M.Sc. Simulation Sciences, Summer Semester 2022

Fast Iterative Solvers

Suggested Work for Multigrid Assignment – Part 2

Restriction Operator

Implement a restriction operation, as discussed in class (Tutorial 11). You will re-use this function when writing a complete multigrid solver!

For validation we approximate a square domain $\Omega = [0, 1]^2$ using a grid as defined in the first work package. We define a "fine mesh" with $N = 2^n$; for some integer $n \geq 1$, and a coarse mesh using $N^c = 2^{n-1}$. This means that points in the coarse mesh will also be points in the fine mesh, while every other point in the fine mesh is deleted.

A function $\mathbf{u}_{2h} = RESTR(\mathbf{u}_h; N^c)$ should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes. Please see the class notes, where we discussed a possible loop layout.

- Test your implementation for two grids, using $n = 4$ and $n = 7$ for the fine grid, respectively.
- initialize $u_{i,j}$ on the *fine* grid such that $u_h[i, j] = u(x_i, y_j)$ for the function $u(x, y) = \sin(2\pi x)\sin(2\pi y)$.
- Then transfer to the coarse grid, and measure $||\mathbf{e}_{2h}||_\infty = \max_{i,j} |u_{2h}[i, j] - u(x_i, y_j)|$. (Note that here (i, j) are *coarse* grid indices and x_i, y_j are the corresponding coarse grid nodes!)

Prolongation Operator

Implement the bilinear prolongation operator, as discussed in class (Tutorial 11). You will re-use this function when writing a complete multigrid solver.

Define fine and coarse grids in the same way as for the restriction operator.

A Function $\mathbf{u}_h = PROLONG(\mathbf{u}_{2h}; N^c)$ should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes. Again, refer to the class notes.

- Test your implementation for two grids, using $n = 4$ and $n = 7$ for the fine grid, respectively.
- initialize $u_{i,j}$ on the *coarse* grid such that $u_{2h}[i, j] = u(x_i, y_j)$ for the function $u(x, y) = \sin(2\pi x)\sin(2\pi y)$.
- Then transfer to the fine grid, and measure $||\mathbf{e}_h||_\infty = \max_{i,j} |u_h[i, j] - u(x_i, y_j)|$. (Note that here (i, j) are *fine* grid indices and x_i, y_j are the corresponding fine grid nodes!)

After you have convinced yourself that restriction and prolongation work, you have all the ingredients. You can now start putting together the multigrid solver as discussed in class.