The Linear Stiffness Matrix Equation is written as:

$$\left[\begin{array}{c|c} K_{\mathrm{ff}} & K_{\mathrm{fs}} \\ \hline K_{\mathrm{sf}} & K_{\mathrm{ss}} \end{array}\right] \cdot \left[\begin{array}{c} u_{\mathrm{f}} \\ \hline u_{\mathrm{s}} \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline 0 \end{array}\right]$$

Solving for the Free DOFs vector $\vec{u_{\mathrm{f}}}$, gives:

$$[K_{\mathrm{ff}}] \cdot \vec{u_{\mathrm{f}}} + [K_{\mathrm{fs}}] \cdot \vec{u_{\mathrm{s}}} = 0$$

Where $\vec{u_{\mathrm{s}}}$ is the known vector of Fixed DOFs (Boundary Conditions). Therefore, the known part of the Equation is moved to the RHS and the Equation is rewritten as:

$$[K_{\mathrm{ff}}] \cdot \vec{u_{\mathrm{f}}} = b_{\mathrm{f}}$$

The Non-Linear Stiffness Equation for the i-th timestep is written as:

$$[K_{\mathrm{T}}] \cdot \Delta u_{\mathrm{f}} = R^i = F_{ext} - F_{int}^i$$

K_{T} is the Tangent Stiffness Matrix, and is defined as $K_{\mathrm{T}} = K_{\mathrm{L}} + K_{\mathrm{NL}}$. K_{L} is the Linear Stiffness Matrix K_{ff}, as previously defined, whereas K_{NL} is the Non-Linear Stiffness Matrix. Both matrices are calculated on a per-element basis, as a 24-by-24 Local Stiffness Matrix as follows:

$$[K_{\mathrm{L}}]_{24\times 24} = \int_V [B_{\mathrm{L}}^T]_{24\times 6}[D]_{6\times 6}[B_{\mathrm{L}}]_{6\times 24}\, dV$$
$$[K_{\mathrm{NL}}]_{24\times 24} = \int_V [B_{\mathrm{NL}}^T]_{24\times 9}[S]^i_{9\times 9}[B_{\mathrm{NL}}]_{9\times 24}\, dV$$

Where D is the Elasticity Matrix, S^i is the 2nd Piola-Kirchhoff Stress Matrix, defined as:

$$\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xx} & \tau_{xy} & \tau_{xz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xy} & \tau_{yy} & \tau_{yz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix}$$

τ_{ij} are the 2nd Piola Stresses. The Linear Deformation Matrix $[B_{\mathrm{L}}]$ and the Non-Linear Deformaiton Matrix $[B_{\mathrm{NL}}]$ are calculated as:

$$[B_{\mathrm{L}}] = \left[\begin{array}{ccc|c} \frac{\partial N}{\partial x} & 0 & 0 & \cdots \\ 0 & \frac{\partial N}{\partial y} & 0 & \cdots \\ 0 & 0 & \frac{\partial N}{\partial z} & \cdots \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} & 0 & \cdots \\ 0 & \frac{\partial N}{\partial z} & \frac{\partial N}{\partial y} & \cdots \\ \frac{\partial N}{\partial z} & 0 & \frac{\partial N}{\partial z} & \cdots \end{array}\right], \ [B_{\mathrm{NL}}] = \left[\begin{array}{ccc|c} \frac{\partial N}{\partial x} & 0 & 0 & \cdots \\ \frac{\partial N}{\partial y} & 0 & 0 & \cdots \\ \frac{\partial N}{\partial z} & 0 & 0 & \cdots \\ 0 & \frac{\partial N}{\partial x} & 0 & \cdots \\ 0 & \frac{\partial N}{\partial y} & 0 & \cdots \\ 0 & \frac{\partial N}{\partial z} & 0 & \cdots \\ 0 & 0 & \frac{\partial N}{\partial x} & \cdots \\ 0 & 0 & \frac{\partial N}{\partial y} & \cdots \\ 0 & 0 & \frac{\partial N}{\partial z} & \cdots \end{array}\right]$$