0) Exam #3

1) OPTIMAL STAGING (SERIES)

2) Rigid Body Dynamics (simple S/C)

$$\Delta V = \sum_{K=1}^{n} -V_{E_K} \ln\left[\varepsilon_K + (1-\varepsilon_K)\pi_K\right]$$

$$\Rightarrow \Delta V + \sum_{K=1}^{n} V_{E_K} \ln\left[\varepsilon_K + (1-\varepsilon_K)\pi_K\right] = 0$$

π_K = Payload ratio $\rightarrow \dfrac{m_{o_{K+1}}}{m_{o_K}}$

$$\varepsilon_K = \frac{m_{s_K}}{(m_{s_K} + m_{p_K})}$$

$$\pi_L = \prod_{K=1}^{n} \pi_K$$

$$\ln \pi_L = \sum_{K=1}^{n} \ln(\pi_K) = \ln(\pi_1) + \ln(\pi_2) + \ln(\pi_3)$$

$$\ln \pi_L = \sum_{K=1}^{n} \ln(\pi_K) + \lambda\left[\Delta V + \sum_{K=1}^{N}\left\{V_{E_K} \ln\left[\varepsilon_K + (1-\varepsilon_K)\pi_K\right]\right\}\right]$$

$$\Rightarrow \frac{\partial \ln(\pi_L)}{\partial \pi_K} = \frac{1}{\pi_K} + \frac{\lambda V_{E_K}(1-\varepsilon_K)}{\varepsilon_K + (1-\varepsilon_K)\pi_K} = 0 \qquad V_{E_K}, \varepsilon_K \text{ const.}$$

$$\pi_K = \frac{-\varepsilon_K}{(1-\varepsilon_K)(1+\lambda V_{E_K})} \qquad \text{where } \lambda \text{ is unknown}$$

$$V_L + \sum_{k=1}^{n} V_{E_k} \ln\left[\varepsilon_k + (1-\varepsilon_k)\,\pi_k\right] = 0$$

$$\Rightarrow V_L + \sum_{k=1}^{n} V_{E_k} \ln\left[\varepsilon_k - \frac{\varepsilon_k}{(1+\lambda V_{E_k})}\right] = 0$$

ONE UNKNOWN $\Rightarrow \lambda$

$\Rightarrow$ Solve for λ

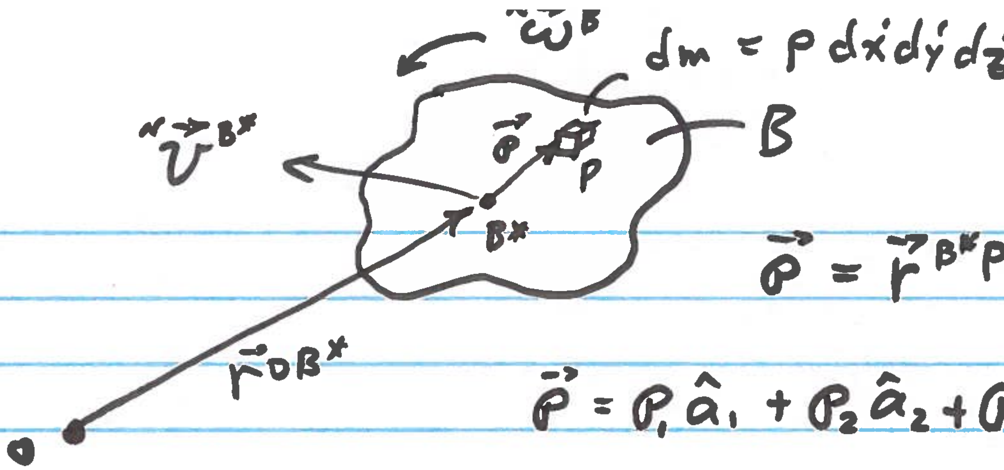
then back to ~~these~~

$$\pi_k = \frac{-\varepsilon_k}{(1-\varepsilon_k)(1+\lambda V_{E_k})}$$

$$\pi_k = \frac{m_{o(k+1)}}{m_{ok}}$$

$$\pi_n = \frac{m_L}{m_L + \underbrace{(m_{s(n-1)} + m_{p(n-1)})}}$$

$$\varepsilon_{n-1} = \frac{m_{s\,n-1}}{m_{s\,n-1} + m_{p\,n-1}}$$



$$\vec{\rho} = \vec{r}^{B^*P}$$

$$\vec{\rho} = \rho_1 \hat{a}_1 + \rho_2 \hat{a}_2 + \rho_3 \hat{a}_3$$

$$ {}^N\vec{\omega}^B = \omega_1 \hat{a}_1 + \omega_2 \hat{a}_2 + \omega_3 \hat{a}_3$$

$$ {}^N\vec{H}^{P/O} \triangleq \vec{r}^{OP} \times m_P\, {}^N\vec{v}^P$$

$$ {}^N\vec{H}^{B/O} = \underbrace{\iiint_B}_{\int_B} \left(\vec{r}^{OP} \times {}^N\vec{v}^P\right) \underbrace{\rho\, \overbrace{dx'dy'dz'}^{dVol}}_{dm}$$

$$ {}^N\vec{H}^{B/O} = \int_B \left\{ \left(\vec{r}^{OB^*} + \vec{\rho}\right) \times \left[{}^N\vec{v}^{B^*} + {}^A\vec{v}^{P/B^*} + {}^N\vec{\omega}^A \times \vec{r}^{B^*P} \right] \right\} \rho\, dVol$$

$$= \int_B \left\{ r^{OB^*} \times {}^N v^{B^*} \right\} \rho\, dVol$$
$$+ \int_B \left\{ \vec{r}^{OB^*} \times \cancelto{0}{{}^A\vec{v}^{P/B^*}} \right\} \rho\, dVol$$
$$+ \int_B \left\{ \vec{r}^{OB^*} \times \left({}^N\vec{\omega}^B \times \vec{\rho} \right) \right\} \rho\, dVol$$
$$+ \int_B \left\{ \vec{\rho} \times {}^N\vec{v}^{B^*} \right\} \rho\, dVol$$
$$+ \int_B \left\{ \vec{\rho} \times \cancelto{0}{{}^A\vec{v}^{P/B^*}} \right\} \rho\, dVol \qquad A \equiv B, \text{ Rigid}$$
$$+ \int_B \left\{ \vec{\rho} \times \left({}^A\vec{\omega}^B \times \vec{\rho} \right) \right\} \rho\, dVol$$

$$ {}^{N}H^{B/O} = (\vec{r}^{\,OB^*} \times {}^{N}\vec{v}^{\,B^*}) \overbrace{\int_B dm}^{m_B} $$

$$ + \, r^{OB^*} \times ({}^{N}\vec{\omega}^{\,B} \times \overbrace{\int_B \vec{P}\, \rho\, dVol}^{0 \text{ BY Definition iff } B^* \text{ is CM}}) $$

$$ + \underbrace{\int_B \hat{P}\, \rho\, dVol}_{0 \text{ iff } B^* \text{ is C.M. of } B} \times {}^{N}\vec{v}^{\,B^*} $$

$$ + \int_B \{ \vec{P} \times (\underset{{}^{N}\vec{H}^{B^*/O}}{{}^{N}\vec{\omega}^{\,B}} \times \vec{P}) \} \rho\, dVol $$

$$ {}^{N}\vec{H}^{B/O} = \overbrace{r^{OB^*} \times m_B \vec{v}^{\,B^*}}^{} $$

$$ + \int_B \{ \vec{P} \times ({}^{N}\vec{\omega}^{\,B} \times \vec{P}) \} \rho\, dVol $$