18.8.23 Preparing graph states with better binomial factors

From 9.8.23 it seems we can't make a convergent sequence for graph states. But we can improve the binomial factors beyond

$|+\rangle\rangle|+\rangle\rangle|+\rangle\rangle \longrightarrow \sum_k \sqrt{\binom{N}{k}}^3 |kkk\rangle$

by doing a better state preparation than). The $H_{123} = S^x_1 + S^x_2 + S^x_3 - N$ Hamiltonian is basically this makes a coupled spin.

Lets first see what state doing iterations of $H^x = S^x_1 + S^x_2 + S^x_3 - N$
$H^z = S^z_1 + S^z_2 + S^z_3 - N$

converge to. we can't get stable convergence for this ↑ but for

$H^x = S^x_1 + S^x_2 + S^x_3$ with even N convergence to $\sum_{perm} \pm |012\rangle$
$H^z = S^z_1 + S^z_2 + S^z_3$

reshuffle (6 terms)

is found

Then we could do $S^z_1 + S^z_2 = 1$ and $S^z_2 + S^z_3 = 1$? we would need to do a modN addition which is a bit more realistic.

So $\xrightarrow{H^x, H^z} |012\rangle + |021\rangle + |102\rangle + |120\rangle + |201\rangle + |210\rangle$

$\longrightarrow |012\rangle + |120\rangle + |201\rangle$ $\quad k_1+k_2+k_3 = 6 \quad (N=4)$

For larger N it is more complicated but it is like
since $S^z_1 + S^z_2 + S^z_3 = 2k_1 + 2k_2 + 2k_3 - 3N = 0$ so $k_1+k_2+k_3 = \frac{3}{2}N = 6$ $(N=4)$ OK

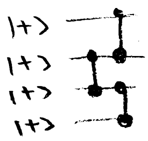
So $\sum_{k_1+k_2+k_3 = \frac{3}{2}N} |k_1 k_2 k_3\rangle a_{k_1k_2k_3}$ more or less similar Order.

For the $M=4$ case we may have found a convergent state!

Using
$H = (S^x_1 - S^x_2 + S^x_3 - S^x_4)$ $\quad H = S^z_1 - S^z_2$
$H = (S^z_1 - S^z_2 + S^z_3 - S^z_4)$ $\quad H = S^z_3 - S^z_4$

No... This is probably just $|EPR\rangle\rangle|EPR\rangle\rangle$ = DUH.
For $N=1$ $(|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$

Better use proper basis. Also this is basically sequential so we should do it like the circuit

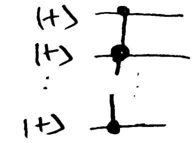


The correlations are
$Z_1 Z_2 \quad X_1 X_2 Z_3 \quad Z_2 X_3 X_4 \quad Z_3 Z_4$
$H = (S^z_1 - S^z_2) \quad H = S^x_1 + S^x_2 + S^z_3 + N \quad H = S^z_2 + S^x_3 + S^x_4 \quad H = S^z_3 - S^z_4$

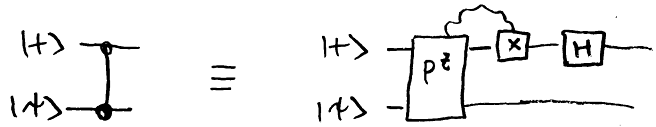
NO The idea is to replace $|+\rangle\rangle^{\otimes M} \rightarrow$ Better initial state before the circuit to remove the binomials

Ok the procedure is:

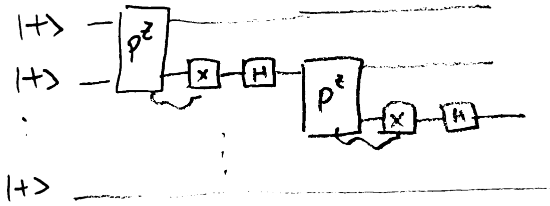
① The Cluster state circuit is



We know that



So lets map



② The thing with above is that when we map to ensembles we will have a lot of binomial factors. Each P^z produced $Q^{z^2} = \sum_k \binom{N}{k} |k\rangle\langle k|$
So the idea is to replace $|+\rangle\rangle^{\otimes M} \longrightarrow$ another state.

③ A particularly clean state would be $\left(\sum_k |k\rangle\right)^{\otimes M}$. But we don't necessarily need a separable state as all correlations will be produced by subsequent QND!

Ok what seems to work is:

Iterate $\quad H^x = S_1^x - S_2^x + S_3^z - N$

$\qquad\qquad H^+ = -S_2^z + S_3^x - S_4^x + N$

This converges <u>deterministically</u> to a state.

Then apply $\quad H_{12} = S_1^z - S_2^z \qquad H_{34} = S_3^z - S_4^z \qquad$ <u>single shot</u>.

What is the convergent state?

Probably $\quad |+\rangle\rangle \left(a_1^{+z} a_2^{+x} + b_1^{+z} b_2^{+x} \right)^N |0\rangle \; |+\rangle\rangle$

For $N=1$ $\quad (|0\rangle + |1\rangle)\left(|0\rangle|+\rangle + |1\rangle|-\rangle \right)(|0\rangle + |1\rangle)$

$= (|0\rangle + |1\rangle)\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle \right)(|0\rangle + |1\rangle)$

so it should have 4 terms with a minus sign

Numerically it seems like $\quad (|0\rangle + |1\rangle)|00\rangle(|0\rangle + |1\rangle) \qquad$ Looks roughly correct

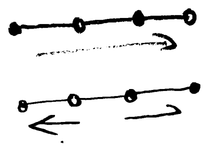
Then $P_{12}^z \; P_{34}^z \longrightarrow |0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle$

So I anticipate the state is $\quad \sum_k \sqrt{\binom{N}{k}} |k\rangle \sum_{k'} |k'\rangle^{(z)} |k'\rangle^{(x)} \sum_{k''} \sqrt{\binom{N}{k''}} |k''\rangle \longrightarrow \sum_{k', k''} \sqrt{\binom{N}{k}} |kk\rangle^{(z)} \, {}^{(z)}\langle k'' | k'\rangle^{(x)} \, |k''\rangle^{(z)} |k''\rangle^{(z)} \sqrt{\binom{N}{k''}}$

This doesn't seem that amazing .. or is it? In the GHZ case what we seemed to do is to prepare $\sum_k \sqrt{\binom{N}{k}} |kkk\rangle$ instead of $\sum_k \sqrt{\binom{N}{k}}^3 |kkk\rangle$

But we could equally do this by doing $|EPR\rangle\rangle |+\rangle \xrightarrow{P^z_{23}} \sum_k \sqrt{\binom{N}{k}} |kkk\rangle$

So why don't we do $|EPR\rangle\rangle |EPR\rangle\rangle \to |C_4\rangle$?

Because we can't apply CZ. Can only grow trees  or

as we are using QND.

But we are just using the z state as an initial state?

Just try $(|0+\rangle + |1-\rangle)(|0+\rangle + |1-\rangle) \xrightarrow{P^z_{23}} |000+\rangle + |100+\rangle + |011-\rangle - |111-\rangle$

$\xrightarrow{H_4} |0000\rangle + |1000\rangle + |0111\rangle - |1111\rangle$

$\xrightarrow{H_2} |0+00\rangle + |1+00\rangle + |0-11\rangle - |1-11\rangle$

$\xrightarrow{P^z_{12}} |0000\rangle + |1100\rangle + |0011\rangle + |1111\rangle$ Arg!

Or try again $(|00\rangle + |11\rangle)(|00\rangle + |11\rangle) \xrightarrow{P^z_{23}} |0000\rangle + |1111\rangle \xrightarrow{H_3} |00+0\rangle + |11-1\rangle$

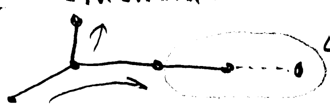
$\xrightarrow{P_{34}} |0000\rangle - |1111\rangle \xrightarrow{P^z_{23}} |+00+\rangle + |-11-\rangle$ same!

Ah .. actually $= (|+0\rangle + |-1\rangle)(|0+\rangle + |1-\rangle)$

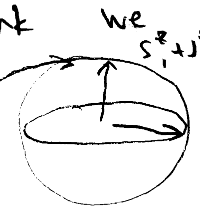
so you get a 4-qubit GHZ either way $\xrightarrow{P^z_{34}} |0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle$

say $|0000\rangle + |1111\rangle \to |00++\rangle + |11--\rangle$

Can't get this to work...

The problem is that we cannot remove the binomials at each step in the tree growth. It would be nice to do  do some iterations to clean up.

The 3 qubit branch cannot be easily implemented

For the $H = S^x_1 + S^x_2 + S^x_3 - N$ I think we can't get convergence because this is basically some spin  $S^z_1 + S^z_2 + S^z_3 = N$ $S^x_1 + S^x_2 + S^x_3 = N$

So this is not going to Have

But what about $S^z_1 + S^z_2 + S^z_3 = 0$? e.g. $\sigma^x_1 + \sigma^x_2 = 2$ $|++\rangle$

but $(\sigma^z_1 + \sigma^z_2)|++\rangle = |+-\rangle + |-+\rangle$

not eigenstate.

Unstable

What happens? $M^z_e |++\rangle = |++\rangle + \varepsilon(|+-\rangle + |-+\rangle)$

More generally we are trying to mimic $|+\rangle|+\rangle|+\rangle|+\rangle \xrightarrow{CZ_{12}\, CZ_{34}} (|+0\rangle + |-1\rangle)(|0+\rangle + |1-\rangle)$

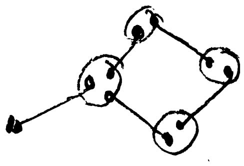
$\xrightarrow{CZ_{23}} |+00+\rangle + |+01-\rangle + |-10+\rangle - |-11-\rangle$

$= |+0\rangle(|0+\rangle + |1-\rangle) + |-1\rangle(|0+\rangle - |1-\rangle)$

or $\quad (|+0\rangle + |-1\rangle)(|+0\rangle + |-1\rangle) \xrightarrow{CZ_{23}} |+0\rangle(|+0\rangle + |-1\rangle) + |-1\rangle(|-0\rangle + |+1\rangle)$

So here CZ_{23} doesn't cause any collapse and is really a unitary ... maybe it's hard with a P^z type operation which always makes a collapse.

Maybe fusion all the way is not too bad...



so for a GHZ 

$$\left(\sum_k |k\rangle^{(z)}|k\rangle^{(x)}\right)\left(\sum_k |k\rangle^{(x)}|k\rangle^{(z)}\right) = \sum_k |k\rangle^{(z)}|kk\rangle^{(x)}|k\rangle^{(z)}$$

$$Q^{z^4}_1\left(a_1^{+(z)} a_{2a}^{+(x)} a_{2b}^{+(x)} a_3^{+(z)} + b_1^{+(z)} b_{2a}^{+(x)} b_{2b}^{+(x)} b_3^{+(z)} \right)^N |0\rangle$$

No binomials.

Also the Δ-factors will be reasonably easy to handle.

But changing basis is a bit harder?

Say we wanted to measure $\langle \sigma_2^z \rangle$ $\quad |\psi\rangle = |0++0\rangle + |1--1\rangle$

Normally: $|0+0\rangle + |1-1\rangle = |000\rangle + |101\rangle + |010\rangle - |111\rangle$

Compare to: $|0++0\rangle + |1--1\rangle = |0000\rangle + |0010\rangle + |0100\rangle + |0110\rangle$
$\qquad\qquad |1001\rangle - |1011\rangle - |1101\rangle + |1111\rangle$

there are like ?

so use a addition basis?

$0 \oplus 0 = 0 \qquad 0 \oplus 1 = 1$
$1 \oplus 1 = 0 \qquad 1 \oplus 0 = 1$

But this would need single atom resolution...

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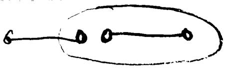
or what about projecting  3 qubit ?

From table in 9.8.23 we could get $|0-1\rangle \quad |1+1\rangle \quad |1-0\rangle \quad (|1-1\rangle)$

with special trick.

want to pick up $|0+0\rangle \quad |0-1\rangle \quad |1-0\rangle \quad |1+1\rangle$

$$\sigma_1^z + \sigma_2^x + \sigma_3^z = -1, 3$$

Ok probably doable, but we need the special trick

OK so how about
1) Prepare $|EPR\rangle|EPR\rangle$
2) Do 3 qubit proj in correct basis
3) Cycle with $H = S_3^z - S_4^z$?

Ok this is the reverse of the previous ideas. OK..

Let's use the code in 2.8.21 and modify to see how it performs

For the qubit case $(|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$

$$\xrightarrow{H_1 H_2} (|++\rangle + |--\rangle)(|00\rangle + |11\rangle) = (|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$$

Then set $-k_1 - k_2 + N = k_3$

$$\longrightarrow |00\rangle|11\rangle \xrightarrow{H_1 H_2} |++\rangle|11\rangle$$

or how about $(|00\rangle + |11\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$

$$\xrightarrow{k_2 + k_3 + k_4 = N} |00\rangle(|01\rangle + |10\rangle) + |11\rangle(|00\rangle + |11\rangle)$$

$\uparrow$ $k_2 + k_3 + k_4 = 3N$.

Actually doing the x-projections last seems to never make sense because the Z_{12} and Z_{34} can be done perfectly, so we may as well do them

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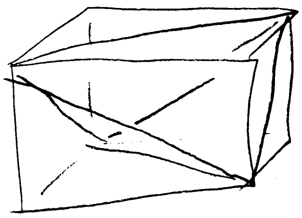
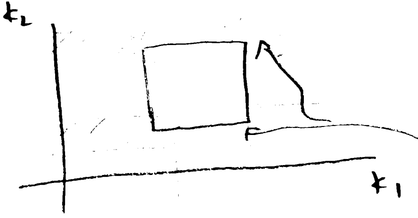
How about weak measurements?

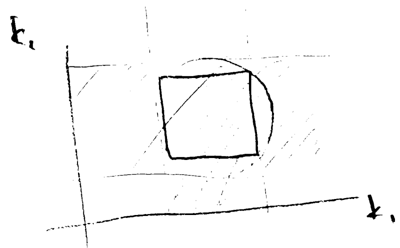


Since we want to make a tetrahedron what if we do a Gaussian along each of the sides and then a Gaussian in the middle?

e.g. say we wanted



No this will make corners



Wait, so for the ideal projection do we actually need all 4 planes? Or would it work without it?

No it seems we need more than one plane

BUT using $-k_1 - k_2 - k_3 + N = \begin{cases} 0 \\ 2 \end{cases}$ both these seems to converge to a reasonable state. The problem is again the success probability...

So aren't there other correlations? Really? The basic thing is

$|00\rangle(|00\rangle + |11\rangle) + |11\rangle(|00\rangle - |11\rangle)$

$\uparrow$

bell states entangled with pairs $|00\rangle, |11\rangle$

Need to rotate $|00\rangle \pm |11\rangle$ such that it's the same QAND form.

$$(\cos\theta|0\rangle + \sin\theta|1\rangle)(\cos\phi|0\rangle + \sin\phi|1\rangle) \pm (\sin\theta|0\rangle - \cos\theta|1\rangle)(\sin\phi|0\rangle - \cos\phi|1\rangle)$$

or $Z_1Z_2 = +1$ $\qquad Y_1Y_2(|00\rangle \pm |11\rangle) = -|00\rangle \mp |11\rangle$

$\left.\begin{matrix} X_1X_2 = \pm 1 \\ Y_1Y_2 = \pm 1 \end{matrix}\right\}$ both are state dependent. Z_1Z_2 is the only independent one

This is why we need 3rd order correlations

So for GHZ if we were to build it branch by branch

$|EPR\rangle = \sum_k |kk\rangle$ ☺

Want $(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger)^N|0\rangle \xrightarrow{x} (a_1^\dagger a_2^\dagger a_3^\dagger + a_1^\dagger b_2^\dagger b_3^\dagger + b_1^\dagger a_2^\dagger b_3^\dagger + b_1^\dagger b_2^\dagger a_3^\dagger)^N|0\rangle$

$$= (a_1^\dagger(a_2^\dagger a_3^\dagger + b_2^\dagger b_3^\dagger) + b_1^\dagger(a_2^\dagger b_3^\dagger + b_2^\dagger a_3^\dagger))^N|0\rangle$$

"Bell states entangled with another spin"

The $S_1^x + S_2^x + S_3^x = -N, -N+4$ should work because $aaa = +3$, $abb, bab, bba = -1$ $(N=1)$

For $N=2$: we will have $(aaa)^2 = +8$ $\quad (aaa)(abb) = +2$ $\quad (abb)^2 = -2$

less important $\qquad$ most important (since 3/4 terms are like this)

As N increases the importance of the $(abb)^N$ terms grows like $(\frac{3}{4})^N$.

No... actually..

Does squeezed GHZ states still have the correlations?

Say $Q_1^{z^2}(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger)^N|0\rangle = \sum_k \sqrt{\binom{N}{k}}|kkk\rangle$

In x basis: $U_{xz} Q_1^{z^2}(\quad '' \quad)^N|0\rangle = U_{xz} Q_1^{z^2} U_{xz}^\dagger U_{xz}(\quad '' \quad)^N|0\rangle$

$$= Q_1^{x^2}(a_1^{\dagger z}(a_2^{\dagger z}a_3^{\dagger z} + b_2^{\dagger z}b_3^{\dagger z}) + b_1^{\dagger z}(a_2^{\dagger z}b_3^{\dagger z} + b_2^{\dagger z}a_3^{\dagger z}))^N|0\rangle$$

Actually NO! The $Q_1^{x^2}$ introduces a lot of non-zero elements. (we can see this from 15.8.23 v5 code).

Actually, found another deterministic scheme for the GHZ.

Project on $k_1 + k_2 + k_3 - N = 2n$. $\quad n = 0, 1, 2, \dots$

No actually the one plane projection ~~DOES~~ work.

Only with $P^x_{123} = \sum_{\substack{k_1,k_2,k_3 \\ \{k_1+k_2+k_3-N\geq 0\}}} |k_1 k_2 k_3\rangle^{(x)}\langle k_1 k_2 k_3|^{(x)}$ and $P^z_{12} = \sum_k |kk\rangle\langle kk|_{12}$, P^z_{23} it works

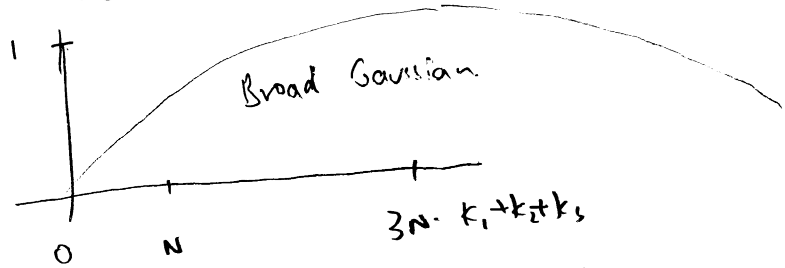
The convergence is slow but it is deterministic $(p=1)$ towards the spinor GHZ state. Just have to kill $|000\rangle^{(x)}$

For $N=1$ it means $k_1+k_2+k_3 \geq 1$.

So actually P^x_{123} does almost nothing on most states. In this way maybe a weak measurement is not a bad idea.

$C_{n_c n_d} \propto \cos^{n_c}(x)\sin^{n_d}(x)$

and we want to probably do $e^{i(S^x_1+S^x_2+S^x_3)\phi}$.



And since the P^x_{123} is the disruptive one, having a weak measurement might be better actually.

What about the 4 site cluster state?

$Z_1Z_2 \quad Z_3Z_4 \quad Z_2X_3X_4 \quad X_1X_2Z_3$

$|00\rangle(|00\rangle+|11\rangle) + |11\rangle(|00\rangle-|11\rangle)$

$|00\rangle(|00\rangle+|11\rangle) + |11\rangle(|01\rangle+|10\rangle)$

$\xrightarrow{H_3H_4} |00\rangle(|00\rangle+|11\rangle) + |11\rangle(|01\rangle+|10\rangle)$

For the GHZ: $|000\rangle+|111\rangle \xrightarrow{H_1H_2H_3} |000\rangle + |011\rangle + |101\rangle + |110\rangle$

$= |0\rangle(|00\rangle+|11\rangle) + |1\rangle(|01\rangle+|10\rangle)$.

OK so with only P^x_{123} and P^x_{234} it's too loose, we need another projector to specify it (there are two unit eigenvalues)

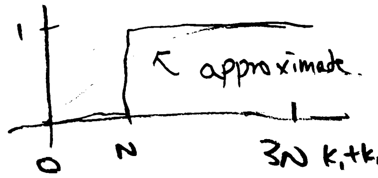
Then doing $P^x_{123} = \sum_{\substack{k_1k_2k_3k_4 \\ \{k_1+k_2+k_3-N\geq 0}} |k_1k_2\rangle^{(x)}|k_3k_4\rangle^{(z)}\langle\cdot\cdot|\langle\cdot\cdot|$ $\quad P^x_{123b} = \sum_{\substack{k_1k_2k_3k_4 \\ \{-k_1+k_2-k_3+N\geq 0\}}} |k_1k_2\rangle^{(x)}|k_3k_4\rangle^{(z)}\langle\cdot\cdot|\langle\cdot\cdot|$

$P^x_{234} = \sum_{\substack{k_1k_2k_3k_4 \\ \{k_2+k_3+k_4-N\geq 0\}}} |k_1k_2\rangle^{(z)}|k_3k_4\rangle^{(x)}\langle\cdot\cdot|\langle\cdot\cdot|$ $\quad P^z = \sum_{k\,k'} |kk\rangle^{(z)}|k'k'\rangle^{(z)}\langle\cdot\cdot|\langle\cdot\cdot|$

Seems to get the spinor $|C_4\rangle\rangle$.

I think the plan now is to use a weak measurement which approximately makes the states,

The ideal projections are like



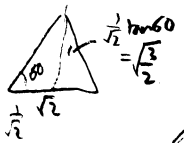
The improvement is that this should be a convergent process for this sequence since $p=1$

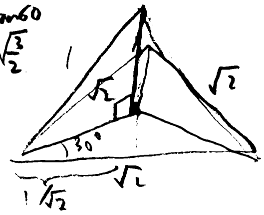
$A=\cancel{\sqrt{2}}\,\frac{1}{2}\sqrt{\frac{3}{\cancel{2}}}\ \frac{\sqrt{3}}{2}$

$h=\sqrt{1-\frac{2}{3}} = \frac{1}{\sqrt{3}}$

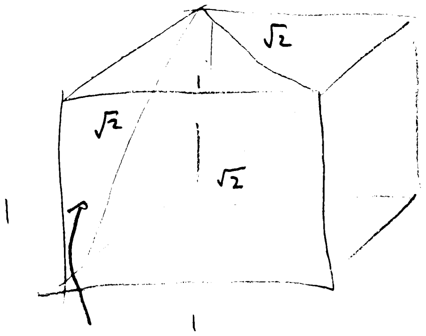
volume $=\frac{1}{3}Ah$

$=\frac{1}{3}\frac{\cancel{\sqrt{3}}}{2}\frac{1}{\cancel{\sqrt{3}}}=\frac{1}{6}$





$\frac{1}{\sqrt{2}}/\underset{\uparrow\ \sqrt{3}/2}{\cos 30^\circ} = \frac{1}{\sqrt{2}}\frac{2}{\sqrt{3}}=\sqrt{\frac{2}{3}}$



$\frac{1}{6}$ th of the points are here