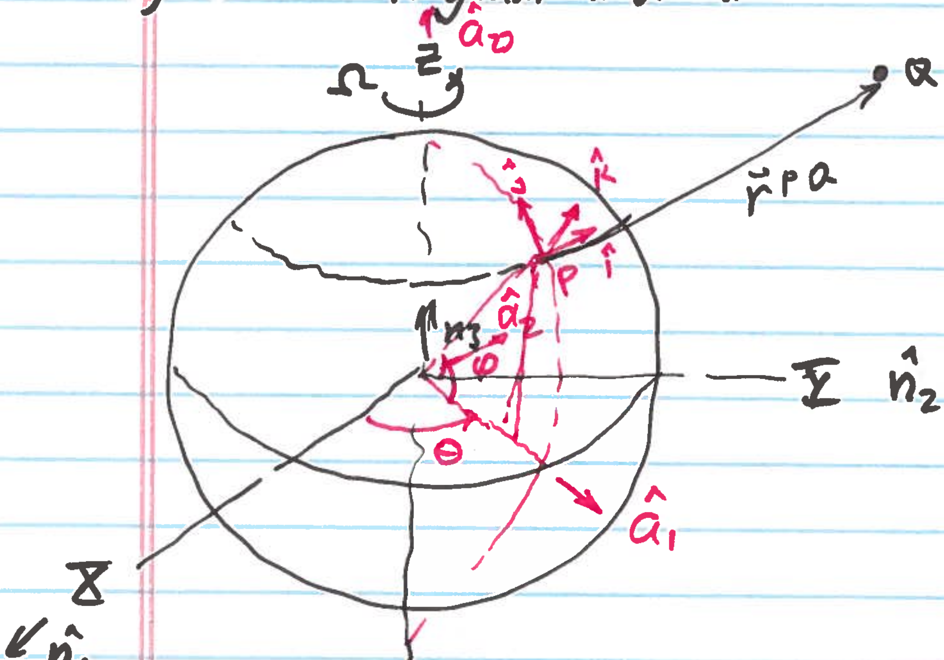
0) Basis transforms

1) orbit "Energy"

2) orbit Angular Moment



$$\Rightarrow \Sigma \vec{F}^Q = \frac{\mu \vec{r}^{OQ}}{(r^{OQ})^3} = {}^{N}\vec{a}^{Q}$$

$$^{N}\vec{a}^{Q} = {}^{N}\vec{a}^{P} + {}^{E}\vec{a}^{Q/P} + \cancel{{}^{N}\vec{\alpha}^{E}}^{\to 0} \times \vec{r}^{PQ} + {}^{N}\vec{\omega}^{E} \times ({}^{N}\vec{\omega}^{E} \times \vec{r}^{PQ}) + 2\,{}^{N}\vec{\omega}^{E} \times {}^{E}\vec{v}^{Q/P}$$

$$^{N}\vec{a}^{P} = {}^{N}\vec{\omega}^{E} \times ({}^{N}\vec{\omega}^{E} \times \vec{r}^{OP})$$

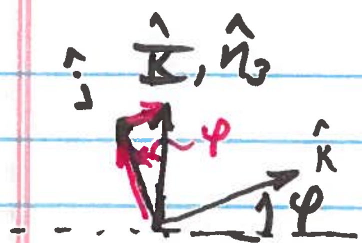
$$^{E}\vec{a}^{Q/P} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k}$$

$$^{E}\vec{v}^{Q/P} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,k$$

$$r^{PQ} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

$$\vec{r}^{OP} = R_E\,\hat{K}$$

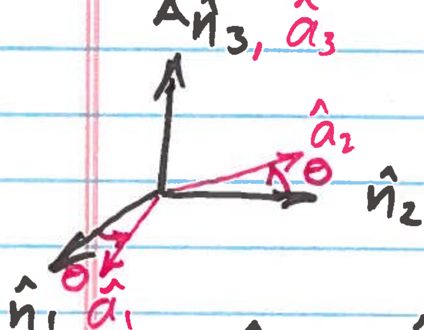
$$^{N}\vec{\omega}^{E} = \Omega\,\hat{n}_3$$



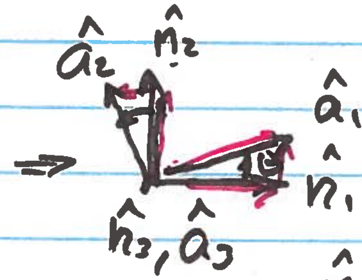
$\hat{n}_3 = C\varphi \, \hat{j} + S\varphi \, \hat{k}$

More General Treatment:

Rot. #1 about $\hat{n}_3$ by amount θ



$\Rightarrow$



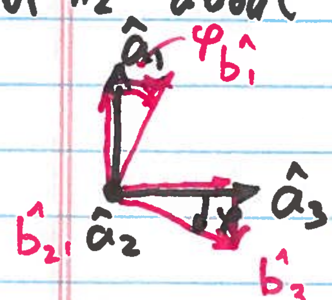
$$\hat{a}_1 = C\theta \, \hat{n}_1 + S\theta \, \hat{n}_2$$
$$\hat{a}_2 = -S\theta \, \hat{n}_1 + C\theta \, \hat{n}_2$$
$$\hat{a}_3 = \hat{n}_3$$

$\Rightarrow$

	$\hat{n}_1$	$\hat{n}_2$	$\hat{n}_3$
$\hat{a}_1$	$C\theta$	$S\theta$	0
$\hat{a}_2$	$-S\theta$	$C\theta$	0
$\hat{a}_3$	0	0	1

$$\text{C\_A\_N} \Rightarrow \quad {}^A\underline{C}^N = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rot #2 about $\hat{a}_2$ by φ



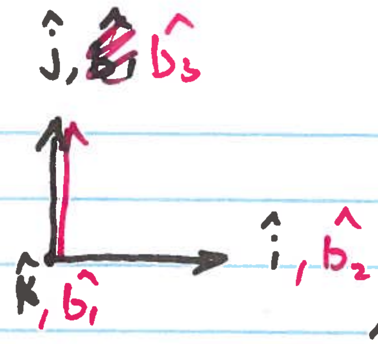
$$\hat{b}_1 = C\varphi \, \hat{a}_1 + S\varphi \, \hat{a}_3$$
$$\hat{b}_2 = \hat{a}_2$$
$$\hat{b}_3 = -S\theta \, \hat{a}_1 + C\theta \, \hat{a}_3$$

$\Rightarrow$

	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$
$\hat{b}_1$	$C\varphi$	0	$S\varphi$
$\hat{b}_2$	0	1	0
$\hat{b}_3$	$-S\varphi$	0	$C\varphi$

$$\text{C\_B\_A} \Rightarrow \quad {}^B C^A = \begin{bmatrix} C\varphi & 0 & S\varphi \\ 0 & 1 & 0 \\ -S\varphi & 0 & C\varphi \end{bmatrix}$$

Final Transform,



$$\begin{aligned} \hat{i} &= \hat{b}_2 \\ \hat{j} &= \hat{b}_3 \\ \hat{k} &= \hat{b}_1 \end{aligned} \Rightarrow \begin{array}{c|ccc} & \hat{b}_1 & \hat{b}_2 & \hat{b}_3 \\ \hline \hat{i} & 0 & 1 & 0 \\ \hat{j} & 0 & 0 & 1 \\ \hat{k} & 1 & 0 & 0 \end{array}$$

$$^{ijk}C^{B} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \vec{V} &= A_1 \hat{a}_1 + A_2 \hat{a}_2 + A_3 \hat{a}_3 \Rightarrow \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_A \\ &= B_1 \hat{b}_1 + B_2 \hat{b}_2 + B_3 \hat{b}_3 \Rightarrow \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_B \\ &= v_1 \hat{n}_1 + v_2 \hat{n}_2 + v_3 \hat{n}_3 \rightarrow \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}_n \end{aligned}$$

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_A = \left[{}^{A}C^{B} \right] \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_B$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix}_N = \left[{}^{N}C^{ijk} \right] \underbrace{\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_{ijk}}_{\vec{r}^{\,op}}$$

$$^{N}\underline{C}^{ijk} = {}^{N}\underline{C}^{A}\, {}^{A}\underline{C}^{B}\, {}^{B}C^{ijk}$$

$$^{N}\underline{C}^{A} = \left({}^{A}C^{N}\right)^{-1} = \underline{\underline{\left({}^{A}C^{N}\right)^{T}}}$$

ORBIT Energy "E" (actually specific Energy ε)

$$W_{1\to 2} \quad \int_{r_1}^{r_2} \vec{F}^{P} \cdot d\vec{r} \qquad {}^{N}v^{P} \triangleq \frac{{}^{N}d}{dt}\vec{r}^{\,OP}$$

$$d\vec{r}^{\,OP} = \underbrace{{}^{N}\vec{v}^{P}}\,dt$$

$$\vec{F}^{P} = m\,{}^{N}\vec{a}^{P} = \underbrace{m_P\,\frac{d\,{}^{N}\vec{v}^{P}}{dt}}$$

FOR Kinetic Energy only "T"

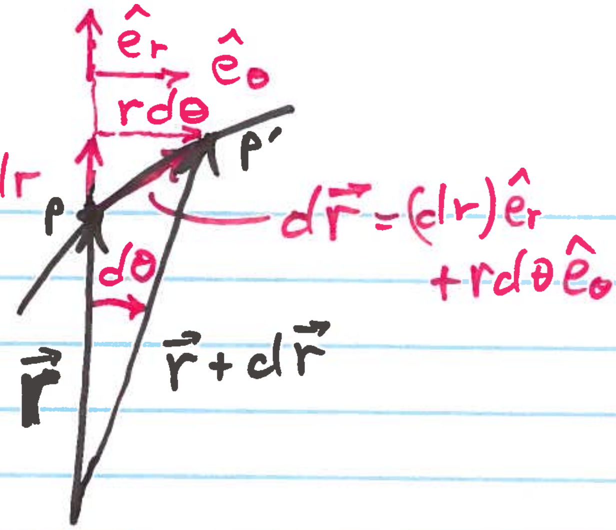
$$W_{1\to 2} = \int_{v_1}^{v_2} m\left(\frac{d\,{}^{N}\vec{v}^{P}}{\cancel{dt}}\right)\cdot {}^{N}\vec{v}^{P}\,\cancel{dt}$$

$$= m\int_{v_1}^{v_2} {}^{N}\vec{v}^{P}\cdot d\,{}^{N}\vec{v}^{P} = \frac{1}{2}\,m_P\underbrace{\left({}^{N}\vec{v}^{P}\cdot{}^{N}\vec{v}^{P}\right)}_{\left({}^{N}v^{P}\right)^2}\Bigg|_{v_1}^{v_2}$$

$$= \frac{1}{2}m\left(v_2^{\,2} - v_1^{\,2}\right)$$

$\frac{v^2}{2}$ (specif K.E.)

Potential Energy U



$$dW = \vec{F} \cdot d\vec{r}$$

$$= \frac{-\mu m\, \vec{r}^{\,op}}{(r^{op})^{\cancel{3}\,2}} \cdot \left(dr\,\hat{e}_r + \cancel{r\,d\theta\,\hat{e}_\theta}^{\,0}\right) \qquad \vec{r}^{\,op} = r^{op}\hat{e}_r$$

$$= \frac{-\mu m}{(r^{op})^2}\,dr$$

$$W_{1\to 2} = -\mu m \int_{r_1}^{r_2} \frac{dr}{r^2} = \cancel{-}\,\mu m \left(\frac{1}{r}\right)\Bigg|_{r_1}^{r_2}$$

$$\Delta U = -\frac{\mu}{r}\Bigg|_{r_1}^{r_2} = -\mu\left[\frac{1}{r_2} - \cancel{\frac{1}{\infty}}^{\,0}\right] \qquad r_1 \triangleq \infty$$

$$U(r) = -\frac{\mu}{r}$$

$$E = \frac{m v^2}{2} - \frac{\mu m}{r} \qquad \text{Orbit Energy}$$

$$\boxed{\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}} \qquad \text{specific orbit Energy}$$

•Q

E

$$E = \frac{m v^2}{2} - \frac{\mu}{\cancel{r}}^{\,0} \qquad r_1 \sim \infty \quad v_1 \sim 0$$

$$E \sim \frac{m(0)^2}{2} - \frac{\mu}{\infty} \sim 0$$

m = sphere Fe ~ 10 m radius
~ 3.01×10^7 Kg

$$E(r) = E = 0 = \frac{m v_i^2}{2} - \frac{\mu m}{r_E}$$

$R_E = 6378$

$\mu = 398600 \ \mathrm{Km^3/s^2}$

$v^2 \rightarrow$ Solving for $v_{impact} \sim 11.2$ Km/s
$\simeq 25{,}030$ mi/hr

Energy we see $= \frac{1}{2} m v_{impact}^2$
$= 1.89 \times 10^{15}$ Joules
$= 450$ Ktonnes

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \begin{cases} > 0 & \text{Hyperbolic} \\ 0 & \text{Parabolic} \\ < 0 & \text{Elliptic} \end{cases}$$

orbital angular Momentum

$$ {}^{N}\vec{H}^{Q/P} \triangleq \vec{r}^{PQ} \times m_Q {}^{N}\vec{v}^{Q} $$

$$ \frac{{}^{N}d\,{}^{N}\vec{H}^{Q/P}}{dt} = \frac{{}^{N}d}{dt}\left(\vec{r}^{PQ} \times m_Q {}^{N}\vec{v}^{Q}\right) $$

$$ = \underbrace{\left(\frac{{}^{N}d}{dt}\vec{r}^{PQ}\right)}_{{}^{N}\vec{v}^{Q}} \times m_Q {}^{N}\vec{v}^{Q} + \vec{r}^{PQ} \times m_Q \underbrace{\frac{{}^{N}d}{dt}{}^{N}\vec{v}^{Q}}_{{}^{N}\vec{a}^{Q}} $$

$$ \frac{{}^{N}d\,{}^{N}\vec{H}^{Q/P}}{dt} = \underbrace{\cancel{{}^{N}\vec{v}^{Q} \times m_Q {}^{N}\vec{v}^{Q}}}_{0} + \underbrace{\vec{r}^{PQ}}_{r^{PQ}\hat{e}_r} \times \underbrace{m_Q {}^{N}\vec{a}^{Q}}_{F_g} $$

$$ F_g = \frac{\mu \vec{r}^{OQ}}{(r^{OQ})^3} = -F_g \hat{e}_r $$

$$ \Rightarrow \frac{{}^{N}d\,{}^{N}H^{Q/P}}{dt} = 0 $$

$$ \Rightarrow {}^{N}\vec{H}^{Q/P} = \text{const} \quad (\text{Ang. Momentum}) $$

$$ {}^{N}h^{Q/P} = \text{const} \quad (\text{spec. Ang. Mom.}) $$