1) Kepler's Eq-ns

2) Grand Canyon Problem
Hints

3) Classic Orbit Elements
(Parameters)

Elliptic traj.

$$\underbrace{\sqrt{\frac{\mu}{a^3}}\,\underbrace{(t - T_0)}_{t'}}_{m,\,n} = E - e\sin E$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{E}{2}\right)$$

Hyperbolic:

$$\sqrt{\frac{\mu}{a^3}}\,(t - T_0) = e\sinh F - F$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{e+1}{e-1}}\,\tanh\left(\frac{F}{2}\right)$$

ELLIPTIC:

$$\sqrt{\frac{\mu}{a^3}}\,(\overbrace{t - T_0}^{t'}) = E - e \sin E$$

$$\Rightarrow \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{E}{2}\right)$$

HYPERBOLIC:

$$\sqrt{\frac{\mu}{a^3}}\,(t - T_0) = e \sinh F - F$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{e+1}{e-1}}\,\tanh\left(\frac{F}{2}\right)$$

$$\vec{r}_0 = r_1\hat{n}_1 + r_2\hat{n}_2 + r_3\hat{n}_3$$
$$\vec{v}_0 = v_1\hat{n}_1 + v_2\hat{n}_2 + v_3\hat{n}_3$$

$$\vec{h} = \vec{h}_0 = \vec{r}_0 \times \vec{v}_0 \qquad \varepsilon = \frac{v_0^2}{2} - \frac{\mu}{r_0} = \frac{-\mu}{2a}$$

$$\vec{e} = \frac{\vec{v}_0 \times \vec{h}_0}{\mu} - \frac{\vec{r}_0}{r_0} \quad \Rightarrow e$$

$$\hat{p} = \frac{\vec{e}}{e}, \quad \hat{w} = \frac{\vec{h}}{h}, \quad \hat{q} \triangleq \hat{w} \times \hat{p}$$

Θ_0 @ Θ_2

$$\left[{}^{PF}C^{N}\right] = \begin{bmatrix} p_1 & p_2 & p_3 \\ \hat{q}_1 & q_2 & q_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$

$$\begin{Bmatrix} x_0 \\ y_0 \\ 0 \end{Bmatrix}_{PF} = \left[{}^{PF}C^{N}\right] \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix}_{N} \qquad \Theta_0 \overset{?}{=} \mathrm{ATAN2}(y_0, x_0)$$

"t" associated with Θ_2 ? (Elliptic)

$$\tan\left(\frac{\Theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$\sqrt{\frac{\mu}{a^3}}(\overset{t=0}{\cancel{t}} - T_0) = E_0 - e\sin E_0 \iff \Theta_0$$

$$\sqrt{\frac{\mu}{a^3}}(t_2 - T_0) = E_2 - e\sin E_2 \Rightarrow \Theta_2$$

$$E_0 = 2\,\mathrm{ATAN}\left\{\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\Theta_0}{2}\right)\right\}$$

HAVE θ_0

$$\Rightarrow \sqrt{\frac{\mu}{a^3}}(-T_0) = E_0 - e\sin E_0$$

Solve for $-T_0$

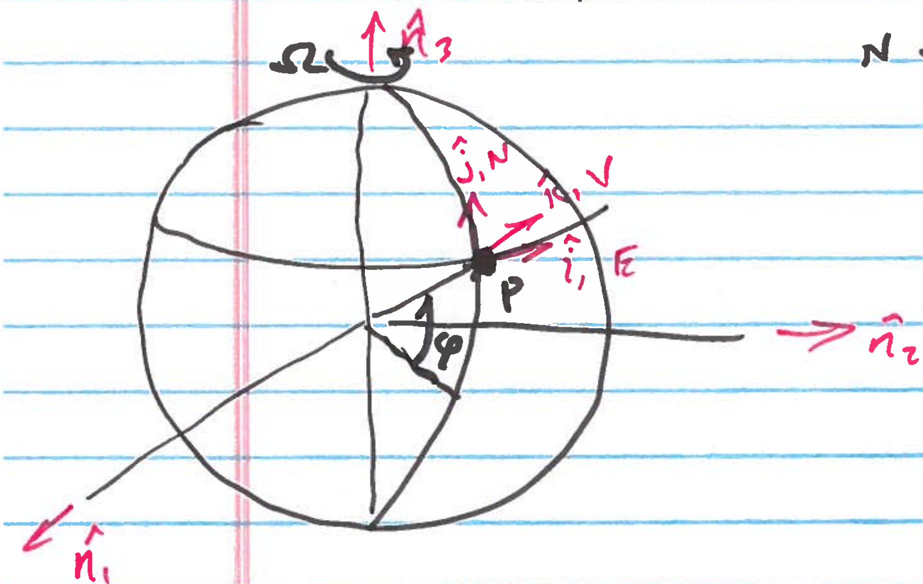
Next θ_2

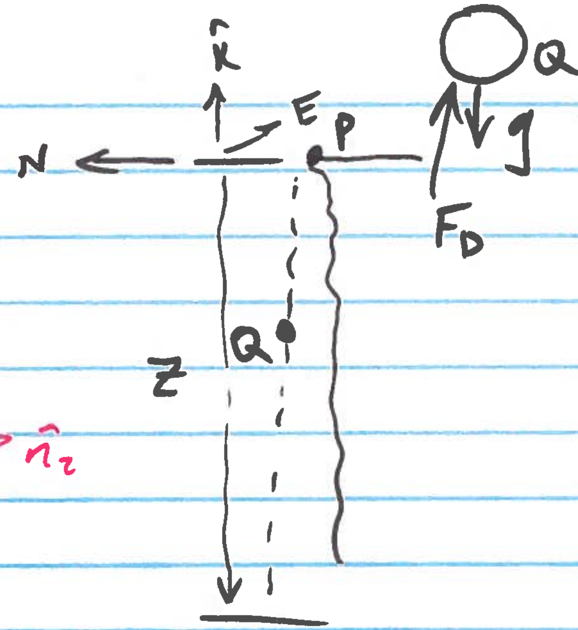
$$E_2 = 2 * \tan^{-1}\left\{\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_2}{2}\right)\right\}$$

$$\Rightarrow \sqrt{\frac{\mu}{a^3}}(t_2 - T_0) = E_2 - e\sin E_2$$

Solve for t_2

Grand Canyon





$$\Sigma \vec{F} = \vec{F}_g + \vec{F}_D$$

$$(\ddot{x}, \ddot{y}, \ddot{z})$$

$$= m_Q \left({}^{N}\vec{a}^{Q} \right)$$

$$= m_Q \left({}^{N}\vec{a}^{P} + {}^{E}\vec{a}^{Q/P} + \overset{0}{{}^{N}\vec{\alpha}^{E}} \times \vec{r}^{PQ} + {}^{N}\vec{\omega}^{E} \times \left({}^{N}\vec{\omega}^{E} \times \vec{r}^{PQ} \right) + 2\, {}^{N}\vec{\omega}^{E} \times {}^{E}\vec{v}^{Q/P} \right)$$

$$\vec{F}_g = \frac{-\mu \vec{r}^{OQ}}{(r^{OQ})^3}$$

$$\vec{r}^{OQ} = \vec{r}^{OP} + \vec{r}^{PQ}$$

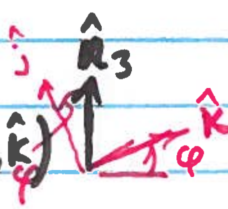
$$\vec{r}^{OP} = (R_E + Alt)\hat{k}$$

$$\vec{r}^{PQ} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F}_D = -\vec{v}\, v \left\{ \tfrac{1}{2} \rho C_D A \right\}$$

$$\vec{v} = {}^{E}\vec{v}^{Q/P} = (\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k})$$

$${}^{N}\vec{\omega}^{E} = \Omega \hat{n}_3 = \Omega (c_\varphi \hat{j} + s_\varphi \hat{k})$$



$$\hat{n}_3 = c_\varphi \hat{j} + s_\varphi \hat{k}$$

$${}^{N}\vec{\omega}^{E} = \Omega(c\varphi\,\hat{j} + s\varphi\,\hat{k})$$

$${}^{N}\vec{a}^{P} = {}^{N}\vec{\omega}^{E}\times\left({}^{N}\vec{\omega}^{E}\times\underbrace{\vec{r}^{OP}}_{(R_E+Alt)\hat{k}}\right)$$

$${}^{E}\vec{a}^{Q/P} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k}$$

$${}^{N}\vec{\omega}^{E}\times\left({}^{N}\vec{\omega}^{E}\times\vec{r}^{PQ}\right) \qquad \vec{r}^{PQ} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

$$2\,{}^{N}\vec{\omega}^{E}\times{}^{E}\vec{v}^{Q/P} \qquad {}^{E}\vec{v}^{Q/P} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$

$$\hat{i}:\ -\frac{1}{2}\rho(\dot{x})\sqrt{(\dot{x})^2+(\dot{y})^2+(\dot{z})^2}\,C_D A = m_Q\left\{-x\Omega^2 + \ddot{x} + 2\Omega(\dot{z}\cos\varphi - \dot{y}\sin\varphi)\right\}$$

$$\hat{j}:\ -\frac{1}{2}\rho(\dot{y})\sqrt{(\dot{x})^2+(\dot{y})^2+(\dot{z})^2}\,C_D A = m_Q\left\{\Omega^2\sin\varphi\left[(R_E+h+z)\cos\varphi - y\sin\varphi\right] + \ddot{y} + 2\Omega\dot{x}\sin\varphi\right\}$$

$$\hat{k}:\ \frac{-\mu m_Q}{(R_E+h+z)^2} - \frac{1}{2}\rho(\dot{z})\sqrt{(\dot{x})^2+(\dot{y})^2+(\dot{z}^2)}\,C_D A = m_Q\left\{-\Omega^2\cos\varphi\left[(R_E+h+z)\cos\varphi - y\sin\varphi\right] + \ddot{z} - 2\Omega\dot{x}\cos\varphi\right\}$$

cross(x, y)

$r^{OP} = [0, 0, 6378{,}000 + h]$

${}^{N}\omega^{E} = \cancel{2}\ \Omega * [0, \cos\varphi, \sin\varphi]$