1) CLASSIC orbit Elements (Parameters)

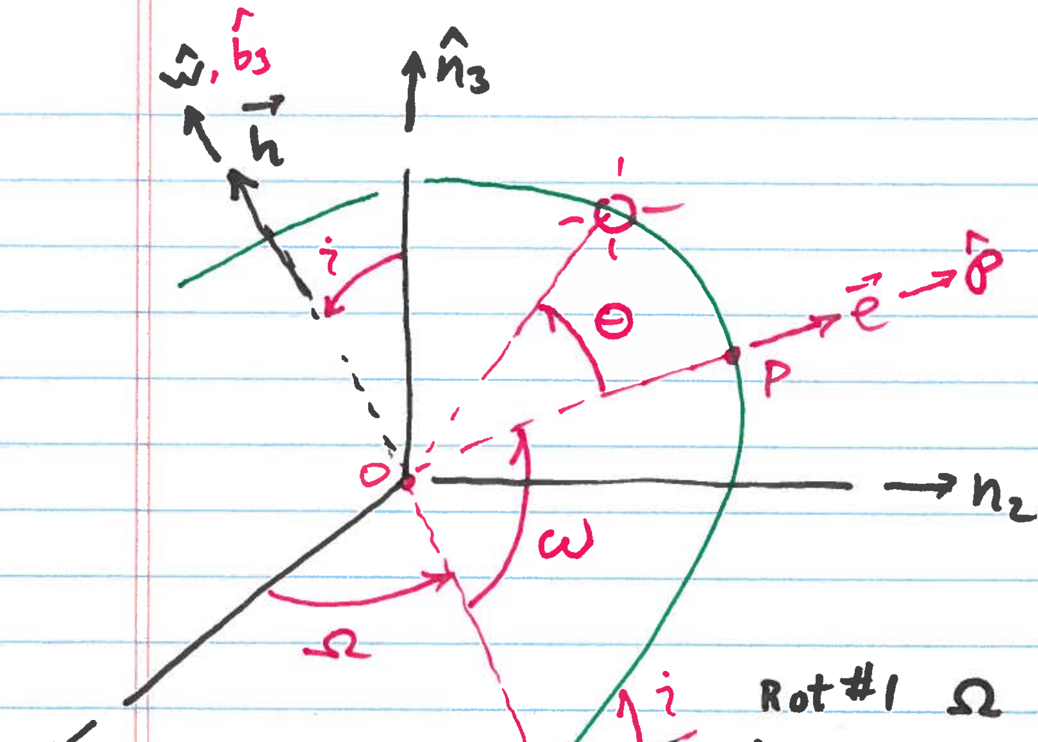
$e, a, \theta_0, \Omega, i, \omega$

$(e, h, T_0, \Omega\ i, \omega)$

2) OBLATENESS -> $J2$

3) Grand Canyon Problem
hints

4) INITIAL
orbit determination
Gibbs Method.



Rot#1 Ω about $\hat{n}_3$

Rot#2 i about $\hat{a}_1$

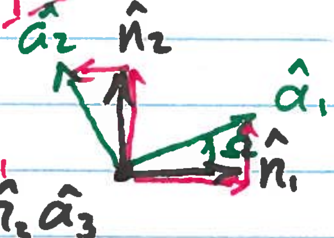
Rot#3 ω about $\hat{b}_3$

Body 313

Body ZXZ

$0 \leq i \leq \pi$

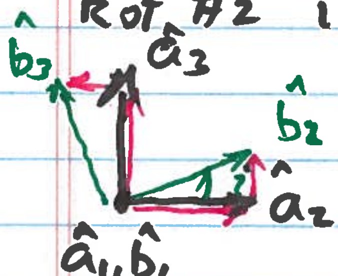
Rot #1 Ω about $\hat{n}_3, \hat{a}_3$



$$\hat{a}_1 = C_\Omega \hat{n}_1 + S_\Omega \hat{n}_2$$
$$\hat{a}_2 = -S_\Omega \hat{n}_1 + C_\Omega \hat{n}_2$$
$$\hat{a}_3 = \hat{n}_3$$

$$\begin{array}{c|ccc} & \hat{n}_1 & \hat{n}_2 & \hat{n}_3 \\ \hline \hat{a}_1 & C_\Omega & S_\Omega & 0 \\ \hat{a}_2 & -S_\Omega & C_\Omega & 0 \\ \hat{a}_3 & 0 & 0 & 1 \end{array} \Rightarrow {}^{A}C^{N} = \begin{bmatrix} C_\Omega & S_\Omega & 0 \\ -S_\Omega & C_\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

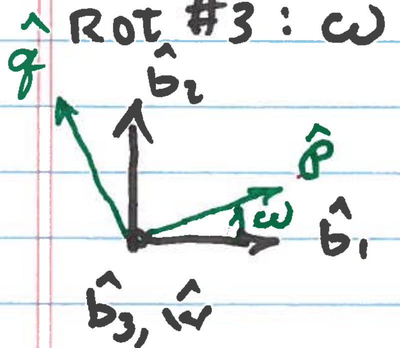
Rot #2 i about $\hat{a}_1$



$$\hat{b}_1 = \hat{a}_1$$
$$\hat{b}_2 = C_i \hat{a}_2 + S_i \hat{a}_3$$
$$\hat{b}_3 = -S_i \hat{a}_2 + C_i \hat{a}_3$$

$$\begin{array}{c|ccc} & \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \\ \hline \hat{b}_1 & 1 & 0 & 0 \\ \hat{b}_2 & 0 & C_i & S_i \\ \hat{b}_3 & 0 & -S_i & C_i \end{array} \Rightarrow {}^{B}C^{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_i & S_i \\ 0 & -S_i & C_i \end{bmatrix}$$

Rot #3 : ω about $\hat{b}_3, \hat{w}$



$$\left[{}^{PF}C^{B}\right] = \begin{bmatrix} c\omega & s\omega & 0 \\ -s\omega & c\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{PF}C^{N}\right] = \left[{}^{PF}C^{B}\right]\left[{}^{B}C^{A}\right]\left[{}^{A}C^{N}\right]$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Ω, i, ω

$e, a, \theta_0, \Omega, i, \omega$

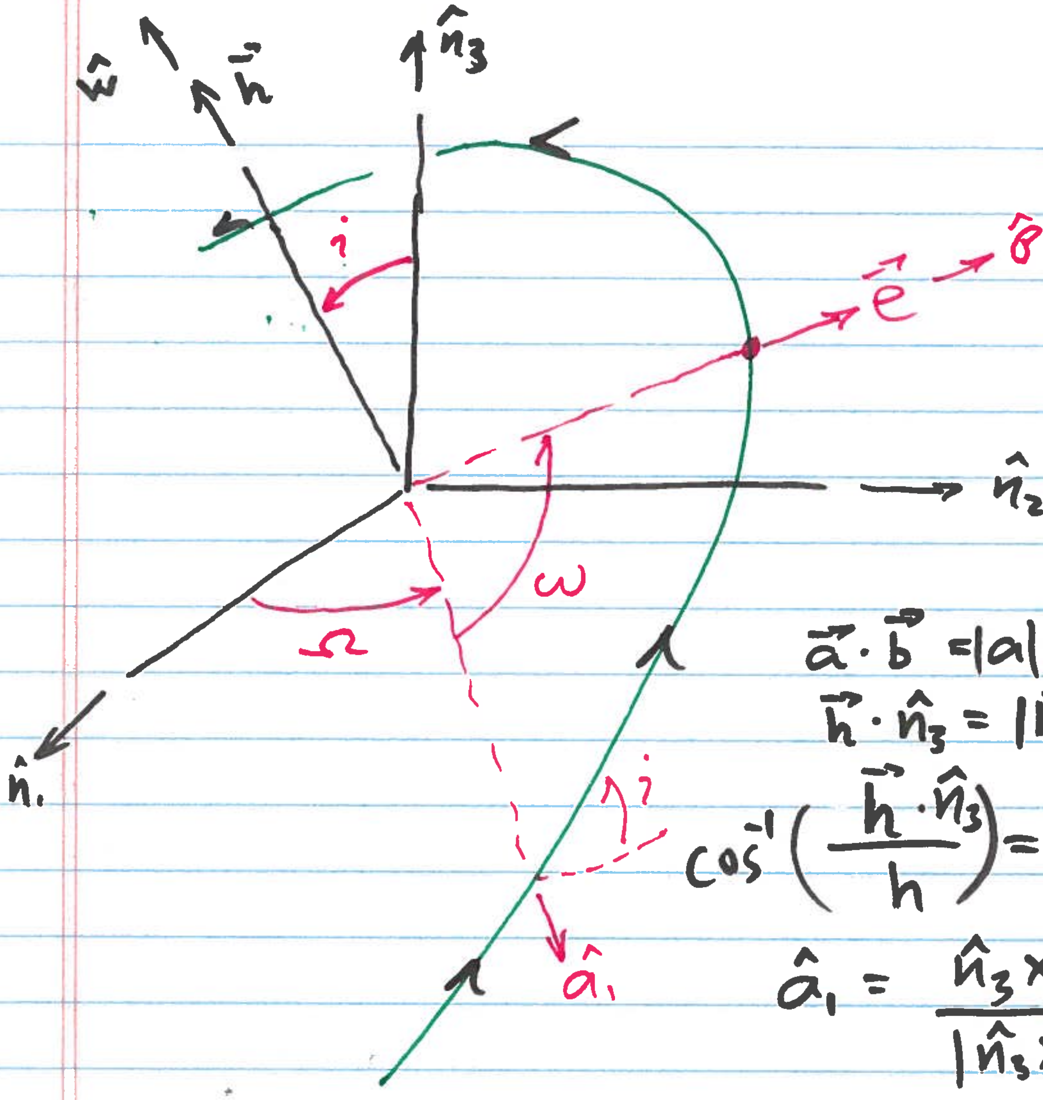
$$\overset{Y}{}\ \overset{X}{}$$

✓ $\omega = \mathrm{ATAN2}(C_{13}, C_{23}) = \mathrm{ATAN2}(S_2 S_3, S_2 C_3)$

$$\frac{\cancel{S_2} S_3}{\cancel{S_2} C_3}$$

✓ $i = \mathrm{ACOS}(C_{33})$

$\Omega = \mathrm{ATAN2}(C_{31}, -C_{32})$



$$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$$

$$\vec{h}\cdot\hat{n}_3 = |\vec{h}||\hat{n}_3|\cos i$$

$$\cos^{-1}\left(\frac{\vec{h}\cdot\hat{n}_3}{h}\right) = i$$

$$\hat{a}_1 = \frac{\hat{n}_3\times\vec{h}}{|\hat{n}_3\times\vec{h}|}$$

$$\hat{a}\cdot\hat{n}_1 = |\hat{a}||\hat{n}_1|\cos\Omega$$

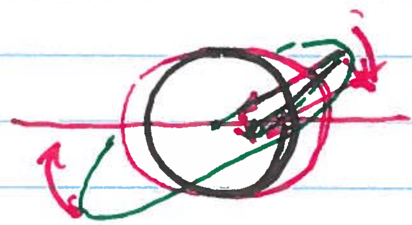
$$\Omega = \cos^{-1}(\hat{a}_1\cdot\hat{n}_1)$$

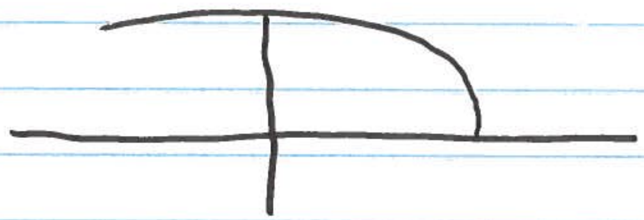
$$\omega = \cos^{-1}(\hat{a}_1\cdot\hat{p})$$

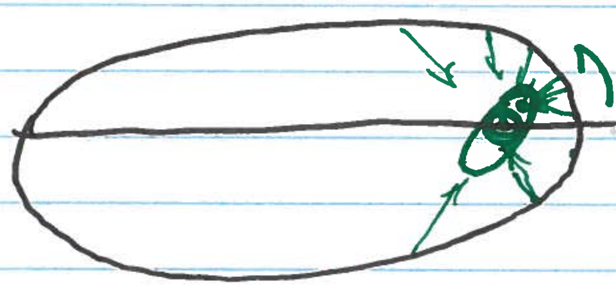
Earth oblateness

$$\text{oblateness} = \frac{\text{Equitorial Radius} - \text{Polar Radius}}{\text{Equitorial radius}}$$

$\Omega \neq$ const







$\omega \neq$ const

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}\,J_2 R^2}{(1-e^2)\,a^{7/2}}\right]\cos i$$

$$\dot{\omega} = \left[-\frac{3}{2}\frac{\sqrt{\mu}\,J_2 R^2}{(1-e^2)\,a^{7/2}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$

$$J_{2_E} = 1.08263 \times 10^{-3} \quad \text{(Earth)}$$

$$\left[{}^{PF}C^{N}\right] = \left[{}^{PF}C^{B}\right]\left[{}^{B}C^{A}\right]\left[{}^{A}C^{N}\right]$$

$$\underset{\vec{r}_0,\ \vec{v}_0}{\uparrow} \qquad \underset{\omega = \omega_0 + \dot{\omega} t}{\uparrow} \qquad \underset{i}{\uparrow} \qquad \underset{\Omega = \Omega_0 + \dot{\Omega} t}{\uparrow}$$