Washington University in St. Louis

JAMES MCKELVEY SCHOOL OF ENGINEERING

Spring 2024 MEMS 412 Design of Thermal Systems

Design Homework #1

Course Instructor: Dr. Patricia Weisensee

I hereby certify that the lab report herein is our original academic work, completed in accordance with the McKelvey School of Engineering and Undergraduate Student academic integrity policies, and submitted to fulfill the requirements of this assignment:

Jacob Rapoza

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Problem Background

This design homework is analyzing the bottom cycle of a combined cycle gas turbine. The cycle it is analyzing is a Rankine Cycle where the system is cooled by a nearby river. The main solution question is at what inlet temperature the highest efficiency occurs where the range of inlet temperatures is from an experimenter-defined minimum to a maximum of 400 $^\circ$ C.

Assumptions

Below are some of the assumptions already given in the design problem statement as well as some additional assumptions that were drawn through literature search and reason.

Given Assumptions.

(1) Rankine cycle is a closed cycle with no mass flow into or out of the system.
(2) The working fluid of the closed Rankine Cycle is water.
(3) The coolant for the condenser is water from a river, lake, or ocean.
(4) The power for the operation of the pump is part of the cost when optimizing the cycle efficiency.
(5) Minor losses from the supplying pipe can be ignored.
(6) Friction losses from flow through the condenser heat exchanger can not be ignored and must be included.
(7) The condenser is a shell and tube condenser.

Additional Assumptions.

(1) Steam turbines can operate safely at 10-12% moisture[1]. This is because steam turbine blades have high rotation speed. Higher moisture content would lead to increased droplet formation which will hit the blades and cause impact erosion over time.
(2) The coolant water inlet temperature is 2 18.3 $^\circ$ C. This is assumed because the water source chosen is the Missouri River at the Holter Dam in Montana. Specifically, it is assumed that the power generation will be occurring in July 2023 when the average water surface temp is about 65° F or 18.3° C [2].
(3) The cooling water discharge temperature is 70° F or 21.1 $^\circ$ C. This is assumed because the EPA says that water temperature discharge can at no point be more than 5° F above the surface water temperature [3].
(4) The difference between the saturation temperature and the temperature of the cooling water is about 34° F. This is assumed because heat flows from hot to cold but you don't want a huge temperature difference split [4, 5]. It is specifically assumed to be 34° F so that the saturation temperature of the liquid at state 1 is a nice round 40° C.
(5) The mass flow rate remains constant throughout the entire Rankine Cycle. This goes along with the system being a closed cycle but clarifies that no mass flow rate will be lost to minor losses, major losses, etc.
(6) The condenser tubing is assumed to be a new copper tubing because these are the most common type due to their good processing performance and moderate cost [6]. Along with this the

assumed surface roughness of the copper pipes is .0015 mm [7].

(7) The state of the water entering the turbine is a super-heated vapor. This is assumed because the quality needs to be high thus we need dry vapor.

(8) The state of the water entering the pump within the Rankine Cycle is saturated liquid. This is assumed because shell and tube condensers work on the basis of pooling saturated liquid at the bottom of their apparatus.

(9) States 1→2 is an adiabatic compression (we will assume ideal so that entropy does not change), states 2 → 3 is an isobaric heating, states 4 → 1 is an isobaric cooling. We assume this because it simplifies everything but the turbine section of the Rankine cycle so that it is ideal.

Boundary Conditions

Given Boundary Conditions.

(1) The turbine has an isentropic efficiency of 90
(2) The water pressure at the pump exit is 1000 kPa.
(3) The high-temperature heat exchanger provides 30 MW of thermal energy.
(4) The upper bound of the steam turbine inlet temperature is 400 °.
(5) The heat transfer rate within the coolant in the condenser is $U = h_{coolant} = 1000\frac{W}{m^2K}$.

Additional Boundary Conditions.

(1) The quality of the steam at the turbine exit is 88%. This is chosen as the quality because it is the lower value for the range of the accepted quality for a utility-scale turbine [1]
(2) The cooling water inlet temperature is 65° F or 18.3 ° C[2].
(3) The cooling water discharge temperature is 70° F or 21.1° C [3].
(4) The temperature split of the condenser is 34° F [6].
(5) The water temperature at the outlet of the condenser (state 1) is 104° F or 40° C.
(6) The lower bound of the steam turbine inlet temperature is 359.1° C because the turbine exit quality is 88%.

Discussion on Environmental Limitations of Cooling Water

The cooling water has environmental regulations affecting it. There are federal guidelines for temperature changes of discharge water compared to the temperature of the water body. For example, the EPA lists the maximum allowable temperature increase of cold water as 5 degrees Fahrenheit [3]. The reason the temperature differences are so low is because discharging much hotter or much colder water can negatively affect the aquatic life within the ocean, lake, river, etc. Because this is a theoretical analysis we can assume a certain time of year when the average water temp is at a desirable temperature. In this case, I chose July because it has higher surface temperatures than other months.

Schematics and Graphs

Below are design schematics and diagrams for the Rankine cycle. In the T-S diagram, the entropy is assumed to be equal between states 1-2. Also, the water at state 1 is assumed to be saturated liquid at 40° C meaning the saturation pressure can be found to be 7.384 kPa through interpolation from the steam tables [8].

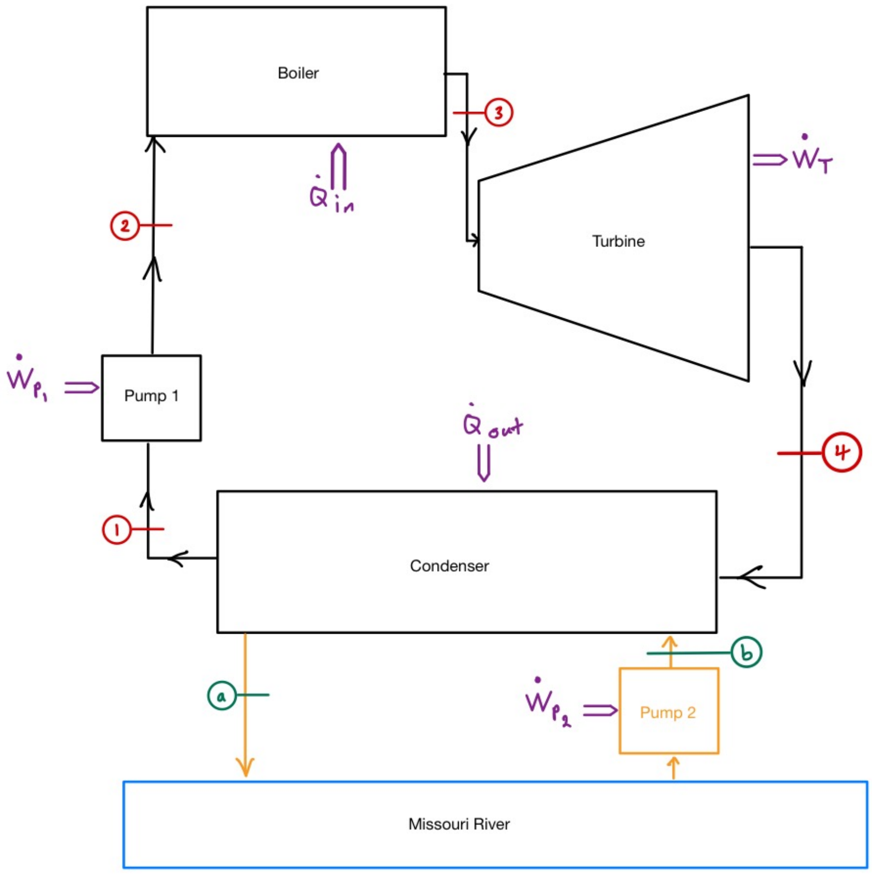

Figure 1 Rankine Cycle schematic including cooling loop.

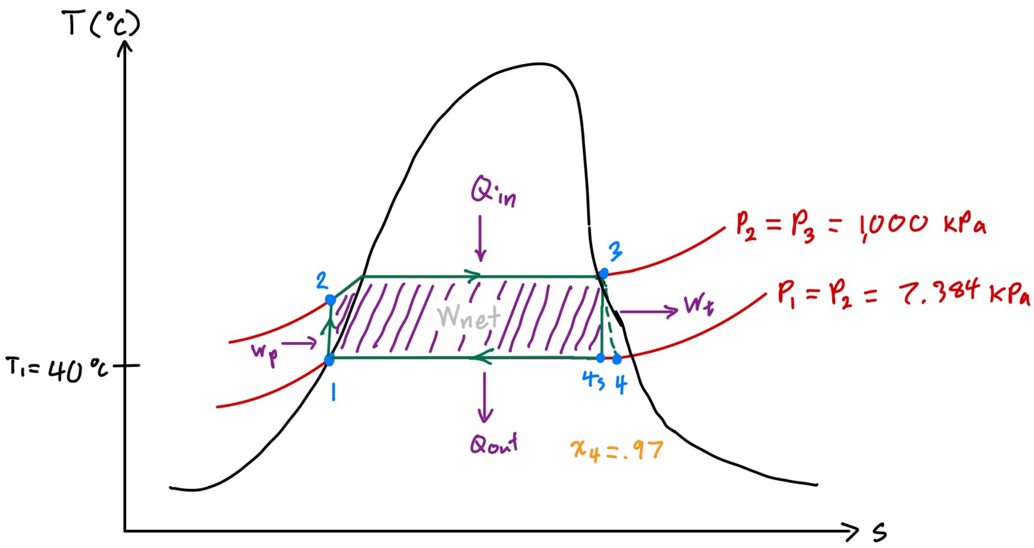

Figure 2 Rankine Cycle T-S diagram.

Equations

Below are the equations that can be used to calculate the efficiency of the closed Rankine cycle.

Solving for the lower temperature bound of the turbine inlet temperature. To find the lower temperature bound for the turbine inlet temperature, one can start by identifying that the pressures for all the states are known. States 1 and 4 have a pressure of 7.384 kPA and states 2 and 3 have a pressure of 1000 kPa. Once this is identified the entropy at state 4 can be calculated using the steam tables to find the saturated vapor and saturated liquid entropies for 7.384 kPA. From these entropies, one can use Equation 1 below to find the entropy for the chosen quality (88%).

$$s_4 = (1 - x_4)s_{4f} + x_4 s_{4g} \tag{1}$$

Once the entropy is found, assuming an ideal case, the temperature for the steam turbine can be found using the calculated entropy and known pressure of 1000 kPa.

Between State 2 and 3: Boiler. Once the range of temperatures for state 3 has been found, it is now possible to solve for the mass flow rate in the Rankine cycle. To solve for the mass flow rate, one can begin by identifying that enthalpy (h_2) and temperature (T_2) at state 2 can be found from the steam tables using the known pressure of 1000 kPa and the entropy of state 1 where it is saturated liquid at 40° C. It is also possible to find the enthalpy of state 3 (h_3) using the known pressure of 1000 kPa and the chosen turbine inlet temperature from the defined range. Once these values have been found from the tables, Equation 2 below and the known boundary condition of 30 MW of power being put into the boiler (Q_{in}) can be used to find the mass flow ($\dot{m}$) rate of the Rankine Cycle for the chosen turbine inlet temperature

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_3 - h_2)} \tag{2}$$

Between state 3 and 4: Turbine. Once the mass flow rate of the Rankine Cycle has been found, the work flow rate of the turbine ($\dot{W}_t$) can be calculated. To begin, the entropy of state 3 can be found by using the chosen turbine inlet temperature, the known pressure of 1000 kPa, and the assumption that the vapor is superheated. Once s_3 is found the ideal enthalpy at the exit of the turbine (h_{4s}) can be found using s_3 and the known pressure of 7.384 kPa. Once h_{4s} is found, the known h_3 from the mass flow rate calculations and the given boundary condition of a 90% isentropic efficiency for the turbine can be used with Equation 3 below to find the true enthalpy at state 4 (h_4).

$$\eta_{s,t} = \frac{h_3 - h_4}{h_3 - h_{4,s}} \tag{3}$$

Once h_4 is known finding the work flow rate of the turbine is as simple as using the calculated $\dot{m}$, and the calculated h_3 with Equation 4 below.

$$\dot{W}_t = \dot{m}(h_4 - h_3) \tag{4}$$

Between state 1 and 2: Rankine Pump. Once ($\dot{W}_t$) has been found all that is needed to do an energy balance of the system is the work flow rate of the pump within the Rankine Cycle ($\dot{W}_{p1}$). Because the $\dot{m}$ has already been calculated finding ($\dot{W}_{p1}$) is as simple as using Equation **??** below with the saturated liquid enthalpy for state 1 (h_1) which can be found from the steam tables as well as h_2 which was found when doing the boiler calculations.

$$\dot{W}_{p1} = \dot{m}(h_2 - h_1) \tag{5}$$

Between state 4 and 1: Condenser. Once the work flow rate from the turbine and Pump 1 have been found an energy balance can be used to solve for the power output into the condenser (Q_{out}) because the work input from the boiler was given as a boundary condition of 30 MW. The equation that represents this is Equation **??** below.

$$Q_{out} = Q_{in} + W_{p1} - W_t \tag{6}$$

Once Q_{out} is found the number of tubes in the condenser can be found using heat transfer. To start, Equations 7 and 8 below can be used with the knowledge that $T_1 = T_{sat} = 40°C$ and $T_{coolant,in} = 18.3°C$ $T_{coolant,out} = 21.1°C$.

$$\Delta T_1 = |T_{sat} - T_{coolant,in}| \tag{7}$$

$$\Delta T_2 = |T_{sat} - T_{coolant,out}| \tag{8}$$

Once ΔT_1 and ΔT_2 are known, ΔT_{lm} can be found using Equation 9 below.

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln(\frac{\Delta T_1}{\Delta T_2})} \tag{9}$$

Once Δ_T is known, Equations 10 and 11 can be used and rearranged into Equation 12 to find the number of tubes required for the heat exchanger (N). D, L, and U are given boundary conditions of 25 mm, 10 m, and $U = h_{coolant} = 1000\frac{W}{m^2K}$ respectively.

$$Q_{out} = U * A * \Delta T_{lm} \tag{10}$$

$$A = N\pi DL \tag{11}$$

$$N = \frac{Q_{out}}{U\pi LD\Delta T_{lm}} \tag{12}$$

Once the number of tubes needed for the condenser is known, the mass flow rate of the cooling water ($\dot{m}_c$) can be calculated using Equation 13 where $C_p = 4.18\frac{J}{g°C}$

$$\dot{m}_c = \frac{Q_{out}}{C_p(T_{coolant,out} - T_{coolant,in})N} \tag{13}$$

Solving for the Major Loss due to friction within the condenser tubes. Once the $\dot{m}_c$ is known the Major Loss due to friction can be calculated. To start, the velocity of the cooling water (V) can be calculated using Equation 14 where $\rho = 999\frac{kg}{m^3}$ and D is the given tube diameter of 25 mm.

$$V = \frac{\dot{m}_c}{\rho\pi\frac{D}{2}^2} \tag{14}$$

Once velocity is calculated, the Reynolds number of the flow can be found using Equation 15 where ρ and D are the same as above and $\mu = 1.12 * 10^{-}3\frac{N}{ms^2}$.

$$Re_D = \frac{\rho VD}{\mu} \tag{15}$$

Once the Reynolds is known which in all of our inline turbine temperatures is above 4000 it can be concluded that the coolant water flow is turbulent. Knowing that the flow is turbulent, the Moody diagram seen below in Fig. 3 can then be referenced along with the assumed surface roughness of ϵ=.0015 mm for new copper pipes. Assuming the calculated Reynolds number and $\frac{\epsilon}{D}$ ratio are within the Moody chart data, Equation 16 below can then be used to find the friction factor f.

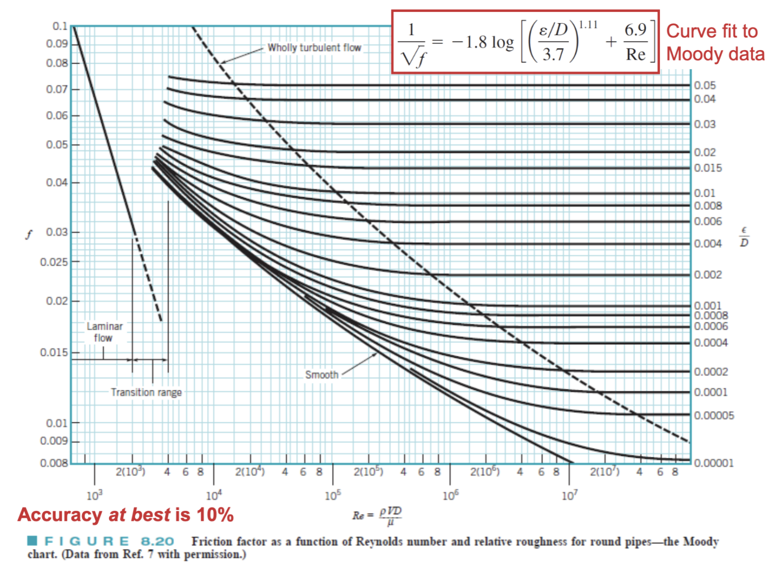

Figure 3 Moody Diagram

$$\frac{1}{\sqrt{f}} = -1.8log[(\frac{\frac{\epsilon}{D}}{3.7})^{1.11} + \frac{6.9}{Re}] \tag{16}$$

Once the friction factor is known, the major loss within each pipe can be calculated using Equation 17 below.

$$\text{Major Loss} = f\frac{L}{D}\frac{\rho V^2}{2} \tag{17}$$

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Between state a and b: Cooling Pump. Once the major loss within each pipe is known the power required to run the cooling pump can be found. To find it the overall pressure loss from states a to b (from the inlet to the outlet of the condenser) has to be found by multiplying the pressure loss from major losses in each tube by the overall number of tubes as seen in Equation 18.

$$\text{Pressure Loss} = \text{Major Loss} * N \tag{18}$$

Once the overall pressure loss is known the power required for the cooling pump can be found using Equation 19.

$$\dot{W}_{p2} = V\pi\frac{D}{2}^2 * \text{Pressure Loss} \tag{19}$$

Overall Rankine Efficiency. Once the power required to run the cooling pump is known, the overall efficiency of the Rankine Cycle can finally be calculated using Equation 20 below.

$$\eta_{th} = \frac{\dot{W}_{p1} + \dot{W}_{p2} - \dot{W}_t}{\dot{Q}_{in}} \tag{20}$$

Results and Discussion Based on the code seen below in the Appendix, the overall Rankine Cycle efficiency as a function of turbine inlet temperature can be seen below in Fig. 4.

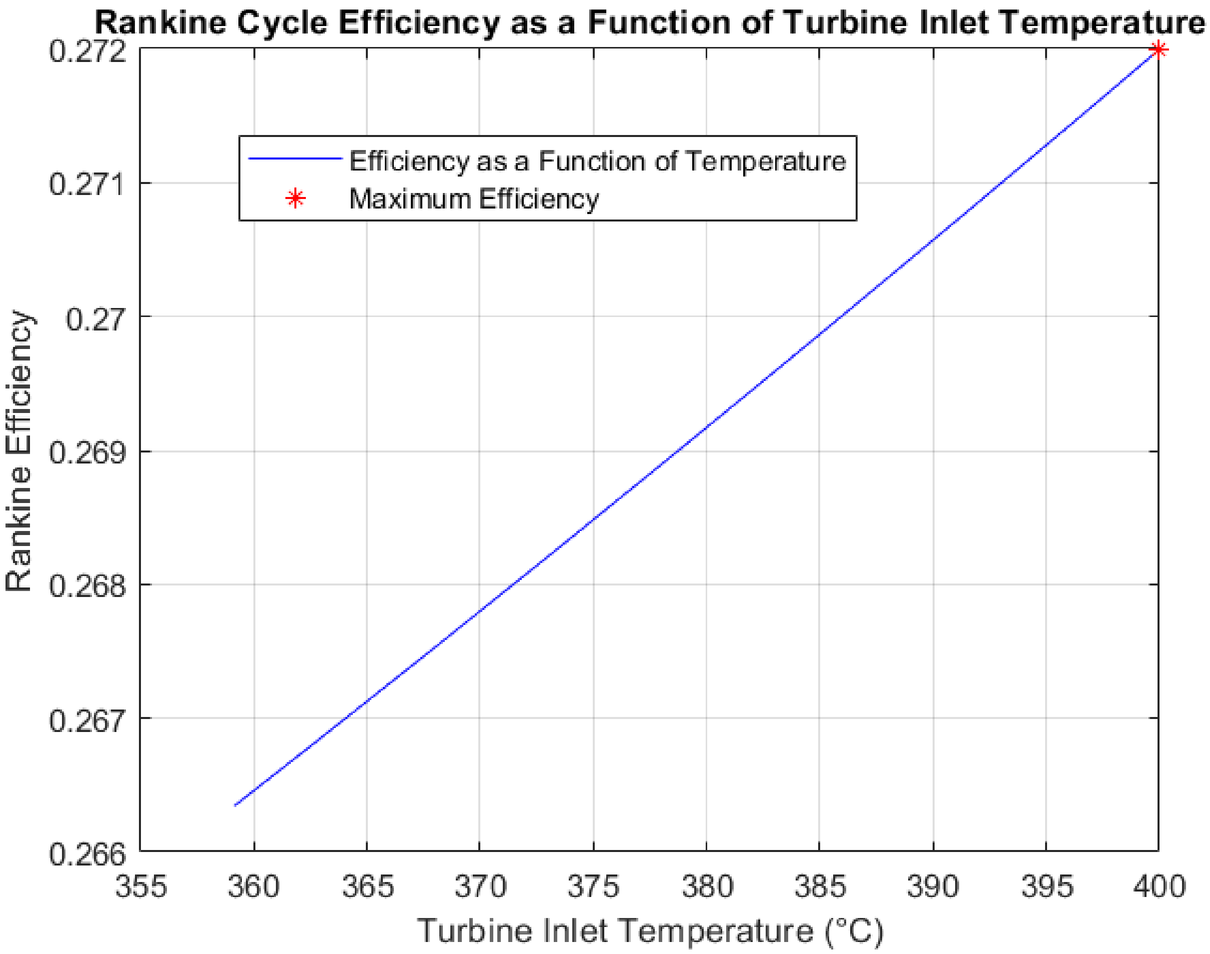

Figure 4 Graph of Rankine Cycle Efficiency as a Function of Turbine Inlet Temperature

The highest efficiency occurs at 400° C with an efficiency of 27.2%. The trend of the graph that higher cycle efficiencies come at higher temperatures makes sense when thinking about the system process as a whole. When the turbine inlet temperature is higher, the mass flow rate of the Rankine Cycle decreases with the decrease in h_3 which in turn lowers $\dot{W}_{p1}$ and $\dot{Q}_c$. By lowering $\dot{Q}_C$ then $\dot{W}_{p2}$ is also, in turn, lowered because the individual mass flow rate in the tubes of the condenser goes down which also brings down the major losses and decreases the work of the pump. Additionally, $\dot{W}_t$ is increased because S_3 increases and the isentropic efficiency of the turbine is known. When both $\dot{W}_{p1}$ and $\dot{W}_{p2}$ are lowered, $\dot{Q}_b$ remains the same, and $\dot{W}_t$ increases then based on Equation 20 the efficiency should go up as well.

Some of the other relevant results at this temperature and efficiency are as follows:

(1) The Rankine Cycle mass flow rate was 9.69 $\frac{kg}{s}$.
(2) The turbine power output was 8.18 MW.
(3) The power input of the pump within the Rankine cycle was 9.45 kW.
(4) The power input of the cooling pump was 10.3 kW.
(5) the number of tubes in the condenser was 1371.39 tubes.

(6) The turbine exit quality was 88%.

The final answers garnered make sense because as seen in literature, the efficiency of a simple Rankine cycle is around 30 % [9, 10]. The answers calculated are slightly lower than those in both articles at 27.2 % but both articles also have slightly different setups. For example, the setup in [10] uses waste heat recovery and also has a turbine isentropic efficiency of "over 93%" while our turbine only has an isentropic efficiency of 90 % [10]. The fact that there is a waste heat recovery system will lead to increased efficiencies [9]. Additionally, based on the schematics of the Rankine cycles in both sources, it appears that neither takes into account the extra pump that is used to operate the cooling fluid flowing through the condenser [9, 10]. Because the two comparison cycles do not appear to take into account the extra cost of a water pump to run the cooling water through the condenser they do not account for the loss in efficiency that would go along with accounting for it. Some other considerations that could lead to the differences are that assumptions were made for our specific calculations such that the pumps were both considered ideal whereas for the cycles in the articles, isentropic efficiencies of the pumps were accounted for.

References

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Appendix: MATLAB Code

Below is the MATLAB for the efficiency calculations.

```
clc
clear all

% known variables
P23=10;
P14=.07384;
x4=.88;
T3U=400;
T1=40;
h1=XSteam('hL_T',T1);
nt=.9;
Qb=30*10^6;
S1=.5724;
D=25*10^-3;
L=10;
U=1000;
Tci=18.3;
Tco=21.1;
epsilon=.0015*10^-3;
Cp=4.18*10^3;
rho=999;
mu=1.12*10^-3;

%solving for lower bound of turbine inlet temp
s4f=XSteam('sL_p',P14);
s4g=XSteam('sV_p',P14);
s4=(1-x4)*s4f+x4*s4g;
T3L=XSteam('T_ps',P23,s4);

%initialize output array
temp=[];
eff=[];
var_min=T3L;
var_max=T3U;
step=.1;

for T3= var_min:step:var_max
    %solving for the boiler
    T2=XSteam('T_ps',P23,S1);
    h2=XSteam('h_ps',P23,S1);
    h3=XSteam('h_pT',P23, T3);
    m_dot=Qb/(h3-h2);

    % solving the turbine portion of the problem
    S3=XSteam('s_pT',P23,T3);
    h4s=XSteam('h_ps',P14,S3);
    h4=nt*(h3-h4s)+h3;
    Wt=m_dot*(h4-h3);

    %solving for Rankine pump
    Wp1=m_dot*(h2-h1);

```

```
    %energy balance to find Qc
    Qc=Qb+Wp1-Wt;

    %solving for condenser
    Δ_T1=abs(T1-Tci);
    Δ_T2=abs(T1-Tco);
    Δ_T=(Δ_T1-Δ_T2)/(log(Δ_T1/Δ_T2));
    N=Qc/(U*Δ_T*pi*D*L);


    %finding major loss and friction coefficient
    m_dot_cooling_total=Qc/(Cp*(Tco-Tci));
    m_dot_cooling=m_dot_cooling_total/N;
    V=(m_dot_cooling)/(rho*pi*(D/2)^2);
    Reynolds= (rho*V*D)/mu;
    f=((1)/(-1.8*log10(((epsilon/D)/3.7)^1.11*(6.9/Reynolds))))^2;
    Maj_loss=((f*L*rho*V^2)*N)/(2*D);

    %cooling pump
    flow_rate=V*pi*(D/2)^2;
    Wp2=flow_rate*Maj_loss;

    %overall rankine efficiency
    nth= (Wt-Wp1-Wp2)/Qb;

    %output array building
    temp=[temp,T3];
    eff=[eff,nth];
end

%efficiency plot
plot(temp,eff, "blue");
hold on
plot(T3,nth,"red*");
legend("Efficiency as a Function of Temperature","Maximum Efficiency")
xlabel("Turbine Inlet Temperature ( C )");
ylabel("Rankine Efficiency");
title("Rankine Cycle Efficiency as a Function of Turbine Inlet Temperature");
grid on;
```