

Problem 1 (11.10). Prove Proposition 11.7 which is stated follows:

Proposition (11.7). Let f be a nonnegative measurable function. Then there is a sequence $\{\phi_n\}$ of simple functions with $\phi_{n+1} \geq \phi_n$ such that $f = \lim_{n \rightarrow \infty} \phi_n$ at each point of X . If f is defined on a σ -finite measure space, then we may choose the functions ϕ_n so that each vanishes outside a set of finite measure.

Problem 2 (11.22). (a) Let (X, \mathcal{B}, μ) be a measure space and g a nonnegative measurable function on X . Set $vE = \int_E g d\mu$. Show that v is a measure on \mathcal{B} .

(b) Let f be a nonnegative measurable function on X . Then

$$\int f dv = \int fg d\mu.$$