} = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$