

Probabilistic Model Checking

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Overview

- Ingredients necessary for model checking
 - representation of systems of interest
 - language to describe properties of these systems
 - a way to automatically check whether the system has the specified property
- From regular MC to probabilistic MC:

Transition systems	→	Probabilistic transition systems
Temporal Logics	→	Variants of temporal logics
(CTL, LTL)	→	(PCTL, LTL)
Algorithms for CTL, LTL	→	Algorithms for PCTL, LTL

- Let's take a peek at the ingredients one at a time

Probabilistic Transition Systems

- Transitions labeled with probabilities.
- Sum of probabilities of outgoing arrows equals one.
- Can be either deterministic OR nondeterministic.
- Both discrete- and continuous-time variants exist.
- May be labeled with costs and/or rewards.

TODO: PICTURE OF DTMC and MDP

Running example will be DTMC (Knuth-Yao algorithm)

Logics for probabilistic systems

- The regular interpretation of CTL and LTL asks questions of the form:

Do all possible traces satisfy φ ?

Does some trace satisfy φ ?

- The probabilistic variants of CTL and LTL ask:

What is the “fraction” of all possible traces satisfying φ ?

- PROBLEMS...
 - How to compute the fraction of an infinite set???
 - All traces are not equally likely...

- Instead the space of traces is augmented with a **probability measure**.
- Given a property of traces one may then ask:

What is the probability of choosing a trace satisfying the property?

- The probability measure should reflect the probability of the system choosing a specific trace.
- The probability measure is generated by "cylinder sets".

Probabilistic Computation Tree Logic

- Probabilistic CTL is a variant of CTL where:
 - Path quantifiers A and E are replaced with probability operators $\mathbb{P}_{<p}(\varphi)$ and $\mathbb{P}_{\leq p}(\varphi)$.
 - Expectation operators $\mathbb{E}_{<c}(\Diamond\Phi)$ and $\mathbb{E}_{\leq c}(\Diamond\Phi)$ may also be defined.
- Semantics:
 - $\mathbb{P}_{\leq p}(\varphi)$ "Is the probability of choosing a path satisfying φ at most p ?"
 - $\mathbb{E}_{\leq c}(\Diamond\Phi)$ "Is the expected cost of reaching a state satisfying Φ at most c ?"
- Apart from these new operators PCTL is like CTL.

Linear Temporal Logic

- LTL describes properties of linear traces.
- Instead of checking whether all(some) linear traces of a system satisfy an LTL formula...
- The probability for choosing a trace satisfying an LTL formula is computed.

TODO: examples of interesting properties PCTL/LTL can express...

Checking PCTL properties of DTMCs

TODO: describe PCTL checking of DTMCs

Checking LTL properties of DTMCs

TODO: Describe idea of LTL checking algorithm

Applications

- Analysis of randomized algorithms
- Analysis of randomized protocols
 - communication
 - security
 - consensus
- systems biology
- reliability engineering

Tool Support

- PRISM
- Probmela
- ...

Conclusion

- active research area:
 - parameter synthesis!!
- qualitative VS quantitative
- absolute VS relative guarantees
- relevance
- usefulness