Problem 1 (4.2)**.** (a) Let f be a bounded function $[a, b]$, and let h be the upper envelope of f (cf. Problem 2.51). Then $R\underline{\int_a^b} f = \int_a^b h$.

(b) Use part (a) to prove Proposition 7 which is stated as follows

Proposition (4.7). A bounded function f on $[a, b]$ is Riemann integrable if and only if the set of points at which f is discontinuous has measure zero.

Problem 2 (4.3)**.** Let f be a nonnegative measurable function. Show that $f = 0$ implies $f = 0$ almost everywhere.

Problem 3 (4.8)**.** Prove the following generalization of Fatuo's Lemma: If f_n is a sequence of nonnegative functions then

$$\int \varliminf_{n\to\infty} f_n \leq \varliminf_{n\to\infty} \int f_n.$$