Wall- and Free-Stream Pressure Processing

Stanford Team

19th August 2025

Abstract

This document formalises, in pseudocode, the processing pipeline. The pipeline estimates a frequency response transfer function (H) between a reference and a treated microphone, then performs coherence-weighted Wiener inverse filtering to recover an estimate of the reference signal from the treated measurements.

1 Notation and Inputs

Let $x[n]$ denote the discrete-time *reference* pressure signal, and $y[n]$ the *treated*, both sampled at $f_s = 2_500$ Hz.

Spectral definitions Welch auto-spectra $S_{xx}[k]$, $S_{yy}[k]$ and cross-spectrum $S_{xy}[k]$ are computed with segment length $N_{\mathrm{seg}} = 4096$, overlap $N_{\mathrm{ov}} = 2048$, and a Hann window $w[\cdot]$. The magnitude-squared coherence is

$$\gamma^2[k] \;=\; \frac{|S_{xy}[k]|^2}{S_{xx}[k]\,S_{yy}[k]} \in [0,1]. \tag{1}$$

The FRF estimate is $H[k] = S_{xy}[k]/S_{xx}[k]$.

2 Algorithms

Algorithm 1 H Transfer-Function Estimation

Inputs: Time series $x[n]$ (reference), $y[n]$ (treated), sampling rate f_s; Welch parameters $N_{\mathrm{seg}}, N_{\mathrm{ov}}, w[\cdot]$.

Outputs: Frequency vector $f[k]$; complex FRF $H[k]$; coherence $\gamma^2[k]$.

Compute $S_{xx}[k]$ and $S_{yy}[k]$ using Welch's method with $(N_{\mathrm{seg}}, N_{\mathrm{ov}}, w)$.
Compute cross-spectrum $S_{xy}[k]$ using the same Welch settings.
$H[k] \leftarrow S_{xy}[k]/S_{xx}[k]$.
$\gamma^2[k] \leftarrow \frac{|S_{xy}[k]|^2}{S_{xx}[k]\,S_{yy}[k]}$, clipped to $[0,1]$.
Return $(f[k], H[k], \gamma^2[k])$.

Algorithm 2 Coherence-weighted Wiener Inverse (γ^2/H)

Inputs: y_r, sampling rate f_s, freq. grid f, transfer $H(f)$, coherence $\gamma^2(f)$
Outputs: $\hat{x}$

$\hat{y_r} \leftarrow \mathcal{F}(y_r, N_{\text{fft}})$
$m \leftarrow |H|; \quad \phi \leftarrow \text{unwrap}(\angle H)$
$m_i \leftarrow \text{interp}_{f \to f_r}(m); \quad \phi_i \leftarrow \text{interp}_{f \to f_r}(\phi); \quad H_i \leftarrow m_i \, e^{j\phi_i}$
$\gamma_i^2 \leftarrow \text{clip}\big(\text{interp}_{f \to f_r}(\gamma^2), \, 0, \, 1\big)$
$\varepsilon \leftarrow$ machine epsilon; $\quad H_{\text{inv}} \leftarrow \gamma_i^2 \cdot H_i^* / \max(m_i^2, \varepsilon)$
$H_{\text{inv}}[0] \leftarrow 0$
$\hat{x} \leftarrow \mathcal{F}^{-1}\big(\hat{y_r} \cdot H_{\text{inv}}, N_{\text{fft}}\big)[0{:}N]$
return y

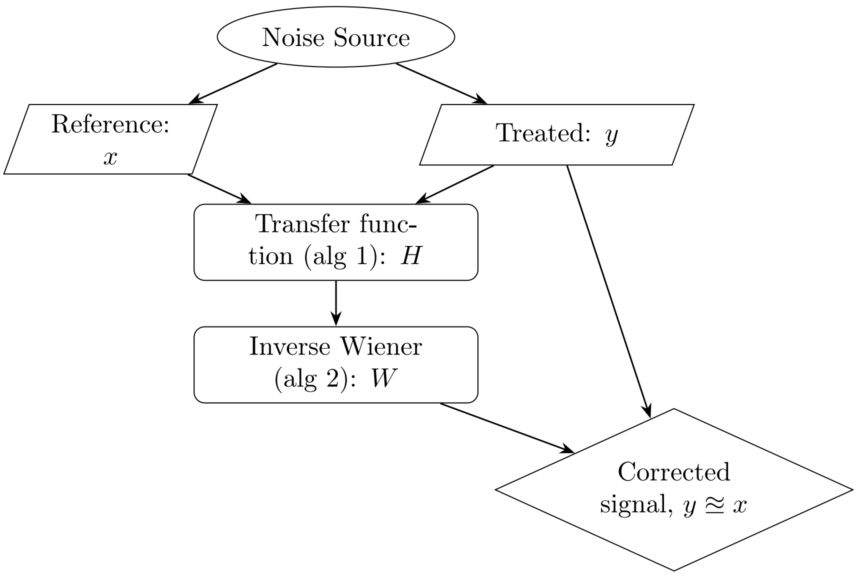

Figure 1: Generic transfer function processing pipeline.

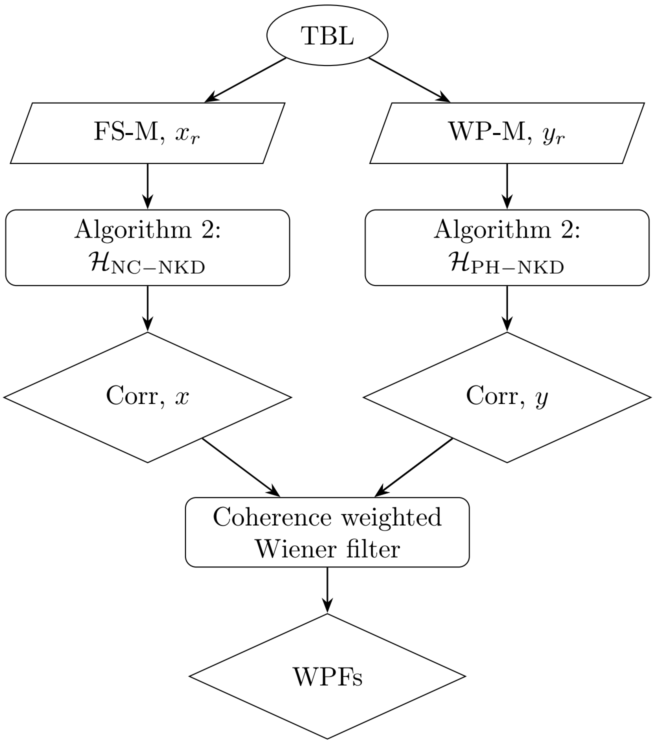

Figure 2: Complete pressure processing pipeline for measurement of the WPFs through a pinhole microphone.

Algorithm 3 Two-microphone Wiener noise cancellation for wall pressure

---

Inputs: Wall mic $y[0{:}N-1]$, free-stream mic $r[0{:}N-1]$, sampling rate f_s
Outputs: Estimate of true wall pressure $\widehat{x}$

$y \leftarrow y - \text{mean}(y); \quad r \leftarrow r - \text{mean}(r)$
Estimate spectra via Welch: $S_{rr}(f)$ and $S_{yr}(f)$ ▷ S_{yr} is cross-PSD
$F(f) \leftarrow \dfrac{S_{yr}(f)}{S_{rr}(f) + \varepsilon}$ ▷ Wiener filter mapping $r \to y$
$R(f) \leftarrow \text{rFFT}(r); \quad Y(f) \leftarrow \text{rFFT}(y)$
$\widehat{D}(f) \leftarrow F(f)\,R(f)$ ▷ Predicted free-stream noise at wall mic
$\widehat{X}(f) \leftarrow Y(f) - \widehat{D}(f)$
$\widehat{x} \leftarrow \text{irFFT}\Big(\widehat{X}(f)\Big)$
return $\widehat{x}$
