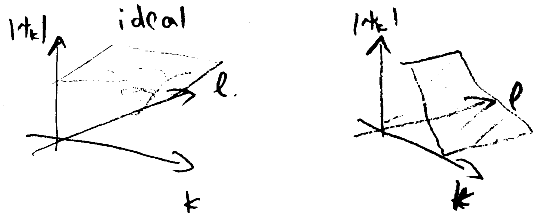
9.8.23

Preparing 4 qubit cluster state with sequence

First lets just try adapting 2.8.21's scheme like with 7.8.23, so that we project onto $k_1+k_2-N=k_3$ and $k_4-k_3+N=k_2$

This doesn't produce something nice. The non-zero coefficients of the cluster state are $\sum_{kl} \psi_{kl} |kkll\rangle$

quite different shape



I think we want something we we grow the cluster state anyway.

How about

$|EPR\rangle$
$H^z = (S_1^z - S_2^z)^2$
$H^x = (S_1^x - S_2^x)^2$

$|GHZ\rangle$
$H^z = (S_1^z - S_2^z)^2 + (S_2^z - S_3^z)^2$
$H^x = (S_1^x + S_2^x + S_3^x + N)^2$

apply some sequence here?. involving 3 qubits since The stabilizers are like Z_1Z_2, Z_2Z_4, $X_1X_2Z_3$, $Z_2X_3X_4$

What does work much better is to use project on all 4 sides of the Tetrahedron.

$$P^z = \sum_{\substack{k_1k_2k_3 \\ \{k_1+k_2-N=k_3,\ -k_1+k_2+N=k_3 \\ -k_1-k_2+N=k_3,\ k_1-k_2+N=k_3\}}} |k_1k_2k_3\rangle^{(x)} \langle k_1k_2k_3|^{(x)}$$

But how to do this? I think the only way is to implement a Hamiltonian like

$$H = (k_1+k_2-N-k_3)^2 + (-k_1+k_2+N-k_3)^2 + (-k_1-k_2+N-k_3)^2 + (k_1-k_2+N-k_3)^2$$

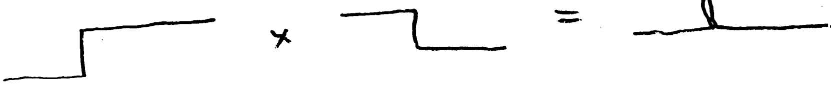
or

$$H = |k_1+k_2-N-k_3| + |-k_1+k_2+N-k_3| + |-k_1-k_2+N-k_3| + |k_1-k_2+N-k_3|$$

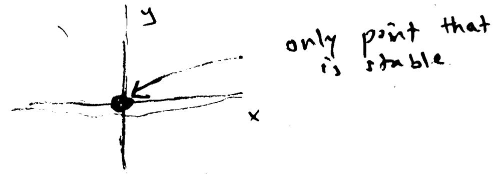
OK what happens if we do e^{-x} and e^{+x} obviously

$e^{-x}e^{+x} = 1$. so everywhere is constant

Ah but if we have



Say we want $H = |x| + |y|$? then



Actually no. We want $x=0$ OR $y=0$

We want $H = xy$ oh .... thats bad.

In this case it is

$H = ((k_1+k_2-N)-k_3)(-(k_1+k_2-N)-k_3)((-k_1+k_2+N)-k_3)((k_1-k_2$

wait in 29.7.21 we have $S^z_1+S^z_2 \le S^z_3+N$, $S^z_1+S^z_2 \ge -(S^z_3+N)$ so $-(S^z_3+N) \le S^z_1+S^z_2 \le S^z_3+N$

also $S^z_3-N \le S^z_1-S^z_2 \le -S^z_3+N$

so $$H = \left((S^z_1+S^z_2)-(S^z_3+N)\right)\left((S^z_1+S^z_2)+(S^z_3+N)\right)\left((S^z_1-S^z_2)-(S^z_3-N)\right)\left((S^z_1-S^z_2)+(S^z_3-N)\right)$$

$$= \left[(S^z_1+S^z_2)^2-(S^z_3+N)^2\right]\left[(S^z_1-S^z_2)^2-(S^z_3-N)^2\right]$$

$$= \left[(S^z_1+S^z_2)(S^z_1-S^z_2)\right]^2 - (S^z_1+S^z_2)^2(S^z_3-N)^2 - (S^z_3+N)^2(S^z_1-S^z_2)^2 + (S^z_3+N)^2(S^z_3-N)^2$$

$$= \left({S^z_1}^2-{S^z_2}^2\right)^2 + \left({S^z_3}^2-N^2\right)^2 - \left((S^z_1+S^z_2)^2(S^z_3-N)^2+(S^z_1-S^z_2)^2(S^z_3+N)^2\right)$$

Too complex

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what basis is the GHZ relative to the cluster state

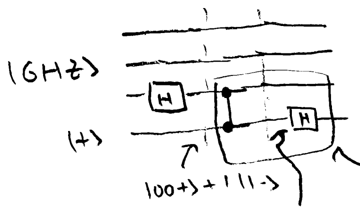
Previously we were working with $(a^\dagger_1 a^\dagger_2 a^\dagger_3 + b^\dagger_1 b^\dagger_2 b^\dagger_3)^N|0\rangle$

which is like Z_1Z_2 Z_2Z_3 $X_1X_2X_3$

Here we have $\left(a^\dagger_1 a^\dagger_2(a^\dagger_3 a^\dagger_4 + b^\dagger_3 b^\dagger_4) + b^\dagger_1 b^\dagger_2(a^\dagger_3 a^\dagger_4 - b^\dagger_3 b^\dagger_4)\right)^N|0\rangle$

Z_1Z_2 Z_2Z_4 $X_1X_2Z_3$ $Z_2X_3X_4$

So if we do



$|00\rangle(|00\rangle+|11\rangle) + |11\rangle(|00\rangle-|11\rangle)$ OK.

this is like P^z

$|000\rangle|+\rangle + |001\rangle|-\rangle + |110\rangle|+\rangle - |111\rangle|-\rangle =$

so we just need to do H_3

Can't stumble upon something that works .. Why did the GHZ work with just one P^x anyway? Qubit case:

$$H^x = (S^x_1+S^x_2+S^x_3+N)^2$$

$$(-S^x_1-S^x_2+S^x_3+N)^2$$

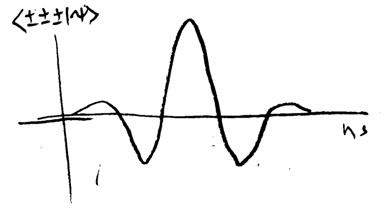
$$(-S^x_1+S^x_2-S^x_3+N)^2$$

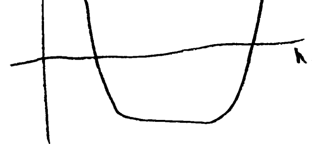
ground states: $a|011\rangle^{(x)} + b|101\rangle^{(x)} + c|110\rangle^{(x)}$

$a|101\rangle + b|011\rangle + c|000\rangle$

$a|000\rangle + b|110\rangle + c|011\rangle$

In the z-basis the eigenstate look like





Lets look at alternative P^x evolutions for the GHZ state.

Playing with $H^x = (S^z_1 + S^z_2 + S^z_3 + l)^2$ ~~It~~ seems $l = N$ is the only one that works well

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So the only thing that really can work is to apply stabilizers in different sites. Let's work in the original basis

<u>M=2</u>

stabilizers : $X_1 Z_2 \quad Z_1 X_2$

Hamiltonians : $H_1 = (S^x_1 - S^z_2)^2 \quad H_2 = (S^z_1 - S^x_2)^2$

$$\longrightarrow (a^{+x}_1 a^{+z}_2 + b^{+x}_1 a^{+z}_2)^N |0\rangle$$
$$= \sum_k |k\rangle^{(x)} |k\rangle^{(z)}$$
$$= \sum_k |k\rangle^{(z)} |k\rangle^{(x)}$$

<u>M=3</u>

stabilizers : $X_1 Z_2 \quad Z_1 X_2 Z_3 \quad Z_2 X_3$

Hamiltonians : $H_1 = (S^x_1 - S^z_2)^2 \quad H_2 = (S^z_1 + S^x_2 + S^z_3 + N)^2 \quad H_3 = (S^z_2 - S^x_3)^2$

$$\longrightarrow \sum_k \sqrt{\binom{N}{k}} |k\rangle^{(x)} |k\rangle^{(z)} |k\rangle^{(x)}$$

<u>M=4</u>

stabilizers : $X_1 Z_2 \quad Z_1 X_2 Z_3 \quad Z_2 X_3 Z_4 \quad Z_3 X_4$

Hamiltonians : $H_1 = (S^x_1 - S^z_2)^2 \quad H_2 = (S^z_1 + S^x_2 + S^z_3 + N)^2 \quad H_3 = (S^z_2 + S^x_3 + S^z_4 + N)^2$

$H_4 = (S^z_3 - S^x_4)^2$

OH... maybe our previous basis was wrong. its XXZ not XXX

OR another method. Do like 26.3.21 and do inside-out.

First prepare $|+\rangle\rangle\, |EPR\rangle\rangle\, |+\rangle\rangle \xrightarrow{H_3} |+\rangle\rangle\, (a^{+z}_2 a^{+x}_3 + b^{+z}_2 b^{+x}_3)^N |+\rangle\rangle$

$$= |+\rangle\rangle \sum_k |k\rangle^{(z)} |k\rangle^{(x)} |+\rangle\rangle \xrightarrow{P^z_{12}} \sum_k \sqrt{\binom{N}{k}} |k\rangle^{(z)} |k\rangle^{(z)} |k\rangle^{(x)} |+\rangle\rangle$$

$$= Q^{z^2} (a^{+z}_1 a^{+z}_2 a^{+x}_3 + b^{+z}_1 b^{+z}_2 b^{+x}_3)^N |+\rangle\rangle$$

$$= Q^{z^2}_1 \left((a^{+z}_1 a^{+x}_2 + b^{+x}_1 b^{+z}_2) a^{+z}_3 + (a^{+z}_1 a^{+z}_2 - b^{+z}_1 b^{+x}_2) b^{+z}_2 \right)^N |+\rangle\rangle$$

$$= Q^{z^2}_1 \left((a^{+z}_1 a^{+x}_2 + b^{+x}_1 b^{+z}_2) a^{+z}_3 + (a^{+z}_1 a^{+x}_2 - b^{+z}_1 b^{+z}_2) b^{+z}_3 b^{+z}_4 \right)^N |0\rangle$$

$$= Q^{z^2}_1 Q^{z^2}_4 \left((a^{+z}_1 a^{+x}_2 + b^{+z}_1 b^{+z}_2) a^{+z}_2 a^{+z}_4 + (a^{+z}_1 a^{+x}_2 - b^{+z}_1 b^{+z}_2) b^{+z}_3 b^{+z}_4 \right)^N |0\rangle$$

Ok lets redo the code to fix up the basis.

Doing a sequence like $P^z_{12} P^z_{34} (P^x_{123} P^x_{234})^n |+_0\rangle$ this superficially seem to work to make a C_4 cluster state $\times \binom{N}{k_1}\binom{N}{k_4}$

BUT! I think the probabilities actually matter. Even for the GHZ case, the probability ~~of~~ of $P^z P^x$ (ie. eigenvalue) is not 1. This means we will not get this outcome with $p=1$

$\Rightarrow$ NOT deterministic

But lets say we think like, combine it with the U_C in ITE. We actually choose which are eigenstates by applying a U_C if its not the desired eigenvalue.

So instead of targetting $S^z_1 + S^x_2 + S^z_3 + N = 0$ why can't we just target all $S^z_1 + S^x_2 + S^z_3 + N \geq 0$? This is just a change in U_C?

How about we 1) measure $S^z_1 + S^x_2 + S^z_3 + N$. 2) Apply U_C if ≤ 0 otherwise do nothing

What kind of operator does this make?

Simpler example. What are the eigenstates of S^z, $U_C(S^z \geq 0)$

This is $M_k = |k\rangle\langle k|$ $\quad U_C = \sum_{k<\frac{N}{2}} |k\rangle^{(z)}\langle k|^{(z)} + \sum_{k\geq \frac{N}{2}} |k\rangle^{(x)}\langle k|^{(z)}$

Then
$$U_C M_k = \begin{cases} |k\rangle^{(x)}\langle k|^{(z)} & k \geq \frac{N}{2} \\ |k\rangle^{(z)}\langle k|^{(z)} & k < \frac{N}{2} \end{cases}$$

Eigenvecto: $U_C M_k |k\rangle^{(x)} = \lambda_k |k\rangle^{(x)}$, $\lambda_k < 1$. $\quad U_C M_k |k\rangle^{(z)} = |k\rangle^{(z)}$, $p = 1$

Ok so actually what will happen is that any $k < \frac{N}{2}$ outcome will collapse to $|k\rangle$. So we will not have a superposition stabilized like $\sum_{k<\frac{N}{2}} \psi_k |k\rangle$.

Doing the above procedure will collapse to a random value of $S^z_1 + S^x_2 + S^z_3 + N$. So it just raises the probability.

Ok lets look at the state again

$$(a^{x+}_1 a^{z+}_2 a^{x+}_3 + b^{x+}_1 b^{z+}_2 b^{x+}_3)^N |0\rangle$$

or $\frac{1}{\sqrt{2}}(|+0+\rangle + |-1-\rangle) = |C_3\rangle$

In this basis

off-diagonal: $Z_1 X_2 Z_3$

diagonal: $X_1 Z_2 \quad Z_2 X_3$

Go to other basis

$|000\rangle + |001\rangle + |100\rangle + |101\rangle$
$+ |010\rangle - |011\rangle - |110\rangle + |111\rangle$
$= |0+0\rangle + |0-1\rangle + |1-0\rangle + |1+1\rangle$

$$(a^{z+}_1 a^{x+}_2 a^{z}_3 + a^{z+}_1 b^{x+}_2 b^{z+}_3 + b^{z+}_1 b^{x+}_2 a^{z+}_3 + b^{z+}_1 a^{x+}_2 b^{z+}_3)^N |0\rangle$$

state	GHZ?	$\sigma^z_1 + \sigma^x_2 + \sigma^z_3$	
0+0	✓	3	
0+1	✗	1	
0−0	✗	1	
0−1	✓	−1	←
1+0	✗	1	
1+1	✓	−1	←
1−0	✓	−1	←
1−1	✗	−3	

Ok targets 3 out of 4 states

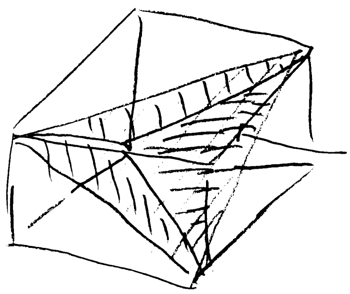
We could also use the natural double peak. and via

$$\sigma^z_1+\sigma^x_2+\sigma^z_2-1 = \pm 2$$

and get all the peaks

OK plotting the function $E = -S^z_1 - S^z_2 - S^z_3 - N$ makes clear why.

7.8.23 works. In the



The function makes one side have $E=0$. Then $E=-2$ so

So this makes a large number, but not all the points be included in the state. We can improve this slightly by doing

$$\boxed{H = -S^z_1 - S^z_2 - S^z_3 - N + 2} = \begin{matrix}-2\\+2\end{matrix}$$

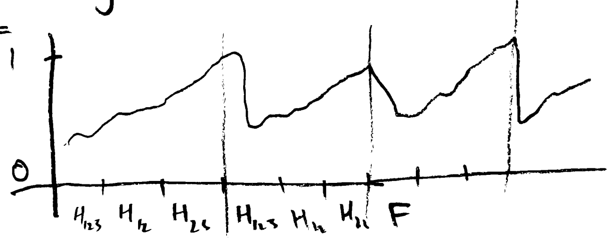
which includes to bottom two layers, not just one

$$H = -2k_1+2k_2+2k_3-4N+2 =$$

$$\text{or } H = -2k_1-2k_2-2k_3+2N+2 = \begin{matrix}-2\\+2\end{matrix}$$

Also tried a projector based ITE but proper convergence is NOT attainted because the H_{123} hamiltonian does NOT have the target state as an eigenstate

what happens is that



so everytime H_{123} comes along this get disrupted.

It does not fully converge since the Hamiltonians don't all have the GHZ state as the target state

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So I think we are stuck! This won't work like the EPR state because:

- We need all Hamiltonians to have a common eigenstate
- We really need 3rd order interactions, like $S^x_1 S^x_2 S^x_3$ for this $\sum_k a_k |kkk\rangle \longrightarrow |k+1\ k+1\ k+1\rangle$
- But we can only make 3rd order with long interaction times, but this is probably hard
- Doing the approximation with $H = -S^z_1 - S^+_2 - S^+_3 - N$ does work

but it is actually not so different to just starting in $|+\rangle|+\rangle|+\rangle$ and then hoping for a $\Delta=0$ outcome in the P_{12} and P_{23} projections. There is an advantage in terms of the binomial factor. $\sum_k \sqrt{\binom{N}{k}}\,|kkk\rangle$ vs $\sum_k \sqrt{\binom{N}{k}}^{3}\,|kkk\rangle$

So the only thing that has come from this is:

- Making $|GHZ\rangle$ and above with a convergent sequence like $|EPR\rangle$ looks impossible without going to long interaction times
- Using a single-shot method, we can improve the binomial coefficient by initially preparing in $S_1^x+S_2^x+S_3^x=N$.

We will need to account for the imperfect $\Delta_{12}\,\Delta_{23}$ outcomes

So we will get $\sum_k |k\rangle|k+\Delta\rangle|k+\Delta'\rangle\, a_k$

NEXT: Let's play with various initializations like $S_1^x+S_2^x+S_3^y=N$ or $S_1^x+S_2^x+S_3^x+S_4^x=0$ to see if we get a good binomial factor a_k.