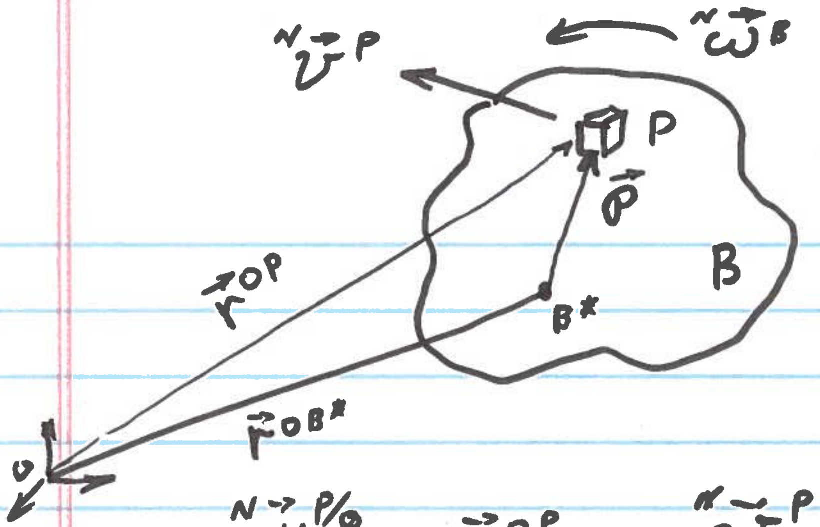

$${}^N\vec{H}^{P/O} = \vec{r}^{OP} \times m_P\, {}^N\vec{v}^P$$

$$\Rightarrow {}^N\vec{H}^{B/O} = \iiint_B dH = \iiint_B (\vec{r}^{OP} \times {}^N\vec{v}^P)\rho\, dVol$$

$$= \iiint_B \Big\{(\underbrace{\vec{r}^{OB^*} \times \vec{\rho}}_{\vec{r}^{OP}}) \times \big[{}^N\vec{v}^{B^*} + \underbrace{{}^B\vec{v}^{P/B^*}}_{0} + {}^N\vec{\omega}^B \times \underbrace{\vec{r}^{OP}}_{\vec{\rho}}\big]\Big\}\rho$$

$$= (\vec{r}^{OB^*} \times \cancel{\iiint_B \vec{\rho}\,\rho\, dVol})\; {}^N\vec{v}^{B^*} \underbrace{\iiint_B \rho\, dVol}_{m_B}$$

$$+ \vec{r}^{OB^*} \times \Big({}^N\vec{\omega}^B \times \underbrace{\iiint \vec{\rho}\,\rho\, dVol}_{\equiv 0 \text{ iff } B^* \text{ is c.m.}}\Big)$$

$$+ \underbrace{\iiint_B \vec{\rho}\,\rho\, dV}_{0} \times {}^N\vec{v}^{B^*}$$

$$+ \iiint \{\vec{\rho} \times ({}^N\vec{\omega}^B \times \vec{\rho})\}\rho\, dVol$$

$${}^N\vec{H}^{B/O} = \vec{r}^{OB^*} \times m_B\, {}^N\vec{v}^{B^*}$$

$$+ \underbrace{\iiint_B \{\vec{\rho} \times ({}^N\vec{\omega}^B \times \vec{\rho})\}\rho\, dVol}$$

$$
{}^N\vec{H}^{B/O} = \vec{r}^{\,OB^*} \times m_B\, {}^N\vec{v}^{B^*} + \vec{\vec{I}}^{B/B^*} \cdot {}^N\vec{\omega}^{B}
$$

$$
\left\{ {}^N\vec{H}^{B/O} \right\} = \overbrace{\left[\vec{r}^{\,OB}\times \right]_B \left\{ m_B\, {}^N\vec{v}^{B^*} \right\}_B}^{{}^N H^{B^*/O}} + \underbrace{\left[I^{B/B^*} \right]_{BB} \left\{ {}^N\vec{\omega}^{B} \right\}_B}_{{}^N H^{B/B^*}}
$$

$$
\left[\omega \times \right] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}
$$

$$
\vec{M}^{B/O} = \frac{{}^N d\, {}^N\vec{H}^{B/O}}{dt} \quad \text{iff} \quad \left\{ \begin{array}{l} {}^N\vec{v}^{O} = \vec{0} \\ {}^N\vec{v}^{B^*} = {}^N\vec{v}^{O} \\ {}^N\vec{v}^{B^*} \parallel {}^N\vec{v}^{O} \end{array} \right.
$$

$$\vec{\rho} = \rho_1 \hat{a}_1 + \rho_2 \hat{a}_2 + \rho_3 \hat{a}_3$$

$$^{N}\vec{\omega}^{B} = \omega_1 \hat{a}_1 + \omega_2 \hat{a}_2 + \omega_3 \hat{a}_3$$

$$\iiint_B \{\vec{\rho} \times (^{N}\vec{\omega}^{B} \times \vec{\rho})\} \rho \, dV = \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B}$$

$$\vec{\vec{I}}^{B/B^*} = I_{11}^{B/B^*} \hat{a}_1 \hat{a}_1 + I_{12}^{B/B^*} \hat{a}_1 \hat{a}_2 + I_{13}^{B/B^*} \hat{a}_1 \hat{a}_3$$

$$I_{21}^{B/B^*} \hat{a}_2 \hat{a}_1 + I_{22} \hat{a}_2 \hat{a}_2 + I_{23} \hat{a}_2 \hat{a}_3$$

$$+ I_{31} \hat{a}_3 \hat{a}_1 + I_{32} \hat{a}_3 \hat{a}_2 + I_{33} \hat{a}_3 \hat{a}_3$$

$$I_{11}^{B/B} \triangleq \iiint_B (\rho_2^2 + \rho_3^2) \rho \, dVol \qquad \vec{\vec{I}}^{B/B^*} \Leftrightarrow [I^{B/}$$

$$I_{22}^{B/B^*} \triangleq \iiint_B (\rho_3^2 + \rho_1^2) \rho \, dVol$$

$$I_{33}^{B/B^*} \triangleq \iiint_B (\rho_1^2 + \rho_2^2) \rho \, dVol$$

$$\underset{i \neq j}{I_{ij}^{B/B^*}} \triangleq - \iiint_B \rho_i \rho_j \rho \, dVol$$

$$[I^{B/B^*}] = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym} & & I_{33} \end{bmatrix}$$

$$\vec{r} = x\hat{n}_1 + y\hat{n}_2 + z n_3 \Rightarrow \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_N$$

$$= r_1\hat{a}_1 + r_2\hat{a}_2 + r_3\hat{a}_3 \Rightarrow \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_A$$

$I_{12} = \#\#$

$I_{31} = \#\#$

$$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ \# & \# & \# \end{bmatrix}$$

$$ {}^{N}\vec{H}^{B/B^*} = \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B} \Leftrightarrow \left[I^{B/B^*} \right]_{BB} \left\{ {}^{N}\omega^{B} \right\} $$

$$ K.E. = \frac{1}{2} m_B \, {}^{N}\vec{v}^{B^*} \cdot {}^{N}\vec{v}^{B^*} $$

$$ + \underbrace{\frac{1}{2} \, {}^{N}\vec{\omega}^{B} \cdot \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B}}_{\text{K.E. of } B \text{ about } B^*} $$

$$ \frac{1}{2} \, {}^{N}\vec{\omega}^{B} \cdot \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B} $$

$$ \Leftrightarrow \frac{1}{2} \left[\omega_1, \omega_2, \omega_3 \right]_B \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym} & & I_{33} \end{bmatrix}_{BB} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix}_B $$

$$ \Sigma \vec{M}^{B/O} = \frac{{}^{N}d \, {}^{N}\vec{H}^{B/O}}{dt} $$

$$ = \frac{{}^{B}d \, {}^{N}\vec{H}^{B/O}}{dt} + {}^{N}\vec{\omega}^{B} \times {}^{N}\vec{H}^{B/O} $$

$$ = \frac{{}^{B}d}{dt} \left(\vec{r}^{OB^*} \times m_B \, {}^{N}\vec{v}^{B^*} + \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B} \right) $$

$$ + {}^{N}\vec{\omega}^{B} \times \left[\vec{r}^{OB^*} \times m_B \, {}^{N}\vec{v}^{B^*} + \vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\omega}^{B} \right] $$

$$ \cancel{\underbrace{\frac{{}^{B}d \, r^{OB^*}}{dt}}_{{}^{N}\vec{v}^{B^*}} \times m_B \, {}^{N}\vec{v}^{B^*}} + \underbrace{\frac{{}^{B}d}{dt} \left(\vec{\vec{I}}^{B^*} \cdot {}^{N}\vec{\omega}^{B} \right)}_{\vec{\vec{I}}^{B/B^*} \cdot {}^{N}\vec{\alpha}^{B}} $$

d $\quad \sum \vec{M}^{B/O} = r^{OB^*} \times m_B \, a^{N \to B^*}$

$$+ \vec{\vec{I}}^{B/B} \cdot \alpha^{N \to B} + \omega^{N \to B} \times \left(\vec{\vec{I}}^{B/B^*} \cdot \omega^{N \to} \right)$$

$$\Rightarrow \sum M^{B/B^*} = \vec{\vec{I}}^{B/B^*} \cdot \alpha^{N \to B} + \omega^{N \to B} \times \left(\vec{\vec{I}}^{B/B^*} \cdot \omega^{N \to} \right)$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix}_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym} & & I_{33} \end{bmatrix}_{BB} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix}_B$$

$$+ \underbrace{\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}}_{[\omega \times]} \left[I^{B/B^*} \right]_{BB} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix}_B$$

Euler's Eqns.

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym} & & I_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}_{\mathcal{E}}$$

Eigen value problem

Principal Directions $\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_n$

$$\vec{\omega} = \omega_1 \hat{\xi}_1 + \omega_2 \hat{\xi}_2 + \omega_3 \hat{\xi}_3$$

$$\vec{M} = M_1 \hat{\xi}_1 + M_2 \hat{\xi}_2 + M_3 \hat{\xi}_3$$

$$M_1 = I_{11}\dot{\omega}_1 - (I_{22} - I_{33})\omega_2\omega_3$$

$$M_2 = I_{22}\dot{\omega}_2 - (I_{33} - I_{11})\omega_3\omega_1$$

$$M_3 = I_{33}\dot{\omega}_3 - (I_{11} - I_{22})\omega_1\omega_2$$

$$\{M\} = [I]\{\dot{\omega}\} + [\omega\times][I]\{\omega\}$$

$$[\dot{\omega}] = [I]^{-1}\{\{M\} - [\omega\times][I]\{\omega\}\}$$