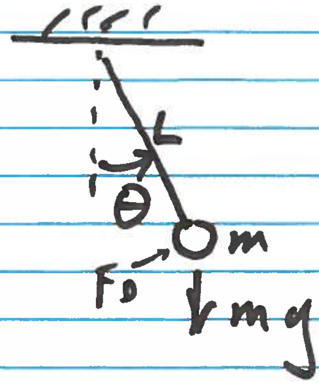
Spaceflight Mechanics:

$= \underbrace{\text{Orbital Mech.}} +$ Spacecraft Attitude Dyn

Orbital Mech:
get from A to B in context of space travel
position vs. time
(velocity, accel.)
Particle model works
$$\Sigma \vec{F}^P = m_p \, {}^N\vec{a}^P \qquad > 90\%$$

Equally important
Spacecraft (S/c) Attitude Dyn. $< 10\%$
(orientation, stability, control)

$\Rightarrow$ Analytic Solution vs. Numeric Solutions



$$\underbrace{mL\ddot{\theta}}_{a} - F_D + mg\sin\theta$$

$$F_D = -\tfrac{1}{2}\rho C_D A (L\dot{\theta})^2 \operatorname{sgn}(-\dot{\theta})$$

$$\ddot{\theta} - \frac{1}{2m}\rho C_D L \dot{\theta}|\dot{\theta}| A + \frac{g}{L}\sin\theta = 0$$

$C_D \sim 0$

$\sin\theta \sim \theta$ $\qquad$ $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5} - \dots$

$$\rightarrow \quad \ddot{\theta} + \frac{g}{l}\theta = 0$$

↳ Analytic Solution

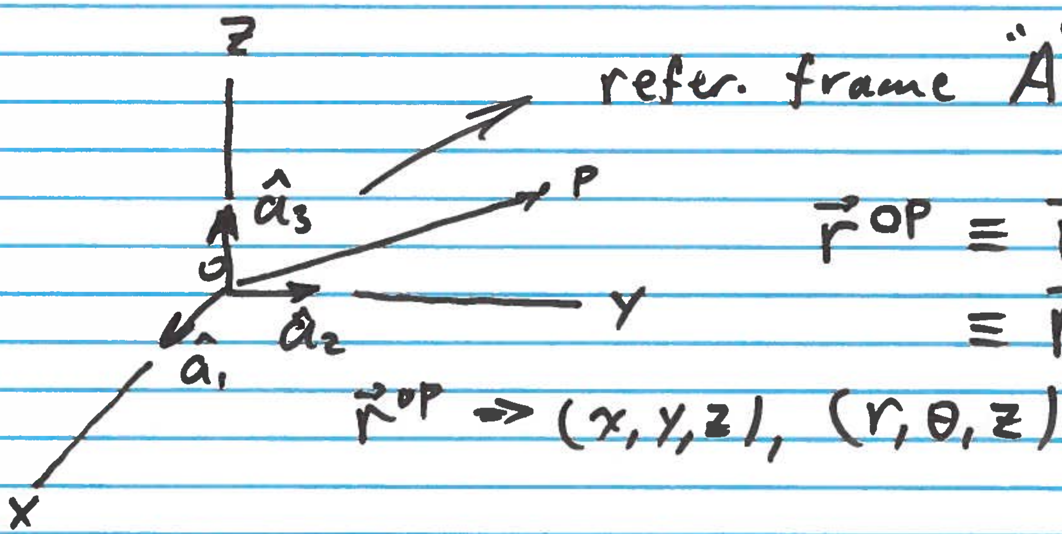
$$\theta(t) = A\sin\omega t + B\cos\omega t$$

$$\omega = \sqrt{g/l}$$

Numeric Solution : Accurate, slow, expensive, little or no insight

$$\vec{F}^P = m_P \, {}^{N}\vec{a}^{P}$$

coordinates & reference frames



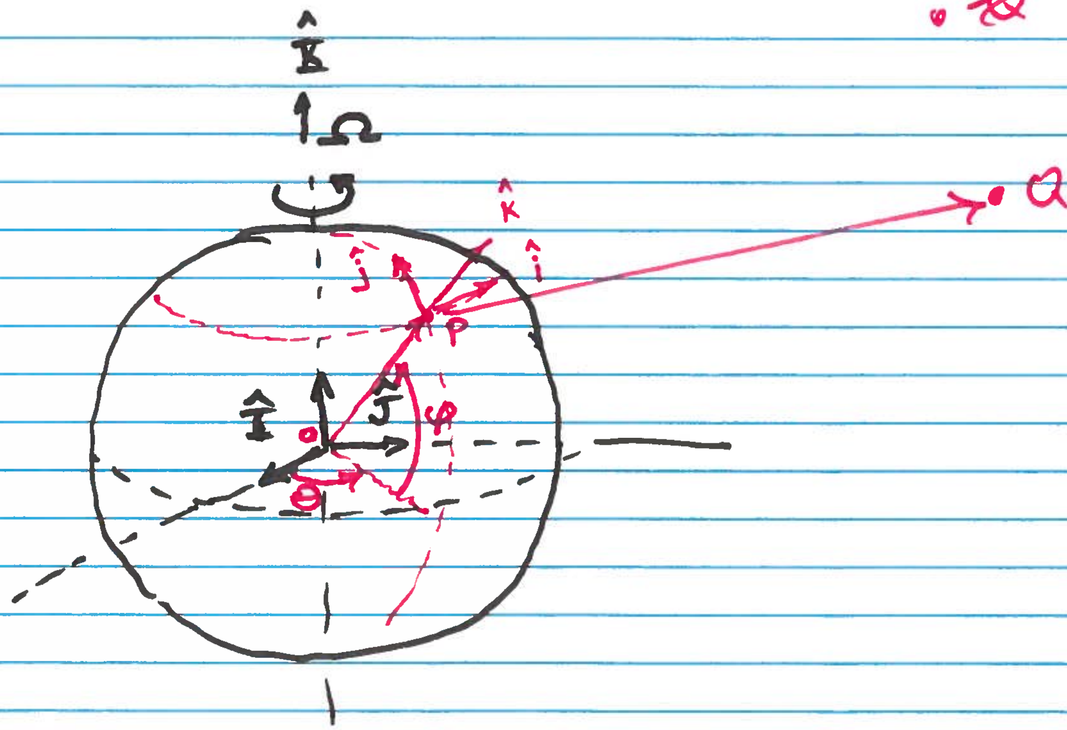
$$\vec{r}^{OP} \equiv \vec{r}^{P} \equiv \vec{r}^{P/O}$$

$$\vec{r}^{OP} \Rightarrow (x, y, z), \; (r, \theta, z), \; (r, \varphi, \theta)$$

$$\vec{r}^{Q} = \vec{r}^{OQ} = \vec{r}^{P} + \underbrace{\vec{r}^{PQ}}_{r^{Q/P}}$$

$$ {}^{N}\vec{v}^{Q/O} = {}^{N}\vec{v}^{Q} = \frac{{}^{N}d\,\vec{r}^{Q/O}}{d} = \overline{{}^{N}\vec{v}^{P/O} + {}^{A}\vec{v}^{Q/P} + {}^{N}\vec{\omega}^{A} \times \vec{r}^{PQ}}$$

$$ {}^{N}\vec{a}^{Q} = {}^{N}\vec{a}^{P} + {}^{A}\vec{a}^{Q/P} + {}^{N}\vec{\alpha}^{A} \times \vec{r}^{PQ} + {}^{N}\vec{\omega}^{A} \times \left({}^{N}\vec{\omega}^{A} \times \vec{r}^{PQ}\right) + 2\, {}^{N}\vec{\omega}^{A} \times {}^{A}\vec{v}^{Q/P}$$



$\hat{I},\hat{J},\hat{K}$ non-rotating geocentric frame

P = groundstation,
Q = S/C or object of interest

have r^{PQ}, ${}^{E}\vec{v}^{Q/P}$.

Need $\vec{r}^{OQ}$, ${}^{N}\vec{v}^{Q/O}$, ${}^{N}\vec{a}^{Q}$

$$\vec{r}^{OQ} = \underbrace{\vec{r}^{OP}}_{R_E\hat{k}} + \underbrace{\vec{r}^{PQ}}_{x\hat{i}+y\hat{j}+z\hat{k}}$$

measured!

$${}^{E}\vec{v}^{Q/P} = \dot{x}\hat{i}+\dot{y}\hat{j}+\dot{z}\hat{K}$$

$${}^{N}\vec{\omega}^{E} = \Omega\hat{K}$$

$$\underbrace{\vec{F}^{Q}}_{\text{gravity}} = m_Q\,{}^{N}\vec{a}^{Q} = m\Big[\,{}^{N}\vec{a}^{P} + {}^{E}\vec{a}^{Q/P} + {}^{N}\vec{\alpha}^{E} \times \vec{r}^{PQ} + {}^{N}\vec{\omega}^{E} \times \left({}^{N}\vec{\omega}^{E} \times \vec{r}^{PQ}\right) + 2\,{}^{N}\vec{\omega}^{E} \times {}^{E}\vec{v}^{Q/P}$$