Wall- and Free-Stream Pressure Processing: Algorithmic Specification

Generated from the provided Python script

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Abstract

This document formalises, in pseudocode, the processing pipeline. The pipeline estimates a frequency response transfer function (H_1) between a reference (free-stream) microphone and a treated (wall) microphone, then performs coherence-weighted Wiener inverse filtering to recover an estimate of the reference signal from the treated measurements.

1 Notation and Inputs

Let $x[n]$ denote the discrete-time *reference* pressure signal, and $y[n]$ the *treated*, both sampled at f_s Hz. Frequency-domain quantities are indexed by k for Welch frequency bins and by r for the dense FFT grid used in deconvolution.

Spectral definitions Welch auto-spectra $S_{xx}[k]$, $S_{yy}[k]$ and cross-spectrum $S_{xy}[k]$ are computed with segment length N_{seg}, overlap N_{ov}, and Hann window $w[\cdot]$. The magnitude-squared coherence is

$$\gamma^2[k] \ = \ \frac{|S_{xy}[k]|^2}{S_{xx}[k]\, S_{yy}[k]} \in [0,1]. \tag{1}$$

The FRF estimate is $H[k] = S_{xy}[k]/S_{xx}[k]$.

2 Algorithms

Algorithm 1 H_1 Transfer-Function Estimation

Inputs: Time series $x[n]$ (reference), $y[n]$ (treated), sampling rate f_s; Welch parameters $N_{\text{seg}}, N_{\text{ov}}, w[\cdot]$.

Outputs: Frequency vector $f[k]$; complex FRF $H[k]$; coherence $\gamma^2[k]$.

1: Optionally de-mean x and y.
2: Compute $S_{xx}[k]$ and $S_{yy}[k]$ using Welch's method with $(N_{\text{seg}}, N_{\text{ov}}, w)$.
3: Compute cross-spectrum $S_{xy}[k]$ using the same Welch settings. ▷ e.g. via a cross-spectral density routine
4: $H[k] \leftarrow S_{xy}[k]/S_{xx}[k]$.
5: $\gamma^2[k] \leftarrow \frac{|S_{xy}[k]|^2}{S_{xx}[k]\, S_{yy}[k]}$, clipped to $[0,1]$.
6: Return $(f[k], H[k], \gamma^2[k])$.

Algorithm 2 Wiener Inverse Filtering for Trace Recovery

Inputs: Output $y[n]$; sampling rate f_s; FRF samples $(f[k], H[k])$; coherence $\gamma^2[k]$; optional band-limit $[f_{\min}, f_{\max}]$; regulariser $\lambda \geq 0$; zero-padding $N_{\text{pad}} \geq 0$.

Outputs: Estimate $\hat{x}[n]$ of the input.

1: De-mean y.

2: $N \leftarrow \text{length}(y)$; $N_{\text{FFT}} \leftarrow N + N_{\text{pad}}$.

3: $Y[r] \leftarrow \text{rFFT}\big(y, N_{\text{FFT}}\big)$; $f_r \leftarrow \text{rFFTFreq}\big(N_{\text{FFT}}, d = 1/f_s\big)$.

4: Interpolate $|H[k]|$ and $\phi[k] = \text{unwrap}(\angle H[k])$ onto f_r to obtain $|H|_r$ and ϕ_r.

5: $H_r \leftarrow |H|_r\, e^{\mathrm{j}\phi_r}$.

6: Interpolate $\gamma^2[k]$ onto f_r, clip to $[0, 1]$ to get γ_r^2.

7: Compute inverse filter (Wiener form)

$$H_r^{-1} \;\leftarrow\; \gamma_r^2 \,\frac{H_r^*}{|H|_r^2 + \lambda}, \tag{2}$$

or equivalently use $|H|_r^2 \leftarrow \max(|H|_r^2, \varepsilon)$ for numerical safety when $\lambda = 0$.

8: **if** $[f_{\min}, f_{\max}]$ provided **then**

9: Zero out H_r^{-1} for all $f_r \notin [f_{\min}, f_{\max}]$. ▷ Band-limited inverse

10: Set DC: $H_r^{-1}[0] \leftarrow 0$.

11: $\hat{x}[n] \leftarrow \text{iRFFT}\big(Y[r] \cdot H_r^{-1}, N_{\text{FFT}}\big)[0{:}N]$.

12: **return** $\hat{x}[n]$.

Algorithm 3 Wall/Free-Stream Pressure Processing Pipeline

Inputs: Index i selecting a calibration file; processing constants $f_s, \nu_0, \rho_0, u_{\tau 0}, \mathrm{W}, \mathrm{He}, L_0, \Delta L_0, U, C$; mode sets $\mathcal{M}, \mathcal{N}, \mathcal{L}$.

Outputs: FRF plot and corrected-trace plot; arrays (f, H, γ^2) and recovered trace $\hat{x}[n]$.

1: Instantiate `WallPressureProcessor` with the given physical and processing parameters.

2: Load calibration test $(p_w[n], p_{fs}[n]) \leftarrow \text{load}(\texttt{fn\_naked\_pressures}[i])$.

3: Set ref $\leftarrow p_w$, trt $\leftarrow p_{fs}$.

4: $(f, H, \gamma^2) \leftarrow$ H1 Transfer-Function Estimation(ref, trt, f_s) ▷ Alg. 1

5: Save a transfer-function figure: magnitude and phase of H vs. f, optionally with coherence overlay.

6: $\hat{x}[n] \leftarrow$ Wiener Inverse Filtering(trt, $f_s, f, H, \gamma^2, [f_{\min}, f_{\max}], \lambda, N_{\text{pad}}$) ▷ Alg. 2

7: Define $t[n] \leftarrow n/f_s$.

8: Save a corrected-trace figure showing $\{\text{ref}(t), \text{trt}(t), \hat{x}(t)\}$.

3 Default Parameters (from the Script)

Unless otherwise stated, the following defaults are used in the reference implementation:

Quantity	Value
Sampling rate f_s	25 000 Hz
Welch defaults	N_{seg}=4096, N_{ov}=2048, w= Hann
Inverse filter band (example)	$(0, 3000]$ Hz *optional*