Problem 1 (6.2)**.** Let f be a bounded measurable function on $[0,1]$. Then $\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$.

Proof. □

Problem 2 (6.8)**.** Young's Inequality

(a) Let $a, b \geq 0$, $1 < p < \infty$, $\dfrac{1}{p} + \dfrac{1}{q} = 1$. Establish Young's inequality

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}.$$

Proof. Not assigned. □

(b) Use Young's inequality to give a proof of the Hölder inequality.

Proof. □

Problem 3 (6.10)**.** Let $\{f_n\}$ be a sequence of functions in L^∞. Prove that $\{f_n\}$ converges to f in L^∞ if and only if there is a set E of measure zero such that f_n converges to f uniformly on E.

Proof. We will need to complete two directions and so let $\{f_n\}$ be a sequence of functions in L^∞.

($\Rightarrow$) First, suppose that $\{f_n\} \to f$.

($\Leftarrow$) Conversely, suppose there exists a set E with $m(E) = 0$ such that $f_n \to f$ uniformly on E.

□