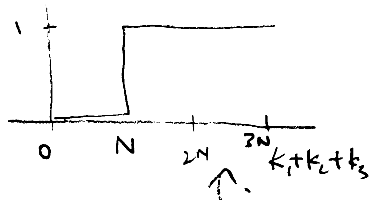
23.8.23

Using weak measurements to make a GHZ state

Lets make a sequence involving an approximation to the step function that we need as part of the GHZ spinor preparation.



We know that doing a sequence of

$$P^z = \sum_k |kkk\rangle\langle kkk| \qquad P^x = \sum_{\substack{k_1 k_2 k_3 \\ \{k_1+k_2+k_3 \geq N\}}} |k_1 k_2 k_3\rangle^{(x)} \langle k_1 k_2 k_3|^{(x)}$$

converges to a spinor GHZ state. Without the P^x, the state converges to $|+\rangle\rangle|+\rangle\rangle|+\rangle\rangle \xrightarrow{P^z} \sum_k \sqrt{\binom{N}{k}}^3 |kkk\rangle$

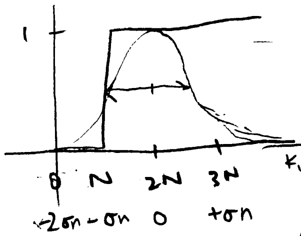
Using the above, we get $(a_1^\dagger a_2^\dagger a_3^\dagger + b_1^\dagger b_2^\dagger b_3^\dagger)^N |0\rangle = \sum_k \sqrt{\binom{N}{k}}^{-1} |kkk\rangle$ (18.8.23)

Using $P^x = \sum_{\{k_1+k_2+k_3=N\}} |k_1 k_2 k_3\rangle^{(x)} \langle k_1 k_2 k_3|^{(x)}$ we get $\approx \sum_k \sqrt{\binom{N}{k}} |kkk\rangle$ (7.8.22)

Now lets try a $P^x = \sum_{k_1 k_2 k_3} C_{n_c n_a}(\tau(k_1+k_2+k_3) + b) |k_1 k_2 k_3\rangle^{(x)} \langle k_1 k_2 k_3|^{(x)}$ ↖ QND C-function

and choose parameters of the C-function so that it is like the step function and just deal with projectors.

First lets not worry about probabilities



Where to fit the Gaussian? How about

$$C_{n_c n_a}(x) \propto \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)$$

$\sigma = \frac{1}{\sqrt{2(n_c+n_a)}}$

$x_0 = \frac{1}{2}\arccos\left(\frac{n_c-\ldots}{n_c+\ldots}\right)$

(from Mani's paper)

so we need $-2\sigma n \leq (k_1+k_2+k_3-2N)\tau \leq \sigma n$

n = no. of sigmas

so $-2\sigma n = -2N\tau$, $\tau = \frac{\sigma n}{N}$

$N\tau = \sigma n$, $\tau = \frac{\sigma n}{N}$ ok.

$$\frac{C_{n_c n_a}(x+x_0)}{C_{n_c n_a}(x_0)} = \exp\left[-\frac{x^2}{2\sigma^2}\right]$$

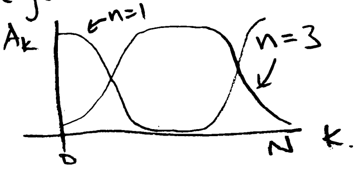
so $x = (k_1+k_2+k_3-2N)\frac{\sigma n}{N}$

$e^{-\frac{4N^2}{2N^2}} = e^{-2} \rightarrow e^{-\frac{1}{2}}$

Use function $\exp\left[-\frac{x^2}{2\sigma^2}\right] = \exp\left[-\frac{n^2(k_1+k_2+k_3-2N)^2}{2N^2}\right]$

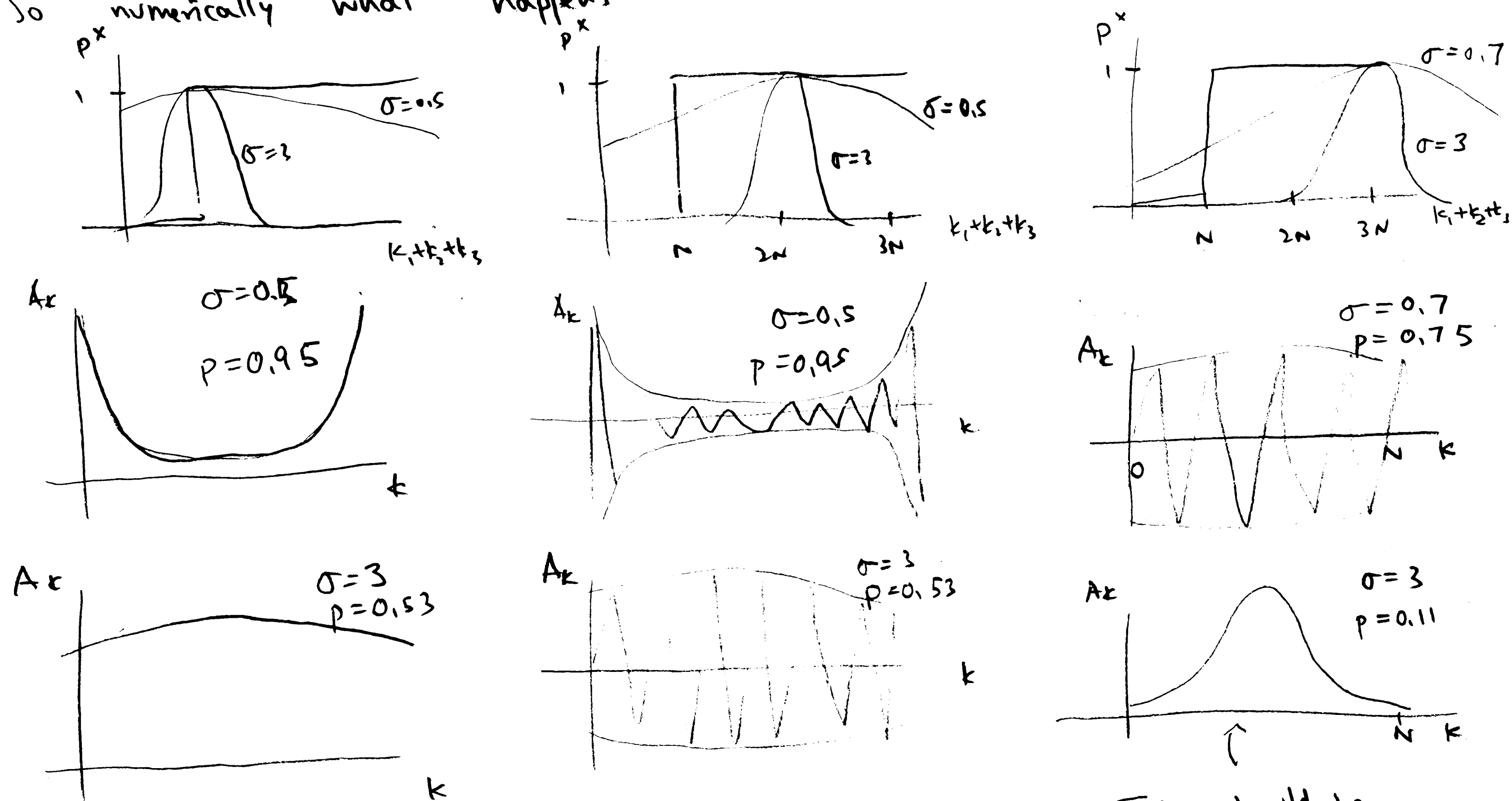
$$\approx \frac{C_{n_c n_a}\left((k_1+k_2+k_3-2N)\frac{\sigma n}{N} + x_0\right)}{C_{n_c n_a}(x_0)}$$

Ok doing this we get convergence to a state of the form $\sum_k A_k |kkk\rangle (-1)^k$ that is like



For some reason there is a factor of $(-1)^k$ which can be eliminated by a $e^{i s_1^z \frac{\pi}{2}}$ rotation.

So numerically what happens is that

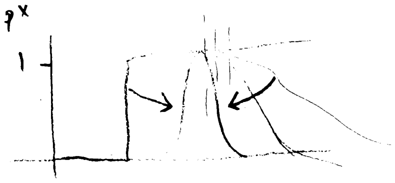


This should be something like $\sum_k^3 \sqrt{\binom{N}{k}}|kkk\rangle$ I guess?

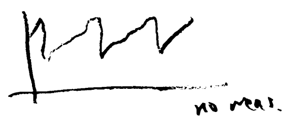
So none of these work spectacularly well. The tighter gaussians produce better states but the probabilities are smaller.

The problem still remains that it will not be determistic because the p^x measurements will come out randomly.

What will happen? After some sequence of measurements, the cumulative effect will result in a Gaussian that is at some position



when its in the right spot we can swap to the p^z but that will be stochastic too. So this will be jumpy F



Perhaps we just stop when F is high? e.g. we keep repeating until we get $\Delta^z=0$ and $\Delta^x=0$ consecutively?

For now lets just look at the ideal case and check it works for all graph states