7.8.23

Variances for various GHZ states

To have the most stable state, probably we want an even distribution of variances. E.g. $|\frac{1}{\sqrt2},\frac{1}{\sqrt2}\rangle\rangle$ has $Var(S^z) = N\,var(\sigma^z) = Var(S^y)$
$= N$
$Var(S^x) = 0$

General GHZ state

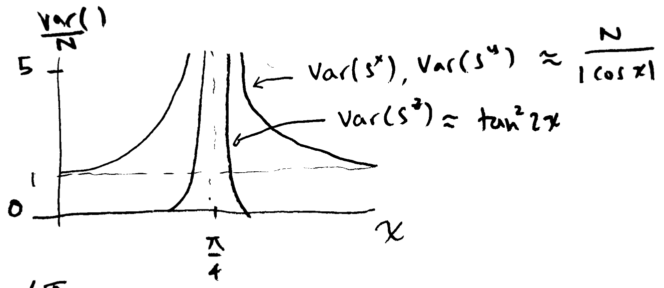
$$\frac{\left(\left(\prod_{n=1}^{M} a_n^\dagger\right) + \left(\prod_{n=1}^{M} b_n^\dagger\right)\right)^N}{\sqrt2}|0\rangle = \frac{1}{\sqrt{2^N}}\sum_{k=0}^{N}\binom{N}{k}\prod_{n=1}^{M} a_n^{\dagger k} b_n^{\dagger N-k}|0\rangle$$

$$= \frac{1}{\sqrt{2^N}}\sum_{k=0}^{N}\binom{N}{k}\sqrt{k!(N-k)!}^{\,M}\,|\underbrace{k\ldots k}_{M}\rangle$$

$$\propto \sum_{k=0}^{N}\sqrt{k!(N-k)!}^{\,M-2}\,|k\ldots k\rangle$$

Normalization $\mathcal{N} = \sum_{k=0}^{N}(k!(N-k)!)^{M-2}$

For $M=2$ we calculated already that



But at $x=\frac{\pi}{4}$ $Var(\tilde{S}^j_n) = \frac{1}{3}N(N+2)$

So although $Var(S^{x,y}) > Var(S^z)$ $x\neq\frac{\pi}{4}$

At $x=\frac{\pi}{4}$ they are the same!

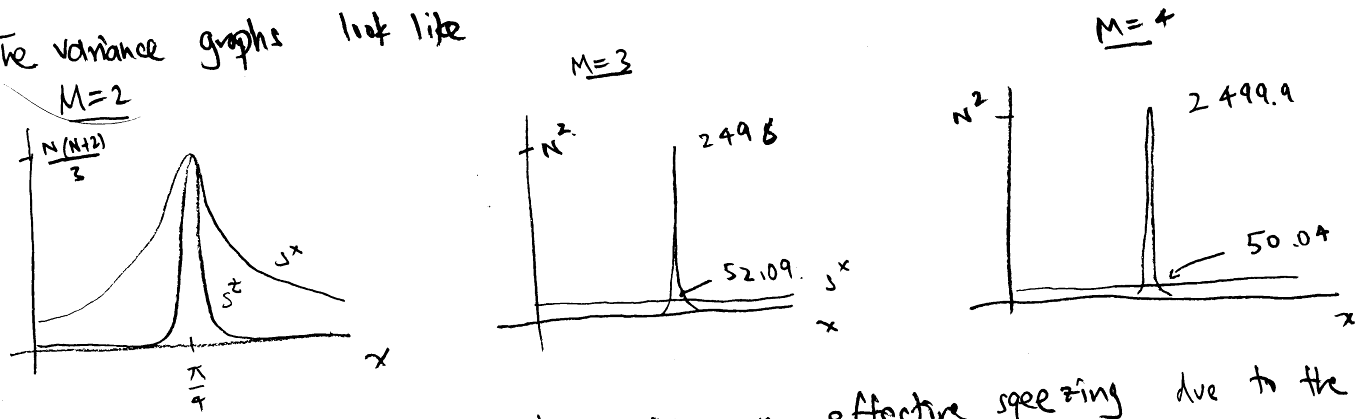
So for the maximally entangled state $M=2$, the spinor state is most symmetrical.

Ok so we need to do $|\theta\rangle\rangle = \left(\cos x\left(\prod_{n=1}^{M} a_n^\dagger\right) + \sin x\left(\prod_{n=1}^{M} b_n^\dagger\right)\right)^N|0\rangle$

$$\propto \sum_{k=0}^{N}\cos^k x\,\sin^{N-k}x\,\sqrt{k!(N-k)!}^{\,M-2}\,|k\ldots k\rangle$$

$$\mathcal{N} = \sum_{k=0}^{N}\cos^{2k}x\,\sin^{2N-2k}x\,[k!(N-k)!]^{M-2}$$

The variance graphs look like



Ok so as suspected there is an effective squeezing due to the bosonic effect.

How to counter the 'bosonic' effect?

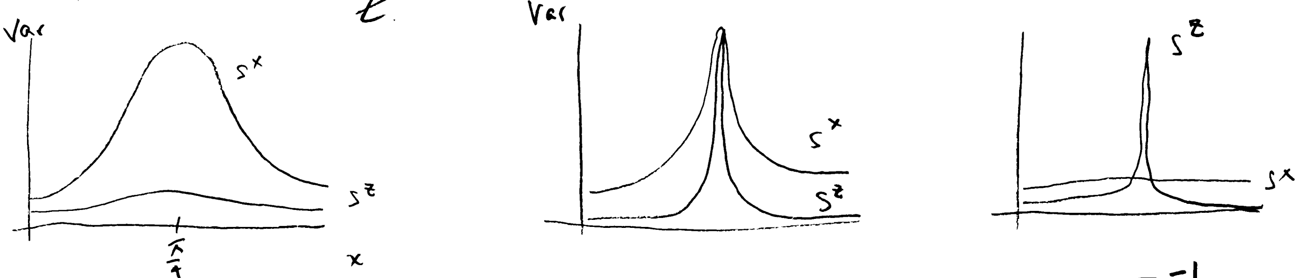
First lets apply $Q^z = \sum_k \sqrt{\binom{N}{k}}|k\rangle\langle k|$. Try $(Q^z)^{M-1} = \sqrt{N!}^{\,M-1}\sum_k \sqrt{k!(N-k)!}^{\,-M+1}|k\rangle\langle k|$

$$= \sqrt{N!}\sum_k \frac{1}{\sqrt{k!(N-k)!}}|k\rangle\langle k|$$

$$Q^{z^{M-1}} |\theta\rangle\rangle \propto \sum_{k=0}^{N} \cos^k x \sin^{N-k} x \sqrt{\binom{N}{k}} |k \cdots k\rangle$$

normalized

For all M:



$\sqrt{\binom{N}{k}}$ $\quad$ 1 $\quad$ $\sqrt{\binom{N}{k}}^{-1}$

$$Q^{z^{M-2}} |\theta\rangle\rangle \propto \sum_{k=0}^{N} \cos^k x \sin^{N-k} x \, |k \cdots k\rangle$$ seems most sensible actually.

Lets look at how to make various cluster states

M=3

: $\frac{1}{\sqrt{2}}(|+0+\rangle + |-1-\rangle)$

According to the above we want $Q_1^x (a_1^{x+} a_2^{z+} a_3^{x+} + b_1^{x+} b_2^{z+} b_3^{x+})^N |0\rangle$
or Q_2^z or Q_3^x

How to make this? If we did

$$\tilde{P}_{12}^z \hat{P}_{23}^z |+\rangle\rangle |+\rangle\rangle |+\rangle\rangle = \sum_k \sqrt{\binom{N}{k}}^3 |kkk\rangle = \sum_k \underbrace{\sqrt{\binom{N}{k}}^3 \frac{1}{\sqrt{k!(N-k)!}^3}}_{\binom{N}{k}^3} (a_1^\dagger a_2^\dagger a_3^\dagger)^k (b_1^\dagger b_2^\dagger b_3^\dagger)^{N-k} |0\rangle$$

$$= Q_1^{z^4} \sum_k \binom{N}{k} (a_1^\dagger a_2^\dagger a_3^\dagger)^k (b_1^\dagger b_2^\dagger b_3^\dagger)^{N-k} |0\rangle$$

$$= Q_1^{z^4} (a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger)^N |0\rangle$$

So each new node gets a Q^{z^2} with respect to the spinor state

Intuitively we should do:
1) Prepare $(a_1^\dagger a_2^\dagger + b_1^\dagger b_2^\dagger)^N |0\rangle = \sum_k |kk\rangle$
2) Add $\sum_k \sqrt{\binom{N}{k}} |k\rangle$
3) $\tilde{P}^z$ $\quad \sum_k \sqrt{\binom{N}{k}} |kkk\rangle$

How to get rid of $\sqrt{\binom{N}{k}}$?

For spinor GHZ $\quad H = (4+N) S_1^z S_2^z + S_1^x S_2^x S_3^x - S_1^y S_2^y S_3^x - S_1^x S_2^y S_3^y - S_1^y S_2^x S_3^y$

$(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger)^N |0\rangle$

One way is to fuse  $\quad \sum_k |kk\rangle \sum_{k'} |k'k'\rangle \xrightarrow{P^z} \sum_k |kkk\rangle$

Want: $\frac{1}{\sqrt{N+1}} \sum_k |k\rangle^{(x)} |k\rangle^{(z)} |k\rangle^{(x)} = Q_1^x (a_1^{x+} a_2^{z+} a_3^{x+} + b_1^{x+} b_2^{z+} b_3^{x+})^N |0\rangle$

Or just change basis for now

$$\frac{1}{\sqrt{N+1}} \sum_k |kkk\rangle = Q_1^z \left(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger \right)^N |0\rangle$$

$$= Q_1^z \sum_k \binom{N}{k} (a_1^\dagger a_2^\dagger a_3^\dagger)^k (b_1^\dagger b_2^\dagger b_3^\dagger)^{N-k} |0\rangle$$

$$= Q_1^z \sum_k \binom{N}{k} \sqrt{k!(N-k)!}^{\,3} |kkk\rangle = \sum_k \sqrt{\binom{N}{k}}^{3} \sqrt{k!(N-k)!}^{\,3} |kkk\rangle$$

$$= \sqrt{N!}^{3} \sum_k |kkk\rangle \qquad \text{OK.}$$

We need some symmetries.

x basis $\quad Q_1^z \left[a_1^\dagger a_2^\dagger a_3^\dagger + a_1^\dagger b_2^\dagger b_3^\dagger + b_1^\dagger a_2^\dagger b_3^\dagger + b_1^\dagger b_2^\dagger a_3^\dagger \right]^N |0\rangle$

This state has a symmetry like $Z_1 Z_2 Z_3 = +1$

or for the spins $\mathrm{mod}_2(k_1+k_2+k_3) = 0$ if we say $a=0$, $b=1$

We would need to do the Manikandan type long interaction method to get this. (have to try to see whether it would work)

So probably 2 ways: ① Fusion of Bell pairs

② Convergent method with high interactions.

It would be interesting to see if ② works but actually ① might be simpler, although more wasteful.

e.g. $$\sum_k |kkkk\rangle = \sum_k \binom{N}{k}^2 (a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger)^k (b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger)^{N-k} |0\rangle$$

$$= Q_1^{z^2} \left(a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger \right)^N |0\rangle$$

Looking back at what I did before: The relevant notes are

29.7.21 Sequential... 10.8.22 Stabilizing... 21.8.22 Making a GHZ

23.9.22 Lanve.. 06.8.21 Making 02.8.21 Sequential..

Of these, there are 2 successful approaches it seems

In 29.7.21 sequential projections to get GHZ state v6, using a projection like

$$P^z = \sum_k |kkk\rangle^{(z)}\langle kkk|^{(z)} \qquad P^x = \sum_{\substack{k_1,k_2,k_3 \\ \in \text{Tetra}}} |k_1 k_2 k_3\rangle\langle k_1 k_2 k_3| \qquad \text{Tetra} \equiv \begin{cases} k_1+k_2-N \le k_3 \\ -k_1+k_2+N \ge k_3 \\ -k_1-k_2+N \le k_3 \\ k_1-k_2+N \ge k_3 \end{cases}$$

converges to $\left(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger \right)^N |0\rangle = \sum_k \sqrt{\binom{N}{k}}^{-1} |kkk\rangle$ EXACTLY. wow.

This also works for a 4 qubit cluster state as shown in 2.8.21.

In 10.8.21 Stabilizing and 21.8.22 Making I do a sequence like $e^{-(S_1^z - S_2^z)^2}\ e^{-\sin^2(\frac{\pi}{a}(S_1^x+S_2^x+S_3^x))}$ and this seems to converge to a state like $\sum_k \psi_k |kkk\rangle$ but there is a lot of freedom in ψ_k still

So lets take the code from 10.8.22 Stabilizing v1 and see what we get for our case

Playing with this, something interesting happens. Iterating

$$\exp\left[-\left((s^z_1-s^z_2)^2+(s^z_1-s^z_3)^2+(s^z_2-s^z_3)^2\right)t\right] \qquad t \gg 1$$

$$\exp\left[-(s^x_1+s^x_2+s^x_3+N)^2 t\right]$$

seems to produce something decent. Maybe approximately

$$\sum_k \binom{N}{k}^{1/4} |kkk\rangle$$

This is doable and maybe ok in terms of decoherence. Also doesn't depend on initial state $|+\rangle^{\otimes 3}$ or $|EPR\rangle|+\rangle$.

Why does this semi-work? Maybe its like the tetrahedron method

One of the criterion is $k_1+k_2+k_3-N \geq 0$

what is similar $\quad s^z_1+s^z_2+s^z_3+N = 2k_1+2k_2+2k_3-3N+N = 2(k_1+k_2+k_3-N)$

Try the other 3 conditions, From 29.7.21

$$-s^z_1-s^z_2+s^z_3+N \geq 0$$
$$-s^z_1+s^z_2-s^z_3+N \geq 0$$
$$+s^z_1+s^z_2+s^z_3+N \geq 0$$
$$+s^z_1-s^z_2-s^z_3+N \geq 0$$

In 29.7.21 I actually say that

$$H = (-s^z_1-s^z_2+s^z_3+N)^2+(-s^z_1+s^z_2-s^z_3+N)^2+(s^z_1+s^z_2+s^z_3+N)^2+(s^z_1-s^z_2-s^z_3+N)^2-3s^z_1s^z_2s^z_3$$

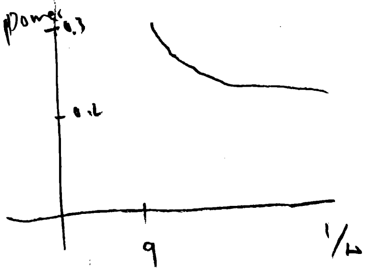
is an OK way to do thinks

It seems using only one of the 4, as well as the z-z projector works best.

N	Fidelity	Similar func
3	0.991	$\binom{N}{k}^{0.175}$
4	0.978	$\binom{N}{k}^{0.22}$
5	0.958	$\binom{N}{k}^{1/4}$
6	0.931	$\binom{N}{k}^{0.26}$
8	0.8	$\binom{N}{k}^{1/2}$
9	?	$\binom{N}{k}^{0.36}$

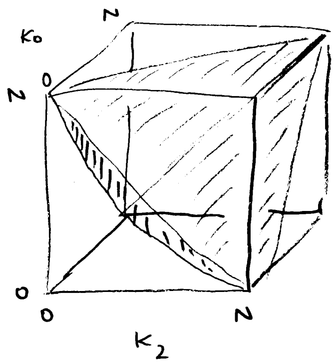
Compared to $\sum_k |kkk\rangle$

7	0.899	$\binom{N}{k}^{0.3}$



Bit hard to see what kind of function the state converges to. But it does take a form like $\sum_{k=0}^{N}\binom{N}{k}^p |kkk\rangle$

Can we derive the state? It must be a state that satisfies
$-S^z_1 - S^z_2 + S^z_3 + N = 0$. From 29.7.21



The way to get it is to find the largest eigenvalue of
$$P^z P^x |\lambda\rangle = \lambda |\lambda\rangle$$

$$P^z = \sum_k |kkk\rangle^{(z)} \langle kkk|^{(z)} \qquad P^x = \sum_{k_1 k_2} |k_1 k_2\ k_3 = N-k_1-k_2\rangle^{(x)} \langle k_1 k_2\ k_3 = N-k_1-k_2|^{(x)}$$

so $$P^z P^x = \sum_{k k_1 k_2} |kkk\rangle^{(z)} \langle k_1 k_2\ k_3 = N-k_1-k_2|^{(x)}\ {}^{(z)}\langle kkk| k_1 k_2\ k_3 = N-k_1-k_2\rangle^{(x)}$$

Applying $$|\lambda\rangle = \sum_{k'} a_{k'} |k'k'k'\rangle^{(z)}$$

So $$P^z P^x |\lambda\rangle = \sum_{\substack{k k_1 \\ k_2 k'}} |kkk\rangle^{(z)} a_{k'} \langle k_1 k_2\ k_3 = N-k_1-k_2|^{(x)} |k'k'k'\rangle^{(z)}\ {}^{(z)}\langle kkk| k_1 k_2 k_3 = N-k_1-k_2\rangle^{(x)}$$

So we need $$\sum_{k_1 k_2 k'} a_{k'} \langle k_1 k_2 k_3 = N-k_1-k_2|^{(x)} k'k'k'\rangle \langle kkk| k_1 k_2 k_3 = N-k_1-k_2\rangle^{(x)} \propto a_k.$$

ie. the matrix $$\sum_{k_1 k_2} {}^{(z)}\langle kkk| k_1 k_2 k_3 = N-k_1-k_2\rangle^{(x)} \langle k_1 k_2 k_3 = N-k_1-k_2| k'k'k'\rangle^{(z)} = A_{kk'}$$

$$A_{kk'} = \sum_{k_1 k_2} {}^{(z)}\langle k|k_1\rangle^{(x)}\ {}^{(x)}\langle k_1|k'\rangle\ {}^{(z)}\langle k|k_2\rangle^{(x)}\ \langle k_2|k'\rangle\ {}^{(z)}\langle k| k_3 = N-k_1-k_2\rangle^{(x)}\ {}^{(x)}\langle k_3 = N-k_1-k_2|k'\rangle$$

Just code up this matrix. OK this is much more efficient
(v8 code) Numerically this seem to approach.

$$\boxed{(P^z P^x)^n |\psi_0\rangle \to \sum_k \sqrt{\binom{N}{k}}\ |kkk\rangle}$$

OK COOL. This is something

$F \approx 0.99982$ at $N=40$

To do: • Imaginary time preparation of this state
• C_4 preparation.

Spechulation: So the sequential method produces $|EPR\rangle = (a^\dagger_1 a^\dagger_2 + b^\dagger_1 b^\dagger_2)^N |0\rangle = \sum_k |kk\rangle$
Just a QND would do $Q_1^{z^2} |EPR\rangle$
For $M=3$ we get $\sum_k \sqrt{\binom{N}{k}} |kkk\rangle = Q_1^{z^2} (a^\dagger_1 a^\dagger_2 a^\dagger_3 + b^\dagger_1 b^\dagger_2 b^\dagger_3)^n |0\rangle$
Just a QND would do $\sum_k \sqrt{\binom{N}{k}}^3 |kkk\rangle = Q_1^{z^4} (a^\dagger_1 a^\dagger_2 a^\dagger_3 + b^\dagger_1 b^\dagger_2 b^\dagger_3)^n |0\rangle$