Problem 1 (5.10)**.** (a) Let f be defined by

$$f(x) = \begin{cases} 0 & x = 0 \\ x^2 \sin\left(\frac{1}{x^2}\right) & x \neq 0. \end{cases}$$

Is f of bounded variation on $[-1, 1]$?

Proof. □

(b) Not assigned.

Problem 2 (5.15)**.** The Cantor ternary function (Problem 2.48) is continuous and monotone but not absolutely continuous.

Proof. □

Problem 3 (5.20)**.** A function f is said to satisfy a Lipschitz condition on an interval if there is a constant M such that $|f(x) - f(y)| \leq M|x - y|$ for all x and y in the interval.

(a) Show that a function satisfying a Lipschitz condition is absolutely continuous.

Proof. Not assigned. □

(b) Show that an absolutely continuous function f satisfies a Lipschitz condition if and only $|f'|$ is bounded.

Proof. □