Washington University in St. Louis

JAMES MCKELVEY SCHOOL OF ENGINEERING

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Hot Dog Heat Transfer

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INTRODUCTION

The objective of this project was to determine the ideal conditions to cook a hot dog using coals in the minimum amount of time based on certain, given constraints and other experimentally controlled variables. The solution itself was done in two ways. It was done analytically and numerically. The analytical solution involved a series solution found in Section 5.6 of the Heat Transfer Textbook [1]. The numerical solution involved a finite difference method with nodal approximation [1]. The setup to cook the hot dog was a coal bed with evenly spaced coals on the bottom and a single long cylindrical hot dog being rotated as seen in Fig. 1 below.

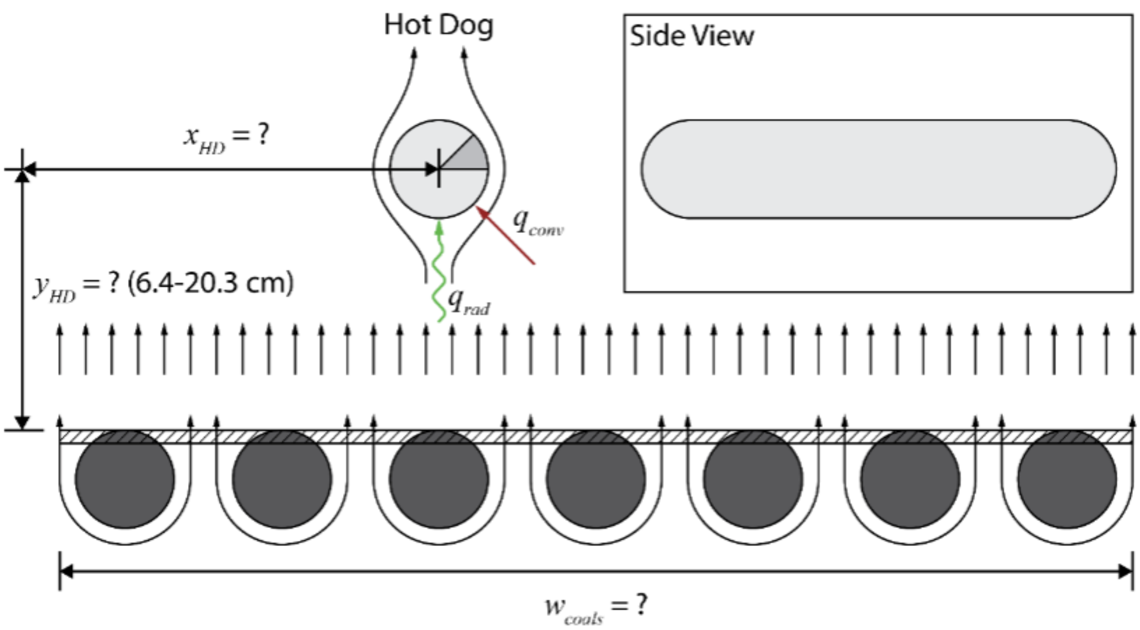

Figure 1 Diagram of Hot Dog on Grill.

The distance on the grill and above the grill will be found based on the ideal cooking conditions to achieve the perfect hot dog.

INITIAL CONDITIONS & ASSUMPTIONS

In order to use the analytical and numerical methods to solve for time, properties of the hot dog, surrounding air, and coals had to be established and assumed. These properties are listed below.

(1) The initial temperature of the hot dog is 10 °C.

(2) The surface temperature of the hot dog can not exceed 100 °C.

(3) The final temperature of the centerline of the hot dog is 68 °C.

(4) When calculating the properties of gas, it is assumed to have air-like behavior.

(5) The air surrounding the hot dog is at 250 °C.

(6) The coals are at 450 °C.

(7) The emissivity of the coals is 0.8.

(8) The emissivity of the hot dog is 0.45.

(9) The hot dog is rotated so that there is uniform heating.

(10) The hot dog had a conduction coefficient of .52 $\frac{W}{m*K}$.

(11) The hot dog had a density of 880 $\frac{kg}{m^3}$.

(12) The hot dog had a c_p value of 3350 $\frac{J}{kg*K}$

(13) The hot dog had a diameter of 2.54 cm.

(14) The coals cover approximately half the surface area of the grill bed.

(15) The hot dog is a gray body.

The user-controlled variables that could be adjusted to attain a solution within the constraints were as follows.

(1) The distance between the charcoal and the hot dog

(2) The hot dog's location over the coals

(3) The cooking time

(4) The diameter of the coals

METHODS

0.1 Analytical Method. The Analytical Solution used a one term approximation for a transient infinite cylinder. We were given placeholder radiation and convective heat transfer coefficients. We solved for the true convective and radiation heat transfer coefficients in the Numerical Solution. These two heat transfer coefficients are combined into the total heat transfer coefficient, h_{tot}. Equation 1 is used to calculate the biot number of the hot dog.

$$Bi = \frac{h_{tot}\frac{D}{2}}{k} \tag{1}$$

In the above equation, h_{tot} is the total heat transfer coefficient, D is the diameter of the hot dog, and k is the thermal conductivity of the hot dog. Equation 2 solves the roots, ζ, of the transient infinite cylinder[1].

$$\zeta_n = Bi\frac{J_0(\zeta_n)}{J_1(\zeta_n)} \tag{2}$$

where Bi is the Biot number calculated in Eq. 1, J_1 and J_0 are Bessel Functions, and ζ_n is the transcendental root we are solving for. Using MATLAB, we solved for ζ_n with a numerical solution solver given an input guess and used 100 ζ_n values for each time step of the solution. Using ζ_n we can solve for the infinite cylinder coefficients, C_n, with Equation 3.

$$C_n = \frac{2}{\zeta_n} * \frac{J_1(\zeta_n)}{J_0^2(\zeta_n) + J_1^2(\zeta_n)} \tag{3}$$

Equation 3 finds the coefficients given ζ_n values for the exact solution of the infinite cylinder at each second of cooking the hot dog. The Fourier number was needed and calculated in Equation 4 for each second of cooking the hot dog.

$$Fo = \frac{kt}{\rho c_p (\frac{D}{2})^2} \tag{4}$$

In Equation 4, k is the hot dog's thermal conductivity, t is the time in seconds, ρ is the density of the hot dog, c_p is the constant pressure heat capacity of the hot dog, and D is the hot dog's diameter. To get the temperature at each time at the surface and center of the hot dog, we can solve for the maximum temperature difference ratio in Equation 5 [1].

$$\theta^* = \sum_{n=1}^{\infty} C_n exp(-\zeta_n^2 Fo) J_o(\zeta_n r^*) \tag{5}$$

where r^* is a dimensionless spatial coordinate that displays the relative length away from the center and can be found by $r^* = \frac{r}{r_o}$. θ is the maximum ratio between air temperature, the initial temperature of the hot dog, and the current temperature of the hot dog and is found with equation 6.

$$\theta^* = \frac{T - T_{\text{inf}}}{T_i - T_{\text{inf}}} \tag{6}$$

T is the current temperature at a location on the hot dog, T_{inf} is the surrounding air temperature of the hot dog, and T_i is the initial temperature of the hot dog before it is cooked. Using the 100 terms of 5, we solved for equations of the surface temperature and center line temperature of the hot dog in Equations 7 and 8.

$$\theta^*_{s,i+1} = \theta^*_{s,i} + C_n exp(-\zeta_n^2 Fo) J_o(\zeta_n) \tag{7}$$

$$\theta^*_{c,i+1} = \theta^*_{c,i} + C_n exp(-\zeta_n^2 Fo) \tag{8}$$

Equation 7 is the surface temperature of the hot dog and r^* is 1. Equation 8 is the center line temperature of the hot dog and r^* is 0. In Matlab, we iterated $\theta^*_{s,i+1}$ and $\theta^*_{c,i+1}$ until we reached our desired internal and external temperature of the hot dog. Using Eq. 6, we calculated the surface and center line temperature of the hot dog for every second.

0.2 Numerical Method. The numerical method of solving involved going through the process with changing heat transfer coefficients for both convection and radiation. First, the heat transfer coefficient for convection was calculated. To do this, we started by finding the properties of air surrounding the coal. The Grashof number, which is a ratio of buoyancy to viscous forces, was calculated, as shown in 9. [1]

$$Gr = \frac{g\beta(T_{coal} - T_\infty) D_{coal}^3}{\nu^2} \tag{9}$$

In equation 9, g is the force due to gravity, β is the inverse of the film temperature of the coal, T_{coal} and T_∞ are the temperatures of the coal and surrounding air, D_{coal} is the approximated diameter of the coal being used, and ν is the kinematic viscosity of the air. The Grashof number is also the square of the Reynold's number, as shown in equation 10.

$$Re_{coal} = \sqrt{Gr} \tag{10}$$

After the Reynold's number was calculated, it was possible to calculate the velocity of the air going around the the coal. This calculation is shown in equation 11.

$$v_{coal} = \frac{Re_{coal}\nu}{D_{coal}} \tag{11}$$

Using the velocity of air going around the coal and the assumption that the velocity of air going around the hot dog is half of that going around the coal (shown in equation 12), it was possible to find the Reynold's number for the air going around the hot dog using the corresponding air properties at each time step, shown in equation 13.

$$v_{hd} = \frac{v_{coal}}{2} \tag{12}$$

$$Re_{hd} = \frac{v_{hd}D_{hd}}{\nu} \tag{13}$$

Knowing the Reynold's number of the air around the hot dog then allowed us to calculate a Nusselt number, shown in equation 14.

$$Nu = 0.3 + \frac{0.62Re_{hd}^{1/2}Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}[1 + (\frac{Re_{hd}}{282,000}^{5/8})]^{4/5} \tag{14}$$

In this equation, Re_{hd} is the Reynold's number calculated for the air surrounding the hot dog, found in equation 13. Pr is the Prandlt number for the air surrounding the hot dog. This was found by setting up a function that changes Prandtl number at each air temperature in accordance with values found in the textbook [1].

Using the Nusselt number, it was possible to find the convection heat transfer coefficient, shown in equation 15.

$$h_{conv} = \frac{Nu \times k_{air}}{D_{hd}} \tag{15}$$

In this equation, Nu is the Nusselt number of the air surrounding the hot dog, D_{hd} is the diameter of the hot dog, and k_{air} is the thermal conductivity coefficient. Due to the fact that the value for k_{air} changes with temperature, it was found by creating a function with a temperature input that output a corresponding k value, similarly to how the Prandtl number was found.

After solving for the heat transfer coefficient due to convection, we next solved for the radiation heat transfer coefficient to find the temperature change over time. Equations 16 the 21 were used to find the radiation heat transfer coefficient.

The hot dog being cooked was modeled as a cylinder and parallel rectangle, allowing the calculation of a view factor, shown in equations 18 and 19. In order to find the view factors, values for s_1 and s_2 first had to be found.

$$s1 = \frac{A_1}{2} \tag{16}$$

$$s2 = -\frac{A_1}{2} \tag{17}$$

In equations 16 and 17, A_1 is the width of the grill that is being used.

$$F_{12} = \frac{r_{hd}}{s_1 - s_2}[tan^{-1}\frac{s_1}{y} - tan^{-1}\frac{s_2}{y}] \tag{18}$$

$$F_{21} = F_{12}\frac{A_1}{A_2} \tag{19}$$

In equation 19, A_2 is the perimeter of the hot dog.

To find the radiation heat transfer coefficient, shown in equation 21, a value for T_2^*, or the effective temperature of the hot dog at each time step, had to be calculated first. [2]

$$T_2^* = \frac{T_2}{(\varepsilon_{coal}F_{21})^{\frac{1}{4}}} \tag{20}$$

In this equation, ε_{coal} is the emissivity value for coal, F_{21} is the view factor, and T_2 is the temperature of the hot dog. Using the assumption that the hot dog is a ”gray” body, a final equation for the radiation heat transfer could be formed, shown in equation 21.

$$h_{rad} = \varepsilon_{hd}\varepsilon_{coal}F_{21}\sigma(T_1 + T_2^*)(T_1^2 + T_2^{*2}) \tag{21}$$

In this equation, ε_{coal} and ε_{hd} are the emissivities for coal and the hot dog, σ is Boltzman's constant ($5.67 * 10^{-8} \frac{W}{m^2 * K}$), and T_1 is the temperature of the coals being used.

The hot dog can be regarded as an infinite cylinder with m finite elements. For all elements except the center element and the boundary element, the energy balance can be written as:

$$\Sigma q_i = \dot{e}V \tag{22}$$

where q_i represents the net heat transfer rate to the element. It can be decomposed into two parts, $q_{m+\frac{1}{2}}$ and $q_{m-\frac{1}{2}}$, which can be further expanded in equations 23 and 24:

$$q_{m-\frac{1}{2}} = k2\pi(m - \frac{1}{2})\Delta r H \frac{T_{m-1} - T_m}{\Delta r} = k\pi H(2m - 1)(T_{m-1} - T_m) \tag{23}$$

$$q_{m+\frac{1}{2}} = k2\pi(m + \frac{1}{2})\Delta r H \frac{T_{m-1} - T_m}{\Delta r} = k\pi H(2m + 1)(T_{m-1} - T_m) \tag{24}$$

$\dot{e}$ is the change in energy with time per unit volume, which can be represented as:

$$\dot{e} = \rho c \frac{T_m^{i+1} - T_m^i}{\Delta t} \tag{25}$$

The volume of the m^{th} element V can be determine using Equation 26, shown below.

$$V = (\pi r_{m+\frac{1}{2}}^2 - \pi r_{m-\frac{1}{2}}^2)H = \pi \Delta r^2[(m + \frac{1}{2})^2 - (m - \frac{1}{2})^2]H = 2m\pi\Delta r^2 H \tag{26}$$

Using the above equations, the general energy balance can then be written as:

$$q_{m-\frac{1}{2}} + q_{m+\frac{1}{2}} = \dot{e}V \tag{27}$$

By expanding Eq. 27 and reorganizing using **the implicit method**, the finite difference of temperature of the m-th element between each time step can be written as:

$$-(1 - \frac{1}{2m})FoT_{m-1}^{i+1} + (2Fo + 1)T_m^{i+1} - (1 + \frac{1}{2m})FoT_{m+1}^{i+1} = T_m^i \tag{28}$$

where Fo is the Fourier number in the finite time difference method:

$$Fo = \frac{\alpha \Delta t}{\Delta r^2} = \frac{k\Delta t}{\rho c \Delta r^2} \tag{29}$$

Using a similar approach, the finite time difference for the center element can be represented using Equation 30:

$$-4FoT_1^{i+1} + (4Fo + 1)T_0^{i+1} = T_0^i \tag{30}$$

The element at the boundary of the hot dog receives heat rates from its surroundings through convection. The energy balance equation is shown in Equation 31.

$$q_{M-\frac{1}{2}} + q_h = \dot{e}V \tag{31}$$

In this equation, M is the total number of finite elements desired and q_h is the total heat transfer from the surroundings. The derivation of $q_{M-\frac{1}{2}}$ is largely similar to Equation 23 and is shown in the appendix (figures 6,7, and 8.) The total heat transfer from the external environment to the cylinder is:

$$q_h = 2\pi M\Delta r H h(T_\infty - T_M) \tag{32}$$

Using equations 25-26 and the implicit method, the energy balance equation can be written as:

$$-4Fo\frac{2M-1}{4M-1}T_{m-1}^{i+1} + \frac{4(2M-1)Fo + 8MFo_h + 4M - 1}{4M-1}T_M^{i+1} = T_M^i + \frac{8M}{4M-1}Fo_hT_\infty \tag{33}$$

In the previous equation, Fo_h is used as a abbreviation for the following expression:

$$Fo_h = \frac{h\Delta t}{\rho c\Delta r} \tag{34}$$

Equations 28, 30, and 33 demonstrate for the coefficients for each finite element in the time step i+1, which can be reorganized such that the entire finite time difference for each element in one time step can be written as a matrix equation like the following:

$$[heatM]\begin{Bmatrix} T_0^{i+1} \\ T_1^{i+1} \\ T_2^{i+1} \\ \dots \\ T_m^{i+1} \end{Bmatrix} = \begin{Bmatrix} T_0^{i} \\ T_1^{i} \\ T_2^{i} \\ \dots \\ T_m^{i} \end{Bmatrix} + \frac{8M}{4M-1}Fo_hT_\infty \tag{35}$$

where $\frac{8M}{4M-1}Fo_hT_\infty$ is the additionally term from Eq.33. Therefore, the temperature of each element in the next time step can be obtained:

$$\begin{Bmatrix} T_0^{i+1} \\ T_1^{i+1} \\ T_2^{i+1} \\ \cdots \\ T_m^{i+1} \end{Bmatrix} = [heatM]^{-1}\left(\begin{Bmatrix} T_0^{i} \\ T_1^{i} \\ T_2^{i} \\ \cdots \\ T_m^{i} \end{Bmatrix} + \frac{8M}{4M-1} Fo_h T_\infty\right) \tag{36}$$

All derivations the finite temperature difference are shown in the Appendix.

RESULTS & DISCUSSION

Heat Transfer Coefficients. The calculated convection heat transfer coefficient is 9.06 $\frac{W}{m^2 \cdot K}$. The calculated radiation heat transfer coefficient is 7.64 $\frac{W}{m^2 \cdot K}$. Using these values, the final results of the analytical and numerical solutions are slightly different.

Analytical Results. The analytical solution found a temperature of 100.06°C at the surface of the hot dog, and 68.02°C at the centerline of the hot dog can be achieved at a cooking time of 455 seconds or about 7.6 minutes as seen below in Fig. 2. The graph shows the centerline and surface temperatures increasing relatively linearly after about 100 seconds of parabolic growth.

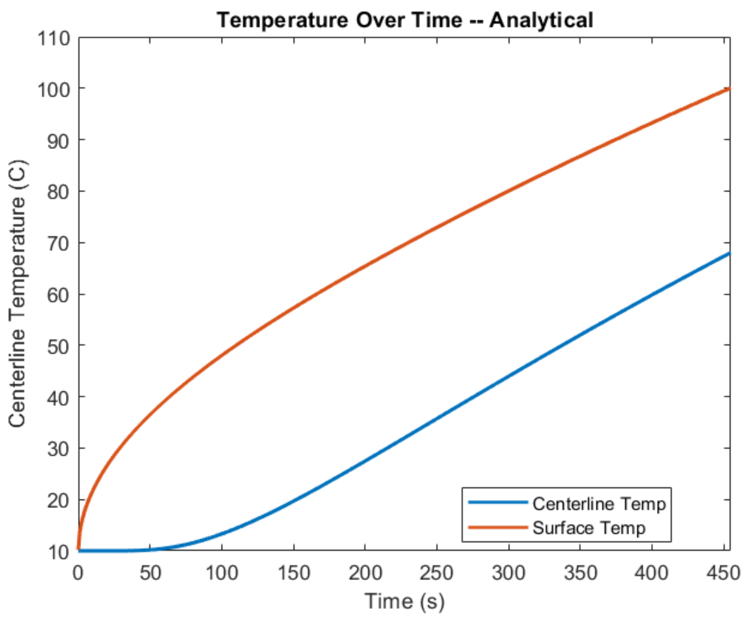

Figure 2 Graph of temperature over time for the centerline and surface for the analytical solution.

Additionally, the analytical solution found that by comparing the temperature distribution along the radius of the hot dog at 30 seconds, 120 seconds, and 455 seconds it is possible to see a clear change in the shape of the temperature distribution with radius as shown in Fig. 3 below. At 30 seconds, it is clear that the center and middle portions of the hot dog still remain pretty cold but the surface is starting to heat up. The 30-second line appears to remain mostly linear at 10°C up until about 0.008 m. At 120 seconds, it is clear that the inner surface temp is heating up but that the center of the hot dog remains around 12°C. The 120-second line appears to begin to slope up smoothly around the 0.002 m mark. At 455 seconds it is clear that the hot dog has been heated all the way through up to the given specifications. The 455-second line has a very slight upwards curvature but is relatively linear in its temperature increase with radius.

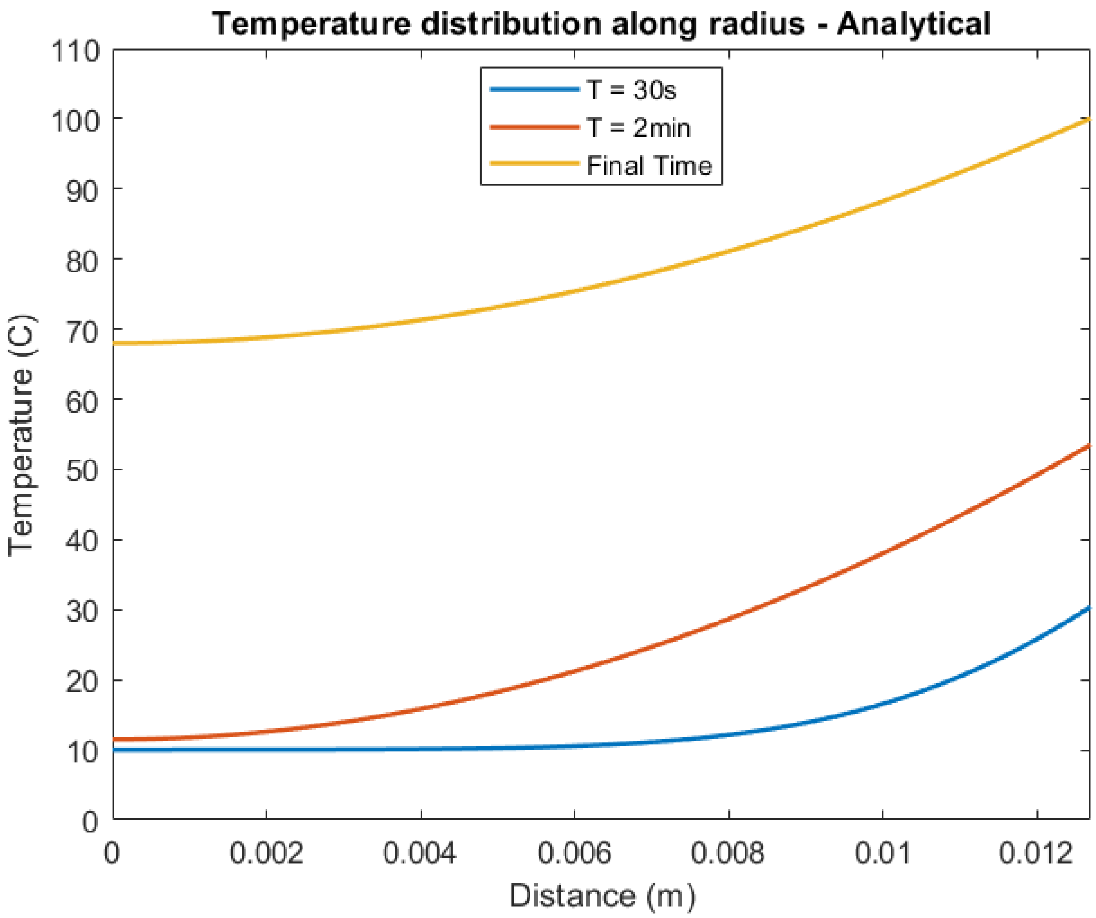

Figure 3 Graph of temperature distribution along radius for the analytical solution.

Table 1 below lists the temperature at the centerline and surface for the given time.

Table 1 Analytical results for the temperature at certain time steps.

Time(s):	Position:	Temperature (°C):
30	Centerline	10.01
30	Surface	30.37
120	Centerline	10.01
120	Surface	53.51
455	Centerline	68
455	Surface	100.07

Numerical Results. The numerical solution found a temperature of 99.86°C at the surface of the hot dog, and 68.08°C at the centerline of the hot dog can be achieved at a cooking time of 472 seconds or about 7.87 minutes as seen below in Fig. 4. Similarly to the analytical solutions graph, after about 100-110 seconds the centerline and surface temperatures increase relatively linearly.

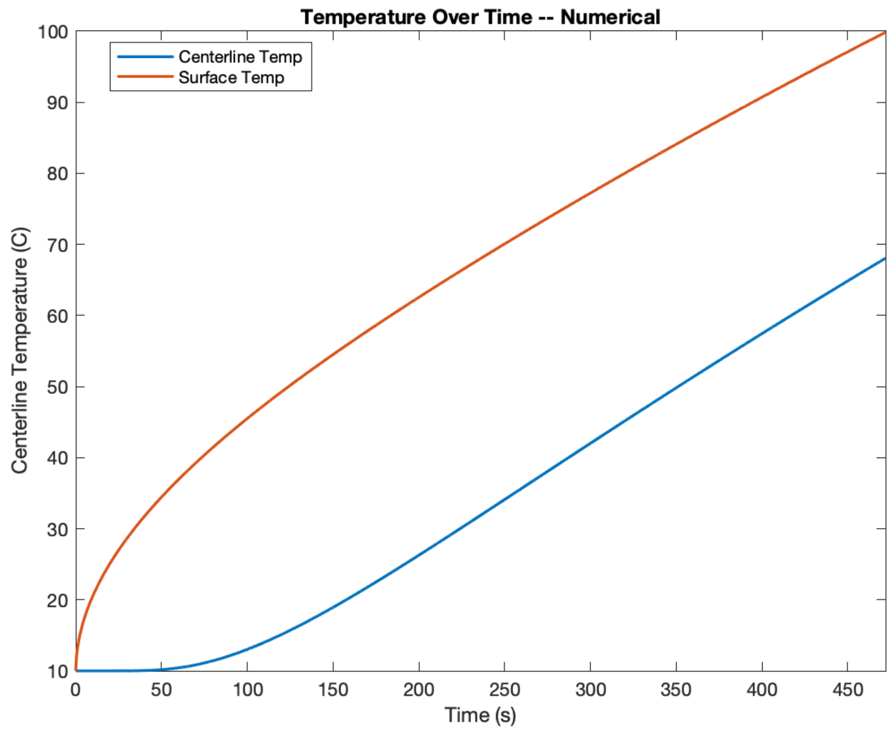

Figure 4 Temperature over time for the centerline and surface for the numerical solution.

Like the analytical solution, the temperature distribution along the radius of the hot dog changes shape with time as seen in Fig. 5 below. At 30 seconds, the center and middle portions of the hot dog up until a radius of 0.008 m remain at 10°C. At 120 seconds, the temperature at the middle and surface portions of the hot dog is roughly linearly increasing with radius. The centerline portion up until .002 m however remains at 15-16°C. At 472 seconds the temperature is at its expected values for the center line and surface. The temperature looks to change rather linearly with its increase in radius experiencing a small upwards curvature.

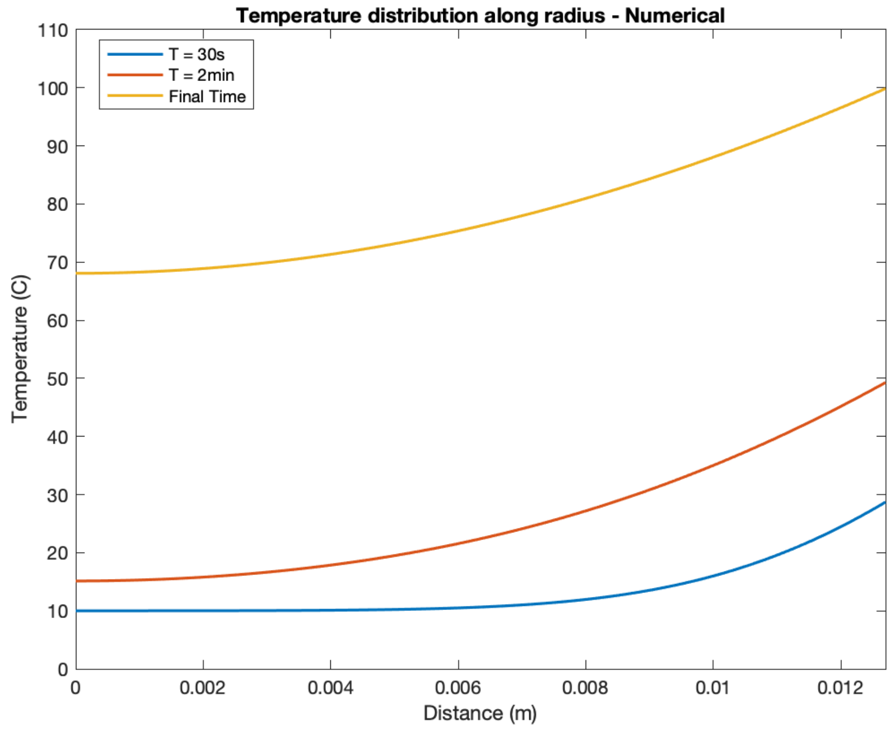

Figure 5 Graph of temperature distribution along radius for the numerical solution.

Table 2 below lists the temperature at the centerline and surface for the given time.

Table 2 Numerical results for the temperature at certain time steps.

Time(s):	Position:	Temperature (°C):
30	Centerline	10.01
30	Surface	28.71
120	Centerline	15.13
120	Surface	49.28
472	Centerline	68.08
472	Surface	99.9

Analytical and Numerical Comparison. Both the analytical and numerical solutions had similar results for the amount of time it would take to reach the desired temperatures of 68°C at the centerline and 100°C at the surface. The numerical solution had a slightly higher estimate at 472 seconds instead of the 455 that the analytical solution arrived at. The 17-second difference that arose is most likely due to the inherent error in both methods of calculation. The analytical method although very good is still a one-term approximation and as such has some small error. The numerical solution also has some error because it is an approximation based on a user-controlled time step. The graph of the temperature distribution along the radius for both of the solution methods was very similar. The only noticeable difference between the two is that for the 120-second line, the centerline temperature is measured to be 15°C whereas the analytical method has it measured at 12°C. This difference most likely comes from the fact that for the analytical method, at 120 seconds the Fourier number was still less than 0.2. The Fourier number being less than 0.2 means that the solution can not necessarily be approximated by a single term.

CONCLUSION

Based on the graphs and results discussed above, the analytical and numerical solutions were very similar. The analytical solution however is more likely to be accurate and as such it should take about 455 seconds at the discussed parameters to cook a hot dog so that its centerline temperature is 68°C and its surface temperature is 100°C. This being said, both methods are still approximations so the experimental solution may differ slightly in reality.

References

[1] Bergman, T., Lavine, A., Incropera, F., and DeWitt, D., 2011, *Fundamentals of Heat and Mass Transfer*, 7th ed., John Wiley Sons, Inc., New York City, New York.

[2] Meacham, M., 2023, "h_{rad} discussion/derivation," Class lecture at Washington University in St. Louis, St. Louis, MO.

APPENDIX

1 MATLAB CODE

Analytical Method Code

```
clc

% Heat Transfer Final Project
%% Given Parameters
% Constants
    rho = 880 ;         % kg/m3
    k = 0.52 ;         % W/mK
    c = 3350 ;         % J/kg-K
    D = .0254  ;        % m
% Iinitial Constraints
    Ti = 10; % C
    Tsmax = 100 ; % C
    Tinf = 250;  % C
    Tcoal = 450; % C
    ecoal = .8;
    ehd = .45;
    time = 0:415;
% Heat Transfer values
    hconv = 12; % W/m^2 * K
    hrad = 6;  % W/m^2 * K
    h_tot = hconv+hrad; % total effective heat transfer rate

%% Calculating Relevant Coefficients
% Biot and Fourier Number
    Bio = (h_tot*(D/2))/k ;
    Fo = ((k/(rho*c))*(time))/((D/2)^2) ;

% Find eta's
    etaLength = 100; % self-defined: multiple eta to derive correct
                      % initial temperature
    etaVec = zeros(1, etaLength);
    for n = 1:etaLength
```

```
        syms eta_n
        eqn = eta_n*besselj(1, eta_n)/besselj(0, eta_n) == Bio;
        guess = 0.1+(n-1)*pi;
        etaVec(n) = vpasolve(eqn, eta_n, guess);
    end

% Find C's
    CVec = zeros(1, etaLength); %vector of coefficients, same length
    for n = 1:etaLength
        CVec(n) = 2/etaVec(n)*besselj(1, etaVec(n))...
                    /(besselj(0, etaVec(n))^2+besselj(1, etaVec(n))^2);
    end

%% Infinite Cylinder Approximate Solution Equations
    Surface_Theta = zeros(1, length(time));
    Center_Theta = zeros(1, length(time));
    for n = 1:length(CVec)
        Surface_Theta = Surface_Theta+CVec(n)*exp(-etaVec(n)^2*Fo)...
                                        *besselj(0, etaVec(n));
        Center_Theta = Center_Theta+CVec(n)*exp((-(etaVec(n)^2)*(Fo)));
    end

% Caluclating Temperatures from thetas
    Surface_T = (Surface_Theta)*(Ti-Tinf) + (Tinf);
    Center_T = (Center_Theta)*(Ti-Tinf) + (Tinf);

%% Temp vs Time Graph
    temp_t_figure = figure;
    centerP = plot(time, Center_T, "LineWidth", 1.5);
    hold on
    surfaceP = plot(time, Surface_T, "LineWidth", 1.5);
    title('Temperature Over Time -- Analytical')
    xlabel('Time (s)')
    xlim([0 time(end)])
    ylabel('Centerline Temperature (C)')
    legend("Centerline Temp", "Surface Temp", "location", "best")
    hold off
    saveas(gcf, 'TempOverTime.png')
```

```
%% Find Relationship between Temp and r
    rStar = 0:0.01:1;
% When t = 30s
    Fo30 = Fo(time==30);
    theta30 = zeros(1, length(rStar)); %find shape distribution
    for n = 1:length(rStar)
        for m = 1:length(CVec)
            theta30(n) = theta30(n)+CVec(m)*exp(-etaVec(m)^2*Fo30)...
                                                *besselj(0, etaVec(m)*rStar(n));
        end
    end
    T30 = theta30*(Ti-Tinf)+Tinf;

% When t = 2min = 120s
    Fo120 = Fo(time==120);
    theta120 = zeros(1, length(rStar)); %find shape distribution
    for n = 1:length(rStar)
        for m = 1:length(etaLength)
            theta120(n) = theta120(n)+CVec(m)*exp(-etaVec(m)^2*Fo120)...
                                                *besselj(0, etaVec(m)*rStar(n));
        end
    end
    T120 = theta120*(Ti-Tinf)+Tinf;

% When t = final time
    FoEnd = Fo(end);
    thetaEnd = zeros(1, length(rStar)); %find shape distribution
    for n = 1:length(rStar)
        for m = 1:length(etaLength)
            thetaEnd(n) = thetaEnd(n)+CVec(m)*exp(-etaVec(m)^2*FoEnd)...
                                                *besselj(0, etaVec(m)*rStar(n));
        end
    end
    TEnd = thetaEnd*(Ti-Tinf)+Tinf;

%% Graph Temp as a function of r
    r = rStar*D/2;
```

```
temp_r_figure = figure;
T30p = plot(r, T30, "LineWidth", 1.5);
hold on
T120p = plot(r, T120, "LineWidth", 1.5);
TEndp = plot(r, TEnd, "LineWidth", 1.5);
title("Temperature distribution along radius - Analytical")
xlabel('Distance (m)')
ylabel('Temperature (C)')
legend("T = 30s", "T = 2min", "Final Time", "location", "best")
xlim([0 r(end)])
ylim([0 110])
saveas(gcf, 'TempDistribution120Seconds.png')
```

Numerical Method Code

```
clear, close all
clc

%% Given Parameters
% Constants
rho = 880 ;         % kg/m3
k = 0.52 ;        % W/mK
c = 3350 ;        % J/kg-K
D = .0254  ;       % m
r = D/2;
alpha = k/c/rho; %thermal diffusivity [m2/s]

% Initial Constraints
Ti = 10; % C
Tsmax = 100 ; % C
Tinf = 250;  % C
Tcoal = 450; % C
ecoal = .8;
ehd = .45;

% Finite elements and iterations
m = 100; % such that finite elements number from 0 to m
dr = r/m;
dt = 0.1;
Tvec = ones(m+1, 1)*Ti; %initial temperature distribution
n = 4720; %number of iteration
time = (1:n)*dt;

%% Initialization
Center_T = zeros(1,n); %Temperature of centerline
Surface_T = zeros(1,n); % Temperature of surface
T30 = zeros(1,m+1); %temperature distribution at 30s
T120 = zeros(1,m+1); %temperature distribution at 120s
TEnd = zeros(1,m+1); %final temperature
hconvvec = zeros(1,n);
```

```
hradvec = zeros(1,n);

% Properties of air around the coal -- used to calculate hconv
g = 9.81;
Tfcoal = (Tinf+Tcoal)/2;
beta = 1/Tfcoal;
nu_coal = findNu(Tfcoal); % interpolate the viscosity of air
D_coal = 0.0254; % diameter of coal -- same as hotdog
Gr = g*beta.*(Tcoal-Tinf)*D_coal^3./nu_coal^2;
Re_coal = Gr^0.5;
vaircoal = Re_coal*nu_coal/D_coal;


for nn = 1:n
    Center_T(nn) = Tvec(1);
    Surface_T(nn) = Tvec(end);%Record temp at this time step

    %% Calculate convection heat transfer coefficient
    % Things happened to the hotdog
    vairdog = vaircoal/2;
    Tfhd = (Tvec(end)+Tinf)/2; %Film temperature around hotdog
    nu_HD = findNu(Tfhd); %vsicosity around HD
    Pr = findPr(Tfhd); %Prandtl number of air
    k_air = findK(Tfhd);
    ReHD = vairdog*D/nu_HD;
    Nu = 0.3+0.62*ReHD^(1/2)*Pr^(1/3)/(1+(0.4/Pr)^(2/3))^(1/4)...
        *(1+(ReHD/282000)^(5/8))^(4/5);
    hconvvec(nn) = Nu*k_air/D;

    %% Calculate Radiation Heat Transfer Coefficients
    T1 = 450+273;
    T2 = Tvec(end)+273;
    A1 = 0.6; % width of grill
    A2 = pi*D; % perimeter of cylinder
    s1 = A1/2;
    s2 = -A1/2;
    y = 0.098; % distance from hot dog to coal
    F12 = r/(s1-s2)*(2*atan(s1/y));
```

```
    F21 = A1*F12/A2;
    T2_star = T2./(ecoal*F21)^(1/4);
    sigma = 5.67e-8;
    hradvec(nn) = ecoal*ehd*F21*sigma*(T1+T2_star).*(T1^2+T2_star.^2);

    h_tot = hconvvec(nn)+hradvec(nn);

    %% Calculate Temperature Change
    Fo = alpha*dt/dr^2;
    Foh = h_tot*dt/rho/c/dr;
    heatM = zeros(m+1,m+1);
    heatM(1, [1 2]) = heatM(1, [1 2])+[4*Fo+1 -4*Fo];
    heatM(m+1, [m m+1]) = [-4*Fo*(2*m-1)/(4*m-1)...
        (4*(2*m-1)*Fo+8*m*Foh+4*m-1)/(4*m-1)];
    for ii = 1:m-1
        heatM(ii+1, ii:ii+2) = [-(1-1/(2*ii))*Fo (2*Fo+1) -(1+1/(2*ii))*Fo];
    end

    Tvec(end) = Tvec(end)+8*m/(4*m-1)*Foh*Tinf;
    Tvec = heatM\Tvec;
    if nn*dt==30
        T30 = Tvec;
    elseif nn*dt == 120
        T120 = Tvec;
    end
end
TEnd = Tvec;
Tvec(1)
Tvec(end)

% %% Temp vs Time Graph
% temp_t_figure = figure;
% centerP = plot(time, Center_T, "LineWidth", 1.5);
% hold on
% surfaceP = plot(time, Surface_T, "LineWidth", 1.5);
% title('Temperature Over Time')
% xlabel('Time (s)')
% xlim([0 time(end)])
```

```
% ylabel('Centerline Temperature (C)')
% legend("Centerline Temp", "Surface Temp", "location", "best")
% hold off
% saveas(gcf, 'TempOverTime_num.png')
%
% %% Graph Temp as a function of r
% rvec = linspace(0,r,m+1);
% temp_r_figure = figure;
% T30p = plot(rvec, T30, "LineWidth", 1.5);
% hold on
% T120p = plot(rvec, T120, "LineWidth", 1.5);
% TEndp = plot(rvec, TEnd, "LineWidth", 1.5);
% xlabel('Distance (m)')
% ylabel('Temperature (C)')
% legend("T = 30s", "T = 2min", "Final Time", "location", "best")
% xlim([0 r])
% ylim([0 110])
% saveas(gcf, 'TempDistribution_num.png')

function nu = findNu(Temp)
    % input Temp has a unit of C and need to be converted to K
    knownTemp = 200:50:700;
    knownNu = [7.59 11.44 15.89 20.92 26.41 32.39 38.79 45.57 52.69...
        60.21 68.1]/10^6;
    nu = interp1(knownTemp, knownNu, Temp+273.15);
end

function k = findK(Temp)
    % input Temp has a unit of C and need to be converted to K
    knownTemp = 200:50:700;
    knownK = [18.1 22.3 26.3 30.0 33.8 37.3 40.7 43.9 46.9 49.7 52.4]/10^3;
    k = interp1(knownTemp, knownK, Temp+273.15);
end

function Pr = findPr(Temp)
    % input Temp has a unit of C and need to be converted to K
    knownTemp = 200:50:700;
    knownPr = [0.737 0.720 0.707 0.700 0.690 0.686 0.684 0.683 0.685...
```

```
        0.690 0.695];
    Pr = interp1(knownTemp, knownPr, Temp+273.15);
end
```

2 Entire derivation of finite temperature difference equations

Fig. 6 shows the derivation to the temperature changes of elements in the middle:

① $\sum q_i + \dot{q}\,\Delta V = \dot{e}\,\Delta V$ in General Case: elements in the middle

↳ $q_{m-\frac{1}{2}} = k\cdot 2\pi(m-\frac{1}{2})\Delta r H \frac{T_{m-1}-T_m}{\Delta r} = k\pi H(2m-1)(T_{m-1}-T_m)$

↳ $q_{m+\frac{1}{2}} = k 2\pi (m+\frac{1}{2})\Delta r H\cdot \frac{T_{m+1}-T_m}{\Delta r} = k\pi H(2m+1)(T_{m+1}-T_m)$

↳ $\dot{q} = 0$

↳ $\dot{e} = \rho c \frac{T_m^{i+1}-T_m^{i}}{\Delta t}$

↳ $\Delta V = (\pi r_{m+\frac{1}{2}}^2 - \pi r_{m-\frac{1}{2}}^2)\cdot H$

$= \left(\pi[(m+\frac{1}{2})\Delta r]^2 - \pi[(m-\frac{1}{2})\Delta r]^2\right)\cdot H$

$= \pi\Delta r^2\left((m+\frac{1}{2})^2-(m-\frac{1}{2})^2\right) H$

$= \pi \Delta r^2\, 2m\cdot H$

$= 2m\pi\Delta r^2 H$

∀ $q_{m-\frac{1}{2}}+q_{m+\frac{1}{2}} = \dot{e}\Delta V$

$\Rightarrow k\pi H(2m-1)(T_{m-1}-T_m)+k\pi H(2m+1)(T_{m+1}-T_m) = 2m\pi H\Delta r^2\cdot\rho c\frac{T_m^{i+1}-T_m^{i}}{\Delta t}$

$(2m-1)(T_{m-1}-T_m)+(2m+1)(T_{m+1}-T_m) = 2m\Delta r^2 \frac{1}{\alpha}\cdot\frac{T_m^{i+1}-T_m^{i}}{\Delta t}$

$(2m-1)T_{m-1}^{i+1} - 4mT_m^{i+1} + (2m+1)T_{m+1}^{i+1} = 2m\Delta r^2\cdot\frac{1}{\alpha} \frac{T_m^{i+1}-T_m^{i}}{\Delta t}$

(i+1 for implicit methods)

$\left[(1-\frac{1}{2m})T_{m-1}^{i+1} - 2T_m^{i+1} + (1+\frac{1}{2m})T_{m+1}^{i+1}\right]F_0 = T_m^{i+1}-T_m^{i}$ [where $F_0=\frac{\alpha\Delta t}{\Delta r^2}$]

$(1-\frac{1}{2m})F_0T_{m-1}^{i+1} - 2F_0T_m^{i+1} - T_m^{i+1} + (1+\frac{1}{2m})F_0T_{m+1}^{i+1} = -T_m^{i}$

$-(1-\frac{1}{2m})F_0T_{m-1}^{i+1} + (2F_0+1)T_m^{i+1} - (1+\frac{1}{2m})F_0T_{m+1}^{i+1} = T_m^{i}$

Figure 6 Equations used to find temperature changes of elements in the middle

Fig. 7 shows the derivation to the temperature changes of the center element:

② Center element : cylinder w/ radius $= \frac{\Delta r}{2}$

$$q_{\dot{s}} = \dot{e} V$$

↳ $q_{\dot{s}} = k 2\pi \left(\frac{\Delta r}{2}\right) H \cdot \frac{T_1 - T_0}{\Delta r}$

↳ $\dot{e} = \rho c \frac{T_m^{i+1} - T_m^{i}}{\Delta t} = \rho c \frac{T_0^{i+1} - T_0^{i}}{\Delta t}$

↳ $V = \pi \left(\frac{\Delta r}{2}\right)^2 \cdot H$

$$\therefore \quad k \cdot \pi \Delta r H \cdot \frac{T_1 - T_0}{\Delta r} = \frac{1}{4} \pi \Delta r^2 H \rho c \frac{T_0^{i+1} - T_0^{i}}{\Delta t}$$

$$k \cdot \frac{T_1 - T_0}{\Delta r} = \frac{1}{4} \Delta r \rho c \frac{T_0^{i+1} - T_0^{i}}{\Delta t}$$

$$4 Fo \left(T_1^{i+1} - T_0^{i+1}\right) = \left(T_0^{i+1} - T_0^{i}\right)$$

$$4 Fo T_1^{i+1} - (4 Fo + 1) T_0^{i+1} = - T_0^{i}$$

$$-4 Fo T_1^{i+1} + (4 Fo + 1) T_0^{i+1} = T_0^{i}$$

Figure 7 Equations used to find temperature changes of the center element

Fig. 8 shows the derivation to the temperature changes of the boundary element:

③ outer boundary : a tube w/ radius $\frac{\Delta r}{2}$

$$q_{M-\frac{1}{2}} + q_{conv} = \dot{e}V$$

$$k\cdot 2\pi\left(M-\tfrac{1}{2}\right)\Delta r H \cdot \frac{T_{M-1}-T_M}{\Delta r} + 2\pi M \Delta r H \cdot h(T_\infty - T_M)$$

$$= \rho c \frac{T_M^{i+1}-T_M^{i}}{\Delta t} \cdot \pi\left((M\Delta r)^2 - \left(M-\tfrac{1}{2}\right)^2 \Delta r^2\right) H$$

$$k(2M-1)\cancel{H}(T_{M-1}-T_M) + 2M\Delta r \cancel{H} h(T_\infty - T_M) = \frac{\rho c}{\Delta t}(T_M^{i+1}-T_M^{i})\left(M-\tfrac{1}{4}\right)\Delta r^2 \cancel{H}$$

multiply $\frac{\Delta t}{\rho c \Delta r^2}$ on both sides:

$$Fo(2M-1)(T_{M-1}-T_M) + \frac{h\Delta t}{\rho c \Delta r} 2M(T_\infty - T_M) = \left(M-\tfrac{1}{4}\right)(T_M^{i+1}-T_M^{i})$$

$Fo = \frac{\alpha \Delta t}{\Delta r^2}$, $Fo_h = \frac{h\Delta t}{\rho c \Delta r}$,

w/

use implicit method

$$Fo(2M-1)\, T_{M-1}^{i+1} - \left[Fo(2M-1) + Fo_h \cdot 2M\right] T_M^{i+1} + Fo_h 2M\, T_\infty = \left(M-\tfrac{1}{4}\right) T_M^{i+1} - \left(M-\tfrac{1}{4}\right) T_m^{i}$$

$$Fo(2M-1) T_{M-1}^{i+1} - \left[Fo(2M-1) + Fo_h 2M + M - \tfrac{1}{4}\right] T_M^{i+1} + Fo_h 2M T_\infty = -\tfrac{1}{4}(4M-1) T_m^{i}$$

$$\underbrace{-4Fo\frac{2M-1}{4M-1} T_{m-1}^{i+1} + \frac{4(2M-1)Fo + 8M Fo_h + 4M-1}{4M-1} T_M^{i+1}}_{} - \frac{8M}{4M-1} Fo_h T_\infty = T_m^{i}$$

$$= T_m^{i} + \frac{8M}{4M-1} Fo_h T_\infty$$

Figure 8 Equations used to find temperature changes of boundary element