Proposition (4.3). Let $E \in \mathfrak{M}$, and let $f : E \to \mathbb{R}$ with $m(E) < \infty$ be a bounded, measurable function. Let ϕ, ψ be simple functions. Then the **Lebesgue integral** of f if

$$\int_E f = \inf \left\{ \int \psi : \psi \geq f \right\} = \sup \left\{ \int \phi : \phi \leq f \right\}.$$

Proof. [1]

□

[1] Proof is on pages 79-80.