- Basic Rocketry

A) Some history

B) Rocket Thrust

C) Basic Rocket Equation

When did it all begin

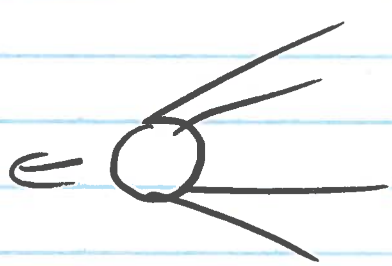
China against monguls ~1232

"Fire Arrow" 

Konstantin Tsiolkovsky 1857

Robert Goddard - 1882

1957



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Appollo Saturn-V

$$\Sigma \vec{F} = m\vec{a} \quad \Rightarrow \quad \Sigma \vec{F} = \frac{d}{dt}\left(m\,{}^{N}\vec{v}^{R}\right)$$

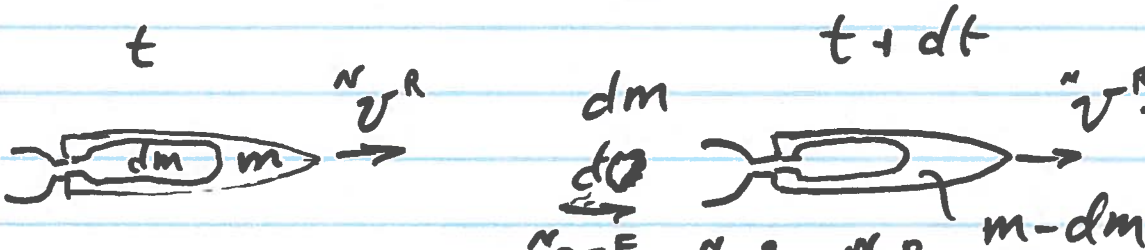
$$\Sigma F = F_g + F_D + \vec{F}_{Thrust} = m(t)\,{}^{N}\vec{a}^{R}$$

gravity

Drag

$$\Sigma \vec{F} = \frac{d}{dt}(mv) = {}^{N}\vec{v}^{R}\underbrace{\left[\frac{dm}{dt}\right]}_{\dot{m}} + m\underbrace{\frac{d\,{}^{N}\vec{v}^{R}}{dt}}_{{}^{N}a^{R}}$$

$$= \underbrace{{}^{N}v^{R}\dot{m}}_{\text{Thrust}} + m_R\,{}^{N}\vec{a}^{R}$$



$${}^{N}v^{R} \qquad {}^{N}v^{R} + d\,{}^{N}v^{R}$$

$${}^{N}v^{E} = {}^{N}v^{R} + d\,{}^{N}v^{R} - {}^{R}v^{E}$$

$$dm\,v =$$

$$\overset{(t+\Delta t)}{\left[(m-dm)({}^{N}v^{R} + d\,{}^{N}v^{R}) + dm({}^{N}v^{R} + d\,{}^{N}v^{R} - {}^{R}v^{E})\right]}$$

$$\overset{t}{-\left[m\,{}^{N}v^{R}\right]}$$

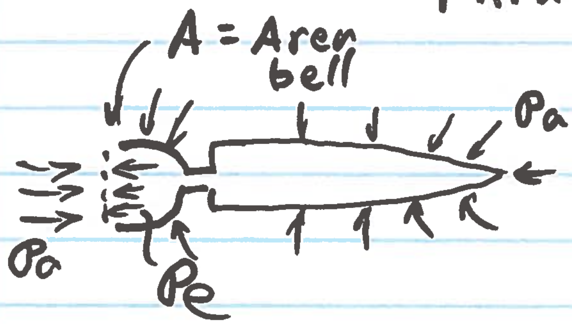
$$= \left[\cancel{m\,{}^{N}v^{R}} + m\,d\,{}^{N}v^{R} - \cancel{dm\,{}^{N}v^{R}} - \cancel{dm\,d\,{}^{N}v^{R}} + \cancel{dm\,{}^{N}v^{R}} + \cancel{dm}\,d({}^{N}v^{R} - dm\,{}^{R}v^{E}\right] - \cancel{m\,{}^{N}v^{R}}$$

$$= m\,d\,{}^{N}v^{R} - (dm)\,{}^{R}v^{E}$$

$$\Sigma F_x\,dt = dP = m\,d\,{}^{N}v^{R} - dm\,{}^{R}v^{E}$$

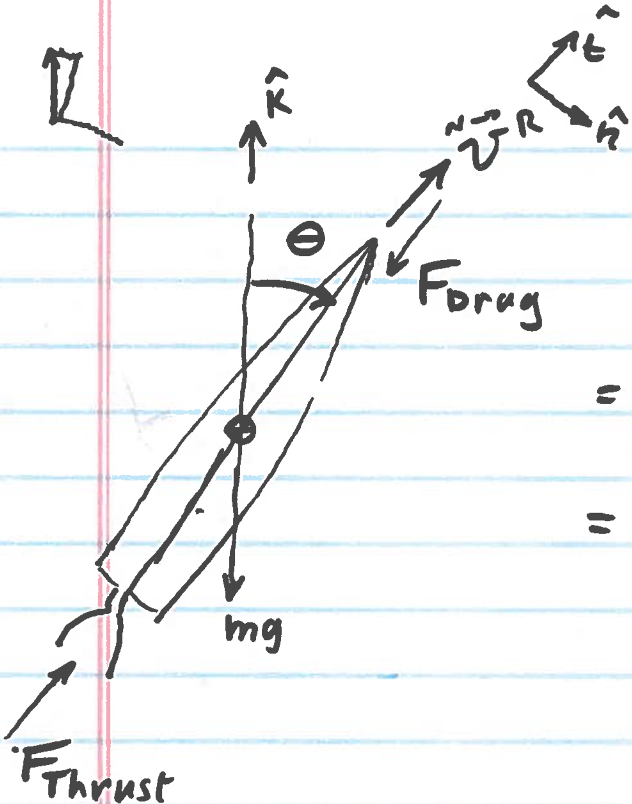
$$\sum F_{EXT} = m\left(\frac{d\,{}^{N}v^{R}}{dt}\right) - \left(\frac{dm}{dt}\right){}^{R}\vec{v}^{E}$$

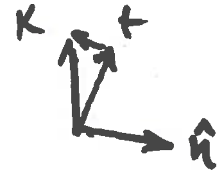
$$\Rightarrow \sum F_{EX} + \underbrace{\underbrace{\left(\frac{dm}{dt}\right)}_{\dot{m}_b}{}^{R}v^{E}}_{\text{Thrust}} = m\underbrace{\left(\frac{d\,{}^{N}v^{R}}{dt}\right)}_{{}^{N}a^{R}}$$



$$\Rightarrow (P_e - P_a)A$$

$$\Rightarrow \sum F_{EXT} = F_g + F_D + \left[\underbrace{(P_e - P_a)A + \dot{m}\,{}^{R}v^{E}}_{\text{static Thrust}}\right] = m\,{}^{N}a^{R}$$





$\hat{K} = C_\theta \hat{t} - S_\theta \hat{n}$

$$\Sigma \vec{F}_{ext} = m(t)\, {}^N\vec{a}^R$$

$$= F_g \hat{K} - \tfrac{1}{2}\rho C_D v^2 A_s \hat{t} + F_T \hat{t}$$

$$= \frac{-m\mu}{(R_E+h)^2}\left[C_\theta \hat{t} - S_\theta \hat{n}\right]$$

$$- \tfrac{1}{2}\rho C_D |v| v A_s \hat{t}$$

$$+ (\dot{m}\, {}^R v^E)\hat{t} = m\, {}^N\vec{a}^R$$

$$= m\left(\dot{v}\hat{t} + \hat{n}\left\{\begin{matrix} \frac{v^2}{\rho} \\ v\dot{\theta} \\ \rho\dot{\theta}^2 \end{matrix}\right.\right.$$

$$\Rightarrow$$

$$\rho\dot{\theta} = v$$

$$\hat{n}: \frac{\mu m S_\theta}{(R_E+h)^2} = \cancel{m} v \dot{\theta}$$

$$\hat{t}: \frac{-m\mu C_\theta}{(R_E+h)^2} - \tfrac{1}{2}\rho C_D |v| v A_s + \underbrace{(\dot{m}\, {}^R v^E)}_{\text{Thrust}}$$

$$= m\dot{v}$$

$$\dot{\theta} = \frac{\mu S_\theta}{v(R_E+h)^2}$$

$$\dot{v} = \frac{1}{m}\left\{\frac{-m\mu C_\theta}{(R_E+h)^2} - \tfrac{1}{2}\rho C_D |v| v A_s + (\dot{m}\, {}^R v^E)\right\}$$

$$\dot{h} = v\cos\theta$$

$$\dot{x} = v\sin\theta$$