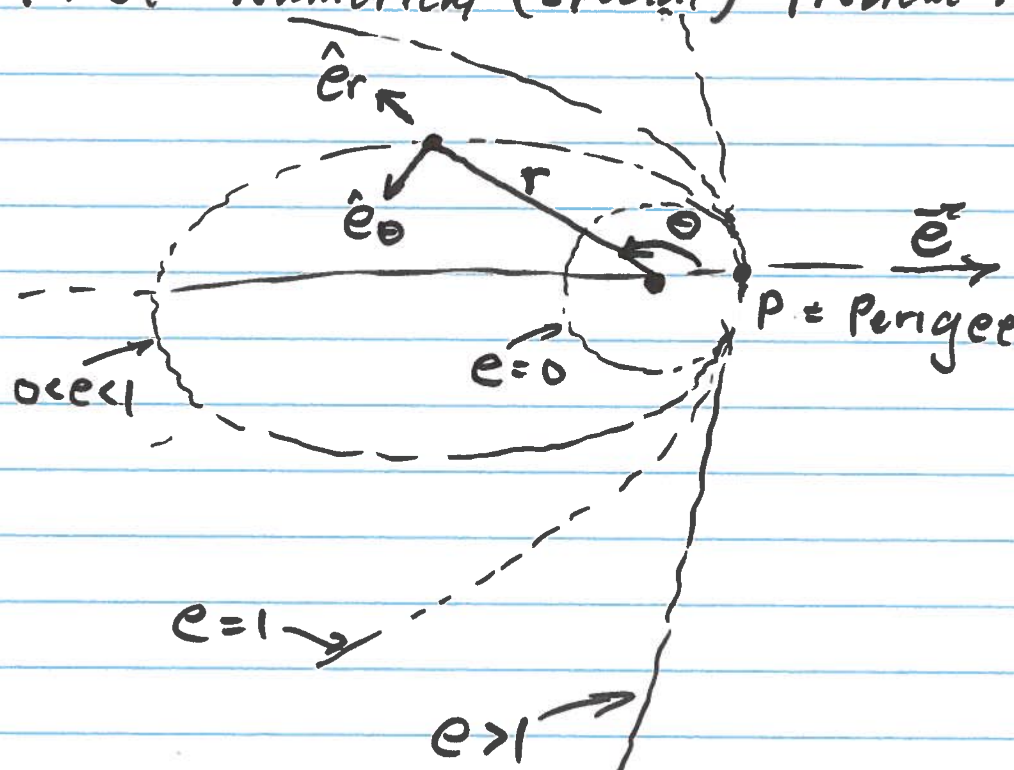
0) Quick review (inventory of useful Eqns).
1) Hyperbolic Trajectories
2) Perifocal Basis (coordinates)
3) First Numerical (special) problem?

$-1 < e < 1$



$$\vec{h} = \vec{r} \times \vec{v} \qquad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \begin{cases} <0 & \text{Ellipse} \\ =0 & \text{parab.} \\ >0 & \text{hyper} \end{cases}$$

$$r(\theta) = \frac{h^2}{\mu(1 + e\cos\theta)}$$

$$v_\theta = \frac{\mu}{h}(1 + e\cos\theta) \qquad v_r(\theta) = \frac{\mu}{h} e \sin\theta$$

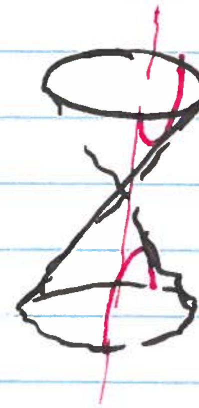
$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}$$

$$r_p = \frac{h^2}{\mu(1+e)} = a(1-e)$$

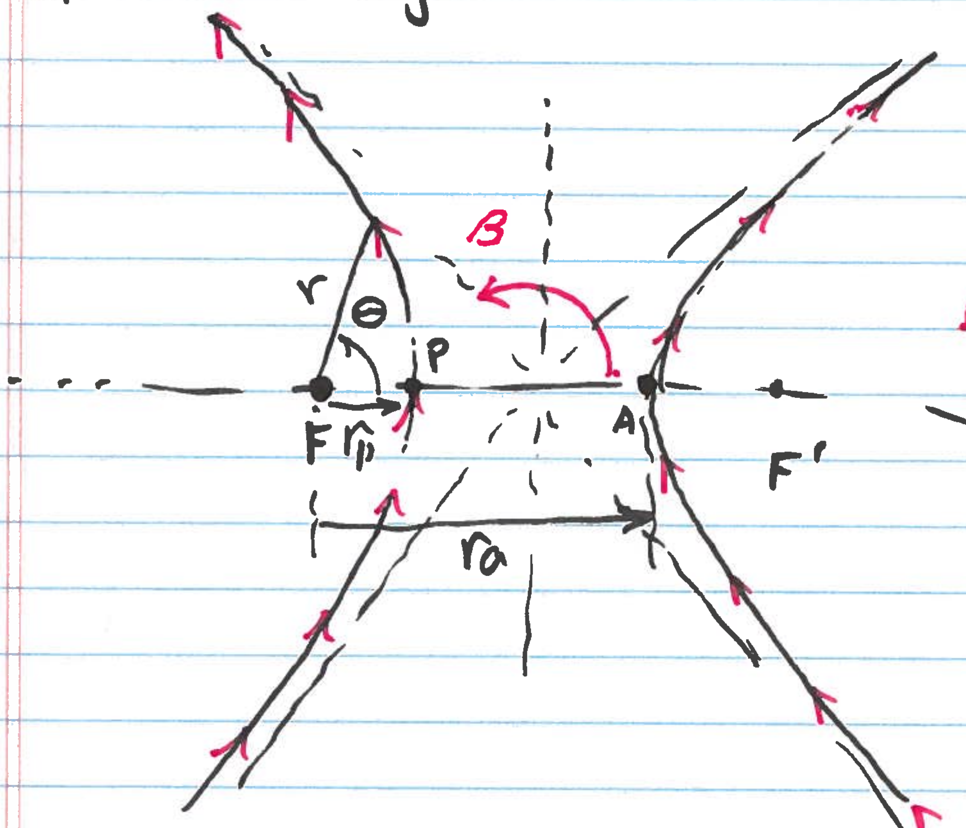
$$r_a = \frac{h^2}{\mu(1-e)} = a(1+e)$$

$$h^2 = \mu a(1-e^2)$$

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$



HYPERBOLIC Traj.



$\theta_\infty = \beta$

$\theta_\infty = \cos^{-1}\left(\frac{-1}{e}\right)$

For our text

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \begin{cases} \text{ELLIPTICAL} \\ \text{parabolic} \end{cases}$$

$$= \frac{\mu}{2a} \quad \{ \text{hyperbolic}$$

a is positive

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

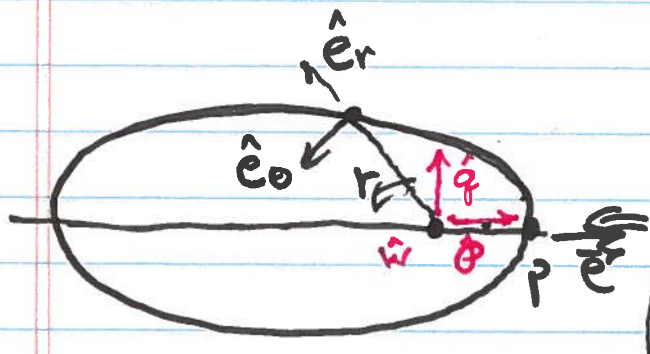
$$r_E(\theta) = \frac{a(1-e^2)}{1+e\cos\theta}$$

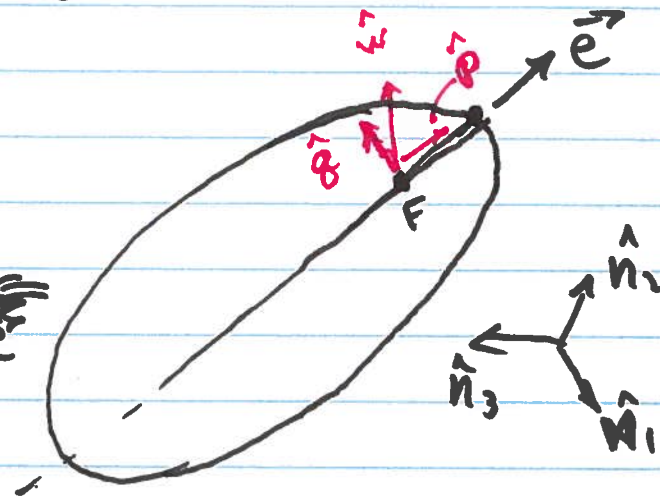
$$r_H(\theta) = \frac{a(e^2-1)}{1+e\cos\theta}$$

$r_{p_E} = a(1-e)$ $\quad$ $r_{p_H} = a(e-1)$

$r_{A_E} = a(1+e)$ $\quad$ $r_{A_H} = a(1+e)$

Perifocal ref. Frames





$$\vec{r} = X\hat{n}_1 + Y\hat{n}_2 + Z\hat{n}_3$$
$$= r_1\hat{a}_1 + r_2\hat{a}_2 + r_3\hat{a}_3$$

$$\vec{v} = \dot{X}\hat{n} + \dot{Y}\hat{n}_2 + \dot{Z}\hat{n}_3$$
$$= v_1\hat{a}_1 + v_2\hat{a}_2 + v_3\hat{a}_3$$

$$\vec{h} = \vec{r} \times \vec{v} = h_1\hat{a}_1 + h_2\hat{a}_2 + h_3\hat{a}_3$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} = e_1\hat{a}_1 + e_2\hat{a}_2 + e_3\hat{a}_3$$

$$\hat{p} \triangleq \frac{\vec{e}}{e} = p_1\hat{a}_1 + p_2\hat{a}_2 + p_3\hat{a}_3$$

$$\hat{w} = \frac{\vec{h}}{h} = w_1\hat{a}_1 + w_2\hat{a}_2 + w_3\hat{a}_3$$

$$\hat{q} \triangleq \hat{w} \times \hat{p} = q_1\hat{a}_1 + q_2\hat{a}_2 + q_3\hat{a}_3$$

$$r = \sqrt{x^2+y^2} \qquad \theta = \mathrm{ATAN2}(\underbrace{x,y}_{?})$$

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta_0)} \qquad v_r = \frac{\mu}{h} e\sin\theta$$

$$v_\theta = \frac{\mu}{h}(1+e\cos\theta)$$

$$\theta_0 = \mathrm{ATAN}(x_0, y_0)$$

time of measure

velocity

$$\vec{v}_F = v_1\hat{a}_1 + v_2\hat{a}_2 + v_3\hat{a}_3 = \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}_A$$

$$= \dot{x}\hat{P} + \dot{y}\hat{Q} + \cancel{\dot{z}}\hat{W}$$

0

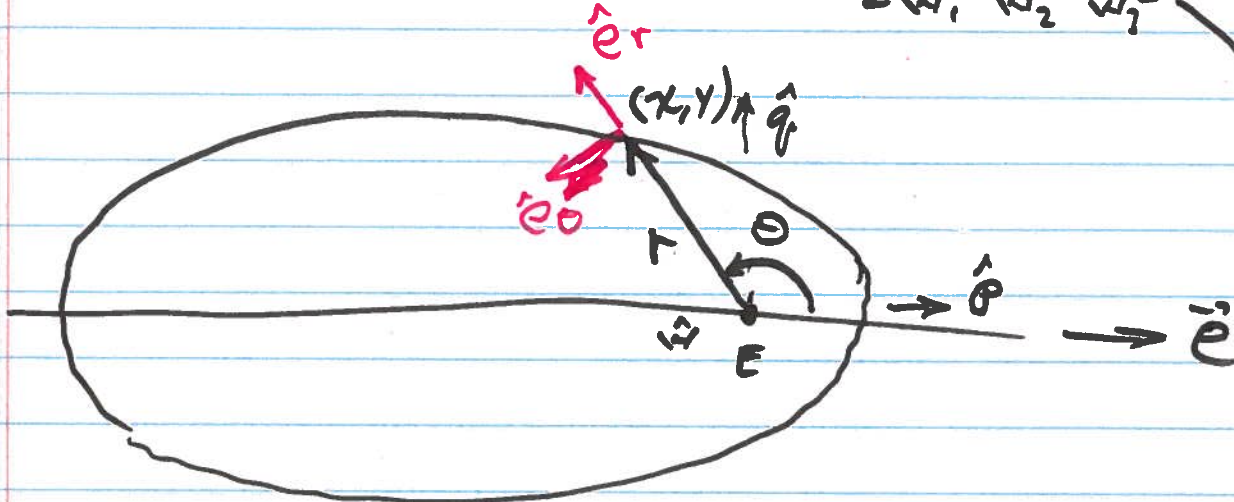
$$\begin{Bmatrix} v_r \\ v_\theta \\ 0 \end{Bmatrix}_{orb} \Leftrightarrow \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} = \left[{}^{PF}C^{A}\right]\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}_A$$

0

$$\hat{p} = p_1\hat{a}_1 + p_2\hat{a}_2 + p_3\hat{a}_3$$
$$\hat{q} = q_1\hat{a}_1 + q_2\hat{a}_2 + q_3\hat{a}_3$$
$$\hat{w} = w_1\hat{a}_1 + w_2\hat{a}_2 + w_3\hat{a}_3$$

$$= \begin{array}{c|ccc} & \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \\ \hline \hat{p} & p_1 & p_2 & p_3 \\ \hat{q}_A & q_1 & q_2 & q_3 \\ \hat{w} & w_1 & w_2 & w_3 \end{array}$$

$$^{P.F.}\underline{C}^{A} = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$



WE HAVE

$$r = r_1\hat{a}_1 + r_2\hat{a}_2 + r_3\hat{a}_3 \Rightarrow \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_A \Rightarrow \begin{Bmatrix} x \\ y \\ \cancel{z} \end{Bmatrix}_{PF}$$
$$= x\hat{p} + y\hat{q} + z\hat{w}$$
$$= r\hat{e}_r$$

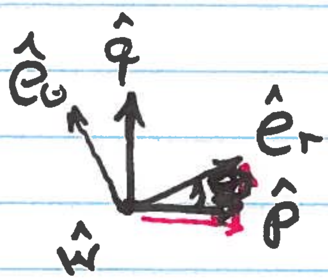
$$\begin{Bmatrix} x \\ y \\ \cancel{z} \end{Bmatrix}_{P.F.} = \left[{}^{PF}C^{A}\right] \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_A$$

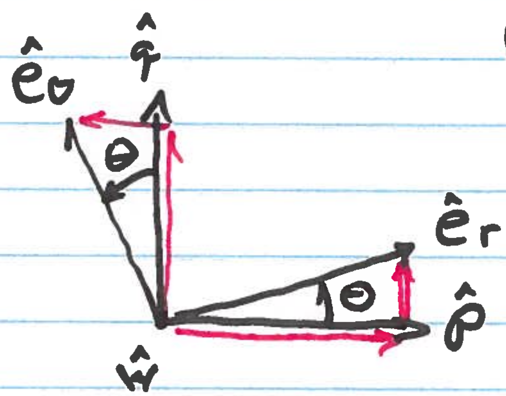
$$r = \frac{h^2}{\mu(1 + e\cos\theta)} \qquad v_r = \frac{\mu}{h} e \sin\theta$$

$$v_\theta = \frac{\mu}{h}(1 + e\cos\theta)$$

$$\begin{Bmatrix} x \\ y \\ 0 \end{Bmatrix}_{PF} = \left[{}^{PF}C^{A} \right] \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}$$

$$\begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}_{pol.} = \left[{}^{Pol}C^{PF} \right] \begin{Bmatrix} x \\ y \\ 0 \end{Bmatrix}_{PF}$$



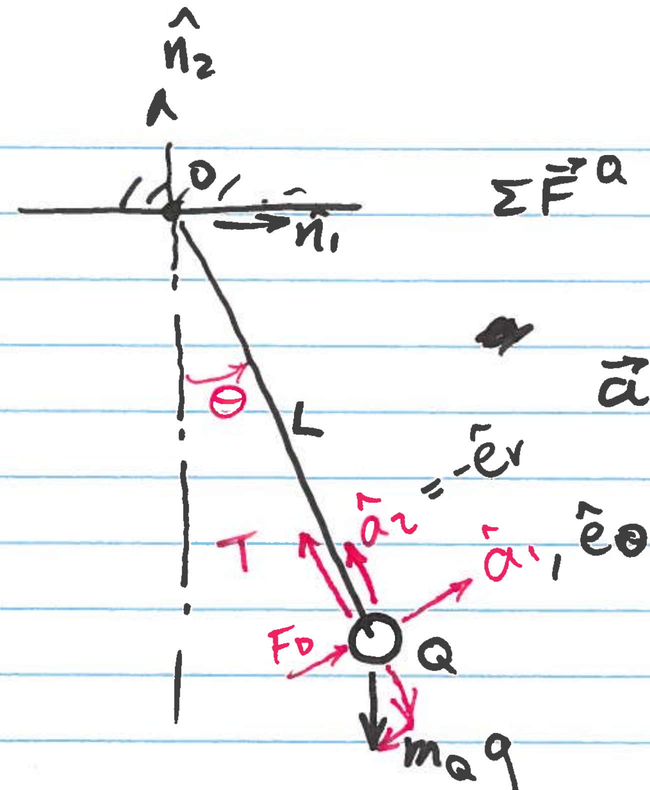


$$\hat{e}_r = c_\theta \hat{p} + s_\theta \hat{q}$$
$$e_\theta = -s_\theta \hat{p} + c_\theta \hat{q}$$
$$\hat{k} = \hat{w}$$

	$\hat{p}$	$\hat{q}$	$\hat{w}$
$\hat{e}_r$	c_θ	s_θ	0
$\hat{e}_\theta$	$-s_\theta$	c_θ	0
$\hat{k}$	0	0	1

$$\left[{}^{Pol}C^{A} \right] = \left[{}^{Pol}C^{PF} \right] \left[{}^{PF}C^{A} \right]$$

$$\left[{}^{Pol}C^{PF} \right] = \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

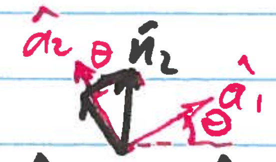


$$\Sigma \vec{F}^Q = m_Q \, {}^N\vec{a}^Q$$

$r = \text{const}$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

0



$$\hat{n}_2 = s_\theta \hat{a}_1 + c_\theta \hat{a}_2$$

$$\vec{F}^Q = T\hat{a}_2 - m_Q g \hat{n}_2 + F_D \hat{a}_1 = m_Q (A_1 \hat{a}_1 + A_2 \hat{a}_2)$$

$$F_D = \tfrac{1}{2}\rho v^2 C_D A \, (-\operatorname{sgn} v)$$

$\operatorname{sign}(v)$

$$\Rightarrow T\hat{a}_2 - mg(s_\theta \hat{a}_1 + c_\theta \hat{a}_2) + \tfrac{1}{2}\rho v^2 C_D A(-\operatorname{sgn} v)$$
$$= m_Q[L\ddot{\theta}\hat{a}_1 + r\dot{\theta}^2 \hat{a}_2]$$

$\hat{a}_1$:

$\hat{a}_2$: