M.Sc. Simulation Sciences, Summer Semester 2023

Fast Iterative Solvers

Work Package for Multigrid Assignment – Part 2

Implement restriction and prolongation operations, as discussed in class (Tutorial 11). You will re-use these function when writing a complete multigrid solver!

After you have convinced yourself that restriction and prolongation work, you have all the ingredients. You can now start putting together the multigrid solver as discussed in class.

Restriction Operator

For validation we approximate a square domain $\Omega = [0,1]^2$ using a grid as defined in the first work package. We define a "fine mesh" with $N = 2^n$; for some integer $n \geq 1$, and a coarse mesh using $N^c = 2^{n-1}$. This means that points in the coarse mesh will also be points in the fine mesh, while every other point in the fine mesh is deleted.

A function $\mathbf{u}_{2h} = RESTR(\mathbf{u}_h; N^c)$ should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes. Please see the class notes, where we discussed a possible loop layout.

- Test your implementation for two grids, using $n = 4$ and $n = 7$ for the fine grid, respectively.
- initialize $u_{i,j}$ on the *fine* grid such that $u_h[i,j] = u(x_i, y_j)$ for the function $u(x,y) = \sin(2\pi x)\sin(2\pi y)$.
- Then transfer to the coarse grid, and measure $||\mathbf{e}_{2h}||_\infty = \max_{i,j} |u_{2h}[i,j] - u(x_i, y_j)|$. (Note that here (i,j) are *coarse* grid indices and x_i, y_j are the corresponding coarse grid nodes!) You should observe roughly an order of magnitude error reduction.

Prolongation Operator

Define fine and coarse grids in the same way as for the restriction operator.

A Function $\mathbf{u}_h = PROLONG(\mathbf{u}_{2h}; N^c)$ should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes. Again, refer to the class notes.

- Test your implementation for two grids, using $n = 4$ and $n = 7$ for the fine grid, respectively.
- initialize $u_{i,j}$ on the *coarse* grid such that $u_{2h}[i,j] = u(x_i, y_j)$ for the function $u(x,y) = \sin(2\pi x)\sin(2\pi y)$.
- Then transfer to the fine grid, and measure $||\mathbf{e}_h||_\infty = \max_{i,j} |u_h[i,j] - u(x_i, y_j)|$. (Note that here (i,j) are *fine* grid indices and x_i, y_j are the corresponding fine grid nodes!) You should observe more than one order of error reduction.