1) Exam #1 Th 2/14

EVERY THING SO FAR

EXCEPT Lagrange Coefs

- restricted 3-Body Problem

Bring a calculator!

$R_E = 6378 \qquad \mu_E = 398600 \; [Km^3/s^2]$

$1 Km = 0.62137 \text{ mi}$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}^{*} \quad \begin{cases} <0 & \text{Elliptic} & a>0 \\ =0 & \text{Parabolic} & a = \infty \\ >0 & \text{hyperb.} & (a<0) \end{cases}$$

$$\vec{h} \triangleq \vec{r} \times \vec{v}$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} \qquad e = |\vec{e}|$$

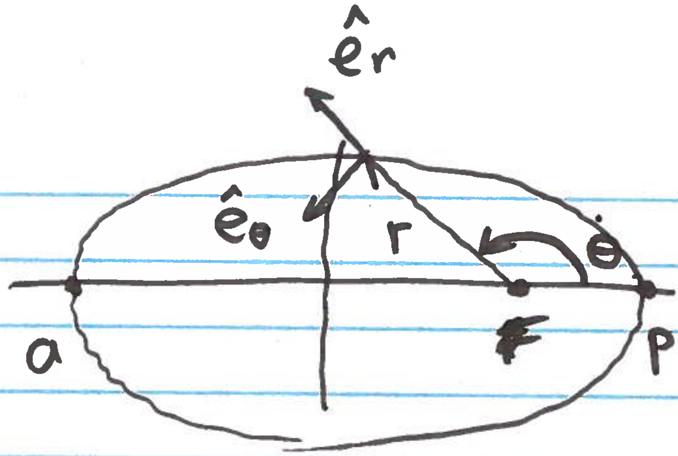
$$r(\theta) = \frac{h^2}{\mu(1 + e\cos\theta)} = \frac{a(1-e^2)}{1 + e\cos\theta}$$

$$r_p = \frac{h^2}{\mu(1+e)} = a(1-e)$$

$$r_a = \frac{h^2}{\mu(1-e)} = a(1+e)$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$v_r(\theta) = \frac{\mu}{h} e \sin\theta \; , \quad v_\theta = \frac{\mu}{h}(1 + e\cos\theta)$$



$$\tau = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$v_{circ} = \sqrt{\frac{\mu}{r}} \qquad \frac{\mu}{r^{\cancel{2}}} = \dot{\theta}^2 r^2 = \underbrace{(\dot{\theta} r)}_{v_{circ}}{}^2$$

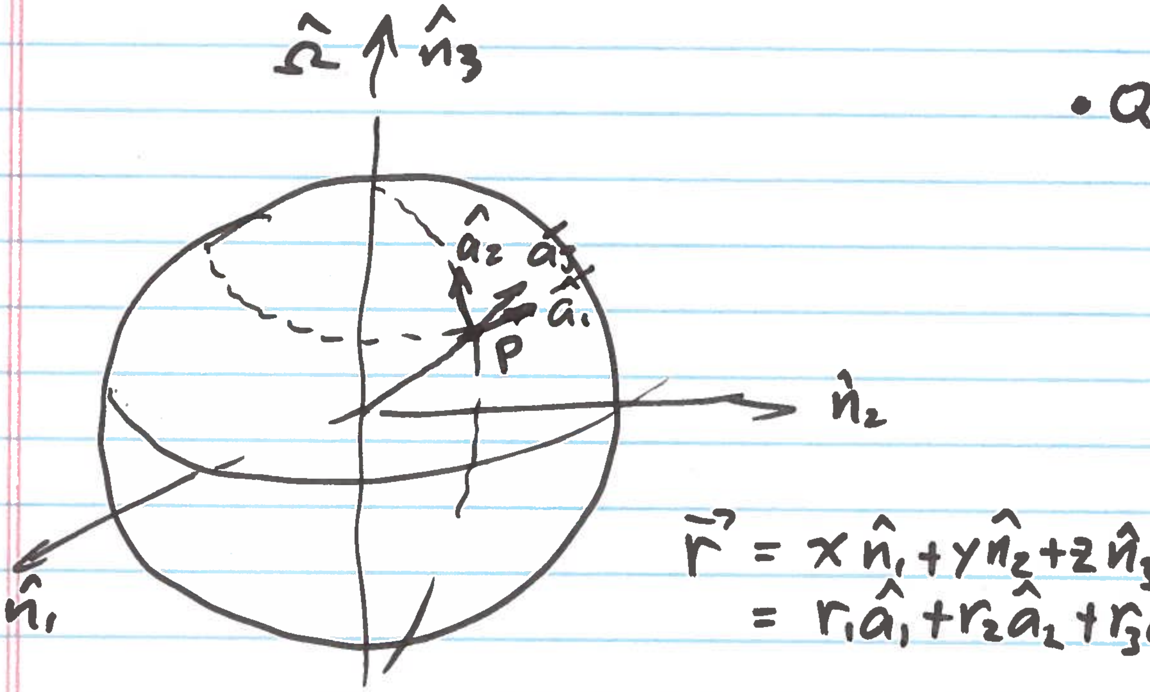
$$v_{Esc} = \sqrt{\frac{2\mu}{r}} = \sqrt{2}\, v_{circ}$$

$$v_\infty = \sqrt{\frac{-\mu}{a}} \quad \left(\sqrt{\frac{\mu}{a}}\right)$$

$$\frac{v_\infty^2}{\cancel{2}} = \frac{-\mu}{\cancel{2}a} \qquad v_\infty = \sqrt{\frac{-\mu}{a}} \left(\sqrt{\frac{\mu}{a}}\right)$$

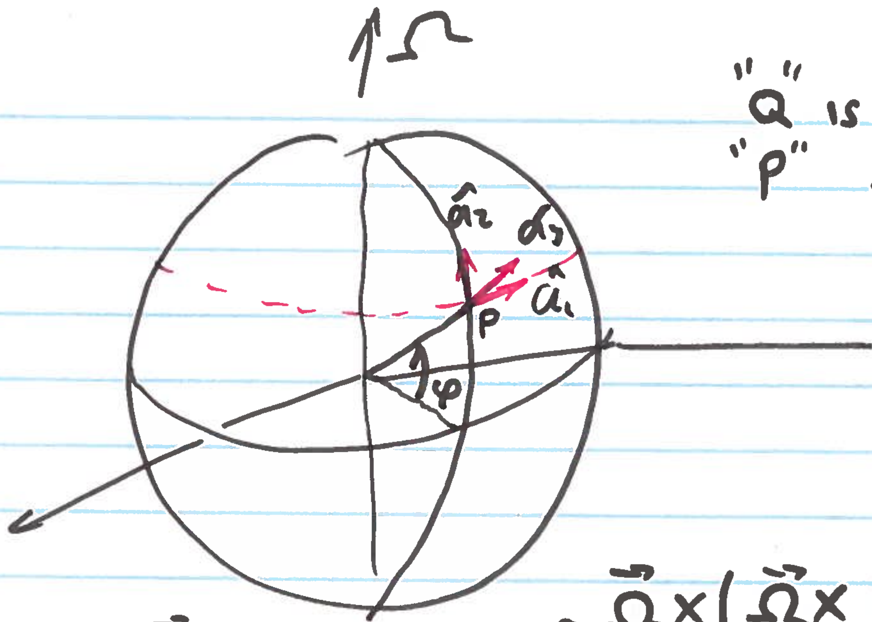
$$\vec{F}_g = \frac{-\mu \vec{r}^{\,OQ}}{(r^{OQ})^3} = {}^N\vec{a}^{\,P} + {}^A\vec{a}^{\,Q/P} + {}^N\vec{\alpha}^{\,A} \times \vec{r}^{\,PQ}$$
$$+ {}^N\vec{\omega}^{\,A} \times ({}^N\vec{\omega}^{\,A} \times \vec{r}^{\,PQ}) + 2\, {}^N\vec{\omega}^{\,A} \times {}^A\vec{v}^{\,Q/P}$$

$$F_g = \frac{\mu m_a}{r^2}$$



$$\vec{r} = x\hat{n}_1 + y\hat{n}_2 + z\hat{n}_3$$
$$= r_1\hat{a}_1 + r_2\hat{a}_2 + r_3\hat{a}_3$$

$$\underbrace{\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_N}_{\text{want}} = \begin{bmatrix} {}^{N}C^{A} \end{bmatrix} \underbrace{\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_A}_{\text{HAVE}}$$



"Q" is on top of "P"

$\vec r^{PQ} = \vec 0$

$$\vec\Omega \times (\vec\Omega \times \vec r^{OP})$$

$$+\vec F_N + \vec F_g = \frac{-\mu \vec r^{OQ}}{(r^{OQ})^3} + \vec F_N = {}^N\vec a^P + \underbrace{\cancel{{}^E\vec a^{Q/P}}}_{0} + \cancel{{}^N\vec\alpha^E}^{\,0} \times \cancel{\vec r^{PQ}}^{\,0}$$

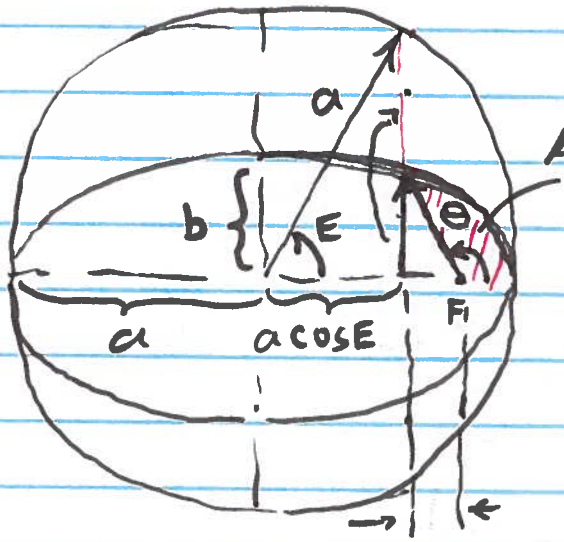
$$+ {}^N\vec\omega^E \times ({}^N\vec\omega^E \times \cancel{\vec r^{PQ}}^{\,0})$$

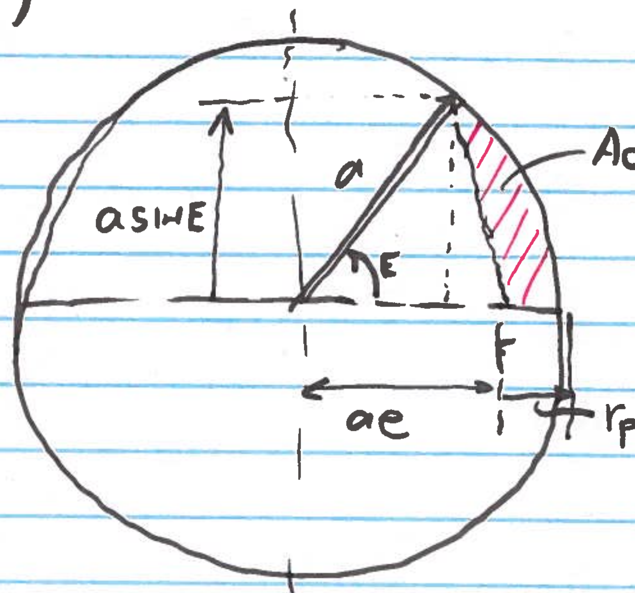
$$+ 2\,{}^N\vec\omega^E \times \underbrace{{}^E\vec v^{Q/P}}_{v\hat a_1}$$

Kepler's Eqn.

$A_e = \frac{b}{a} A_c$

$\left(\frac{a}{b}\right)$





$r_p = a(1-e)$

$$\frac{dA_e}{dt} = \text{const}$$

θ = Actual Anomaly

E = Eccentric Anomaly

$$A_c = \tfrac{1}{2}a(aE) - \tfrac{1}{2}ae(a\sin E)$$
$$= \tfrac{1}{2}a^2\left[E - e\sin E\right]$$

$b = a(1-e^2)$

$$\frac{dA_E}{dt} = \frac{\pi a b}{\tau} = \frac{\pi a b}{\frac{2\pi}{\sqrt{\mu}} a^{3/2}} = \frac{ab}{2}\sqrt{\frac{\mu}{a^3}}$$

$$\int_0^{A_E} dA_E = \frac{ab}{2}\sqrt{\frac{\mu}{a^3}} \int_{T_0}^{t} dt$$

$A_E = \frac{b}{a} A_c$

$$A_E = \frac{a\cancel{b}}{2}\sqrt{\frac{\mu}{a^3}}\,(t - T_0)$$
$$= \frac{\cancel{b}}{a}\left[\tfrac{1}{2}a^2\{E - e\sin E\}\right]$$

$$\Rightarrow \boxed{\underbrace{\sqrt{\frac{\mu}{a^3}}}_{n,\ M_e}(t - T_0) = E - e\sin E}$$

FOR ELLIPTIC ORBIT

$$\mathrm{TAN}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\ \mathrm{TAN}\left(\frac{E}{2}\right)$$

FOR ELLIPTIC ORBITS

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FOR HYPERBOLIC TRAJECTORIES

$$\sqrt{\frac{\mu}{a^3}}^{\,*}(t - T_0) = e\sinh F - F$$

F = "HYPERBOLLIC anomally"

$$\mathrm{TAN}\left(\frac{\theta}{2}\right) = \sqrt{\frac{e+1}{e-1}}\ \cancel{\sqrt{\frac{1+e}{1-e}}}\ \tanh\left(\frac{F}{2}\right)$$