

Problem 1 (6.21). (a) Let g be an integrable function on $[0, 1]$. Show that there is a bounded measurable function f such that $\|f\| \neq 0$

$$\int fg = \|g\|_1 \cdot \|f\|_\infty.$$

(b) Let g be a bounded measurable function. Show that for each $\varepsilon > 0$, there is an integrable function f such that

$$\int fg \geq (\|g\|_\infty - \varepsilon) \|f\|_1.$$

Problem 2 (11.3). (a) Show that $\mu(E_1 \triangle E_2) = 0$ implies $\mu(E_1) = \mu(E_2)$ provided that $E_1, E_2 \in \mathcal{B}$.

(b) Not assigned.