2.8.21

Sequential projection to get 4 BEC cluster state

The state we want is $\left[a_1^\dagger a_2^\dagger \left(a_3^\dagger a_4^\dagger + b_3^\dagger b_4^\dagger\right) + b_1^\dagger b_2^\dagger \left(a_3^\dagger a_4^\dagger - b_3^\dagger b_4^\dagger\right)\right]^N |0\rangle$

with qubit stabilizers $Z_1 Z_2$, $Z_3 Z_4$, $X_1 X_2 Z_3$, $Z_2 X_3 X_4$

So obviously the $\Pi^{(z)}$ projection is $\Pi^{(z)} = \sum_{kk'} |k k k' k'\rangle\langle k k k' k'|$

which we can do with $\exp\left((S_1^z - S_2^z)^2 t + (S_3^z - S_4^z)^2 t \right)$.

Rotate to x-basis

$$a_3^\dagger a_4^\dagger \pm b_3^\dagger b_4^\dagger \to \frac{1}{2}(a_3^\dagger + b_3^\dagger)(a_4^\dagger + b_4^\dagger) \pm \frac{1}{2}(a_3^\dagger - b_3^\dagger)(a_4^\dagger - b_4^\dagger)$$

$$= \begin{cases} a_3^\dagger a_4^\dagger + b_3^\dagger b_4^\dagger & (+) \\ a_3^\dagger b_4^\dagger + b_3^\dagger a_4^\dagger & (-) \end{cases}$$

Rotating only 34

$$\to \left[a_1^\dagger a_2^\dagger (a_3^\dagger a_4^\dagger + b_3^\dagger b_4^\dagger) + b_1^\dagger b_2^\dagger (a_3^\dagger b_4^\dagger + b_3^\dagger a_4^\dagger) \right]^N |0\rangle$$

Rotate only 12

$$\to \left[(a_1^\dagger a_2^\dagger + b_1^\dagger b_2^\dagger) a_3^\dagger a_4^\dagger + (a_1^\dagger b_2^\dagger + b_1^\dagger a_2^\dagger) b_3^\dagger b_4^\dagger \right]^N |0\rangle$$

Rotate 12 and 34

$$\to \left[\frac{1}{2}\left((a_1^\dagger a_2^\dagger + b_1^\dagger b_2^\dagger)(a_3^\dagger + b_3^\dagger)(a_4^\dagger + b_4^\dagger) + (a_1^\dagger b_2^\dagger + b_1^\dagger a_2^\dagger)(a_3^\dagger - b_3^\dagger)(a_4^\dagger - b_4^\dagger) \right) \right]^N |0\rangle$$

(all 16 terms)

Probably the "only 34" and "only 12" are more useful. These have only 4 out of 16 potential terms

These would have the $Z_2 Z_3 Z_4$ and $Z_1 Z_2 Z_3$ type of stabilizers just like the GHZ states!

Actually it has the same form but with either BEC 1 or 4 repeated.

e.g. GHZ $\left(a_1^\dagger a_2^\dagger a_3^\dagger + a_1^\dagger b_2^\dagger b_3^\dagger + b_1^\dagger a_2^\dagger b_3^\dagger + b_1^\dagger b_2^\dagger a_3^\dagger \right)^N |0\rangle$

$\downarrow$

"only 12": $\left(a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger + a_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger + b_1^\dagger a_2^\dagger b_3^\dagger b_4^\dagger + b_1^\dagger b_2^\dagger a_3^\dagger a_4^\dagger \right)^N |0\rangle$

same same same same.

The projector sequence for this then must be.

$$\Pi^{(X_{12})} = \sum_{\substack{k_1 k_2 k_3 k_4 \\ \left\{ \begin{smallmatrix} (-k_1-k_2+k_3+N\geq 0) \text{ AND } (-k_1+k_2-k_3+N\geq 0) \\ \text{AND } (k_1+k_2+k_3-N\geq 0) \text{ AND } (k_1-k_2-k_3+N\geq 0) \end{smallmatrix} \right\}}} |k_1 k_2 k_3 k_4\rangle\langle k_1 k_2 k_3 k_4|$$

$$\Pi^{(X_{34})} = \sum_{\substack{k_1 k_2 k_3 k_4 \\ \left\{ \begin{smallmatrix} (-k_4-k_3+k_2+N\geq 0) \text{ AND } (-k_4+k_3-k_2+N\geq 0) \\ \text{AND } (k_4+k_3+k_2-N\geq 0) \text{ AND } (k_4-k_3-k_2+N\geq 0) \end{smallmatrix} \right\}}} |k_1 k_2 k_3 k_4\rangle\langle k_1 k_2 k_3 k_4|$$

$(k_3 \to k_2 \quad k_4 \to k_1$
$k_1 \to k_4 \quad k_2 \to k_3)$

Then try $\left(\Pi^{(z)}\Pi^{(X_{34})}\Pi^{(X_{12})}\right)^h \Pi^{(z)} \, |\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\rangle^{\otimes 4}$.

What is the cluster state in the Fock basis?

$$\sum_k \binom{N}{k} (a_1^\dagger a_2^\dagger)^k (b_1^\dagger b_2^\dagger)^{N-k} (a_3^\dagger a_4^\dagger + b_3^\dagger b_4^\dagger)^k (a_3^\dagger a_4^\dagger - b_3^\dagger b_4^\dagger)^{N-k} |0\rangle$$

$$= \sum_k \left(\sum_l \binom{k}{l} (a_3^\dagger a_4^\dagger)^l (b_3^\dagger b_4^\dagger)^{k-l} \right) \left(\sum_{l'} \binom{N-k}{l'} (a_3^\dagger a_4^\dagger)^{l'} (-1)^{N-k-l'} (b_3^\dagger b_4^\dagger)^{N-k-l'} \right) |kk00\rangle$$

$$= \sum_k \sum_{ll'} \binom{k}{l}\binom{N-k}{l'} (a_3^\dagger a_4^\dagger)^{l+l'} (b_3^\dagger b_4^\dagger)^{N-l-l'} (-1)^{N-k-l'} |kk00\rangle$$

$$= \sum_{kll'} \binom{k}{l}\binom{N-k}{l'} (l+l')! \, (N-l-l')! \, (-1)^{N-k-l'} |k, k, l+l', l+l'\rangle$$

$$= \sum_{kll'} \frac{\binom{k}{l}\binom{N-k}{l'}}{\binom{N}{l+l'}} (-1)^{N-k-l'} |k, k, l+l', l+l'\rangle$$

$k=k_1 \quad k_2=k=k_1 \quad k_3=l+l' \quad k_4=k_3$
$l'=k_3-l$.

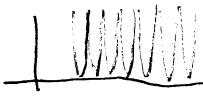
matrix element. $\langle k_1 k_2 k_3 k_4| \times$.

$$C_{k_1k_2k_3k_4} = \sum_l \frac{\binom{k_1}{l}\binom{N-k_1}{k_3-l}}{\binom{N}{k_3}} (-1)^{N-k_1-k_3+l} \, \delta_{k_1k_2}\delta_{k_3k_4}$$

See also (22.7.21)

Trying this, this totally works! COOL!

Then it's just a matter of finding a good way of implementing the projectors $\Pi^{(X_{12})}$ & $\Pi^{(X_{34})}$. How to do this? The problem is that we need to project on a <u>volume</u> not a plane. We want 

Try  $E_k = \sum_l a(k-k_l)^2$? $a \gg 1$

NO! its impossible to get this shape with only quadratic