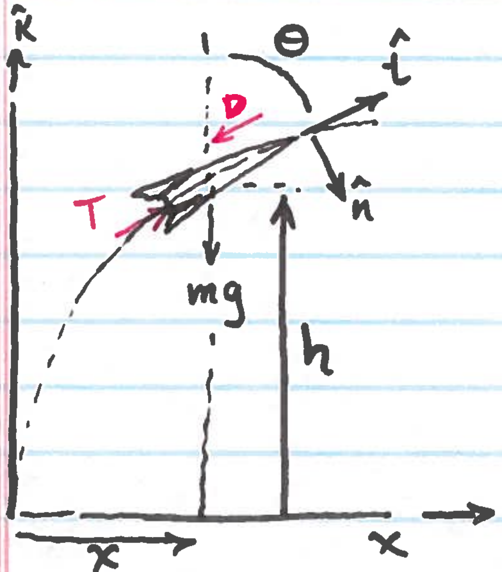
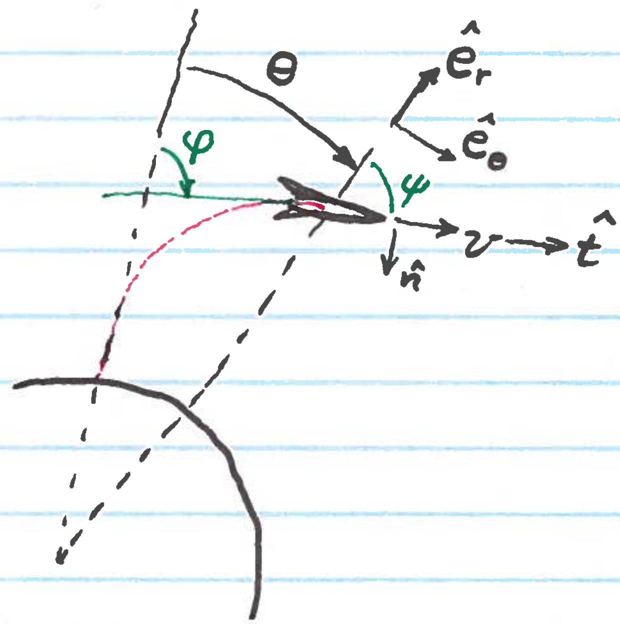
1) Basic Rocket Equation
2) ~ Analytic solution
3) Isp, specific Impulse
4) some Chemical Kinetics
5) payload Ratio, structure ratio, staging

FLAT EARTH



CYLINDRICAL EARTH



$$\Sigma \vec{F} = m\vec{a}^{\,N/R}$$

$$(\vec{T} - \vec{D} - m\vec{g}) = m\vec{a}$$

$$\left[(P_E - P_A)A + \dot{m}_B v_E - \tfrac{1}{2}\rho C_D v^2 A_s\right]\hat{t} + \frac{m_R \mu}{(R_E+h)^2}\left(-\cos\theta\,\hat{t} + \sin\theta\,\hat{n}\right)$$

$$= m\left(\dot{v}\hat{t} + v\dot{\theta}\hat{n}\right)$$

$\hat{t}$: $$\dot{v} = \left[(P_E - P_A)A + \dot{m}_B v_E - \tfrac{1}{2}\rho v^2 C_D A_s - \frac{m_R\mu}{(R_E+h)^2}\cos\theta\right]\Big/ m_R$$

$\hat{n}$: $$\dot{\theta} = \left[\frac{\cancel{m_R}\,\mu \sin\theta}{(R_E+h)^2}\right]\Big/ \cancel{m_R}\, v$$

$$\dot{h} = v\cos\theta\,, \quad \dot{x} = v\sin\theta$$

$\hat{n}$: $$\dot{\varphi} = \frac{\mu}{(R_E+h)^2}\sin\psi\left(\frac{1}{v}\right)$$

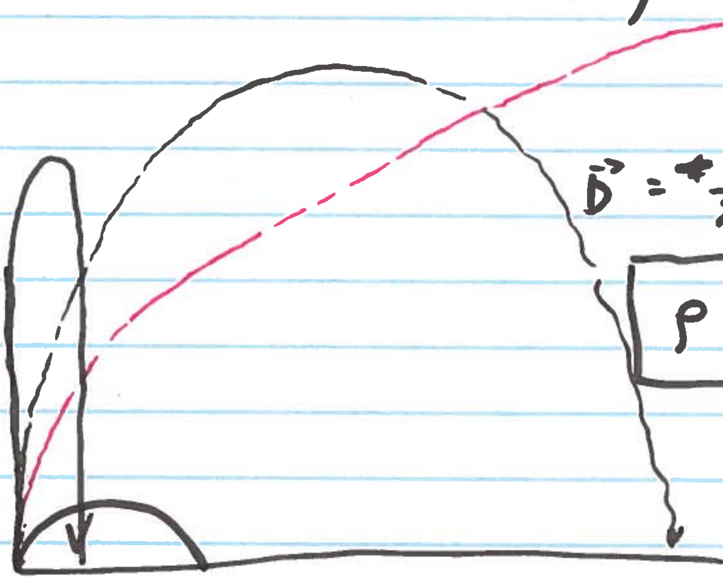
$\hat{t}$: $$\dot{v} = \left[(P_E - P_A)A + \dot{m}_B v_E - \tfrac{1}{2}\rho v^2 C_D A_s - \frac{m_R\mu}{(R_E+h)^2}\cos\psi\right]\Big/ m$$

$$\dot{h} = v\cos\psi$$

$$\dot{\theta} = \frac{v\sin\psi}{(R_E+h)} = \left(\frac{v_\theta}{r}\right)$$

$$\dot{\psi} = \dot{\varphi} - \dot{\theta}$$

$$mg = \left(\frac{m\mu}{(R_E+h)^2} - \frac{v_x^2}{(R_E+h)} \right)$$

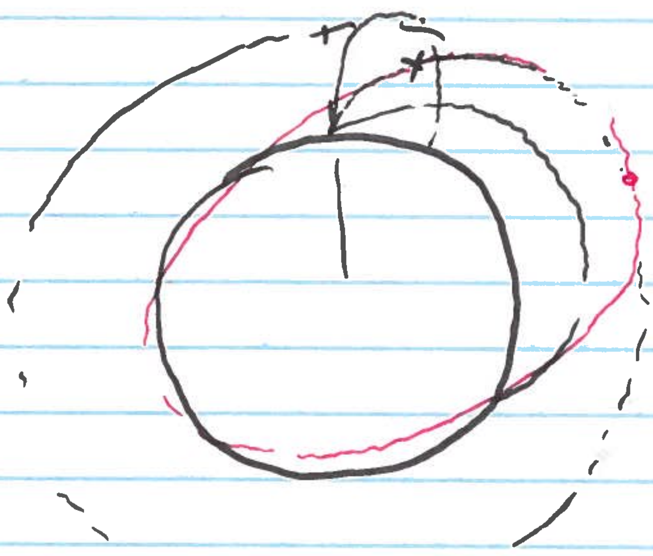


$$\vec{D} = -\frac{1}{2} \rho v^2 C_D A_s \hat{t}$$

$$\boxed{\rho = \rho_0 e^{-h/h_0}}$$

$h_0 \sim 7500$ m

8500 m



$$\dot{m}_B v_E - m_R g \cos\theta = m_R a$$

Vertical: $$\left(\frac{dm_B}{dt}\right) v_E - m_R g_\circ \cos\theta = m_R \left(\frac{dv_R}{dt}\right)$$

$$\left(\frac{dm_B}{m_R}\right) v_E - g \cos\theta \, dt = dv_R$$

$$\dot{m}_B = -\dot{m}_R$$

$$-\left(\frac{dm_R}{m_R}\right) V_E - g \cos dt = dv_R$$

$$\Rightarrow V_E \int_{m_0}^{m_f} -\left\{\frac{dm_R}{m_R}\right\} - g\cos\theta \int_{t_0}^{t_f} dt = \int_{v_0}^{v_f} dv_R$$

$$V_E \ln\left(\frac{m_0}{m_f}\right) - g_{Avg} \cos\theta_{Avg} \underbrace{(t_f - t_0)}_{\Delta t}$$

$$= v_f - v_0 = \Delta v_R$$

Integrate again

$$\Rightarrow V \ln\left(\frac{m_0}{m_0 - \dot{m} t}\right) - g_{Avg} \cos\theta_{Avg} (t - t_0)$$
$$= v_y - v_0$$

$$h(t) = v_0 t - \tfrac{1}{2} g_{Avg} \cos\theta_{Avg} t^2$$
$$+ v_E t \left[1 - \frac{m}{m_0 - m} \ln\left(m_0/m\right)\right]$$

$$m = m(t) = m_0 - \dot{m}_B t$$

$$\cancel{I_{sp} - \frac{V_E\,dm}{dm} = V_E}$$

$$\Delta V = V_E \ln\left(\frac{m_0}{m}\right)$$

$$I_{sp} = \frac{V_E(dm)}{(g_0\,dm)} = \frac{V_E}{g_0} \quad [s]$$

g_0 ← standard gravity
not local gravity

V_E [Km/s]	I_{sp} [s]	Technology
1.6 - 2.1	170 - 220	solid
1.9 - 3.4	200 - 350	Hydrocarbon (RP-1)
4.4	455	LH_2 + LOx
7.0+	700+	Nuclear + H_2
	3000 → ? 10000+	Electric (Ion)

$$E_{CH} \sim \frac{1}{2} m_e V_E^2 \sim \frac{5}{2} KT$$

$$V_E \sim \sqrt{\frac{2E_{CH}}{m_e}}$$

m_e = 18 for H_2O
= 1 for H
= 44 for CO_2

$$T \sim \frac{m_E V_E^2}{5K}$$

6000°C

$$\Delta V = V_E \ln\left(\frac{m_0}{m}\right) \qquad V_E = I_{sp}\, g_0$$

	$\frac{m_0}{m}$
ΔV	1
$1 V_E$	2.72
$2 V_E$	7.4
$3 V_E$	20.1
$4 V_E$	54.6
$5 V_E$	148.4
$6 V_E$	403.