c) Resuelva por series para $-1 \leq x \leq 1$, con $y(0)=0$ y $y'(0)=0$

$$y=\sum_{n=0}^{\infty} C_n x^n \; ; \quad y'=\sum_{n=1}^{\infty} C_n \cdot n \cdot x^{n-1} \; ; \quad y''=\sum_{n=2}^{\infty} C_n \cdot n \cdot (n-1) \cdot x^{n-2} \; ; \; y(0)=C_0=0$$

$$y'(0)=C_1=0$$

$$(1-x^2)y''+xy'+(x^2-1/4)y=xe^x$$

$$\Rightarrow +\sum_{n=2}^{\infty} C_n \cdot n \cdot (n-1)x^{n-2} - x^2 \cdot \sum_{n=2}^{\infty} C_n \cdot n \cdot (n-1)x^{n-2} + x \cdot \sum_{n=1}^{\infty} C_n \cdot n \cdot x^{n-1} + x^2 \cdot \sum_{n=0}^{\infty} C_n x^n$$

$$-\frac{1}{4}\sum_{n=0}^{\infty} C_n x^n = xe^x$$

$$\Rightarrow \sum_{n=2}^{\infty} 4C_n \cdot n\,(n-1)x^{n-2} - \sum_{n=2}^{\infty} C_n \cdot n \cdot (n-1)x^n + \sum_{n=0}^{\infty} C_n n\, x^n + \sum_{n=0}^{\infty} C_n x^{n+2} - \sum_{n=0}^{\infty} \frac{C_n}{4} x^n = xe^x$$

- $\displaystyle\sum_{n=2}^{\infty} 4C_n \cdot n \cdot (n-1)x^{n-2} = 4 \cdot C_2 \cdot 2 \cdot 1 + 4C_3 \cdot 3 \cdot 2 \cdot x + \sum_{n=4} 4C_n \cdot n(n-1)x^{n-2}$
- $\displaystyle\sum_{n=0}^{\infty} C_n \cdot n x^n = C_1 x + \sum_{n=2}^{\infty} C_n \cdot n \cdot x^n$
- $\displaystyle\sum_{n=0}^{\infty} \frac{C_n}{4} x^n = \frac{C_0}{4} + \frac{C_1}{4}x + \sum_{n=2} \frac{C_n}{4} x^n$

$$\Rightarrow 8C_2 + 24C_3 \cdot x + \sum_{n=4}^{\infty} 4C_n \cdot n(n-1)x^{n-2} - \sum_{n=0} C_{n+2}(n+2)(n+1)x^{n+2} + C_1 x + \sum_{n=2} C_n n x^n$$

$$+\sum_{n=0}^{\infty} C_n x^{n+2} - \frac{C_0}{4} - \frac{C_1}{4}x - \sum_{n=2} \frac{C_n}{4} x^n = xe^x$$

$$\Rightarrow 8C_2 + 24C_3 \cdot x + C_1 x - \frac{C_0}{4} - \frac{C_1}{4}x + \sum_{n=0}^{\infty} 4 \cdot C_{n+4} \cdot (n+4)(n+3)x^{n+2} - \sum_{n=0} C_{n+2}(n+2)(n+1)x^{n+2}$$

$$+\sum_{n=0}^{\infty} C_{n+2} \cdot (n+2)x^{n+2} + \sum_{n=0}^{\infty} C_n x^{n+2} - \sum_{n=0}^{\infty} \frac{C_{n+2}}{4} x^{n+2} = xe^x$$

$$\Rightarrow x\left[24C_3 - \cancel{\frac{C_1}{4}} + \cancel{C_1}\right] + 8C_2 - \cancel{\frac{C_0}{4}} + \sum_{n=0}^{\infty}\left[4(n+4)(n+3)C_{n+4} - (n+2)(n+1)C_{n+2} + (n+2)C_{n+2}\right.$$

$$\left. + C_n - \frac{C_{n+2}}{4}\right]x^{n+2} = xe^x$$

$(n+2)C_{n+2} - (n+2)(n+1)C_{n+2}$

$(n+2)C_{n+2}\left[1-(n+1)\right]$

$1-n-1$

$-(n+2) \cdot n \cdot C_{n+2}$

Scanné avec CamScanner

Como está igualado a una función, se deben comparar los términos

$$\Rightarrow 24C_3x + 8C_2 + \sum_{n=0}^{\infty}\left[4\cdot(n+2)(n+3)C_{n+4} - n(n+2)C_{n+2} + C_n - \frac{C_{n+2}}{4}\right]x^{n+2} = x\left(1 + x + \frac{x^2}{2!} + \dots\right)$$

$$\Rightarrow 24C_3x + 8C_2 + \sum_{n=0}^{\infty}\left[4(n+2)(n+3)C_{n+4} - n(n+2)C_{n+2} + C_n - \frac{C_{n+2}}{4}\right]x^{n+2} = x\cdot\frac{x^n}{n!} = \frac{x^{n+1}}{n!}$$

Igualando coeficientes

$$y = C_0 + C_1x + C_2x^2 + \dots C_ix^i$$

$$\left.\begin{array}{l} C_0 = 0 \\ C_1 = 0 \end{array}\right\} \text{condiciones iniciales.}$$

Para x^0:

$$8C_2 = 0$$
$$C_2 = 0$$

Para x^1:

$$24C_3 = 1$$
$$C_3 = 1/24$$

Para x^2:

$$4\cdot 2\cdot 3\cdot C_4 + \cancel{C_0}^{\,0} - \frac{\cancel{C_2}^{\,0}}{4} = 1$$
$$24C_4 = 1$$
$$C_4 = 1/24$$

Para x^3: $48C_5 - 3\cdot\frac{1}{24} - \frac{1}{24}\cdot\frac{1}{4} = \frac{1}{2}$

$$48C_5 - \frac{1}{8} - \frac{1}{96} = \frac{1}{2}$$

$$C_5 = \frac{\frac{1}{8} + \frac{1}{96} + \frac{1}{2}}{48} = \frac{61}{4608}$$

Para x^4: $C_6 = \frac{7}{768}$

$$4(4)(5)C_6 - 2(4)C_4 + C_2 - \frac{C_4}{2} = \frac{1}{6}$$

$$80C_6 - 8C_4 + C_2 - \frac{C_4}{2} = \frac{1}{6}$$

$$80C_6 - 8\cdot\frac{1}{24} + 0 - \frac{1}{24}\cdot\frac{1}{2} = \frac{1}{6}$$

$$C_6 = \frac{\frac{1}{6} + \frac{1}{24\cdot 2} + \frac{1}{3}}{80}$$