Efficient Control For a Domestic Hot Water Heating System

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***Abstract*— Significant cost and energy savings can be realized by optimizing the controller for a domestic hot water heating system. As a result, a typical heating system was modelled, and this model was used to determine what type of control scheme is best able to preserve system functionality in an efficient manner. When it comes to a hot water heating system, the control problem is to maintain the temperature at the top of the system's storage tank at approximately 60^oC while minimizing cost and energy use. In total, four controllers were designed; an industry standard On/Off controller, a constant level controller, a state feedback controller with an integrator, and a linear quadratic regulator (LQR) controller with an integrator. These controllers were then tested through simulations on the developed model over 48 hour test periods. Ultimately, it was determined that the LQR controller has the best performance, using the least energy of all explored controllers while consistently maintaining the target temperature at the top of the hot water heating system's storage tank.**

I. INTRODUCTION

Approximately a quarter of domestic energy use in Canada goes towards heating water [1]. As a result, it is clearly beneficial from both a cost and energy standpoint for households to reduce their hot water usage. Further cost and energy savings can also be attained by utilizing a low energy heating method such as an electric heat pump. These pumps provide more efficient heating than other sources, such as natural gas, and also have the added benefit of being easier to control through typical means [2]. That being said, the majority of domestic heating systems incorporating a heat pump still use an On/Off controller to satisfy the household's hot water demands [3]. This leaves plenty of room for improvement, and indicates that by optimizing the system's controller a significant reduction in energy use and the corresponding cost could be realized. With that in mind, the goal of this paper is to accurately model a domestic hot water heating system, and use this model to design, test, and compare multiple control schemes to determine which is best suited for this application.

II. SYSTEM DESCRIPTION

A. Hot Water Heating System

The domestic hot water heating system consists of two main components. The first is the thermal storage tank (TST), while the second is the heat pump. In this setup the TST acts as a battery, storing hot water for later use, while the heat pump adds heat to the system. Two additional components are the temperature sensors, denoted in figure 1 by the thermometers, and the circulation pump. The temperature sensors are necessary in order to monitor and control the heat level in the TST while the circulation pump, which operates at a constant optimal flow rate, brings cooler water from the bottom of the TST to the heat pump, and replaces it with heated water. A diagram summarizing the various components of the system can be seen in Figure 1.

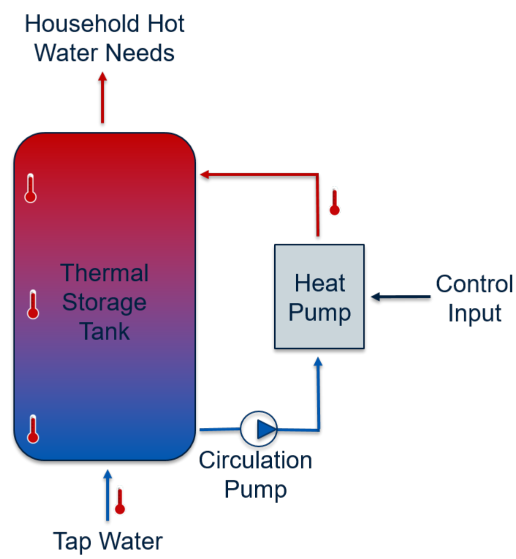

Fig. 1. Domestic hot water heating system configuration

In this system the domestic hot water load is taken from the top of the TST and replaced with relatively cold tap water at the bottom, such that the volume of water in the tank remains constant at all times. This represents one of the two forms of heat loss from the system, with the other being the heat loss that occurs through the walls of the TST to the air. In order to offset these losses the heat pump adds heat to the system in the form of thermal energy, the amount of which generated can be regulated with a control input signal that sets the heat pump's compressor speed ratio.

B. External Factors

In addition to the system itself, there are two key external factors that must be considered. The first is the fact that domestic hot water demand varies over the course of a day, but follows a general day to day pattern. This pattern is depicted in Figure 2, which shows the typical daily hot water load for the average household in Canada [4].

The second external factor of importance is the cost of electricity in Canada. Different Provinces use different pricing schemes so this paper will only consider the one applied in British Columbia (BC). This actually simplifies things as BC's pricing method is constant over the course of a day, unlike other Provinces like Ontario. In BC the

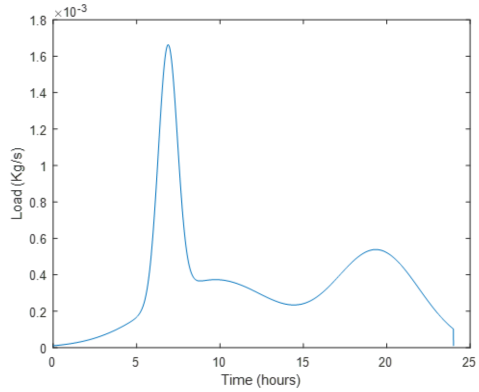

Fig. 2. Average daily domestic hot water demand for Canadian households in liters per hour

rates only change once a household passes a certain monthly threshold of electricity usage [5]. This means the amount of energy used in a hot water heating system in BC is linearly correlated to cost, regardless of when it is used. As a result, minimizing the cost of heating water is equivalent to minimizing the energy used to heat it, and it is not necessary to directly consider electrical rates.

III. PROBLEM DEFENITION

The control problem at hand given the described hot water heating system is to maintain the temperature at the top of the TST at approximately 60^oC while minimizing the amount of energy used. This is the standard household hot water temperature, which is hot enough to kill bacteria but not too hot to burn skin [6]. As mentioned, the controller in the system sets the compressor speed ratio of the heat pump in order to regulate the amount of thermal energy added to the TST. Note that no observer is needed for this system as all temperature states are known thanks to the temperature sensors seen in Figure 1.

IV. STATE SPACE MODELLING

The thermal system described above is clearly a distributed system that could be accurately described with a number of partial differential equations. This complicates things from a control perspective however, and so a lumped approximation will be used instead. This lumped approximation is depicted in Figure 3 and consists of the TST being divided into three components, each with assumed uniform temperature. In this simplified model the system is comprised of three states and one input. Here the states are the temperatures in each part of the TST and the input is the compressor speed ratio of the heat pump. To round out the variables in Figure 3, both $\dot{m}_{HP}$ and T_{room} are constant, and $\dot{m}_{load}$ is described by Figure 2 and treated as a disturbance.

Thanks to the lumped approximation the system can now be modelled in the form of a nonlineaer state space equation, which is derived from a series of ordinary differential equations (ODE). The most important ODE in this case is the first

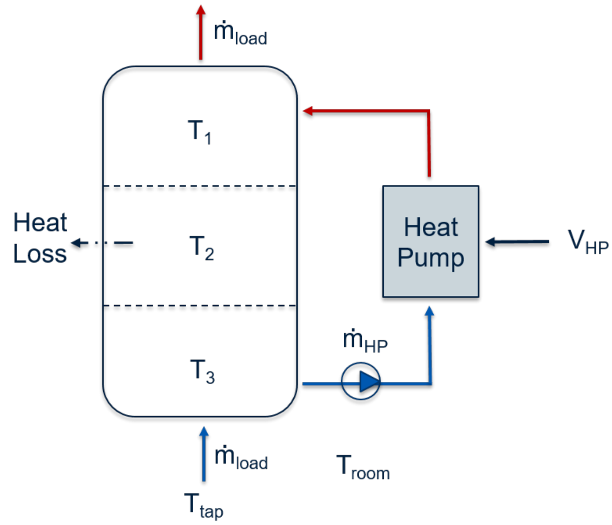

Fig. 3. Lumped approximation to hot water heating system

law of thermodynamics, which is essentially a conservation of energy equation.

$$MC_p\dot{T} = \dot{Q}_{in} - \dot{Q}_{out} + \dot{m}h(T_{inlet}) - \dot{m}h(T_{outlet}) \quad (1)$$

Here M is the mass of the working fluid in kg, C_p is the specific heat capacity of the working fluid in $\frac{J}{K}$, T is the temperature of a given component of the system in K and $\dot{Q}$ is the rate of change of total energy in $\frac{kJ}{s}$ with $\dot{Q}_{in}$ and $\dot{Q}_{out}$ representing the energy entering and leaving a given component of the system respectively. Further, $\dot{m}$ is the flow rate entering or exiting a certain part of the system in $\frac{kg}{s}$ and $h(T)$ is the enthalpy of the fluid at temperature T.

The next ODE is a governing equation for the heat pump.

$$T_{HP,out} = T_{HP,in} + \frac{\dot{Q}_{HP}}{\dot{m}_{HP}C_{p,HP}} \quad (2)$$

This equation depicts the relationship between the temperature of the fluid entering the heat pump and the temperature of the fluid leaving. Note that in this case $\dot{Q}_{HP}$ is the rate of change of thermal energy added by the heat pump, which can be further modelled by equations (3) and (4).

$$\dot{Q}_{HP} = \dot{Q}_{HP,m} * P_{\dot{Q}_{HP}} \quad (3)$$

$$P_{\dot{Q}_{HP}} = -0.3498 * V_{HP}^2 + 1.35 * V_{HP} \quad (4)$$

Here $P_{\dot{Q}_{HP}}$ is the performance ratio of the heat pump, $\dot{Q}_{HP,m}$ is the maximum possible rate of thermal energy added to the system by the heat pump, and V_{HP} is the compressor speed ratio, which is also the control input for the system. Note that all heat pump equations were originally obtained from the heat pump data sheet [7].

Finally, by using equations (1) through (4) the nonlinear state space model of the system can be derived. The form of the model is displayed below in equations (5) and (6).

$$\dot{x} = f(x,u,w) = \begin{bmatrix} f_1(x,u,w) \\ f_2(x,u,w) \\ f_3(x,u,w) \end{bmatrix}, \ y = T_1 \quad (5)$$

$$x = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}, \; u = V_{HP}, \; w = \dot{m}_{load} \qquad (6)$$

V. ANALYSIS AND DESIGN

A. Model Analysis

Before designing controllers for the system it is important to examine the model. With that in mind two initial simulations were carried out. One of which explores the system response over time with a constant control input of 0, while the other examines the system step response. The results of the first test can be seen in Figure 4.

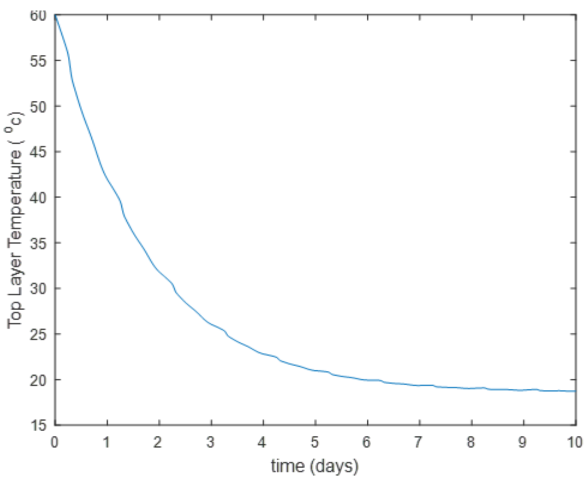

Fig. 4. System response with no input

Here the model seems to behave correctly, with the top layer temperature steadily decreasing until leveling out around 18^oC. This makes sense since the room temperature is assumed to be 20^oC while the incoming tap temperature is taken as a standard 15^oC. The results of the step response can then be seen in Figure 5.

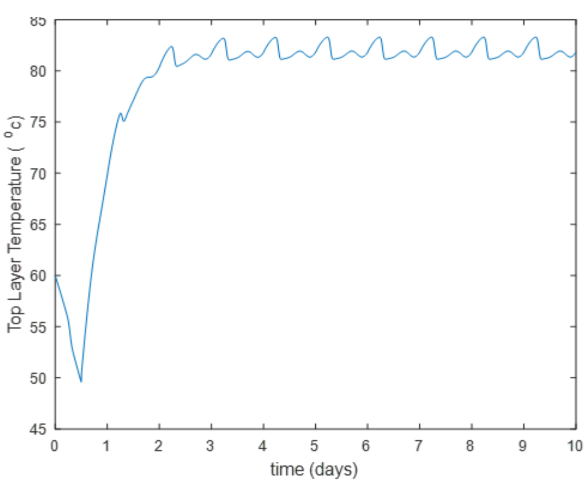

Fig. 5. Step response of the system

Again, the model appears to be fairly accurate, with the top layer temperature decreasing until the heat pump is turned on with a control input of 1. At this point the temperature begins to steadily rise until plateauing at around 82^oC.

B. Controller Design

Multiple controllers were designed and tested to determine which best suits this application. The various controller types and the corresponding design work completed is detailed ahead.

1) On/Off Controller: For comparison purposes the first controller designed was the industry standard On/Off controller. This controller simply switches between a control input of 0 and 1, with the heat pump being turned on when the temperature at the top of the TST dips below 55^oC and being turned off when the top layer temperature rises above 65^oC.

2) Constant Controller: A simple yet rarely used alternative to the On/Off controller is the constant controller, which sets the control input to some constant optimal level. This is an example of open loop control, and so does not see much use due to its inability to reject disturbances. In this case though, it can be effective, since the range of possible hot water demand is fairly well classified. Here it was found that the optimal constant control input, which is the input leading to an average top layer temperature of 60^oC, is $u = 0.29$.

3) State Feedback with Integrator: Next a state feedback controller was designed. An integrator was also included in order for the controller to track the reference temperature of 60^0C. Before designing this controller it was first necessary to linearize the nonlinear state space model into the form seen below.

$$\dot{x} = Ax + Bu, \; y = Cx \qquad (6)$$

This is a continuous-time linear state space model where A is a $3x3$ matrix, B is a $3x1$ matrix and C is a $1x3$ matrix. This was done around the following set point.

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 60^oC \\ 60^oC \\ 60^oC \end{bmatrix}, \; VHp = 0, \; \dot{m}_{load} = \text{avg}(\dot{m}_{load}) \qquad (7)$$

Next, given that there is an integrator, it is necessary to augment the A and B matrices in order to place the poles of the system.

$$A_{aug} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \; B_{aug} = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad (8)$$

It was determined that this set of augmented matrices is controllable and thus any pole placement is possible. Nonetheless, the placement was still tuned a little through trial and error based on total energy use. The best found pole positions are displayed in equation (9).

$$\text{Poles} = \begin{bmatrix} -0.001 \\ -0.002 \\ -0.005 + 0.001i \\ -0.005 - 0.001i \end{bmatrix} \tag{9}$$

Lastly, because the controller is not actually continuous, each calculated control input is assumed to be applied at a constant level for 10 seconds, before the control signal is updated. This time interval is essentially negligible when compared to the rate at which temperatures change in the system, as seen in Figures 4 and 5, thus it should not have a negative impact on performance.

4) Linear Quadratic Regulator (LQR) with Integrator: Finally, an LQR controller was designed. In this case a discrete-time LQR was used, and it again included an integrator. In order to complete this design a discrete-time linear state space model was obtained by discretizing the continuous-time linear state space model described in equation (6) using the zero-order-hold method.

Now, the main benefit of an LQR controller is that the control inputs are realized by minimizing a cost function that accounts for the control input levels as well as the reference error. In this situation that means the energy use is directly taken into account in the controller design. The cost function is described below.

$$J(u) = \sum_{k=0}^{\text{inf}} (r - y[k])^\top Q (r - y[k]) + \\ (u[k+1] - u[k])^\top R (u[k+1] - u[k]) \tag{10}$$

subj. to

$$x[k+1] = A_d x[k] + B_d u[k],\ y[k] = C_d x[k] \tag{11}$$

Here Q and R are tuning parameters that determine how much error and large inputs are penalized respectively. Ultimately, the tuning parameters used in the design were $Q = 1$ and $R = 20$. This puts a much larger penalty on high input levels than it does on reference error.

VI. Simulation Results

A. Controller Results

All of the controllers outlined in the previous section were tested on the system model over a trial simulation period of 48 hours. The results can be seen in Figures 6 and 7.

Looking at Figure 6, all controllers do a reasonable job at maintaining the temperature at the top of the tank at approximately 60^oC, though the state feedback and LQR controllers do a much better job than the other two. Figure 7 then demonstrates the different levels of energy exerted under the various control schemes. By looking at the picture it is clear that the On/Off controller uses much more energy than is necessary, while the other three seem to be doing a reasonably efficient job at maintaining the target temperature in the TST. In each case the curves in Figure 7 were integrated over the 48 hour period to determine the total

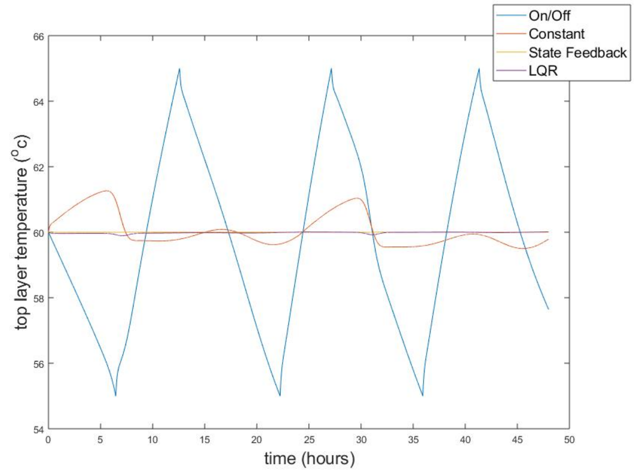

Fig. 6. Top layer TST temperature over the 48 hour control period

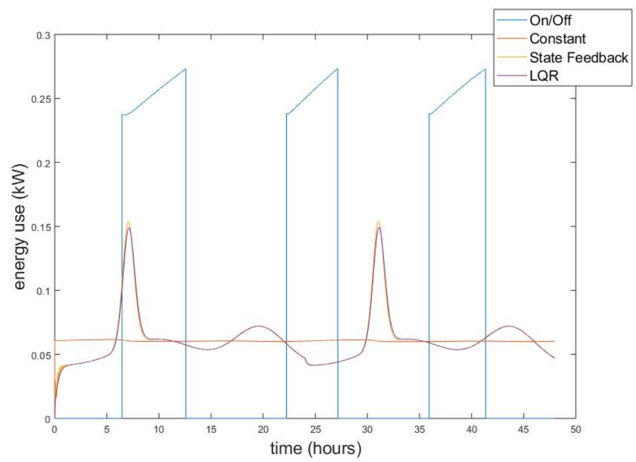

Fig. 7. Energy used over the 48 hour control period

energy used by each scheme. The results can be seen in Table 1.

TABLE I

TOTAL ENERGY USED OVER 48 HOUR TEST PERIOD

Control Scheme	Total Energy Used (kWh)
On/Off	4.194
Constant	2.955
State Feedback	2.959
LQR	2.904

This shows that the LQR controller is the most energy efficient, followed by the constant controller, the state feedback controller, and finally the On/Off controller. The table further depicts a 30.75% reduction in energy use for the LQR controller compared to the industry standard On/Off controller.

As a whole these results demonstrate that LQR control is the best option for this application due to a combination of its reference tracking accuracy and low energy use.

VII. CONCLUSION

Four separate control schemes were designed for the efficient control of a hot water heating system that was modelled using a lumped approximation and a state space equation consisting of three states. These schemes were compared based on their ability to maintain the temperature in the top of the TST at 60^oC as well as on their energy efficiency. In the end, it was determined that the best controller option for this thermal system is a discrete time LQR controller with an integrator, which performs much better than the industry standard On/Off controller. As such, the addition of LQR control to the hot water heating systems found in households across Canada would produce significant cost and energy savings while maintaining consistent hot water temperatures for every households' hot water needs.

REFERENCES

[1] "What is the breakdown of household electricity use?," Hydro Quebec. [Online]. Available: http://www.hydroquebec.com/residential/customer-space/electricity-use/electricity-consumption-by-use.html. [Accessed: 17-Mar-2019].

[2] "Should you get a heat pump?," BC Hydro. [Online]. Available: https://www.bchydro.com/news/conservation/2015/heat-pump.html. [Accessed: 17-Mar-2019].

[3] Honeywell, "Temperature On/Off Control - Honeywell Heating Controls," Honeywell UK. [Online]. Available: https://heatingcontrols.honeywellhome.com/products/temperature-controls/temperature-onoff-control/. [Accessed: 18-Mar-2019].

[4] I. Knight, N. Kreutzer, M. Manning, M. Swinton, and H. Ribberink, "European and Canadian non-HVAC electric and DHW load profiles for use in simulating the performance of residential cogeneration systems," May 2007.

[5] "Residential Rates," BC Hydro. [Online]. Available: https://app.bchydro.com/accounts-billing/rates-energy-use/electricity-rates/residential-rates.html. [Accessed: 10-Apr-2019].

[6] "Heated Debate About Hot Water – Canada Safety Council," Canada Safety Council, 02-Feb-2018. [Online]. Available: https://canadasafetycouncil.org/heated-debate-about-hot-water/. [Accessed: 17-Mar-2019].

[7] H. Druck, "Multiport store - model for trnsys," Institut fur Thermodynamik und Warmetechnik (ITW), Tech. Rep., 2006.