Multigrid Method

Fast Iterative Solvers, Project 2

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1 Remarks on used architecture and compiler options

The code was written in C++, compiled using the Clang compiler and executed on an Apple Silicon M1 Pro Chip featuring the ARM64 architecture. To measure timings, the high resolution clock of the std chrono library was employed. The following flags where passed to the compiler to enhance code performance of the aforementioned architecture: -march=native, -O2.

2 Multigrid Method - MG

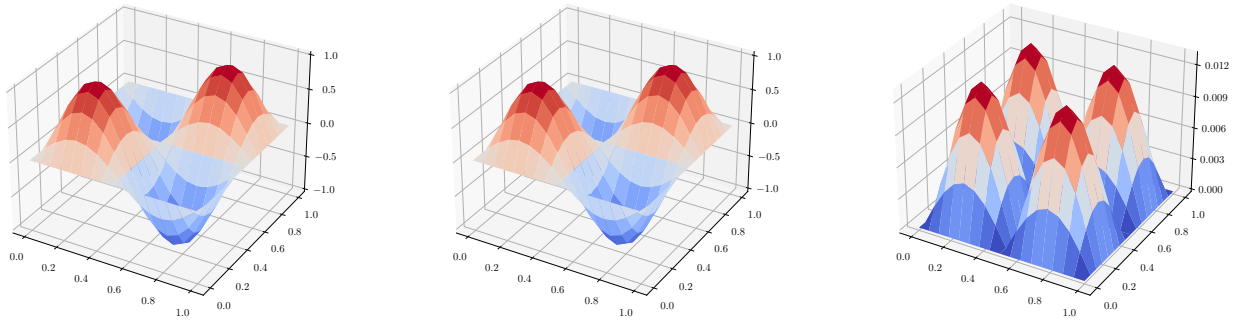
For the Multigrid iterations, W-cycles ($\gamma = 2$) were used. An iteration was considered as converged with the criterion

$$\frac{||r^{(m)}||_\infty}{||r^{(0)}||_\infty} < 10^{-10}, \tag{1}$$

were $r^{(m)}$ denotes the residual of the m-th iteration and $r^{(0)}$ the initial residual using $\mathbf{u} = \mathbf{0}$ as an initial guess for the solution vector.

2.1 Validation of MG Method

The solution obtained by the multigrid method is compared to the exact solution $\mathbf{U}_{ex}(x, y) = sin(2\pi x)sin(2\pi y)$ in Fig. 1 for $n = 4$ and in Fig. 2 for $n = 7$. The estimated solution is in good agreement with the exact solution, the max error for $n = 4$ is $\max_{i,j} e_{i,j} = 1.295 \cdot 10^{-2}$ and for $n = 7$ the max error measures $\max_{i,j} e_{i,j} = 2.01 \cdot 10^{-4}$.



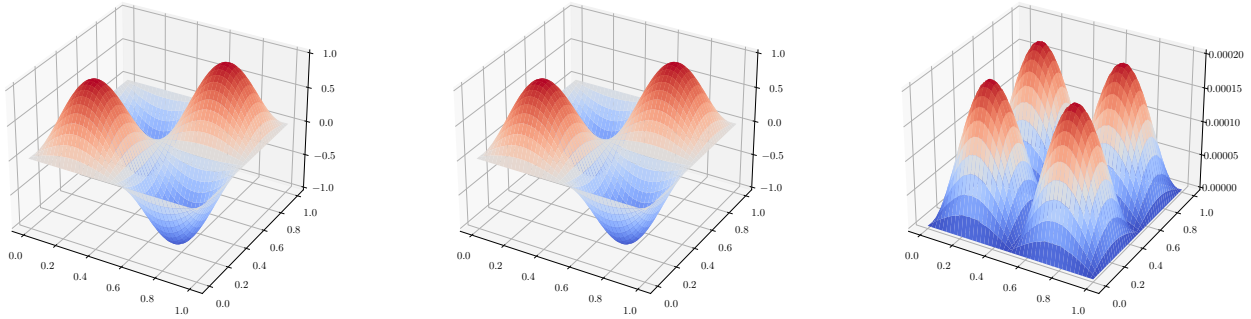
(a) $\mathbf{U}_{ex}$ (b) $\mathbf{U}_{est}$ (c) Error $\mathbf{e} = |\mathbf{U}_{ex} - \mathbf{U}_{est}|$

Figure 1: Plots of exact solution, approximated solution and corresponding error for $n = 4$

2.2 Convergence Plots

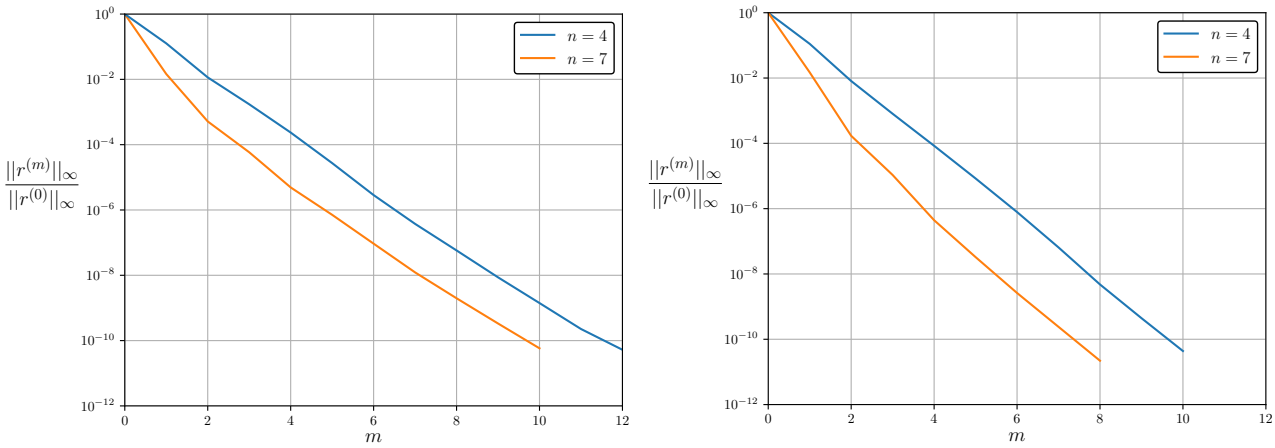
2.3 Convergence for different ν_1

2.4 Convergence for different γ



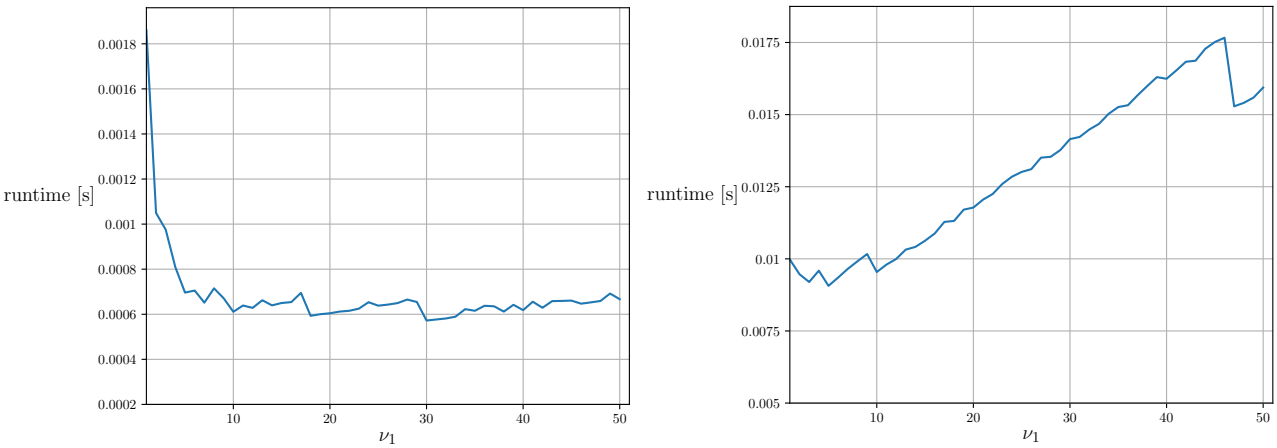
(a) $\mathbf{U}_{ex}$ (b) $\mathbf{U}_{est}$ (c) Error $\mathbf{e} = |\mathbf{U}_{ex} - \mathbf{U}_{est}|$

Figure 2: Plots of exact solution, approximated solution and corresponding error for $n = 7$



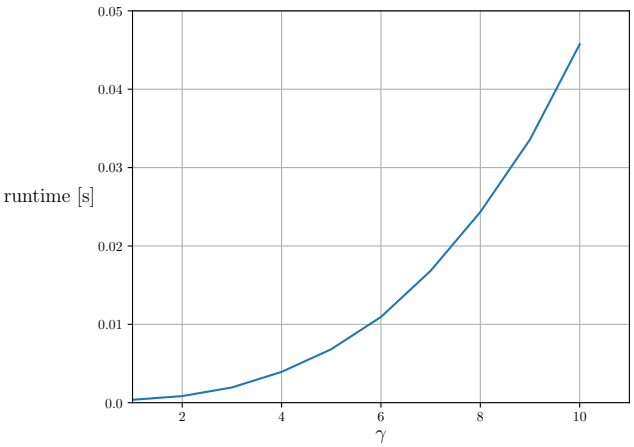
(a) $\nu_1 = \nu_2 = 1$ (b) $\nu_1 = 2, \nu_2 = 1$

Figure 3: Convergence against multigrid iterations m for meshes $n = 4$ and $n = 7$

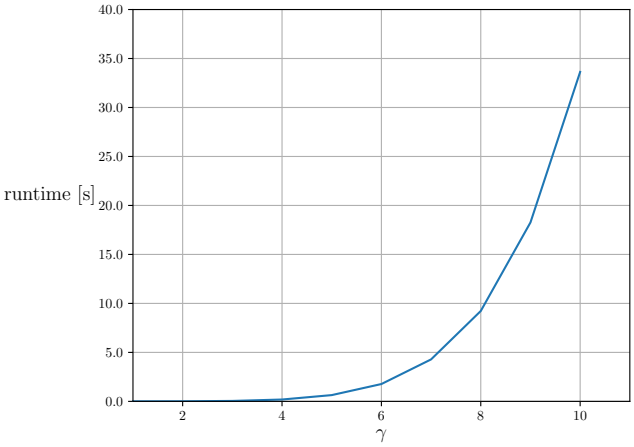


(a) $n = 4$ (b) $n = 7$

Figure 4: Runtime against number of smoothing iterations $\nu_1 = 1...50$ for meshes $n = 4$ and $n = 7$



(a) $n = 4$



(b) $n = 7$

Figure 5: Runtime against number of γ-cycles for $\gamma = 1...10$ for meshes $n = 4$ and $n = 7$