Assignment 1: CS205

Joshua Ross
RUID: 185008511
Rutgers University - New Brunswick
jjr276@scarletmail.rutgers.edu

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1.1

32c)

Problem: Construct a truth table for $p \bigoplus (p \bigvee q)$

p	q	$p \bigvee q$	$p \bigoplus (p \bigvee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

32f)

Problem: Construct a truth table for $(p \bigoplus q) \bigwedge (p \bigoplus \neg q)$

p	q	$\neg q$	$p \bigoplus q$	$p \bigoplus \neg q$	$(p \bigoplus q) \bigwedge (p \bigoplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

36c)

Problem: Construct a truth table for $(p \bigvee q) \bigvee r$

p	q	r	$p \bigvee q$	$(p \bigvee q) \bigvee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

36f)

Problem: Construct a truth table for $(p \bigwedge q) \bigvee r$

p	q	r	$\neg r$	$p \bigwedge q$	$(p \bigwedge q) \bigvee \neg r$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

1.3

26)

Problem: Use boolean algebra to show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \bigvee r)$ are logically equivalent

- $\neg p \rightarrow (q \rightarrow r) \equiv \neg p \rightarrow (\neg q \bigvee r) \equiv p \bigvee (\neg q \bigvee r)$ by the implication law
- $p \bigvee (\neg q \bigvee r) \equiv \neg q \bigvee (p \bigvee r)$ by the commutative and associative laws
- $\neg q \bigvee (p \bigvee r) \equiv q \rightarrow (p \bigvee q)$ by the implication law

30)

Problem: Use boolean algebra to show that $(p \bigvee q) \bigwedge (\neg p \bigvee r) \rightarrow (q \bigvee r)$ is a tautology

- $(p \bigvee q) \bigwedge (\neg p \bigvee r) \rightarrow (q \bigvee r) \equiv \neg((p \bigvee q) \bigwedge (\neg p \bigvee r)) \bigvee (q \bigvee r)$ by the implication law
- $\neg((p \bigvee q) \bigwedge (\neg p \bigvee r)) \bigvee (q \bigvee r) \equiv \neg(p \bigvee q) \bigvee \neg(\neg p \bigvee r) \bigvee (q \bigvee r) \equiv (\neg p \bigwedge \neg q) \bigvee (p \bigwedge \neg r) \bigvee (q \bigvee r)$ by De Morgan law

- $(\neg p \bigwedge \neg q) \bigvee (p \bigwedge \neg r) \bigvee (q \bigvee r) \equiv ((\neg p \bigwedge \neg q) \bigvee q) \bigvee ((p \bigwedge \neg r) \bigvee r)$ by commutative and associative laws
- $((\neg p \bigwedge \neg q) \bigvee q) \bigvee ((p \bigwedge \neg r) \bigvee r) \equiv ((\neg p \bigvee q) \bigwedge (\neg q \bigvee q)) \bigvee ((p \bigvee r) \bigwedge (\neg r \bigvee r))$ by the distributive law
- $((\neg p \bigvee q) \bigwedge (\neg q \bigvee q)) \bigvee ((p \bigvee r) \bigwedge (\neg r \bigvee r)) \equiv ((\neg p \bigvee q) \bigwedge T) \bigvee ((p \bigvee r) \bigwedge T)) \equiv \neg p \bigvee q \bigvee p \bigvee r$ by the negation and domination laws respectively
- $\neg p \bigvee q \bigvee p \bigvee r \equiv (p \bigvee \neg p) \bigvee r \bigvee q$ by commutative and associative laws
- $(\neg p \bigvee p) \bigvee q \bigvee r \equiv T \bigvee q \bigvee r \equiv T$ by negation and domination laws respectively, proving that this is a tautology

1.4

36a)

Problem: Find a counterexample (if possible) to $\forall x(x^2 \neq x)$ where the domain of x is all real numbers
For $x = 1$, $(1)^2 = (1)$, meaning $x^2 = x$, providing a counterexample to $\forall x(x^2 \neq x)$

36b)

Problem: Find a counterexample (if possible) to $\forall x(x^2 \neq 2)$ where the domain of x is all real numbers
For $x = 1$, $(\sqrt{2})^2 = 2$, meaning $x^2 = 2$, providing a counterexample to $\forall x(x^2 \neq 2)$

36c)

Problem: Find a counterexample (if possible) to $\forall x(|x| > 0)$ where the domain of x is all real numbers
For $x = 0$, $|0| = 0$, meaning $|x| = 0$, providing a counterexample to $\forall x(|x| > 0)$