A) $\sum \vec{F}^{P} = m_p \, {}^{N}\vec{a}^{\,a}$ Ex 1

B) $\vec{r}_0, \vec{v}_0 \rightleftarrows a, \vec{e}, \vec{h}, \theta_0, \Omega, i, \omega,$ Ex 2

C) Basis/coordinate transformation DCM

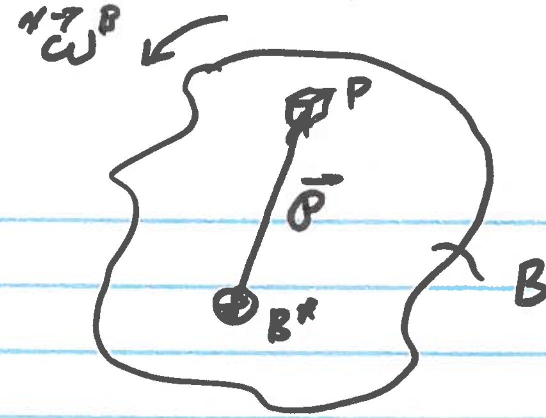
D) Keplers Eqn.

E) Transfers [Hohmann, Bi-Elliptic, inclin. changes phasing. etc.]

F) Basic Rocket Problem

G) simple s/c attitude Dynamics

$$\vec{\rho} = \rho_1 \hat{a}_1 + \rho_2 \hat{a}_2 + \rho_3 \hat{a}_3$$



$$\vec{\vec{I}}^{B/B^*} = I_{11}\,\hat{a}_1\hat{a}_1 + I_{12}\,\hat{a}_1\hat{a}_2 + I_{13}\,\hat{a}_1\hat{a}_3 + \cdots + I_{32}\,\hat{a}_3\hat{a}_2 + I_{33}\,\hat{a}_3\hat{a}_3$$

$$\hat{a}_i\hat{a}_j \neq \hat{a}_j a_i$$

$$\Rightarrow \left[I^{B/B^*}\right] = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}_{aa}$$

$$I_{11} \triangleq \iiint_B (\rho_2^2 + \rho_3^2)\,\rho\, dVol$$

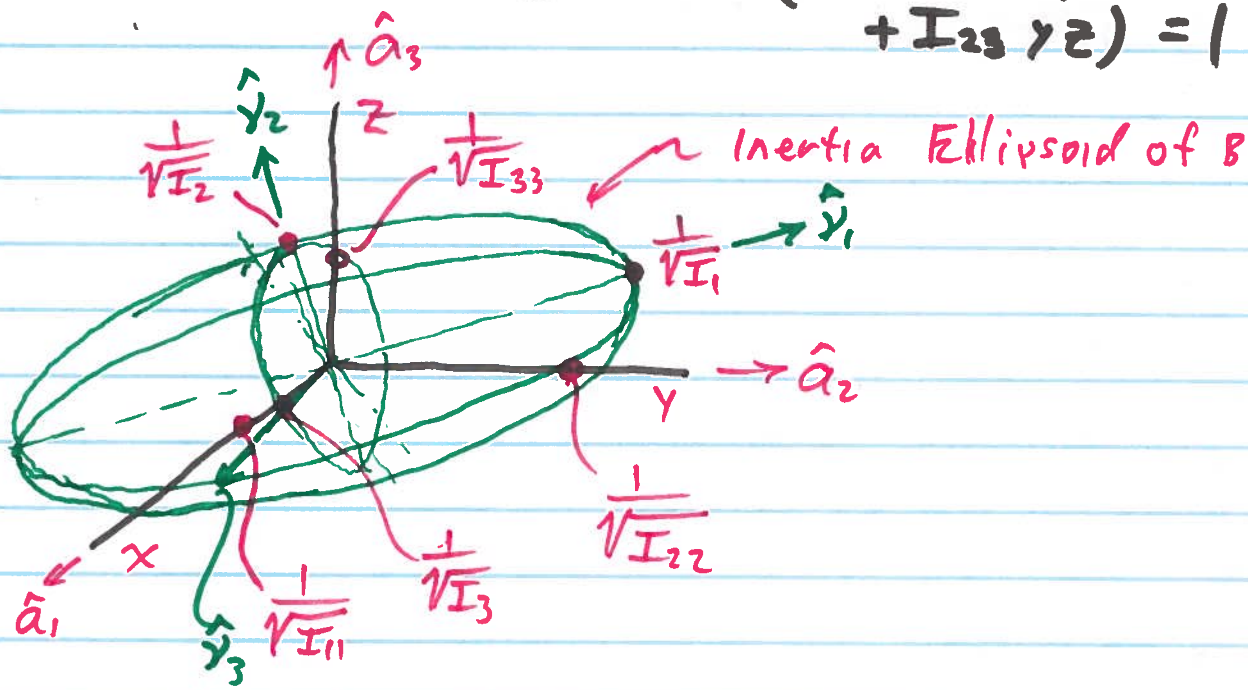
$$I_{22} \triangleq \iiint_B (\rho_3^2 + \rho_1^2)\,\rho\, dVol$$

$$I_{33} \triangleq \iiint_B (\rho_1^2 + \rho_2^2)\,\rho\, dVol$$

$$\underset{i\neq j}{I_{ij}} \triangleq -\iiint_B \rho_i\rho_j\,\rho\, dV$$

$$[x, y, z]_a \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ & I_{22} & I_{23} \\ \text{sym} & & I_{33} \end{bmatrix}_{aa} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_a = 1$$

$$I_{11} x^2 + I_{22} y^2 + I_{33} z^2 + 2(I_{12} xy + I_{13} xz + I_{23} yz) = 1$$



$$[I]_{aa} \Longleftrightarrow \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}_{yy}$$

$$[U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\left[[I^{B/B^*}] - [U]\lambda_i \right]}\{\hat{\nu}_i\} = 0$$

$$\{\nu_i\} = \{0\}$$

$$\text{DET}\left[[I^{B/B^*}] - [\lambda] \right] = 0$$

$$\mathrm{DET}\begin{bmatrix} (I_{11}-\lambda_i) & I_{12} & I_{13} \\ I_{12} & (I_{22}-\lambda_i) & I_{23} \\ I_{13} & I_{23} & (I_{33}-\lambda_i) \end{bmatrix} = 0$$

$$A\lambda_i^3 + B\lambda_i^2 + C\lambda_i + D = 0$$

Solve for $\lambda_i \Rightarrow \lambda_1 = I_1$

$\lambda_2 = I_2$

$\lambda_3 = I_3$

$I_1 < I_2 < I_3$

or $I_1 > I_2 > I_3$

$${}^{N}\vec{\omega}^{B} = \omega_1 \hat{\nu}_1 + \omega_2 \hat{\nu}_2 + \omega_3 \nu_3$$

$$\left[I^{B/B^*} \right]_{\nu\nu} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}_{\nu\nu}$$

$$\left\{ \Sigma M^{B} \right\}_{\nu} = \left[I^{B/B} \right]_{\nu\nu} \left\{ \dot{\omega} \right\}_{\nu} + \left[{}^{N}\omega^{B}\times \right]_{\nu} \left[I \right]_{\nu\nu} \left\{ {}^{N}\omega^{B} \right\}_{\nu}$$

$$\boxed{\begin{aligned} \Rightarrow M_1 &= I_1 \dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 \\ M_2 &= I_2 \dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 \\ M_3 &= I_3 \dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 \end{aligned}}$$

Stability Analysis $\Rightarrow \sum \vec{M} = \vec{0}$

$$\omega_1 = \Omega + \delta\omega_1 \qquad \Omega = \text{const.} \quad \delta\omega_1 \ll 1$$

$$\dot{\omega}_1 = \delta\dot{\omega}_1$$

$$\omega_2 = \delta\omega_2 \qquad \delta\omega_2 \ll 1$$

$$\omega_3 = \delta\omega_3 \ll 1$$

$$\Rightarrow \delta\dot{\omega}_1 = \left(\frac{I_2 - I_3}{I_1}\right)\cancel{\delta\omega_2\delta\omega_3}^{\sim 0} = 0$$

$$\Rightarrow \delta\dot{\omega}_2 = \left(\frac{I_3 - I_1}{I_2}\right)\delta\omega_3(\Omega + \delta\omega_1)$$

$$\cong \left(\frac{I_3 - I_1}{I_2}\right)\Omega\,\delta\omega_3$$

$$\Rightarrow \delta\dot{\omega}_3 = \left(\frac{I_1 - I_2}{I_3}\right)(\Omega + \delta\omega_1)\delta\omega_2$$

$$\cong \left(\frac{I_1 - I_2}{I_3}\right)\Omega\,\delta\omega_2$$

$$\delta\ddot{\omega}_3 = \left(\frac{I_1 - I_2}{I_3}\right)\Omega\,\delta\dot{\omega}_2$$

$$\delta\ddot{\omega}_3 = \left(\frac{I_1 - I_2}{I_3}\right)\Omega\left(\frac{I_3 - I_1}{I_2}\right)\Omega\,\delta\omega_3$$

$$\delta\ddot{\omega}_3 + \left(\frac{(I_1 - I_2)(I_1 - I_3)}{I_2 I_3}\right)\Omega^2\,\delta\omega_3 = 0$$

A) $I_1 < I_2 , I_3$ $\Rightarrow \delta\omega_3(t) = A\sin pt + B\cos pt$
"stable"

B) $I_1 > I_2 , I_3$ $\Rightarrow$ stable

c) $I_2 < I_1 < I_3$ or $I_2 > I_1 > I_3$
$\delta\omega_3(t) = Ae^{\lambda t} + Be^{-\lambda t}$
"unstable"