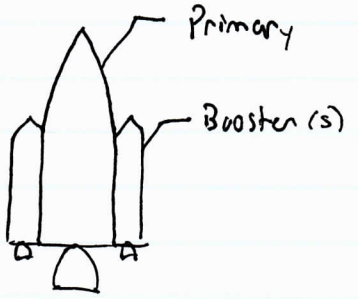
A few notes on Parallel Staging

Paul McKee



Stage 0: primary AND boosters firing until boosters run out

drop boosters, then...

Stage 1: primary fires with whatever fuel it has left

Stage 0: $\Delta V_0 = \underbrace{I_{sp,//}\, g_0}_{V_{E,//}} \ln\left(\frac{m_{i,0}}{m_{f,0}}\right)$

$$V_{E,//} = \frac{\sum V_{Ei}\,\dot{m}_i}{\sum \dot{m}_i}$$ } weighted sum of V_E over mass flow rate for each thruster

$$m_{i,0} = m_L + m_{sp} + m_{fp} + m_{sb} + m_{fb}$$

(m_L: payload; m_{sp}: structure of primary; m_{fp}: fuel of primary; m_{sb}: structure of boosters; m_{fb}: fuel of boosters)

$$m_{f,0} = m_L + m_{sp} + m_{fpr} + m_{sb}$$

(m_{fpr}: remaining primary stage fuel → calculate from burn rate, time of stage 0)

Stage 1: $\Delta V_1 = \underbrace{I_{sp}\, g_0}_{\text{primary only}} \ln\left(\frac{m_L + m_{sp} + m_{fpr}}{m_L + m_{sp}}\right)$

Brief Example:

1 primary
2 boosters

$V_{E,p} = 3000$ m/s $\quad \dot{m}_p = 40$ kg/s

$V_{E,b} = 2000$ m/s $\quad \dot{m}_b = 100$ kg/s

$$V_{E,//} = \frac{3000(40) + 2(2000)(100)}{40 + 100 + 100} = 2166.67 \text{ m/s}$$

for stage 0