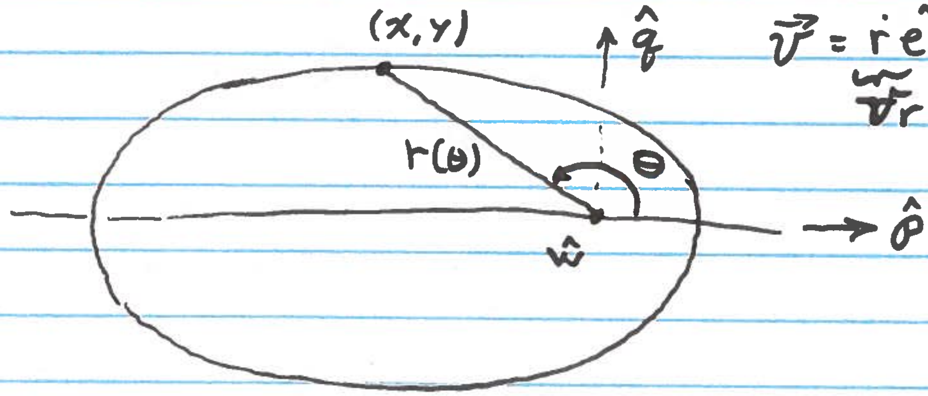
1) Lagrange-Gibbs Coefficients

2) A) Restricted 3-Body Problem

B) "Lagrange Points"



$$\vec{v} = \underbrace{\dot{r}}_{v_r}\hat{e}_r + \underbrace{r\dot{\theta}}_{v_\theta}\hat{e}_\theta$$

$$r(\theta) = \frac{h^2}{\mu(1+e\cos\theta)}$$

$$\vec{r} = x\hat{p} + y\hat{q}$$
$$= r\cos\theta\,\hat{p} + \underbrace{r\sin\theta}_{y}\,\hat{q}$$

$$\vec{v} = \frac{d}{dt}\vec{r} = \dot{x}\hat{p} + x\cancel{\dot{\hat{p}}}^{\,0} + \dot{y}\hat{q} + y\cancel{\dot{\hat{q}}}$$
$$= \dot{x}\hat{p} + \dot{y}\hat{q}$$

$$= \underbrace{(\dot{r}\cos\theta - r\dot{\theta}\sin\theta)}_{\dot{x}}\hat{p} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\hat{q}$$

$$\vec{v} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$$

$$\dot{r} = v_r = \frac{\mu}{h}e\sin\theta, \quad v_\theta = \frac{\mu}{h}(1+e\cos\theta)$$

$$\dot{x} = \underbrace{\dot{r}}_{v_r}\cos\theta - \underbrace{r\dot{\theta}}_{v_\theta}\sin\theta$$

$$\dot{x} = \frac{\mu}{h}e\,\cancel{\sin\theta\cos\theta} - \frac{\mu}{h}(1+e\cos\theta)\sin\theta$$
$$= -\frac{\mu}{h}\sin\theta$$

$$\dot{y} = \underbrace{\dot{r}\sin\theta}_{v_r} + \underbrace{r\dot{\theta}\cos\theta}_{v_\theta} = \frac{\mu}{h}(\cos\theta + e)$$

$$\vec{r}_0 = x_0\hat{p} + y_0\hat{q} \Rightarrow$$

$$\vec{v}_0 = \dot{x}_0\hat{p} + \dot{y}_0\hat{q}$$

$$\vec{h} = \vec{h} \triangleq \vec{r}_0 \times \vec{v}_0 = (x_0\hat{p} + y_0\hat{q}) \times (\dot{x}_0\hat{p} + \dot{y}_0\hat{q})$$

$$= \underbrace{(x_0\dot{y}_0 - y_0\dot{x}_0)}_{h = h_0}\hat{w}$$

solve $\vec{r}_0$ for $\hat{q}$

$$\hat{q} = \frac{\vec{r}_0 - x_0\hat{p}}{y_0}$$

$$\vec{v}_0 = \dot{x}_0\hat{p} + \dot{y}_0\hat{q}$$

$$= \dot{x}_0\hat{p} + \dot{y}_0\left(\frac{r_0 - x_0\hat{p}}{y_0}\right)$$

$$= \frac{(y_0\dot{x}_0 - x_0\dot{y}_0)}{y_0}\hat{p} + \frac{\dot{y}_0}{y_0}\vec{r}_0 = \frac{-h}{y_0}\hat{p} + \frac{\dot{y}_0}{y_0}\vec{r}_0$$

solve this for $\hat{p} = \frac{\dot{y}_0}{h}\vec{r}_0 - \frac{y_0}{h}\vec{v}_0$

$$\vec{r} = x\hat{p} + y\hat{q}$$

$$= \underbrace{\left(\frac{x\dot{y}_0 - y\dot{x}_0}{h}\right)}_{f}\vec{r}_0 + \underbrace{\left(\frac{yx_0 - xy_0}{h}\right)}_{g}\vec{v}_0$$

$$\vec{v} = \dot{x}\,\hat{p} + \dot{y}\,\hat{q}$$

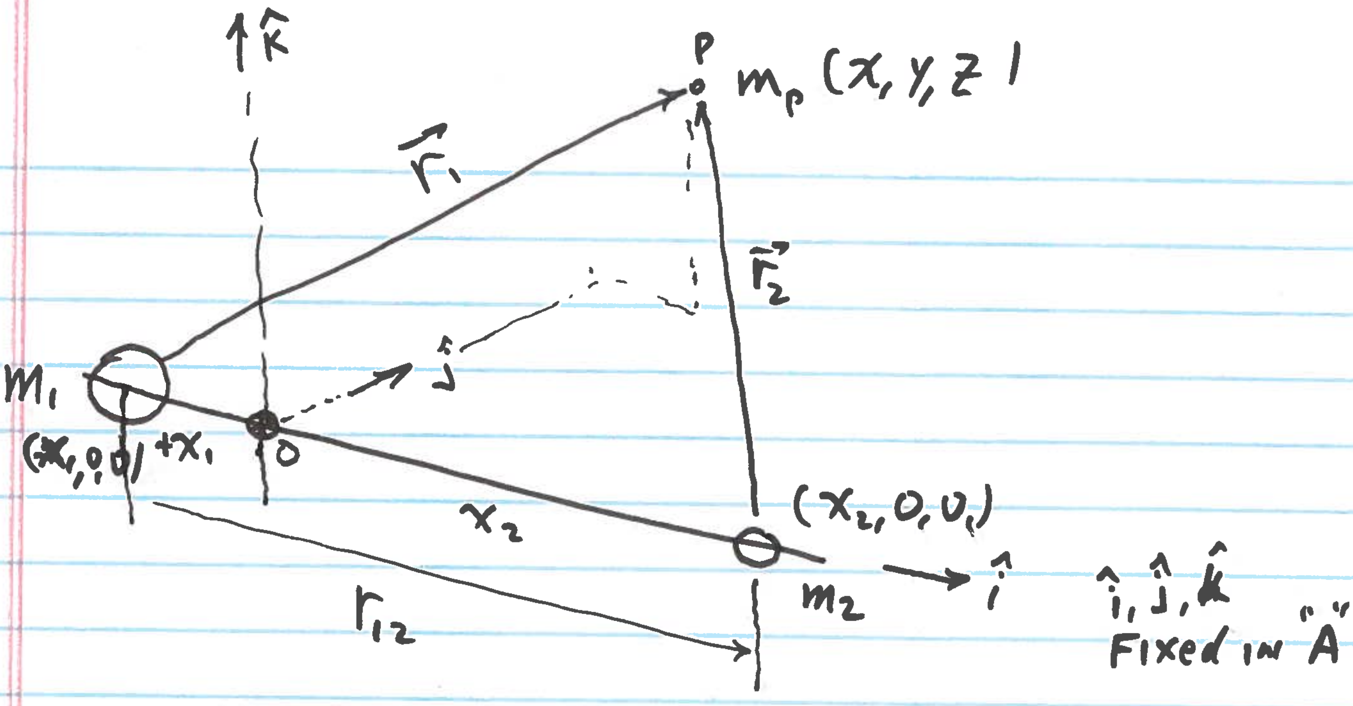
$$= \underbrace{\left(\frac{\dot{x}\dot{y}_0 - \dot{y}\dot{x}_0}{h}\right)}_{\dot{f}}\vec{r}_0 + \underbrace{\left(\frac{\dot{y}x_0 - \dot{x}y_0}{h}\right)}_{\dot{g}}\vec{v}_0$$

$$\Rightarrow \quad \vec{r} = f\,\vec{r}_0 + g\,\vec{v}_0$$

$$\vec{v} = \dot{f}\,\vec{r}_0 + \dot{g}\,\vec{v}_0$$

$f, g \Rightarrow$ Lagrange Coéfficient

$$\frac{\mu}{r^2} = r\dot{\theta}^2 \qquad \underbrace{\dot{\theta}}_{\Omega} = \sqrt{\frac{\mu}{r^3}}$$



$$m_1 x_1 = m x_2 \qquad x_2 - x_1 = r_{12}$$

$$x_2 = r_{12} + x_1$$

$$x_1 = \underbrace{\left(\frac{-m_2}{m_1+m_2}\right)}_{\pi_2} r_{12} \qquad x_2 = \underbrace{\left(\frac{m_1}{m_1+m_2}\right)}_{\pi_1} r_{12}$$

$$\pi_1 + \pi_2 = 1$$

$$\vec{r}_1 = (x - x_1)\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}_2 = (x - x_2)\hat{i} + y\hat{j} + z\hat{k}$$

$${}^N\vec{\omega}^A = \Omega\hat{k}$$

$$\Omega = \sqrt{\frac{\mu}{r_{12}^3}}$$

$$\Sigma F_g^P = m_p\, {}^N\vec{a}^P$$

$$m_p\Big[\cancel{{}^N\vec{a}^O} + \underbrace{{}^A a^{P/O}}_{\ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}} + \cancel{{}^N\vec{\alpha}^A} \times \vec{r}^{\,OP} + {}^N\vec{\omega}^A \times ({}^N\vec{\omega}^A \times \vec{r}^{\,OP}) + 2\,{}^N\vec{\omega}^A \times \underbrace{{}^A\vec{v}^{P/O}}_{\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}}\Big]$$

$$\mu = G(M_1 + m_2) \qquad \mu_1 = \pi_1 \mu$$

$$\mu_2 = \pi_2 \mu$$

$$\sum F_g^P = \frac{-\mu_1 \vec{r}_1}{r_1^3} - \frac{\mu_2 \vec{r}_2}{r_2^3}$$

specific gravity

$$= \frac{-\mu_1 [(x+x_1)\hat{i} + y\hat{j} + z\hat{k}}{[(x-x_1)^2 + (y^2) + (z^2)]^{3/2}}$$

$$- \frac{\mu_2 [(x-x_2)\hat{i} + y\hat{j} + z\hat{k}]}{[(x-x_2)^2 + y^2 + z^2]^{3/2}}$$

$$\hat{i} : \ddot{x} - \Omega^2 x - 2\Omega\dot{y} = \frac{-\mu_1 (x + \underbrace{\pi_2 r_{12}}_{x_1})}{r_1^3} - \frac{\mu_2 (x - \pi_1 r_{12})}{r_2^3}$$

$$\hat{j} : \ddot{y} + \Omega^2 y + 2\Omega\dot{x} = -\left\{ \frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3} \right\} y$$

$$\hat{k} : \ddot{z} = -\left\{ \frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3} \right\} z$$