1) QUIZ #2 $\quad$ Body 313 $\quad \Omega, \left[{}^{PF}C^{N}\right]$

$\Omega \quad i \quad \omega$

2) Exam #2

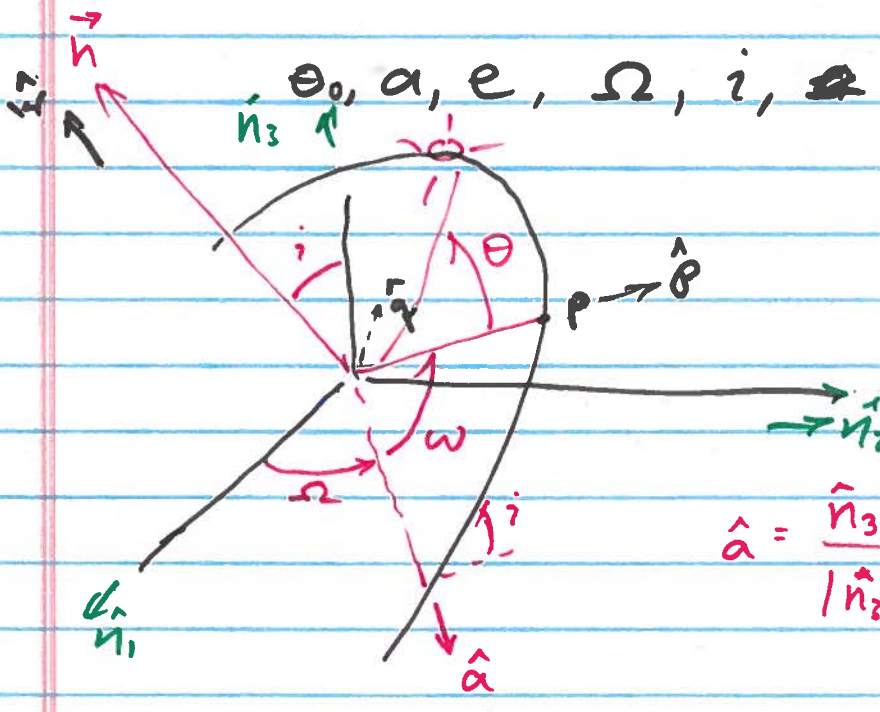
3) PHASING MANUEVER

4) Non-Hohmann Transfers

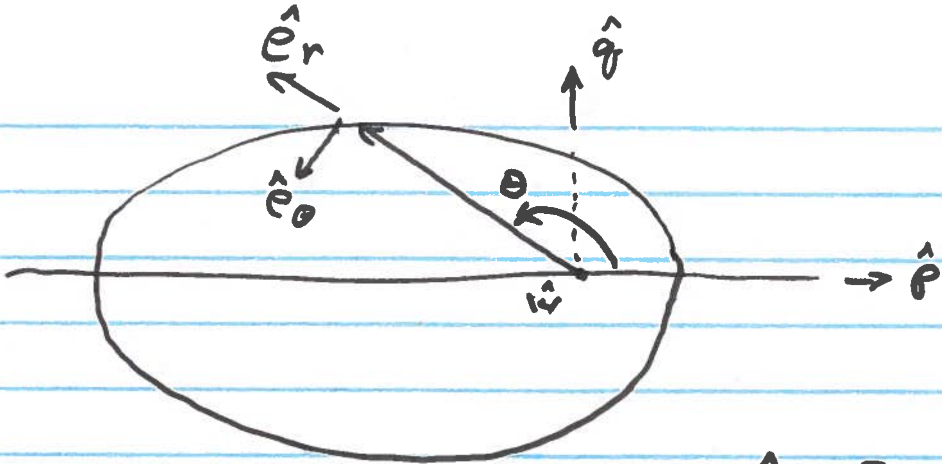
$$\vec{r}_0 = r_1 \hat{n}_1 + r_2 \hat{n}_2 + r_3 \hat{n}_3$$

$$\vec{V}_0 = V_1 \hat{n}_1 + V_2 \hat{n}_2 + V_3 \hat{n}_3$$

$$\theta_0, a, e, \Omega, i, \omega$$



$$\hat{a} = \frac{\hat{n}_3 \times \vec{h}}{|\hat{n}_3 \times \vec{h}|}$$



$$\vec{r}_0 = r_1 \hat{n}_1 + r_2 \hat{n}_2 + r_3 \hat{n}_3$$
$$\vec{V}_0 = V_1 \hat{n}_1 + V_2 \hat{n}_2 + V_3 \hat{n}_3$$

$\Rightarrow \hat{h}, \ \vec{e}, \ \theta_0$

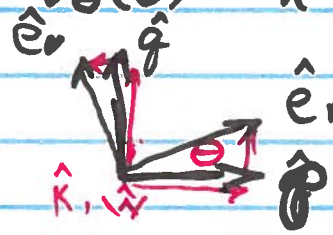
$\hat{p}, \hat{w}, \hat{q} = [\,C\,]^{PF\ N}$

$$\{\ \}_N = [{}^{N}C^{PF}]\{\ \}_{PF}$$

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

$$v_r(\theta) = \frac{\mu}{h} e \sin\theta$$

$$v_\theta(\theta) = \frac{\mu}{h}(1+e\cos\theta)$$



Polar orb

$$C^{PF} = \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{e}_r = c_\theta \hat{p} + s_\theta \hat{q}$$
$$\hat{e}_\theta = -s_\theta \hat{p} + c_\theta \hat{q}$$
$$\hat{K} = \hat{w}$$

	$\hat{p}$	$\hat{q}$	$\hat{w}$
$\hat{e}_r$	c_θ	s_θ	0
$\hat{e}_\theta$	$-s_\theta$	c_θ	0
K	0	0	1

$\Omega, i, \omega \Rightarrow$ BODY313 $\Rightarrow$ BODY ZXZ

$$\omega = \mathrm{ATAN2}(C_{13}, C_{23}) \qquad \left(\underset{Y}{-} \quad \underset{X}{+}\right)$$

$$i = \mathrm{ACOS}(C_{33})$$

$$\Omega = \mathrm{ATAN2}(C_{31}, -C_{32})$$

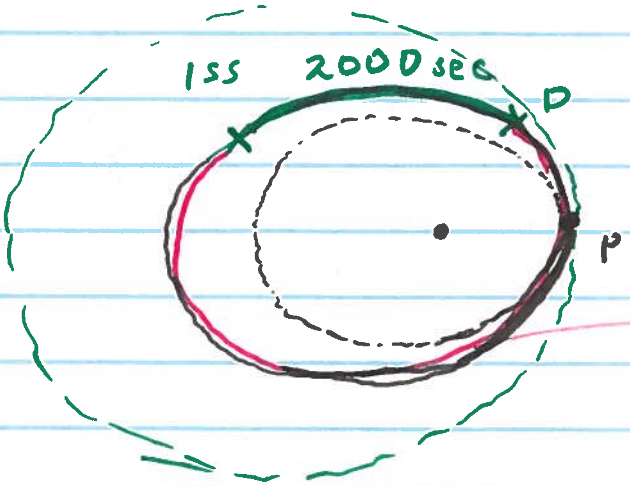
$$\left[{}^{PF}C^{N}\right] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$

~~B~~ space 123

$$\sqrt{\frac{\mu}{a^3}}\,(t - T_0) = E - e\sin E$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{E}{2}\right)$$

PHASING MANUEVERS



$5471 - 2000 = 3471$

SPACE STATION (ISS)

$h_p = 319.2$ Km
$h_A = 346.9$

$\Rightarrow r_p = 6697.2$ Km
$r_A = 6724.9$ Km

ISS LEADS CHASE VEHICLE BY 2000SEC

$$\tau_{ISS} = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(r_p + r_A)^3}{8\mu}}$$

$$= \pi\sqrt{\frac{(r_p + r_A)^3}{2\mu}} \cong 5471 \text{ SEC}$$

OPTION 1) $\tau_{REND} = 3471/n$

OPTION 2) $\tau_{REND} = \tau_{ISS} + \dfrac{3471}{n}$

n = # of orbit to accomplish manuever

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$

a

$$\Rightarrow a = \left[\left(\frac{\tau}{2\pi}\right)^2 \mu\right]^{1/3} \qquad \tau_{rend}$$

1 - orbit

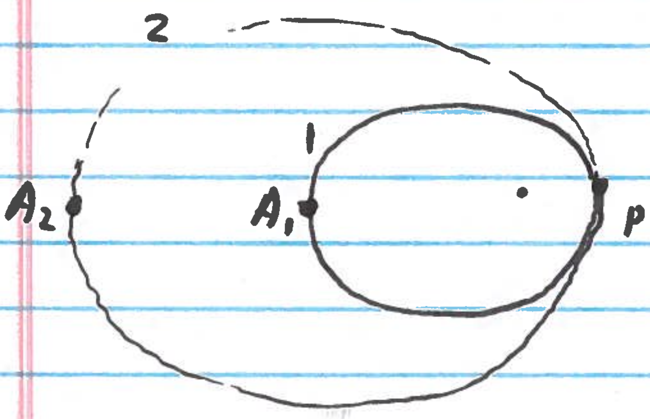
$$\tau_{rend} = \tau_{ISS} - 2000 = 3471$$

$$a = \left[\left(\frac{3471}{2\pi}\right)^2 (398600)\right]^{1/3} = \underbrace{4954.8\ Km}_{a < R_E}$$

$$\tau_{rend} = \tau_{ISS} + \frac{3471}{n} \sim 8942\ SEC$$

$$\Rightarrow a = \left[\left(\frac{8942}{2\pi}\right)^2 (398600)\right]^{1/3} = 9311.4\ Km$$

$$= \left(\frac{r_P + r_A}{2}\right) \Rightarrow 2a - r_P = r_{A_{req}} = 11925\ Km$$



$$v_{P_1} = \sqrt{\frac{2\mu r_{A_1}}{r_{P_1}(r_{P_1} + r_{A_1})}} = 7.707$$

$$v_{P_2} = \sqrt{\frac{2\mu r_{A_2}}{r_{P_2}(r_{P_2} + r_{A_2})}} = 8.731$$

$$\Delta V_{P_{rend}} = 8.731 - 7.707 = 1.024\ Km/s$$

$$r_{P_1} = r_{P_2} = 6697.2\ Km$$

$$r_{A_1} = 6724.9\ Km$$

$$r_{A_2} = 11925\ Km$$

$$\Delta V_{TOTAL} = 2\Delta V_P$$

10 orbits:

High orbit $\tau = \underbrace{\tau_{ISS}}_{5471} + \frac{3471}{10}$

$$a_{req} = \left[\left(\frac{5471 + \left(\frac{3471}{10}\right)}{2\pi} \right)^2 \underbrace{(398600)}_{\mu} \right]^{1/3} = 6991.6 \text{ Km}$$

$$r_{A_{req}} = 2(a_{req}) - r_p$$

$$r_{A_{req}} = 2(6991.6) - 6697.2 = 7286.1 \text{ Km}$$

$$v_{P_1} = 7.707 \text{ Km/s}$$

$$v_{P_2} = \sqrt{\frac{2 \mu r_{A_2}}{r_{P_2}(r_{P_2} + r_{A_2})}} = \sqrt{\frac{2(398600)(7286)}{6697(6697+7286)}}$$

$r_{A_2} = 7286.1$ Km $\qquad = 7.87$

$r_{P_2} = r_{P_1} = 6697.2$ Km

$$\Delta V_p \sim 7.87 - 7.71 = 0.16 \text{ Km/s}$$

10 orbits

$$\Delta V_{TOTAL} = 2 \Delta V_p = 0.32 \text{ Km/s}$$