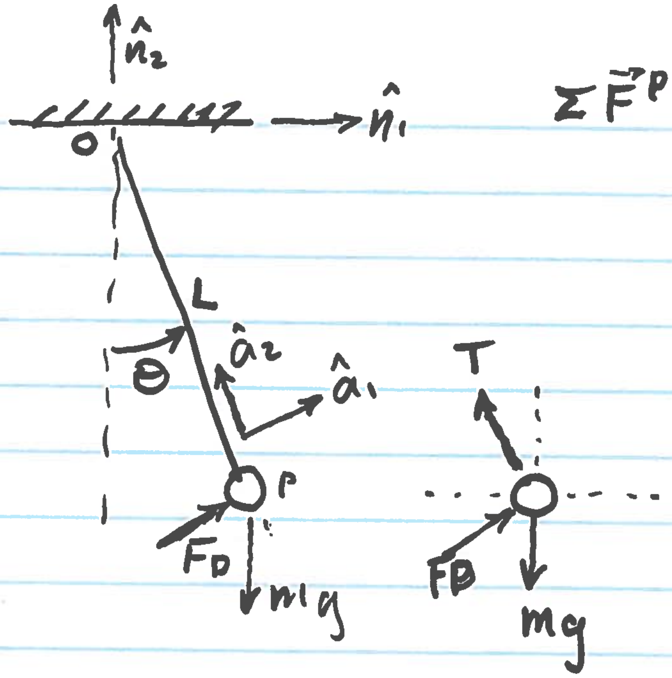
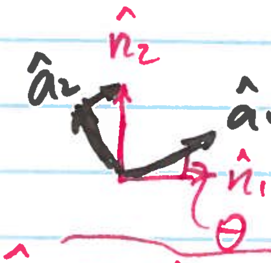


$$\Sigma \vec{F}^P = T\hat{a}_2 + F_D\hat{a}_1 - m_p g \hat{n}_2$$
$$= m_p \, {}^N\vec{a}^P$$
$$= m_p (L\ddot{\theta}\hat{a}_1 + L\dot{\theta}^2\hat{a}_2)$$



$$\hat{n}_2 = C_\theta \hat{a}_2 + S_\theta \hat{a}_1$$

$$\hat{a}_1: \quad m_p L\ddot{\theta} = F_D - m_p g S_\theta$$
$$\hat{a}_2: \quad m_p L\dot{\theta}^2 = T - m_p g C_\theta$$

$$F_D = -\mathrm{sgn}(\underbrace{v}_{L\dot{\theta}}) \tfrac{1}{2}\rho C_D \cancel{v}^2 A \quad (L\dot{\theta})^2$$

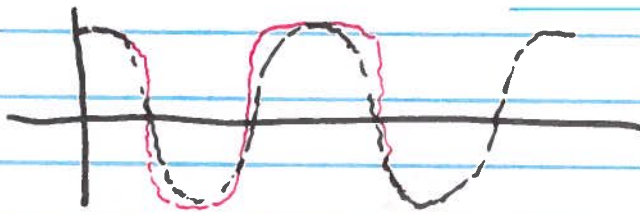
$$v = L\dot{\theta}$$
$$\vec{v} = \cancel{\dot{r}}\hat{e}_r + \underbrace{r}_{L}\dot{\theta}\hat{e}_\theta \quad 0$$

$$m_p L\ddot{\theta} + \mathrm{sgn}(\dot{\theta})\tfrac{1}{2}\rho C_D (L^2\dot{\theta}^2) A + m_p g \sin\theta = 0$$

sign( )

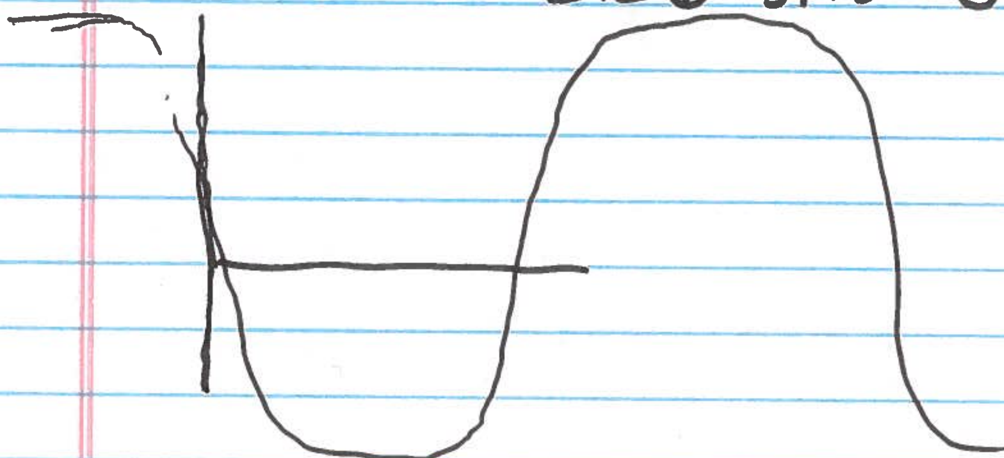
$$\ddot{\theta} + \mathrm{sign}(\dot{\theta})\left(\frac{1}{2m_p}\right)\rho C_D L\dot{\theta}^2 A + \frac{g}{L}\sin\theta = 0$$

$$C_D \sim 0 \quad \Rightarrow \ddot{\theta} + \frac{g}{L}\sin\theta = 0$$
$$\theta \ll 1 \quad \Rightarrow \ddot{\theta} + \left(\frac{g}{L}\right)\theta = 0$$
$$\Rightarrow \theta(t) = A\sin\sqrt{\frac{g}{L}}\,t + B\cos\sqrt{\frac{g}{L}}\,t$$



$\Theta(t) = \Theta_0 \cos\sqrt{\frac{g}{L}}\, t$

$\cancel{\sin\Theta}$ $\sin\Theta \sim \Theta - \frac{\Theta^3}{3!} + \frac{\Theta^5}{5!} + \cdots$



$\Theta \Leftrightarrow x_1$

$\dot{\Theta} \Leftrightarrow x_2$

$\Theta \Rightarrow \dot{x}(1)$

$\dot{\Theta} \Rightarrow x(2)$

$\ddot{\Theta} =$

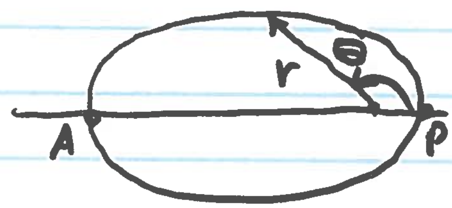
1) TAKE INVENTOR

$$\vec{h} = \vec{r} \times \vec{v} = \overrightarrow{\text{const}}$$

$$\frac{v^2}{2} - \frac{\mu}{r} = \mathcal{E} = -\frac{\mu}{2a} \quad *$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} \qquad e = |\vec{e}|$$

$$r(\theta) = \frac{h^2}{\mu(1 + e\cos\theta)}$$



$$v_r = \frac{\mu}{h} e \sin\theta$$

$$v_\theta = \frac{\mu}{h}(1 + e\cos\theta)$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$r_p = a(1-e) \quad *$$

$$r_a = a(1+e) \quad *$$

$$h^2 = a\mu(1-e^2) \quad *$$

$$r(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta} \quad *$$

$t = t_0 \Rightarrow \vec{r}_0 = 8182\,\hat{i} - 6865.9\,\hat{j}$ [Km]

$\vec{v}_0 = 0.47572\,\hat{i} + 8.8116\,\hat{j}$ [Km/s]

$\varepsilon = ?$, $e = ?$, $\theta_0 ?$
$v @ \theta_0 + 120^\circ$, $r(\theta_0 + 120^\circ)$

$$\vec{h} = \vec{h}_0 = \vec{r}_0 \times \vec{v}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8182 & -6865 & 0 \\ 0.475 & 8.81 & 0 \end{vmatrix} = 75366\,\hat{k}$$

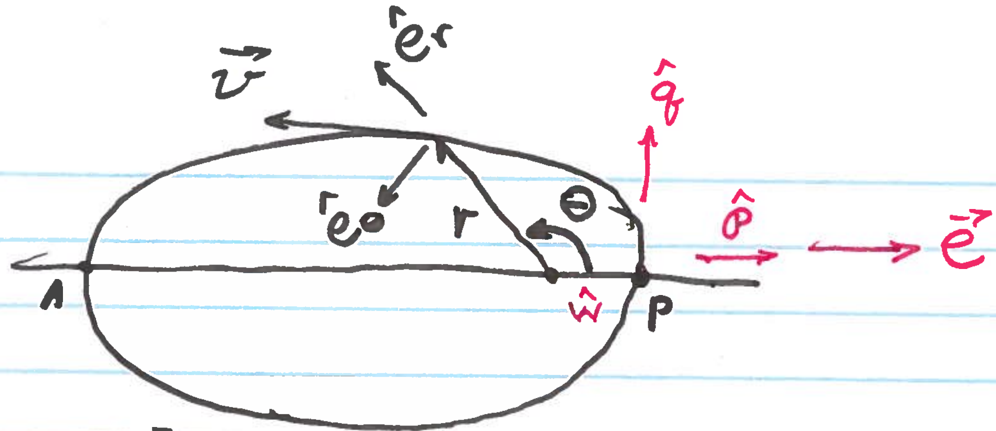
$$r_0 = |\vec{r}_0| = \sqrt{(8182)^2 + (-6865.9)^2} = 10681 \text{ Km}$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{\vec{v}_0 \cdot \vec{v}_0}{2} - \frac{398600}{10681} = \quad \text{hyp.}$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} = 0.900\,\hat{i} + 0.553\,\hat{j}$$

$$e = |\vec{e}| = 1.0562$$

$$\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}_{ijk} = \left[C \right]^{ijk\ orb} \begin{Bmatrix} v_r \\ v_\theta \\ 0 \end{Bmatrix}_{orb}$$



$$\hat{p} \triangleq \frac{\vec{e}}{e} = 0.852\,\hat{i} + 0.523\,\hat{j}$$

$$\hat{w} \triangleq \frac{\vec{h}}{h} = k$$

$$\hat{q} \triangleq \hat{w} \times \hat{p} = -0.523\,\hat{i} + 0.852\,\hat{j}$$

$$\begin{array}{c|ccc} & \hat{i} & \hat{j} & \hat{k} \\ \hline \hat{p} & 0.852 & 0.523 & 0 \\ \hat{q} & -0.523 & 0.852 & 0 \\ w & 0 & 0 & 1 \end{array}$$

$$\Downarrow$$

$$C^{PF}_{ijk} = \begin{bmatrix} 0.852 & 0.523 & 0 \\ -0.523 & 0.852 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} x \\ Y \\ z \end{Bmatrix}_{PF} = \begin{Bmatrix} 3379 \\ -10133 \\ 0 \end{Bmatrix}_{PF} = \left[C^{PF}_{ijk} \right] \begin{Bmatrix} 8182 \\ -6865 \\ 0 \end{Bmatrix}_{ijk}$$

$$\theta_0 = \text{ATAN2}(Y_0, X_0) = -1.249 \text{ rad.}$$

$$v_r(\theta = \theta_0 + 120°) = \frac{\mu}{h} e \sin\theta = 4.180 \text{ km/s}$$

$$v_\theta = \frac{\mu}{h}(1 + e\cos\theta) = 8.995 \text{ km/s}$$

$$r(\theta) = \frac{h^2}{\mu(1 + e\cos\theta)} = 8379$$

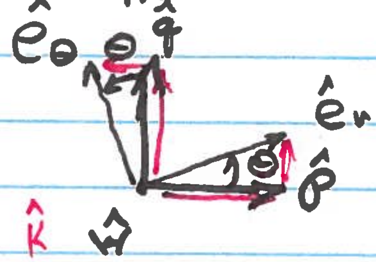
want

$$\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}_{ijk} = \left[{}^{ijk}C^{orb} \right] \begin{Bmatrix} v_r \\ v_\theta \\ 0 \end{Bmatrix}_{orb}$$

Have

$$\left[{}^{PF}C^{ijk} \right] \Rightarrow \left[{}^{ijk}C^{PF} \right] = \left[{}^{PF}C^{ijk} \right]^T$$

HAVE



$$\Rightarrow \quad \hat{e}_r = C\theta\,\hat{p} + S\theta\,\hat{q}$$

$$\hat{e}_\theta = -S\theta\,\hat{p} + C\theta\,\hat{q}$$

$$\hat{k} = \hat{w}$$

	$\hat{p}$	$\hat{q}$	$\hat{w}$
$\hat{e}_r$	$C\theta$	$S\theta$	0
$\hat{e}_\theta$	$-S\theta$	$C\theta$	0
$\hat{k}$	0	0	1

$$\Rightarrow {}^{Pol}C^{PF} = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

orb $\triangleq$ polar

$${}^{ijk}C^{pol} = \left[{}^{ijk}C^{PF} \right] \left[{}^{PF}C^{pol} \right]$$

$8379 = r(\theta_0+120^\circ)$

$$\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_{ijk} = \left[\,{}^{ijk}C^{pol}\,\right] \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}_{pol}$$

@ $\theta = \theta_0 + 120^\circ$

$$\vec{r} = r\hat{e}_r$$