Assignment 9: Spectra of non-periodic Signals

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May 6, 2022

Abstract

In this assignment we are supposed to:

- Compute the spectra of non-periodic signals
- To try out the impact of hamming window in making the DFT of non-periodic functions better.
- Visualize the plots of the DFT and seek information about them.

For the computation of the DFT for the given signals we use the `nupy.fft()` package.

1 Introduction

We shall try computing the Digital Fourier transform of non-periodic signals for the case without windowing and with windowing. We use the principle of windowing to make a non-periodic signal as a square integrable. In this process we convert a infinite long signal to a finite signal, since to take DFT we need finite aperiodic signal.

In the process of windowing, it creates some other new frequencies other than the frequency of the signal that is being considered. This is called Spectral Leakage. So we need to choose a proper windowing function for the specified applications. The window function we use is called Hamming window, which is generally used in naarow-band applications.

2 Spectrum of $sin(\sqrt{2}\pi)$

2.1 Without Hamming window

In this part, we plot the spectrum of $sin(\sqrt{2}\pi)$ without the effect of Hamming window, the plot of spectrum looks like:

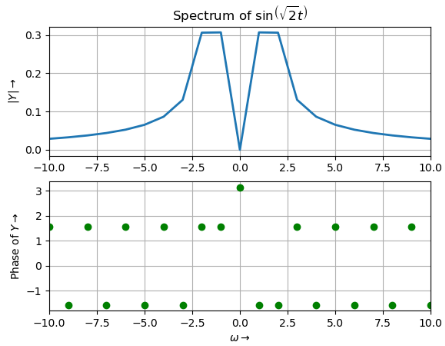

Figure 1: Spectrum of $sin(\sqrt{2}\pi)$ without hamming window

2.2 With Hamming window

In this part, we plot the Spectrum of $sin(\sqrt{2}\pi)$ with the effect of Hamming window, the plot of Spectrum looks like:

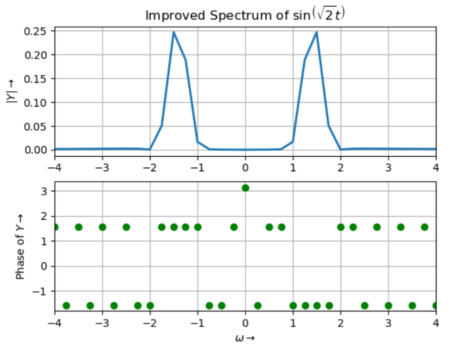

Figure 2: Spectrum of $sin(\sqrt{2}\pi)$ with hamming window

3 Spectrum of $cos^3(\omega_0 t)$

3.1 Without Hamming window

In this part, we plot the spectrum of $cos^3(\omega_0 t)$ (for the case of $\omega = 0.86$) without the effect of Hamming window, the plot of spectrum looks like:

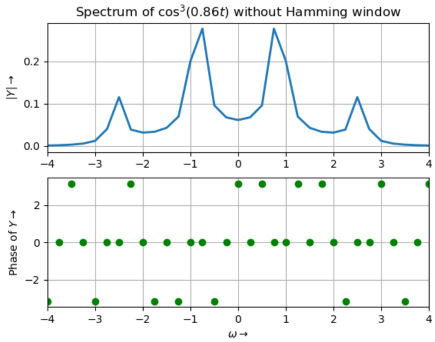

Figure 3: Spectrum of $cos^3(0.86t)$ without hamming window

3.2 With Hamming window

In this part, we plot the Spectrum of $cos^3(\omega_0 t)$ (for the case of $\omega = 0.86$), with the effect of Hamming window, the plot of Spectrum looks like:

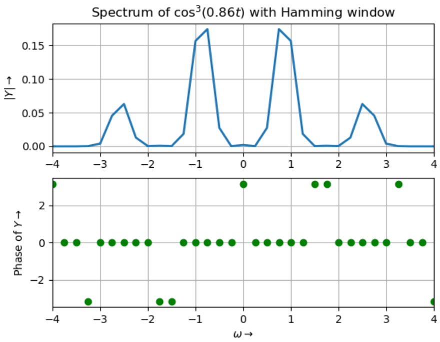

Figure 4: Spectrum of $cos^3(0.86t)$ with hamming window

4 Estimation of ω_0 and δ for $cos(\omega_0 t + \delta)$

As given in the question, we try estimating the value of ω_0 and δ in the expression of $cos(\omega_0 t + \delta)$, such that two peaks appear at the frequencies of $\pm\omega$. We do this for the frequency in the range of (0.5, 1.5), and the values of t goes from -π to π, with and without adding noise to the signal.

4.1 Without adding noise

- Estimated value of ω_0 without noise : 1.4730276250507859
- Estimated value of δ without noise : 0.5018760117245951

The plot of the spectra of $cos(\omega_0 t + \delta)$ for the respective input values is as shown:

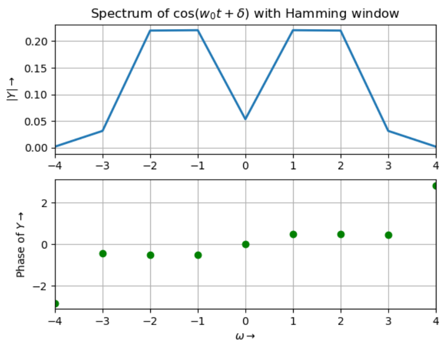

Figure 5: Spectrum of $cos(\omega_0 t + \delta)$ without noise

4.2 With added white noise

In this part to the existing signal of $cos(\omega_0 t + \delta)$ by adding **white gaussian noise**, this is done using the package of **randn()** in python. The extent of this noise is 0.1 in amplitude (i.e., *0.1randn(N)*, where N is the number of samples).

- Estimated value of ω_0 with noise : 2.0387968138662727
- Estimated value of δ with noise : 0.5066228952148905

The plot of the spectra of $cos(\omega_0 t + \delta)$ for the respective input values is as shown:

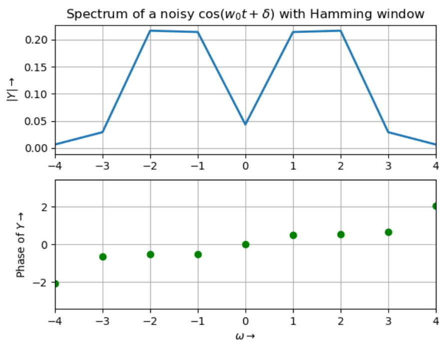

Figure 6: Spectrum of $cos(\omega_0 t + \delta)$ with noise

5 Spectrum of the chirped signal

In this section we analyse the given chirped signal $cos(16t(1.5+\frac{t}{2\pi}))$, where the variable t is in the range -π to π in 1024 steps. We now plot the spectra of the chirped for the two cases, one without hamming window and other with hamming window.

5.1 Without Hamming window

The plot of the spectrum of the chirped signal $cos(16t(1.5+\frac{t}{2\pi}))$ without Hamming window looks like:

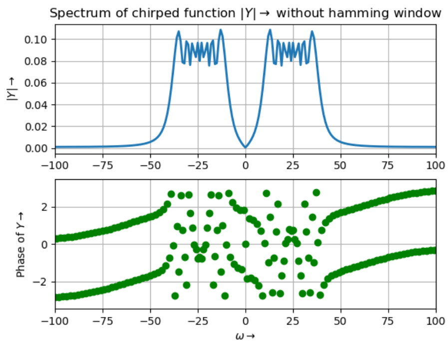

Figure 7: Spectrum of $cos(16t(1.5+\frac{t}{2\pi}))$ without Hamming window

5.2 With Hamming window

The plot of the spectrum of the chirped signal $cos(16t(1.5+\frac{t}{2\pi}))$ with Hamming window looks like:

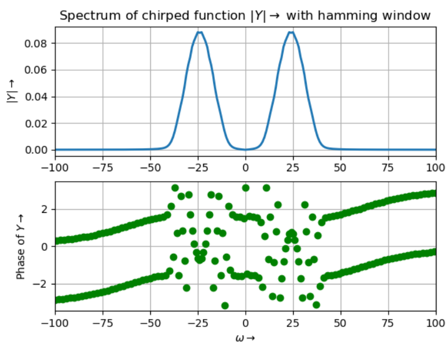

Figure 8: Spectrum of $cos(16t(1.5+\frac{t}{2\pi}))$ with Hamming window

5.3 Surface plot of chirped signal

In this subsection we split the chirp in the time interval $[-\pi,\ \pi]$ into smaller intervals of time, and observe how the frequency of the signal varies with time. Initially we had a 1024 length vector with the values of the chirp signal. We shall split it into 64-length vectors, take the DFT of these localized vectors, and plot a **time-frequency** surface plot to observe the variation of the frequency with time.

5.4 Without Hamming window

The 3D surface plot of the magnitude of the chirped signal with time and frequency for the case without hamming window looks like:

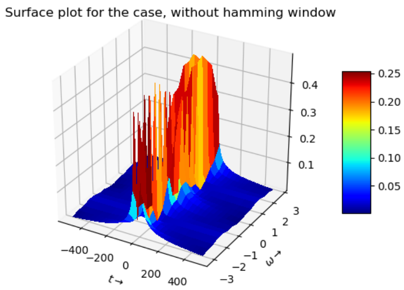

Figure 9: Surface plot for the case, without Hamming window

5.5 With Hamming window

The 3D surface plot of the magnitude of the chirped signal with time and frequency for the case with hamming window looks like:

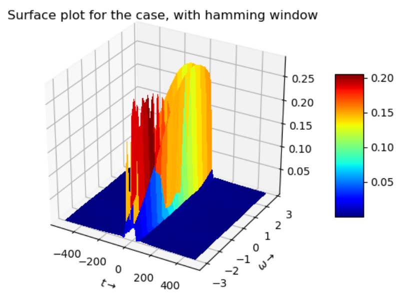

Figure 10: Surface plot for the case, with Hamming window

Conclusion

In order to conclude, we have seen the following in this assignment,

1. We have computed the Discrete Fourier transform of non-periodic functions.
2. We have also tried finding the effect of the Hamming window on the DFT of non-periodic functions. The error in the computation of Spectrum of the non-periodic signals is reduced after the addition of a hamming window.
3. We have also plotted the 3D surface plot of the magnitude of the spectrum of chirped signal with time-frequency axes.