Modelling of Fire Contagion with Application in Farm Insurance

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UQÀM | co-operators
Co-operators Chair in
Actuarial Risk Analysis

Paper accepted in Variance

About the Project

- Project done for Co-operators;
- Results obtained in Summer 2021;
- Paper submitted in Fall 2021;
- ... still not published in Variance.
- Generalizations of the project began in June 2023.

Authors

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- Andra Crainic, Co-operators;
- Alexandre LeBlanc Co-operators;
- Vincent Masse, Co-operators.

Modeling of Fire Contagion in Farms Insurance

A Preprint

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Abstract

In a farm, a fire that starts in any structure can spread to all other structures of that same farm, to barns, granaries, silos, etc.. Intuitively, we then expect that a farm with more structures will be more at risk of fire propagation than a smaller one. From an actuarial perspective, as the total premium for farm insurance is the sum of premiums of each structure of that farm, it is therefore necessary to propose a way to compute each premium by considering the risk of fire propagation. Based on the distances between structures on the same farm, we propose a new pricing approach that considers fire propagation. The proposed model makes it possible to analytically compute the probability of fire propagation as a function of the fire origin. This can in turn be used to price all individual structures of any given farm. A practical application of the model based on insurance data and satellite images is given.

Keywords Fire Contagion, Farm Insurance, Ratemaking, Exponential Distributions

1 Introduction

A single fire can sometimes generate large losses due to the fact that the aforementioned can spread to buildings near to the origin. Naturally, such a risk is of interest to actuaries. In this paper, we propose a new ratemaking model where fire contagion is a main concern. The model is applied to farm insurance, where a single farm may have several types of structures: dwellings, barns, granaries, silos, etc. As these structures can be close to each other, all structures of a single farm can be touched by a single fire. Indeed, as we will see in the numerical illustration shown in this paper, summary statistics show that a fire beginning in one structure can often spread to other structures of the same farm.

To estimate the risk of fire damages, the characteristics of each farm and structures of that farm should be used for modelling. However, given the limited literature and the limited data on the subject at the present time, we will instead focus on studying fire contagion mainly as a function of the distances between structures. Indeed, it is more likely to expect a fire to propagate when a farm's structures are located near each other. Using satellite images of 39 different farms as well as policy and claim information from 2014-2019, we propose a model of fire contagion having the objective of pricing each structure of a given farm. We believe that the proposed approach is an interesting first step in developing a more general approach for the modeling of fire propagation in farm insurance, or even in commercial or home insurance.

Challenge

Concerns for the insurance company

A fire can spread to all other structures of a farm (barn, granaries, silos, etc.):

- We expect that a farm with more structures will be more at risk of fire propagation than a smaller one;
- If all structures on a farm are close to each other, the fire risk will be larger.

How to model the risk of fire contagion?

Types of Farms in Canada

Types of Farms in Canada

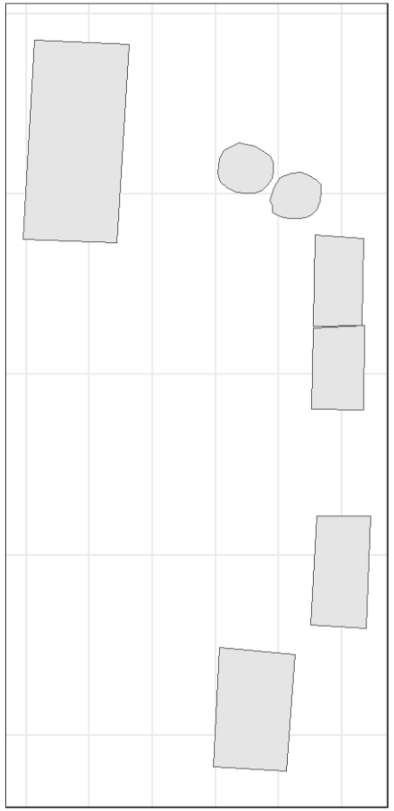
Types of Farms in Canada

Part I: Creation of the Dataset

Google Earth

- We identified the physical disposition of farms using the address of the farm and images from *Google Earth*;
- Images allowed us to find the size of each structure of the farm, and the distance between all structures;
- Based on these distances between structures on the same farm, we then propose a new pricing approach that considers fire propagation.

KML File



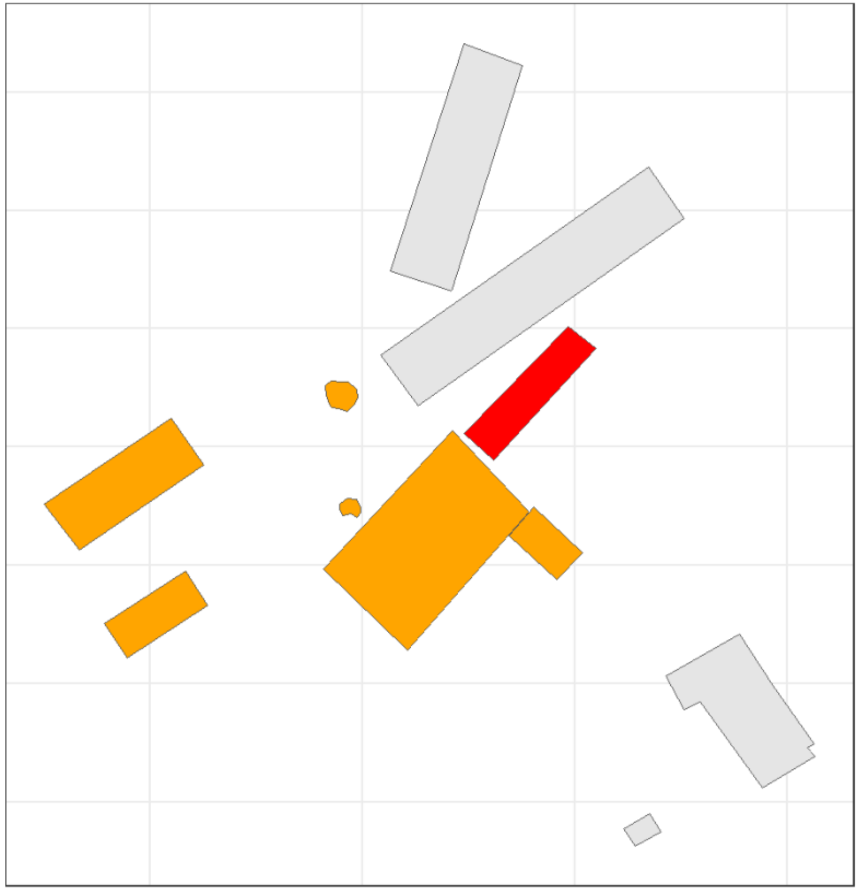
The *sf* package

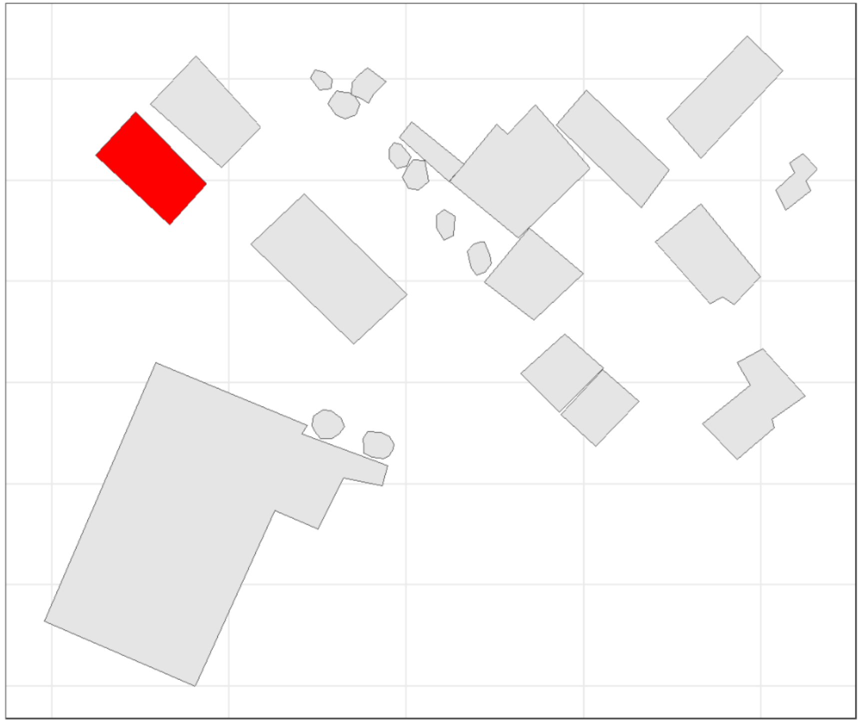
We can also found all distances between the structures of a single farm with the *sf* package in R:

0.000	11.851	18.556	24.328	26.435	41.730	51.496
11.851	0.000	0.772	8.470	18.078	40.548	56.093
18.556	0.772	0.000	2.370	13.591	36.757	53.512
24.328	8.470	2.370	0.000	0.053	23.356	40.462
26.435	18.078	13.591	0.053	0.000	13.087	30.284
41.730	40.548	36.757	23.356	13.087	0.000	4.154
51.496	56.093	53.512	40.462	30.284	4.154	0.000

Analyzing each Claim

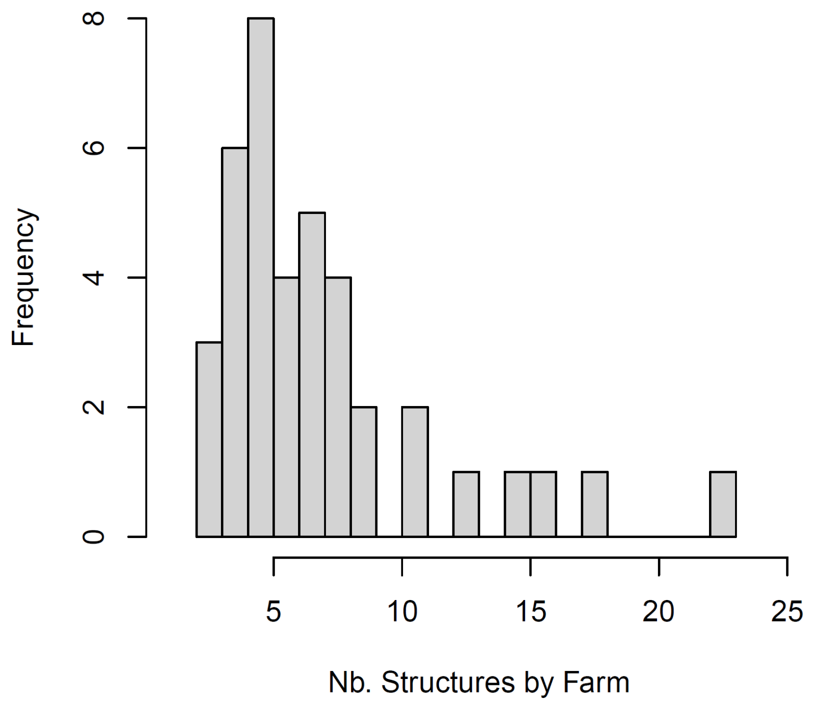
By reading all the notes of each claim, it became possible to identify the fire origin as well as all structures touched by the fire:

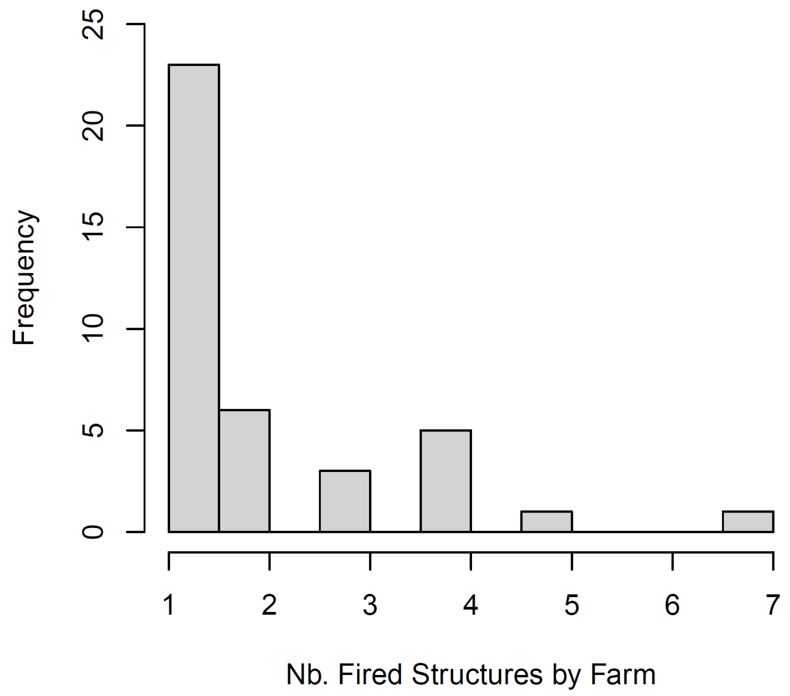




Sample of Data

In total, for the first phase of the project, we studied a sample of 39 farms with fire damages.





Part II: Modelling Approach

Risk of Fire

Fire Origin

To model the risk of fire of a stucture, two possibilities must be studied:

1) The fire starts directly from that structure;

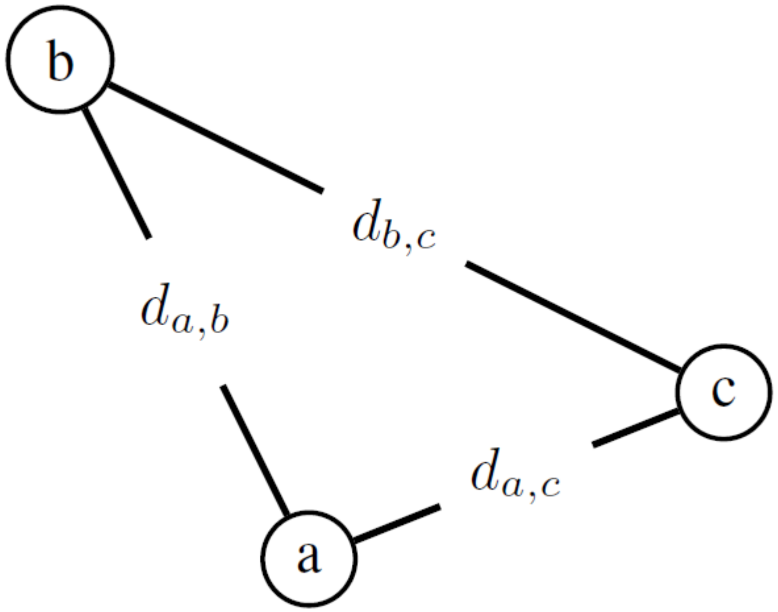
2) The fire starts in another structure but *somehow* propagates to that structure.

Contagion Processes

There are many ways the fire starting in structure b can spread to structure a:

- *Level 1 contagion*, where the fire spread directly from structure b to structure a;
- *Level 2 contagion*, where the fire spread first from structure b to another structure, and from that structure spread to structure a;
- Any other levels of contagions, where, more generally, for a farm with J structures, there are potentially up to $J-1$ levels of contagion, where each successive level of contagion has longer and longer paths from structure b to structure a.

Example with 3 Structures



Example with 3 Structures

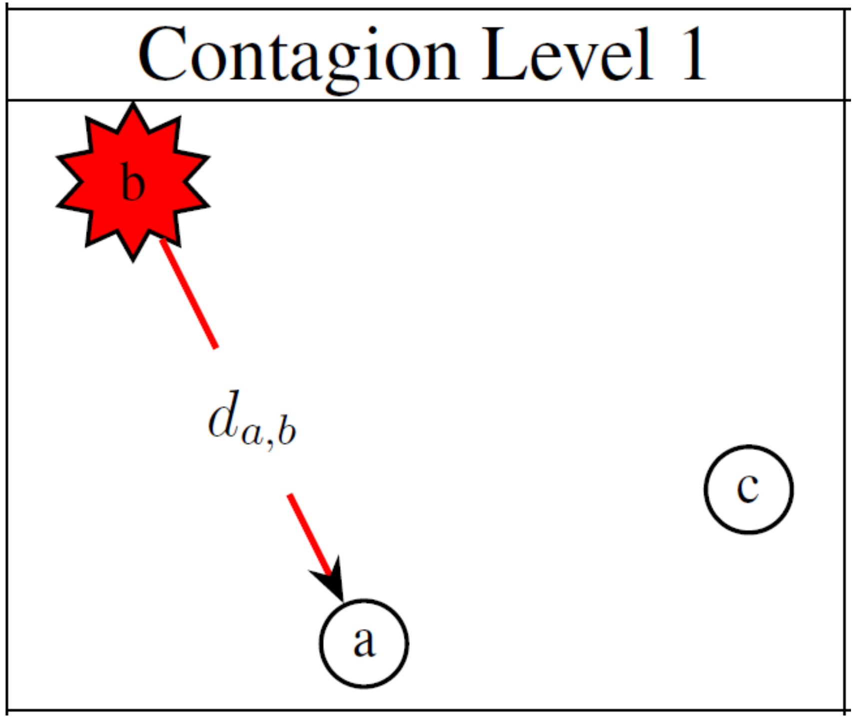
If we want to rate structure a, 2 contagions have to be considered:

1) A fire that starts in structure b:

- **A level 1 contagion, where the fire passes directly from b to a;**
- A level 2 contagion, where the fire passes from b to c, and then goes from c to a.

2) A fire that starts in structure c:

- A level 1 contagion, where the fire passes directly from c to a;
- A level 2 contagion, where the fire passes from c to b, and then goes from b to a.



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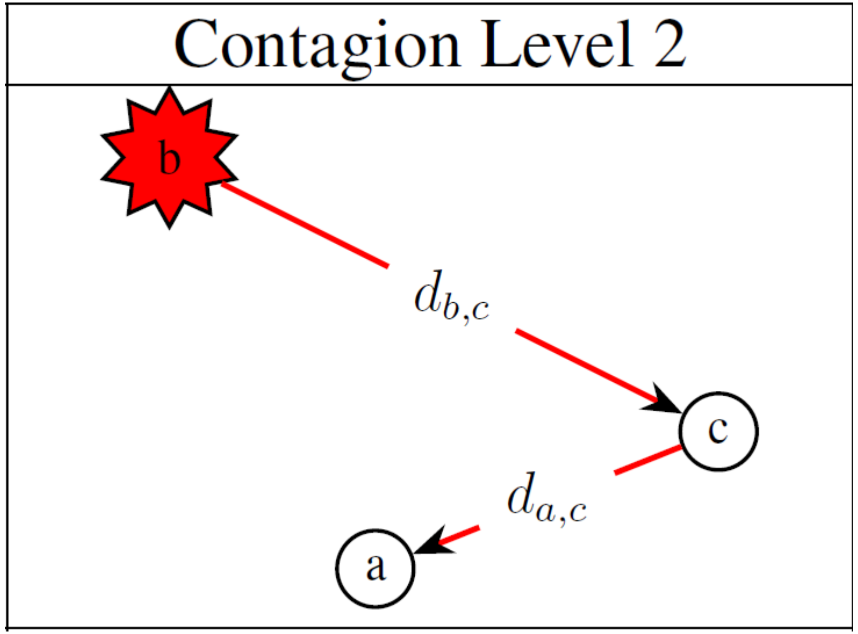

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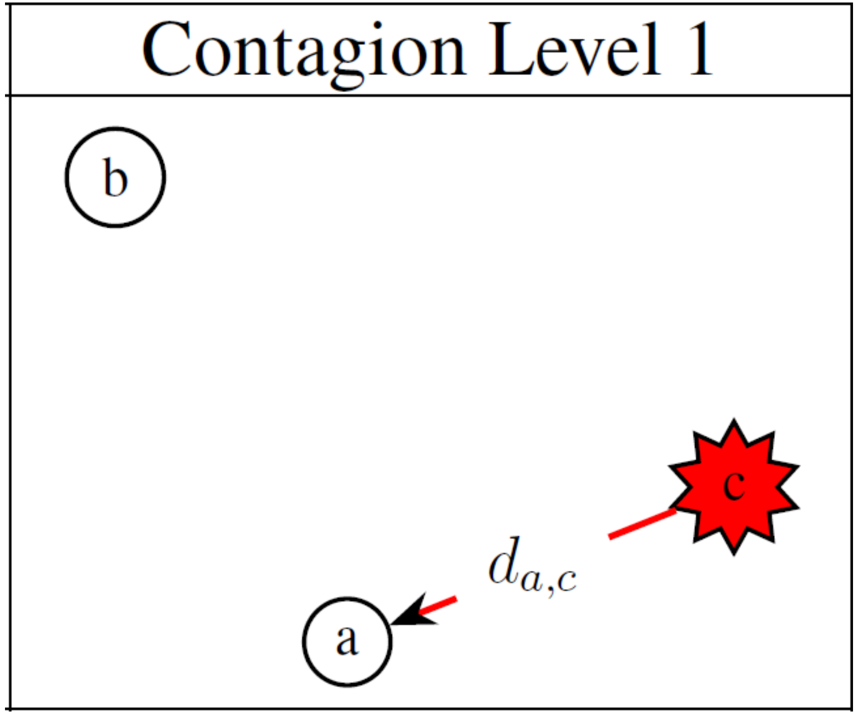

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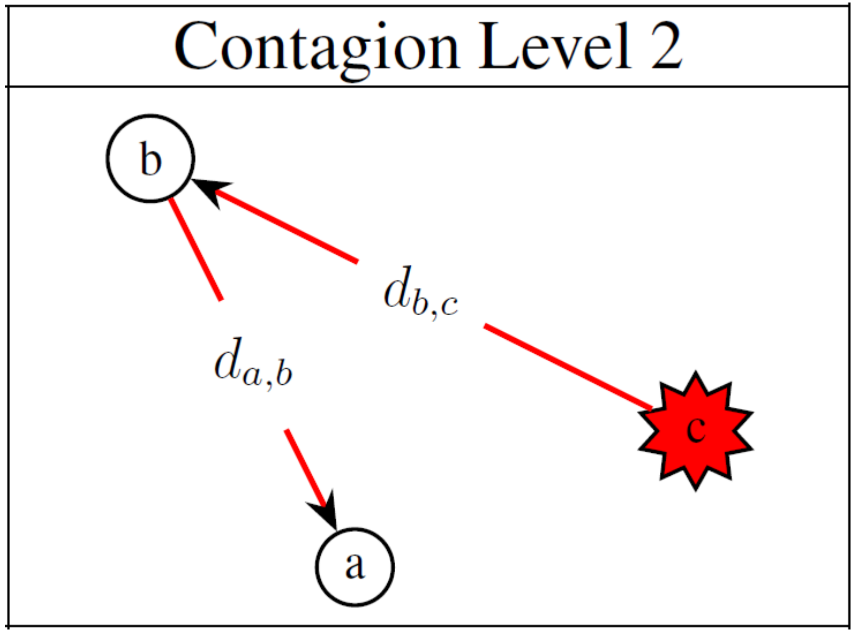


Contagion Processes

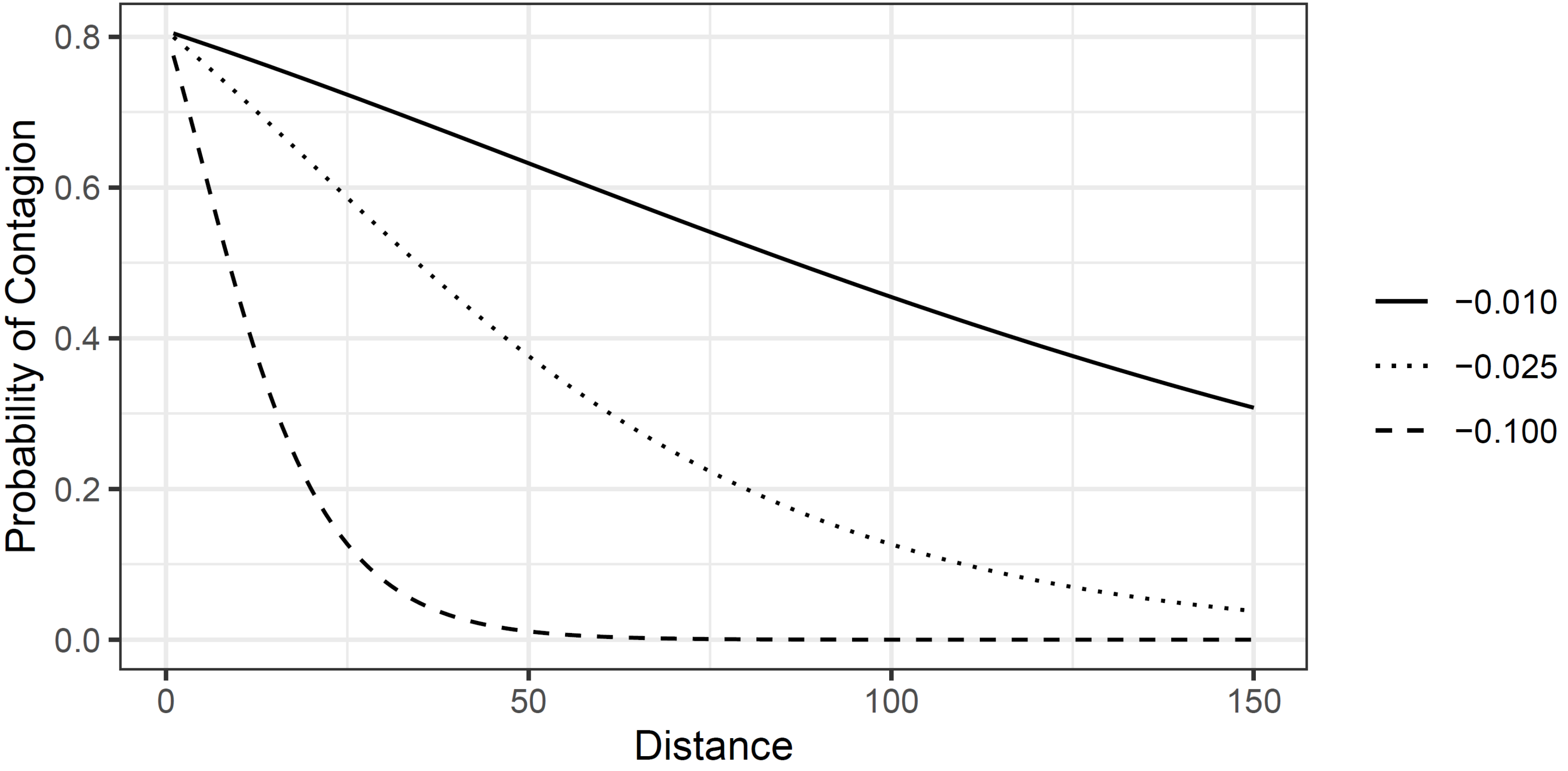
- Each contagion process is an exponential random variables noted $\epsilon(\lambda)$, where λ depends on the distance between structures.
- A structure will catch fire when ϵ will be less than a certain threshold ϕ. The probability of a specific type of contagion is then expressed as:

$$\Pr(I = 1) = \Pr(\epsilon(\lambda) \leq \phi = 1) = 1 - \exp(-\lambda)$$

- For a level 1 contagion, we could use $\lambda = \exp(\beta_0 + \beta_1 d_1)$, where d_1 is the distance between the structure studied and the structure where the fire originated,
- For a level 2 contagion, we could use $\lambda = \exp(\beta_0 + \beta_1 d_1 + \beta_2 d_2)$, where:
    - d_1 is the distance between the intermediate structure and the structure where the fire originated;
    - d_2 is the distance between the intermediate structure and the structure studied.
- etc.

Impact of Parameters

The figure illustrates the probability of contagion for a level 1 contagion, depending on the distance d_1, for various values of β_1.

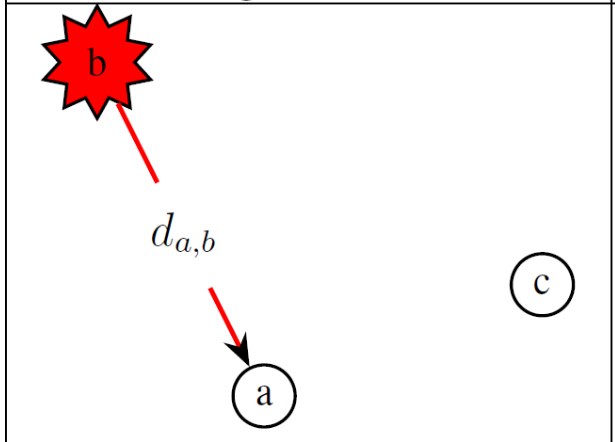
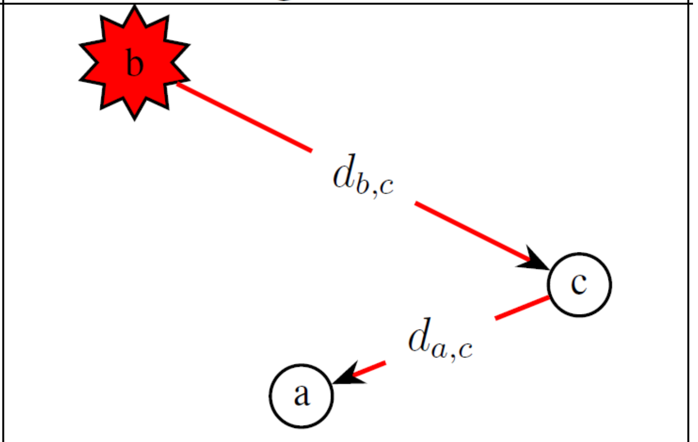


Competition between Processes

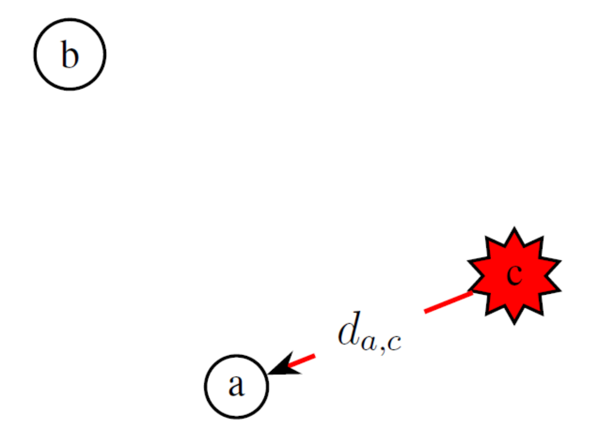
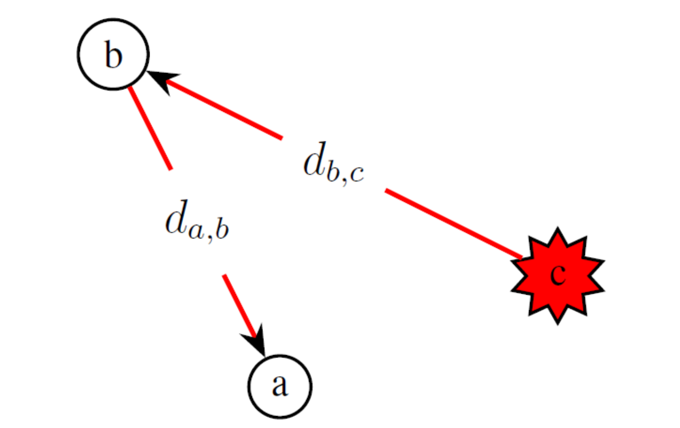
- Each potential path of a fire is a contagion process and among all contagion processes, at most one fire contagion is realized.
- This can be seen as a competition between all contagion processes to determine which one will be the cause of the fire reaching a structure.

> Therefore, the contagion process that is realized can be considered the minimum value of all processes involved. Because each process is an exponential random variable, it can be shown that the minimum value of all processes is an exponential variable with parameter equal to the sum of all parameters.

Summary: Fire starts in Structure b

<table>
<tr><th colspan="2">Fire Origin: Structure b</th></tr>
<tr><th>Contagion Level 1</th><th>Contagion Level 2</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\epsilon^{(a)}_{b:(.)} \sim \text{Exp.}(\lambda^{(a)}_{b:(.)})$
$\lambda^{(a)}_{b:(.)} = \exp(\beta_0 + \beta_1 d_{a,b})$</td><td>$\epsilon^{(a)}_{b:(c)} \sim \text{Exp.}(\lambda^{(a)}_{b:(c)})$
$\lambda^{(a)}_{b:(c)} = \exp(\beta_0 + \beta_1 d_{b,c} + \beta_2 d_{a,c})$</td></tr>
<tr><td colspan="2">$\epsilon^{(a)}_{b} \sim \text{Exp.}(\lambda^{(a)}_{b})$
$\lambda^{(a)}_{b} = \lambda^{(a)}_{b:(.)} + \lambda^{(a)}_{b:(c)}$</td></tr>
</table>

Summary: Fire starts in Structure c

Fire Origin: Structure c	
Contagion Level 1	Contagion Level 2
	
$\epsilon^{(a)}_{c:(.)} \sim \text{Exp.}(\lambda^{(a)}_{c:(.)})$ $\lambda^{(a)}_{c:(.)} = \exp(\beta_0 + \beta_1 d_{a,c})$	$\epsilon^{(a)}_{c:(b)} \sim \text{Exp.}(\lambda^{(a)}_{c:(b)})$ $\lambda^{(a)}_{c:(b)} = \exp(\beta_0 + \beta_1 d_{b,c} + \beta_2 d_{a,b})$
$\epsilon^{(a)}_{c} \sim \text{Exp.}(\lambda^{(a)}_{c})$ $\lambda^{(a)}_{c} = \lambda^{(a)}_{c:(.)} + \lambda^{(a)}_{c:(b)}$	

Premium from Fire Contagion

In summary, to price the structure a, we obtain the following two probabilities of contagion, based on the fire origin (in subscript):

$$\Pr(I_b^{(a)} = 1) = F_{\epsilon_b^{(a)}}(1; \lambda_b^{(a)}) = 1 - \exp(-\lambda_b^{(a)})$$
$$\Pr(I_c^{(a)} = 1) = F_{\epsilon_c^{(a)}}(1; \lambda_c^{(a)}) = 1 - \exp(-\lambda_c^{(a)})$$

Part IV: Empirical Illustration

Inference: Example

A fire on a farm implies 2 types of contagion processes:

1- Observed processes in competition;
2- Unobserved processes that not *activated*.

Example

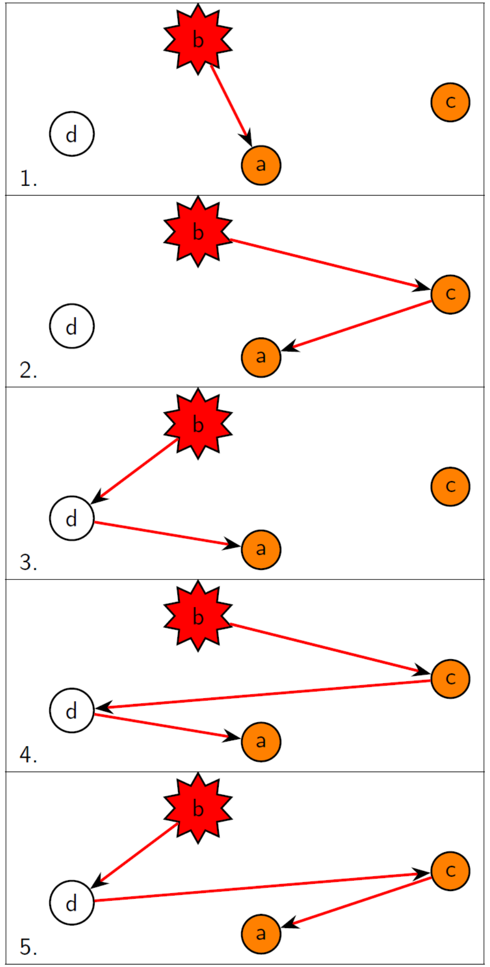
Suppose a farm with 4 structures with:

i) The origin of the fire is structure b;

ii) Structures a and c are affected by the fire (we don't know in what order);

iii) Structure d is spared and is not affected by the fire.

We want to analyse what happened to structure a from the contagion processes.



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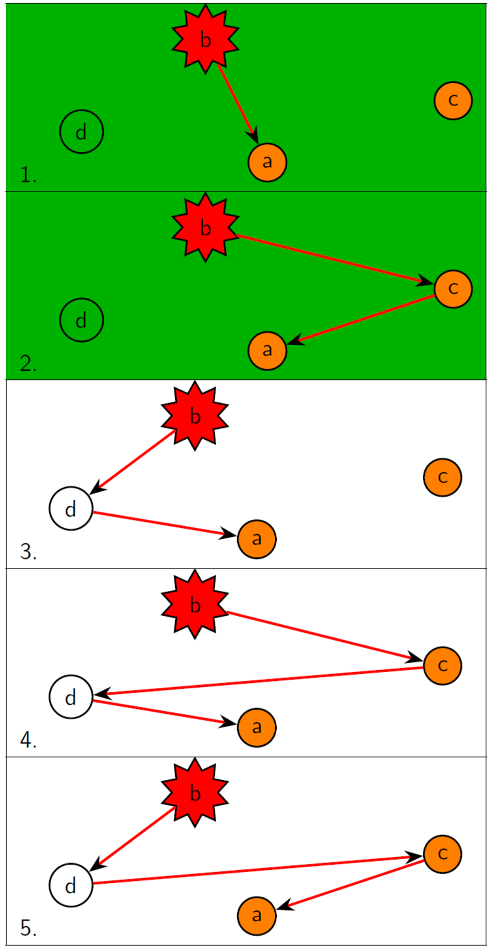


Loglikelihood

With our sample of 39 fires, we found $25,907$ contagion processes for contagion levels 1,2 and 3:

- $25,618$ processes are considered impossible:

$$\ell_1 = \sum_{i=1}^{25{,}618} -\lambda_i$$

- For the remaining 289 possible processes, we obtained 37 competitions between processes:

$$\ell_2 = \sum_{i=1}^{37} \log(1 - \exp(-\lambda_i))$$

where λ_i corresponds to the sum of λ from possible processes.

To estimate the parameters of the model, we then obtain the total log-likelihood of $\ell = \ell_1 + \ell_2$.

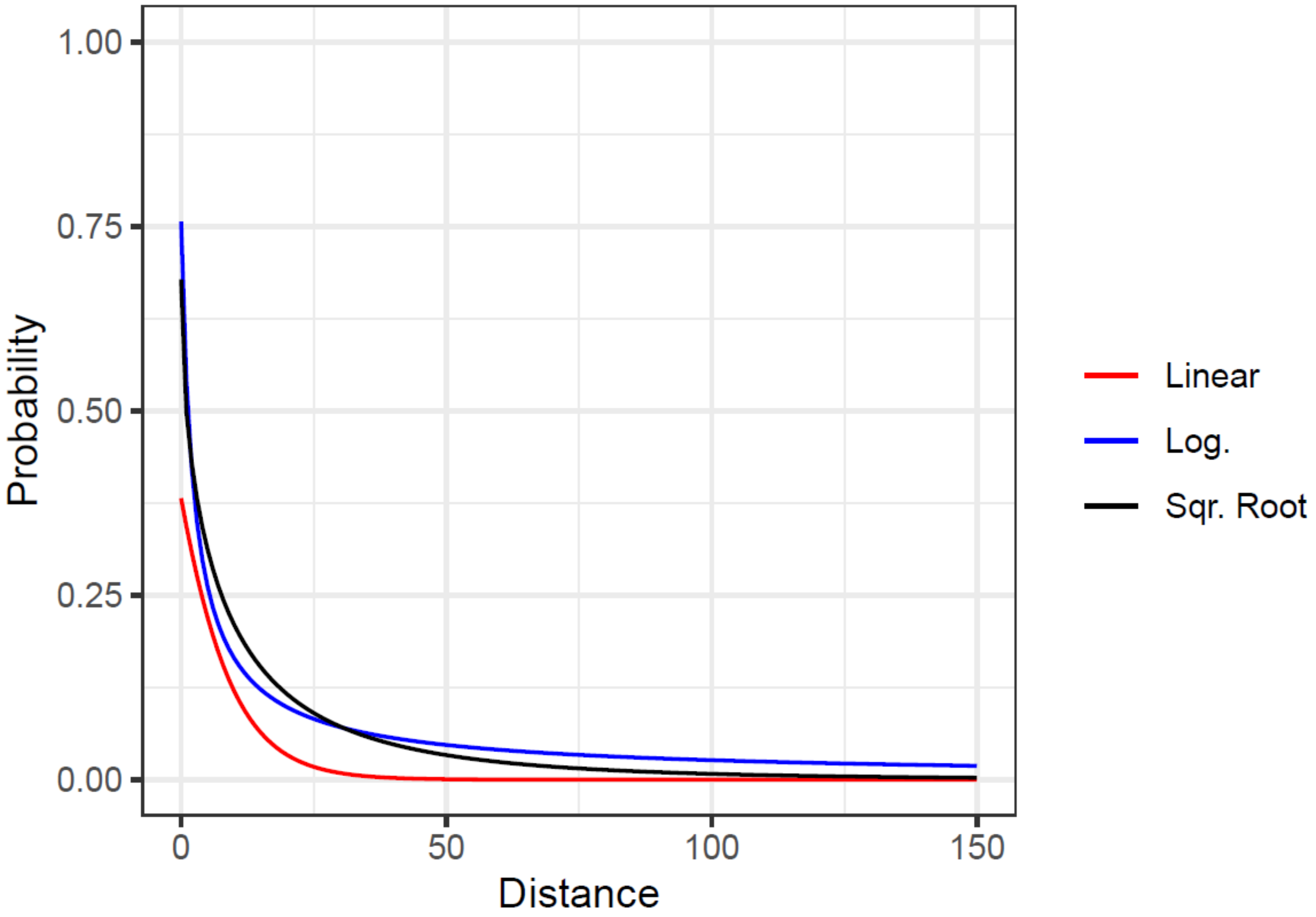
Estimated Parameters

Parameter	Linear Est.	Linear (s.e)	Logarithmic Est.	Logarithmic (s.e)	Square root Est.	Square root (s.e)
β_0	-0.0396	(0.2625)	1.0389	(0.2973)	0.8184	(0.2879)
β_1	-0.1323	(0.0288)	-0.8591	(0.1237)	-0.4964	(0.0804)
β_2	-0.0977	(0.0244)	-83.4250	(103.1140)	-8.6023	(8.7921)
β_3	-48.0926	(100.4818)	-227.8673	(1899.7283)	-24.2324	(137.3874)
Loglikelihood	-94.1097		-85.5895		-84.7147	

with:

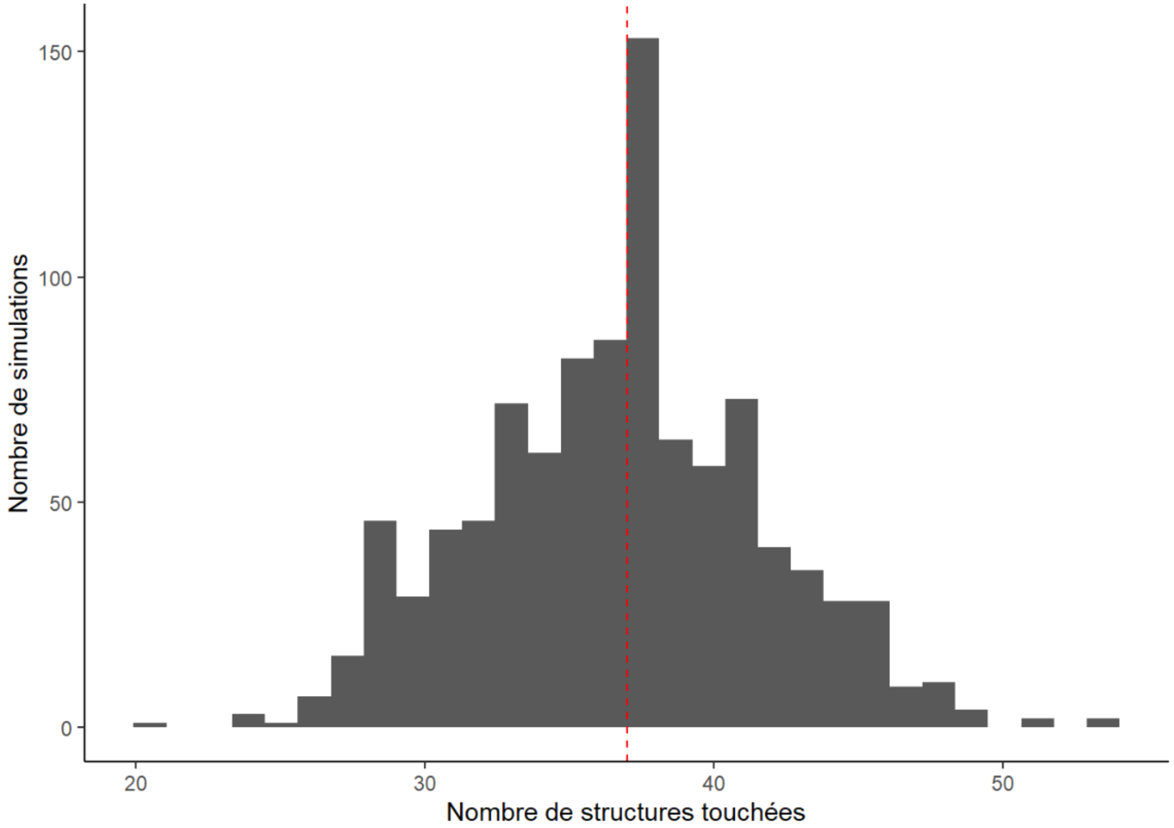
$$\lambda^{lin} = \exp(\beta_0^{lin} + \beta_1^{lin} d_1 + \beta_2^{lin} d_2 + \beta_3^{lin} d_3)$$
$$\lambda^{log} = \exp(\beta_0^{log} + \beta_1^{log} \log(d_1) + \beta_2^{log} \log(d_2) + \beta_3^{log} \log(d_3))$$
$$\lambda^{sqr} = \exp(\beta_0^{sqr} + \beta_1^{sqr} \sqrt{d_1} + \beta_2^{sqr} \sqrt{d_2} + \beta_3^{sqr} \sqrt{d_3})$$

Distance and Contagion

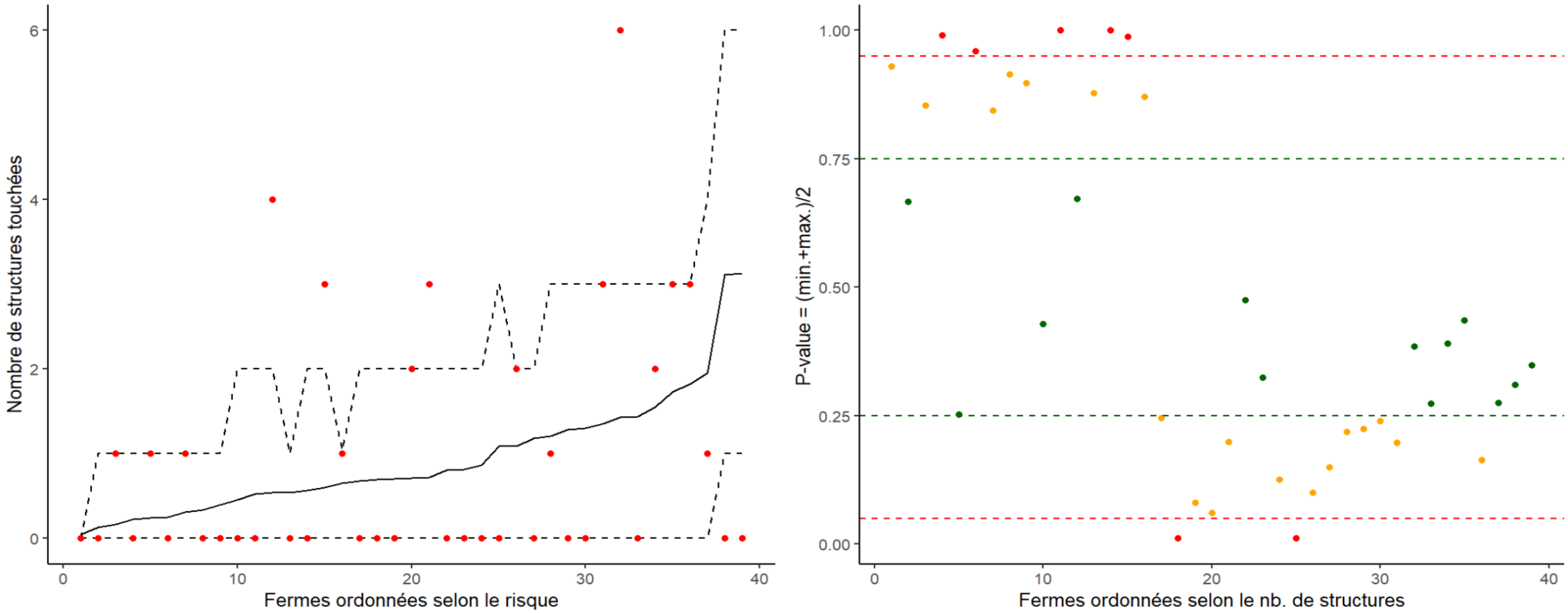


Verifications

To verify the fit of the model, we simulated 10,000 times the fire contagion model of the 39 farms analyzed. We kept the same fire origin for each farm but put in competition all the contagion processes.

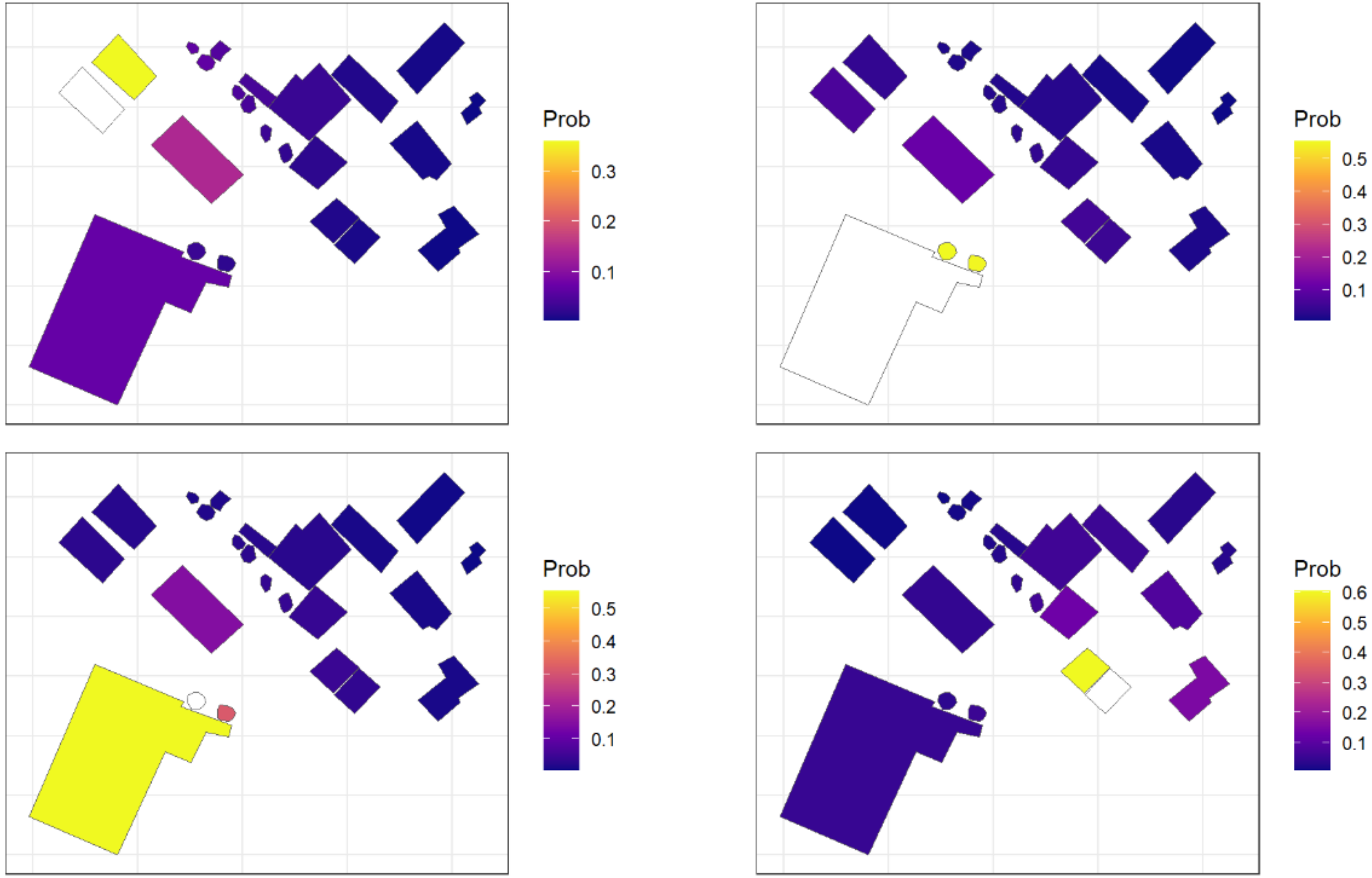


Expected vs Observed



Part IV: Ratemaking

Example of Contagion



Premium

The premium for structure a is then computed as:

$$\pi^{(a)} = E[S^{(a)}] = E[N_a]E[Y^{(a)}] + \alpha \sum_{k=1, k\neq a}^{J} E[N_k]E[I_k^{(a)}]E[Y^{(a)}]$$

where:

1) $I_k^{(a)}$ is an indicator that a fire that starts in structure k touches structure a;

2) N_k, the number of fires starting in structure k (per year), a function of the area of the structure, with:

$$E[N_k] = \text{area of the structure in } m^2/500000.$$

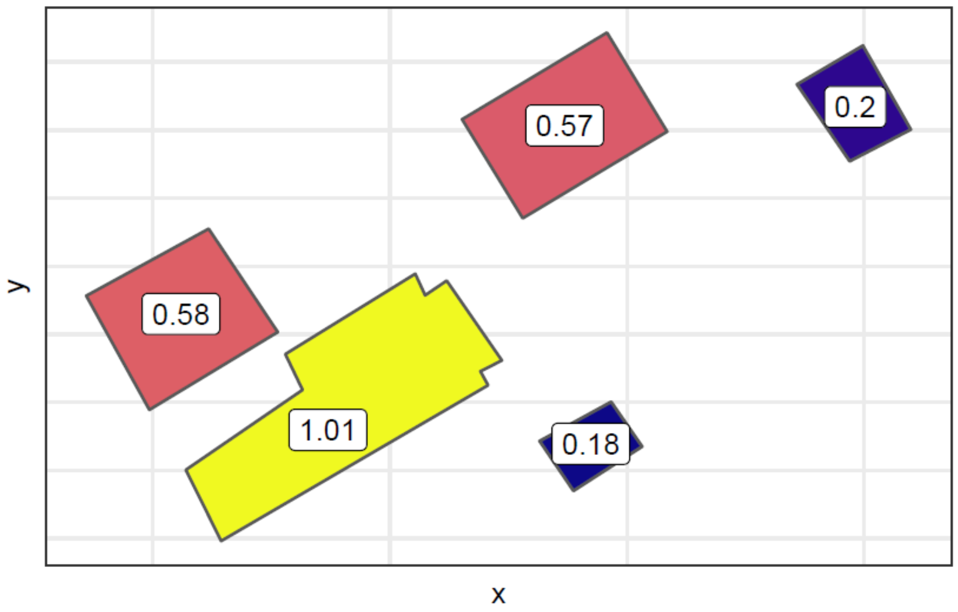
3) $Y^{(a)}$, the severity of a fire in structure a is a function of the area of the structure, with:

$$E[Y^{(a)}] = 5000 \times \sqrt{\text{area of the structure in } m^2}$$

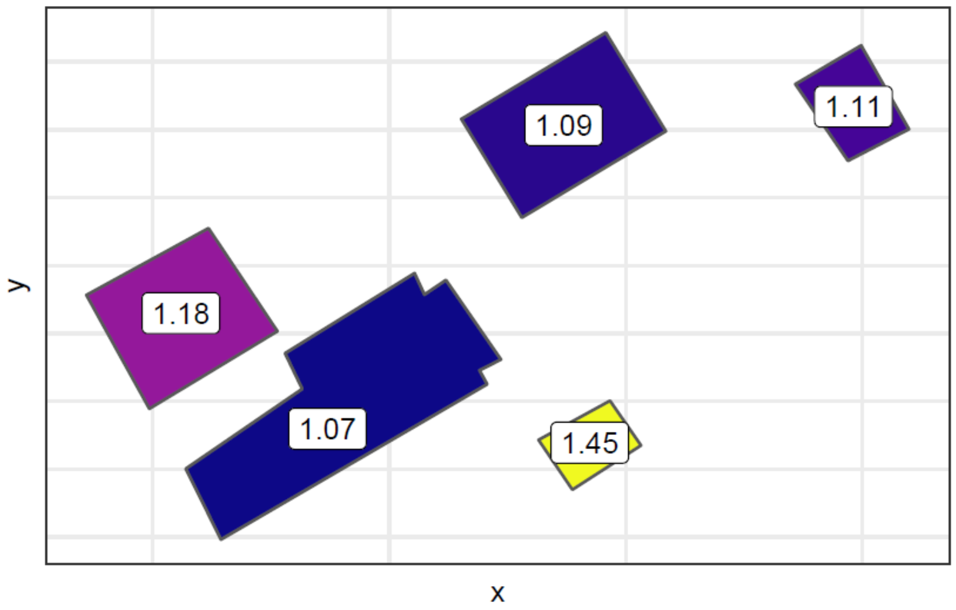
4) If the fire comes from a contagion, we assume that the damage will be $0.25 \times Y^{(a)}$.

Impact of Contagion

Premium for each structure without contagion.



Increase of premium from the contagion.



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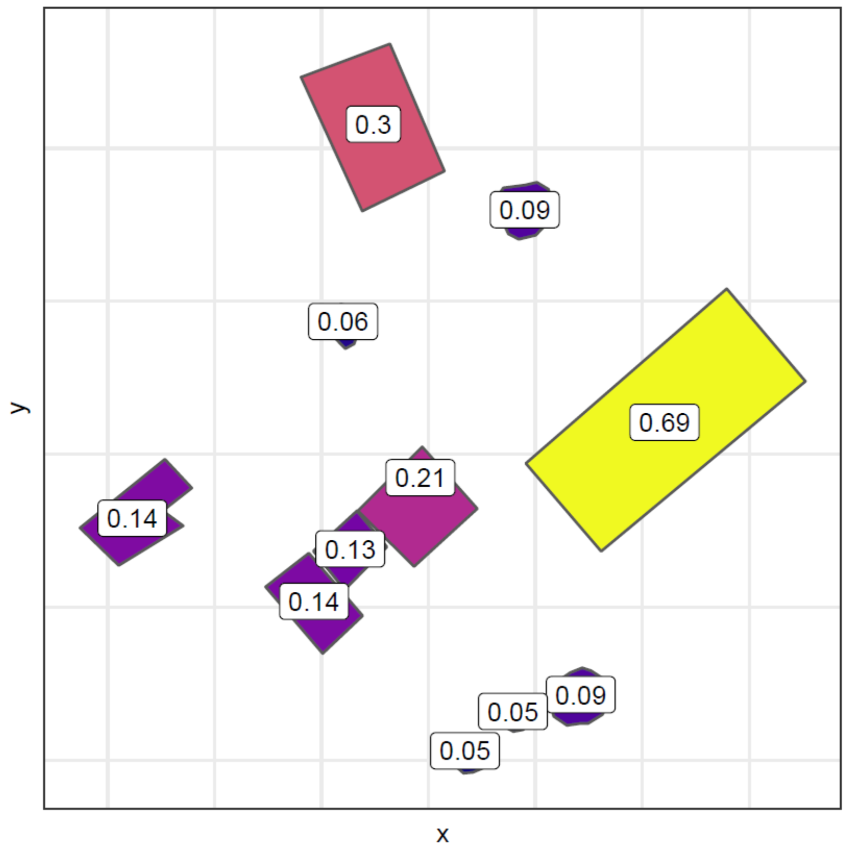

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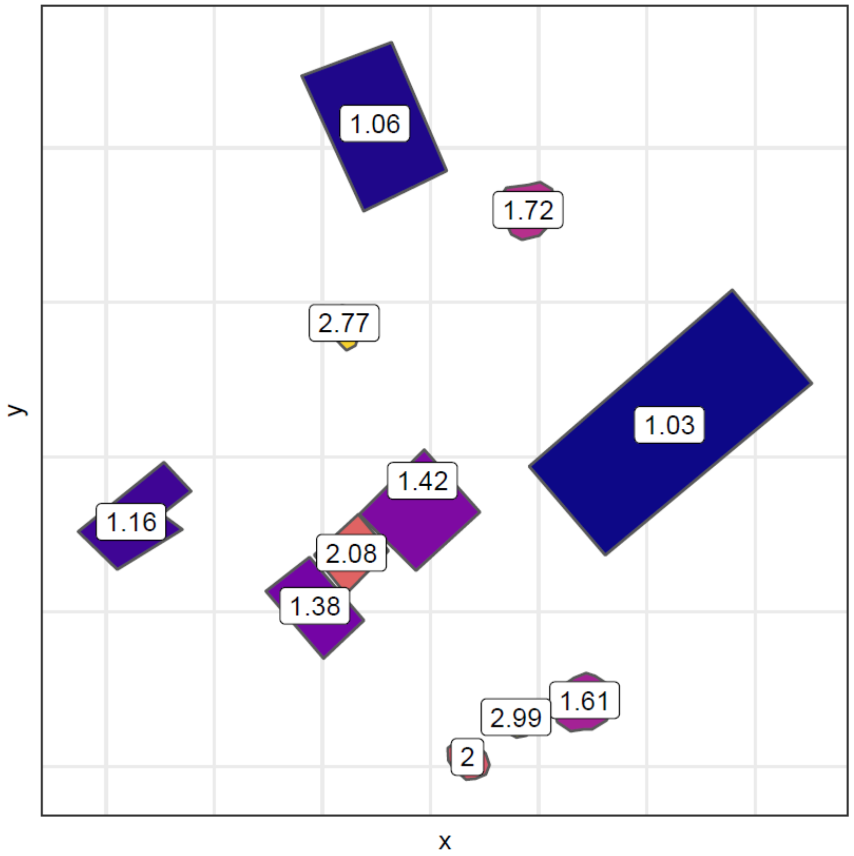


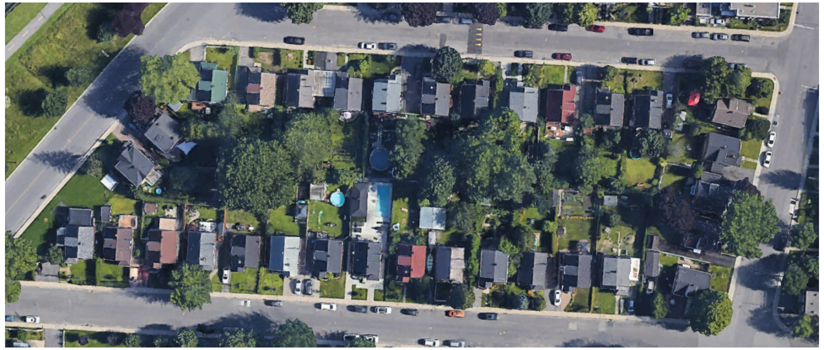
Impact of Contagion

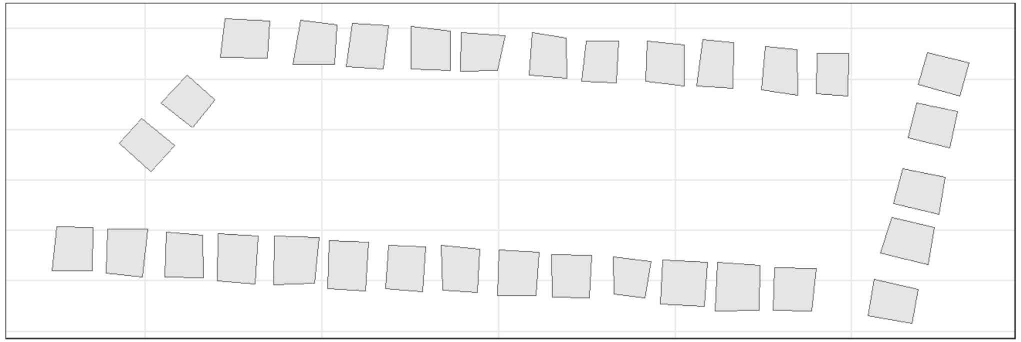
Farm #	# Structures	$\sum_{j=1}^{J} E[S_s]$	$\sum_{j=1}^{J} E[S^{(s)}]$	Impact
1	5	2.269	2.550	112.3 %
2	23	6.000	9.279	154.6 %
3	11	3.173	4.214	132.8 %
4	11	1.545	1.929	124.9 %
5	18	8.979	13.424	149.5 %
6	4	1.562	1.665	106.7 %
7	7	1.374	1.692	123.1 %
8	15	4.959	7.322	147.7 %
9	3	0.742	0.782	105.4 %

Part V: Conclusion

Conclusion

- The approach can be seen as a first step in modeling fire contagion between structures - in this case, in farm insurance;
- Improvements are proposed in the paper to generalize the proposed model;
- Application with larger datasets could be interesting:
    - To check some of the hypothesis;
    - To verify if the sample used in this project is representative;
    - To consider all type of fire;
    - etc.
- Using the fire contagion model for home insurance is also a possibility:





Source, data and references

Website of the research Chair

You can check the website of the Co-operators Chair in Actuarial Risk Analysis for publications, fundings, etc.

> https://chairecara.uqam.ca/en/

Github

This presentation (including the R scripts, but not the dataset) can be found on my *github* page:

> https://github.com/J-PBoucher

Thanks

Finally, a special thank to my (former) Ph.D. student **Francis Duval** who created this nice *xaringan* template with RMarkdown. A new template that uses Quarto should be soon developed.

Announcement

Ph.D. Scholarship:

The Co-operators Chair in Actuarial Risk Analysis (CARA) is looking for doctoral (Ph.D.) students to work on research projects on developing P&C insurance models.

Successful applicants will receive funding for at least three years (including equipment and conference travel). The order of magnitude of this funding is $30,000/year (grant) and will be adjusted according to inflation, etc.

Visit the CARA website for more details:

- https://chairecara.uqam.ca/en/