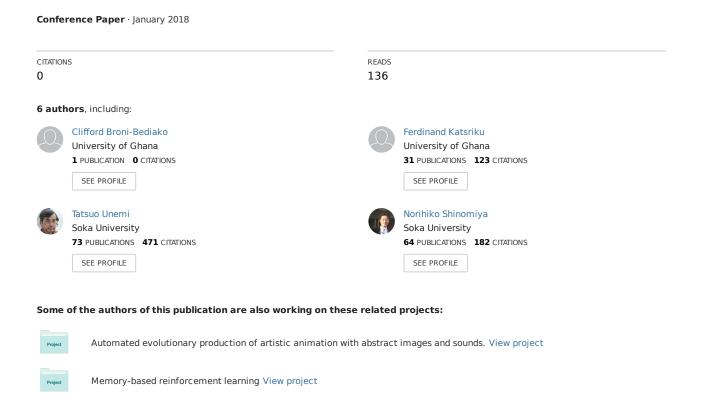
El niño-southern oscillation forecasting using complex networks analysis of LSTM neural networks



El niño-southern oscillation forecasting using complex networks analysis of LSTM neural networks

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Abstract: Arguably, El Niño-Southern Oscillation (ENSO) is the most influential climatological phenomenon that has been intensively researched during the past years. Currently, the scientific community knows much about the underlying processes of ENSO phenomenon, however, its predictability for longer horizons, which is very important for human society and the natural environment is still a challenge in the scientific community. Here we show an approach based on using various complex networks metrics extracted from climate networks with long short-term memory (LSTM) neural network to forecast ENSO phenomenon. The preliminary experiments show that training LSTM model on network metrics time series dataset provides great potential for forecasting ENSO phenomenon longer multiple steps ahead.

Keywords: Complex networks, ENSO forecasting, LSTM neural networks, time series forecasting

1 INTRODUCTION

The El Niño-Southern Oscillation (ENSO) represents heterogeneous climate conditions that have their source in the tropical Pacific ocean and is associated with severe rainfalls, floods and droughts affecting regions far from the tropics, and with disastrous socioeconomic and environmental impact on many countries [1]. To mitigate the adverse impact of ENSO, decision and policy-makers will need reliable and accurate information from the scientific community that forecasts with a high degree of precision the occurrence of the event. Such information is usually based on skilful multiple horizon forecasting. However, despite great effort and significant progress, forecasts, skillfully performed can only provide up to 6-month forecast horizon. Thus far, for longer multiple steps ahead forecasting, the performance achieved is low and unreliable [2], [3]. And this is because time series forecasting over a long multiple steps ahead is still an open challenge [4].

There are many conventional models for forecasting ENSO event, both dynamical and statistical models [2]. The dynamical models use physical equations of the ocean and atmosphere to forecast ENSO event, which are computationally very expensive and not available outside the atmospheric scientific community. While, the statistical models use mathematical formulations to learn from observed data to forecast ENSO event. And it has been indicated that, the average performance of dynamical models slightly exceeds that of statistical models [2]. Another significant reason posited in literature as the factor limiting ENSO predictability is its nonlinear characteristics [5], which is difficult for conventional statistical models to capture.

Some studies have suggested that long short-term memory (LSTM) neural network, a novel nonlinear machine learning algorithm, capable of learning long-term temporal dependencies of complex phenomenon [6], can be

a better alternative to the traditional neural networks methods for forecasting climatological time series with a good degree of accuracy [7].

Although LSTM models have not received much attention in climatological forecasting task [8], compared to disciplines like natural language translation, image and video captioning [9], they have demonstrated potential capability in forecasting climate variability, e.g., forecasting sea surface temperatures (SSTs) [8] and tropical cyclones [10]. And as ENSO phenomenon relates to SSTs anomalies [11] and tropical cyclones [12], we argue that, LSTM models have strong capability to capturing long-term temporal dependencies of ENSO, and model its nonlinear characteristic

In 2004, Tsonis and Roebber [13] introduced complex network theory [14] into climate science, and proposed the term *climate network* (CN) as a complex network constructed from climatological time series dataset. Since then, CN analysis has provided appreciable new and valuable insights into the climate variability [15], [16], including forecasting ENSO phenomena [3].

Both neural networks and complex networks techniques, generally share common goals and address similar problems, that is, for encoding and extracting insightful information from complex systems such as the brain and earth's climate system, each with its own strength and weakness [17]. Rarely in the literature, do we see the scientific community applying both techniques to draw on their complementary strengths to solve a problem [17].

In 2010, Steinhaeuser et. al. [15] suggested pushing CN analysis beyond descriptive analysis and toward predictive models for climate variability. To this end, clusters extracted from climate networks were used as inputs into a predictive model for land climate, and successfully demonstrated its predictive power [18]. Likewise, climate indices obtained by community detection methods as predictors,

achieved a mean absolute error of 5.4 % for forecasting Indian monsoon rainfall, which is superior to the existing Indian meteorological department models, using generalized regression neural networks [19]; and network motifs as discriminatory signatures, classified hurricane tracks at their formation stages in West Africa region with more than 90% accuracy for 10-15 days in advance, for those that will hit the land of the North Atlantic region [20].

In contrast, in this paper, we address multiple steps ahead forecasting problem of ENSO events to complement existing models. We aim at forecasting ENSO event 6-month and more multiple steps in advance using LSTM neural networks and several network metrics as predictors. Conceptually, our approach relates to, but distinctively different from, recent studies in granular media [21] and brain networks [22]. To the best of our knowledge, no literature has addressed this problem using this approach.

The remainder of this paper is organized as follows. Section 2 defines the ENSO phenomenon; while Section 3 presents the climatological datasets we used and describe the proposed method. The experimental results are reported in Section 4, and finally, Section 5 concludes the paper.

2 THE ENSO PHENOMENON

The ENSO emerges in the tropical Pacific ocean and it is the largest natural interannual climate variation signal that has very severe impact on the global climate system. The phenomenon alternates between El Niño (La Niña) events in Pacific ocean extending from the coast of Peru and Ecuador to the central, near the international date line. It is characterized by a five consecutive 3-month running mean of SSTs anomalies in the Niño 3.4 region (5°S–5°N and 170°W–120°W) (Fig. 1.) that is above or below the threshold of +0.5°C or -0.5°C respectively. The Oceanic Niño Index (ONI) is the standard index for monitoring ENSO [23].

The ENSO has three phases: El Niño, La Niña, and neutral. And the intensity of the phases is categorized as: strong El Niño (La Niña), weak El Niño (La Niña), and neutral ENSO based on approximately 1.0°C (-0.1°C) differences of the SSTs anomaly Niño 3.4 index [3, 24]. Current ENSO predictions, which is updated monthly is available at http://iri.columbia.edu/our-expertise/climate/foreca sts/enso/current/.

4 DATA AND METHOD

3.1 Climatological datasets

We use the National Oceanic and Atmospheric Administration's (NOAA's) first global reanalysis daily surface air temperatures (SATs) time series, jointly provided by National Centers for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) [25] to construct the CNs. The dataset start from January 1950 to February 2017. It consists of 10,512 data points at an equally gridded space, in the region bounded by 30°S–30°N and 120°E–60°W, which covers Niño3.4 region (5°S–5°N and 170°W–120°W), with spatial resolution of 2.5° by 2.5°, as shown in **Fig. 1**., and retrieved from https://www.esrl. noaa.gov/psd/data/gridded/data.ncep.reanalysis.html.

ENSO events, we use the NOAA's ONI, an Extended Reconstructed SST version 5 (ERSSTv5) derived from the International Comprehensive Ocean Atmosphere Dataset (ICOADS) [26]. A 3-month running average of monthly mean SST anomalies averaged time series over the Niño3.4 region from January 1950 to October 2017, retrieved from http://www.cpc.ncep.noaa.gov/. The Niño3.4 index is the predictand.

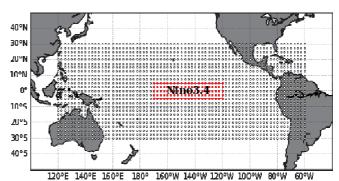


Fig. 1. The 73 by 144 NCEP/NCAR Reanalysis-1 SATs data points, covering the Niño3.4 basin, (red grid points), at a spatial resolution of 2.5° by 2.5°

3.2 An overview of the proposed approach

The proposed approach has two phases (**Fig. 2.**). The first phase basically delivers networks metrics as feature set to the second phase. The phase one involves constructing 3-month evolving climate networks (evolving-CNs) [27] from the SATs time series dataset using complex network methods, then extracts multiscale network metrics [15], [16] from each evolving-CN. We systematically map these multiscale network metrics to a multivariate time series as feature matrix that are used as input variables (predictors) together with the Nino3.4 index time series (predictand) into the LSTM models. In the second phase, LSTM neural network models are applied to determine which of the feature subset of the multiscale complex network metrics that best generalize and forecast ENSO events for 6-, 9-, and 12-month multiple steps ahead.

3.3 MIMO time series forecasting

Multiple steps time series forecasting is predicting two or more steps ahead using single-output or multiple-output strategy [4]. The multiple-output strategy is known as multiple-input multiple-output (MIMO) time series forecasting, as it models time series data as a multiple-input multiple-output function. Given time series, $T = \{\phi^1, \phi^2, ..., \phi^N\}$, MIMO models T as finding the relationship between the historical time series, $X = \{\phi^{t-n+1}, \phi^{t-1}, ..., \phi^t\}$, and the future observation, $y = \{\phi^{t+1}, \phi^{t+2}, ..., \phi^{t+H}\}$, where H > 1 is the forecasting horizon [4].

3.4 LSTM neural networks

The LSTM [28] is a novel recurrent neural network (RNN) variant architecture which is capable of capturing long-term temporal dependencies, and has successfully addressed the vanishing and exploding gradients problem, which limits standard RNN [6]. It is also superior to feed-forward neural networks in solving time series tasks [29].

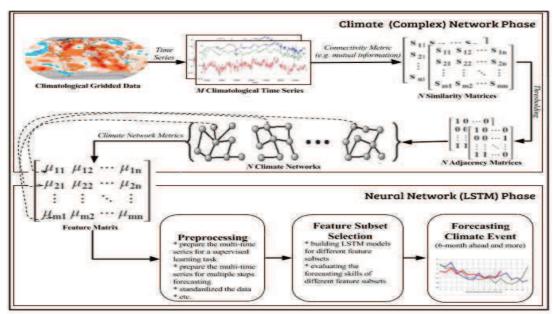


Fig. 2. The schematic representation of the proposed approach for forecasting ENSO event

Basically, LSTM RNNs Sequence-to-Sequence prediction is a MIMO prediction. The LSTM aims to estimate the conditional probability

 $p(y_1, y_2, ..., y_{T^1}|x_1, x_2, ..., x_T)$ where $(x_1, x_2, ..., x_T)$ is the input sequence of predictors and $(y_1, y_2, ..., y_{T^1})$ is its target output sequence whose length T^1 may differ from T. The LSTM computes this conditional probability by first obtaining the fixeddimensional representation v of the input sequence $(x_1, x_2, ..., x_T)$ given by the last hidden state of the LSTM, and then computing the probability of $(y_1, y_2, ..., y_{T^1})$ with a standard LSTM-LM formulation whose initial hidden state is set to the representation v of $(x_1, x_2, ..., x_T)$ [30]: $p(y_1, y_2, ..., y_{T^1} | x_1, x_2, ..., x_T) =$

$$p(y_1, y_2, ..., y_{T^1} | x_1, x_2, ..., x_T) = \prod_{t=1}^{T^1} p(y_t | v, x_1, x_2, ..., x_{t-1})$$
(2)

3.4 Climate network construction

Mathematically, a complex network is a graph, G = (V, E), which has a finite nonempty set of nodes or vertices $V = \{v_1, v_2, ..., v_n\}$ and a set of links or edges $E = \{e_1, e_2, ..., e_m\}$ between the vertices, where $E \subseteq$ $\{(v,u)|v,u\in V\}.$

In the context of CN, nodes are spatial locations that climatological time series data are observed, and if there is a statistical similarity between two time series observed at different spatial locations, then a link exist between the nodes [16]. In this study, (Fig. 1.), each evolving-CN consists of 10,512 nodes of 3-month daily SATs time series data. Donges, Zou, Marwan et al. has posited that "the observed similarity of Pearson correlation (linear) and mutual (nonlinear) information networks can be considered statistically significant" [31]. To reduce computational cost, Pearson correlation is used to determine the links between the nodes. Then, using 3-month window sliding (to align with 3-month running average of the target data, Nino3.4 index) to represent evolving phenomenon state within several consecutive windows in time [27], a total of 804 similarity matrices were constructed from January 1950 to February 2017 SATs time series.

In literature, typically, a global threshold, $\tau = 0.5$, suggested by Tsonis and Roebber [13] is the choice [27]. Thus, we applied the threshold constraint τ to each similarity matrices to determine the links that are statistically relevant, and obtained binary representation of 804 samples of evolving-CNs. The threshold parameter τ can be tuned [18], but we did not consider. Then, 12 multiscale complex network metrics (Table 1.) were extracted from each evolving-CN as feature sets. A Python library package pyunicorn [32] was used to construct the CNs.

3.5 The network metrics

In the last decade, many different network metrics have been reported in the literature to describe the topological features of a given network G = (V, E), at the micro, meso-, and macro scale [33]. We employed 12 of these metrics at multiscale, based on their frequency of used in 95 CN related literature we reviewed (Table 1.). The metrics are categorized into 3 groups: least-used (L), averagely-used (A), and mostly-used (M) network metrics. We averaged all metrics not at macro scale to macro scale to form a feature matrix of dimension 804 by 12.

Table 1. The selected network metrics used for the study (feature set).

Freq.	Metrics (scale)		
L	Edge betweenness (meso), Assortativity (macro), Global efficiency (macro), Diameter (macro)		
	Closeness centrality (meso), Area weighted connectivity (meso), Average path length (macro), Characteristic path length (macro), Transitivity coefficient (macro)		
М	Clustering coefficient (meso), Degree (micro), Betweenness (meso)		

3.6 Data preparation, training and testing

The predictors (network metrics feature set) was transformed to 3-dimension (W, H, F) shape, where W is the window size as the training sequence, H is the multiple time steps (horizon) and F is the number of features, and each window sample was normalized to a range of -1 and 1. The predictand (Nino3.4 index) was also transformed to 2-dimension (1, H) shape. A window size of 4 years was used, as on average ENSO phenomenon occurs every 4 years. The training sequences are sliding windows, that is, shifted by 1 each time to obtain an overlap with the previous windows. The 6-, 9-, 12-month horizons yielded 757, 755, and 752 samples windows respectively. Each sample was divided into 85% training set and 15% testing set, and 10% of the training set was used as validation set. To maintain time dependency, the validation set is always the last 10% samples of the training set.

As a preliminary study to find out if complex network metrics have a potential predictive skills to forecast ENSO in a longer multiple steps ahead, a simple one layer LSTM model with 50 units was trained at 200 epochs and tested on the dataset. The traditional optimization technique, stochastic gradient descent (SGD), was employed with arbitrary learning rate of 0.0005 and momentum of 0.0. Based on the number of samples of the training and validation sets, the possible training batch sizes were 2, 4, and 8, and 4 was selected arbitrarily. No hyperparameters were tuned at this preliminary study. The preliminary forecasts of ENSO was evaluated using mean absolute error (MAE) and root mean square error (RMSE) metrics as used in the recent analysis of skills of various ENSO models [2], also available at https://www.climate.gov/news-features/blogs /enso/how-good-have-enso-forecasts-been-lately. Keras, a Python deep learning library package, available at https://keras.io/, was used to build the LSTM models.

4 EXPERIMENTAL RESULTS

The results obtained from the preliminary study are significant though not outstanding, as it shows which of the network metrics (i.e. mostly-, averagely-, and least-used) is/are likely predictor(s) for ENSO phenomenon in multiple steps ahead, and the capability of LSTM neural networks to learn the long-term temporal dependencies of the phenomenon.

Tables 2, 3, 4, & 5 show the results. The boldface values in the Tables 2 & 4 are the smallest RMSE and MAE average horizons, which represent the best performance for forecasting ENSO in multiple steps ahead. From the results, it can be seen that the LSTM neural networks may have the power to learn the long-term temporal dependencies of the ENSO phenomenon, and model its nonlinear characteristics, as the performance differences between 6-,

9-, 12-month horizons are not significant. This means that at longer multiple steps ahead, LSTM has the potential to forecast ENSO phenomenon just as the short multiple steps ahead. Also, with the exception of 9-month lead forecast, that the averagely-used network metrics performed best, the results show that generally using all the 12 multiscale network metrics together may yield better prediction skills for forecasting ENSO phenomenon.

Table 2. The results of all the 12 network metrics for forecasting ENSO

	6-month lead	9-month lead	12-month lead
RMSE	0.8897	0.9017	0.8995
MAE	0.6376	0.6374	0.6428

Table 3. The results of the 3 mostly-used network metrics for forecasting ENSO

	8		
	6-month lead	9-month lead	12-month lead
RMSE	0.9318	0.9095	0.9221
MAE	0.6690	0.6442	0.6772

Table 4. The results of the 5 averagely-used network metrics for forecasting ENSO

	6-month lead	9-month lead	12-month lead
RMSE	0.8906	0.8969	0.9089
MAE	0.6547	0.6588	0.6781

Table 5. The results of the 4 least-used network metrics for forecasting ENSO

	6-month lead	9-month lead	12-month lead
RMSE	0.9292	0.9337	0.9365
MAE	0.7163	0.7164	0.7183

Fig.5 shows 6-, 9-, and 12-month lead forecast of ENSO (i.e. 3-month season SST anomaly in the Niño3.4 region of the tropical Pacific) using all the 12 multiscale network metrics. This is an example showing that training LSTM neural networks on network metrics time series have a potential to generalized and forecast ENSO phenomenon in a long multiple steps ahead. The true values (i.e. Niño3.4 index) are represented in red lines, spanning from JAS 2013 through OND 2014 seasons. The LSTM model forecasts, span from SON 2013 through FMA 2014 for the 6-month lead; from SON 2013 through MJJ 2014 for the 9-month lead; and from SON 2013 through ASO 2014 for the 12-month lead.

5 CONCLUSION

In this paper, we conducted a preliminary study of the proposed approach of combining complex network and neural network methods for forecasting ENSO phenomenon in a long multiple steps ahead. A simple one layer LSTM model was trained on a relatively small data sample (network metrics), with no hyperparameters tuned, which

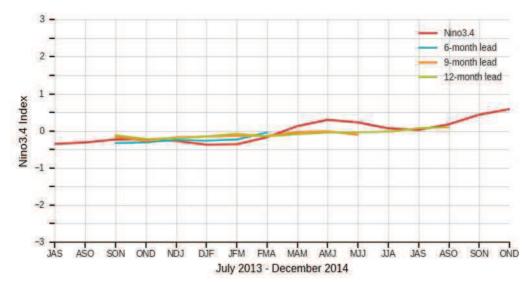


Fig. 3. The ENSO prediction using all the 12 network metrics spanning from 3-month season SON 2013 to ASO 2014

possibly would have enhanced the performance of the model. The results of the study shows that, with more data sample and a quite complex LSTM neural network model, the approach has a greater potential of forecasting ENSO phenomenon in a longer multiple steps ahead. To the best of our knowledge, this is the first time this approach has been applied to forecast ENSO phenomenon. We believe this approach has great potential performance skills to augment the ENSO forecasting activity of climate scientists and meteorologists. And our vision is to improve the model by increasing the data sample size and the complexity of the LSTM architecture. The gridded global SST anomalies dataset from the United Kingdom meteorological office spanning from 1856 to the present, and a multilayer LSTM with attention mechanism will be used to improve the performance.

REFERENCES

- [1] World Meteorological Organization (2014), El Niño-Southern Oscillation. WMO-No.1145
- [2] Barnston AG, Tippett MK, L'Heureux ML, Li S, DeWitt DG (2012), Skill of Real-Time Seasonal ENSO Model Predictions during 2002–11: Is Our Capability Increasing?. Bull. Amer. Meteor. Soc. 93:5, pp 631–651
- [3] Ludescher J, Gozolchiani A, Bogachev MI, Bunde A, Havlin S, Schellnhuber HJ (2014), Very early warning of next El Niño. Proc Natl Acad Sci USA 111:6, pp.2064–2066
- [4] Ben Taieb S, Bontempi G, Atiya AF, Sorjamaa A (2012), A review and comparison of strategies for multistep ahead time series forecasting based on the NN5 forecasting competition. Expert Syst Appl 39:8, pp.7067–7083
- [5] Zebiak SE, Cane MA (1987), A Model El Niñ-Southern Oscillation. Monthly Weather Review 115
- [6] Gers FA, Schmidhuber J, Cummins F (2000), Learning to forget: continual prediction with LSTM. Neural Comput, 12:10, pp.2451–2471
 - [7] Akram Zaytar M, El Amrani C (2016), Sequence to

- Sequence Weather Forecasting with Long Short-Term Memory Recurrent Neural Networks. IJCA, 143:11, pp.7–11
- [8] Zhang Q, Wang H, Dong J, Zhong G, Sun X (2017), Prediction of Sea Surface Temperature using Long Short-Term Memory. IEEE Geoscience and Remote Sensing Letters
- [9] Lipton ZC, Berkowitz J, Elkan C (2015), A Critical Review of Recurrent Neural Networks for Sequence Learning. arXiv:1506.00019v4
- [10] Li Y, Yang R, Yang C, Yu M, Hu F, Jiang Y (2017), Leveraging LSTM for rapid intensifications prediction of tropical cyclones. ISPRS Ann. Photogramm. Remote Sens. Spatial Inf. Sci. IV-4/W2, pp.101–105
- [11] Hong C, Cho K-D, Kim H-J (2001), The relationship between ENSO events and sea surface temperature in the East (Japan) Sea. Progress in Oceanography 49:1-4, pp. 21–40
- [12] Catto JL, Nicholls N, Jakob C (2012), North Australian sea surface temperatures and the el niño-southern oscillation in observations and models. J Clim 25:14, pp.5011–5029
- [13] Tsonis AA, Roebber PJ (2004), The architecture of the climate network. Physica A: Statistical Mechanics and its Applications 333, pp.497–504
- [14] Albert R, Barabási A-L (2002), Statistical mechanics of complex networks. Rev. Mod. Phys. 74:1, pp.47–97
- [15] Steinhaeuser K, Chawla NV, Ganguly AR (2010), Complex networks in climate science: progress, opportunities and challenges. In Proceedings of the 2010 Conference on Intelligent Data Understanding, USA, 2010, pp.16–26
- [16] Donner RV, Wiedermann M, Donges JF (2017), Complex network techniques for climatological data analysis. In: Franzke CLE, O'Kane TJ (eds), Nonlinear and Stochastic Climate Dynamics, Cambridge, UK: Cambridge University Press, pp.159–183.
- [17] Zanin M, Papo D, Sousa PA, Menasalvas E, Nicchi A, Kubik E, Boccaletti S (2016), Combining complex

- networks and data mining: Why and How. Physics Reports 635, pp.1–44
- [18] Steinhaeuser K, Chawla NV, Ganguly AR (2011), Complex networks as a unified framework for descriptive analysis and predictive modeling in climate science. Stat Anal Data Min 4:5, pp.497–511.
- [19] Saha M, Mitra P (2015), Climate network based index discovery for prediction of Indian monsoon. In: Kryszkiewicz M, Bandyopadhyay S, Rybinski H, Pal SK (eds), Pattern recognition and machine intelligence 9124, Cham:Springer International Publishing, pp.554–564
- [20] Sencan H, Chen Z, Hendrix W, Pansombut T, Semazzi F, Choudhary A, Kumar V, Melechko AV, Samatova NF (2011), Classification of emerging extreme event tracks in multivariate spatio-temporal physical systems using dynamic network structures: application to hurricane track prediction. In IJCAI'11 Proceedings of the Twenty-Second international joint conference on Artificial Intelligence, Barcelona, Spain, 2011, 2, pp.1478–1484
- [21] van der Linden JH, Narsilio GA, Tordesillas A (2016), Machine learning framework for analysis of transport through complex networks in porous, granular media: A focus on permeability. Phys Rev E 94:2-1, p.022904
- [22] Jamal W, Das S, Oprescu I-A, Maharatna K, Apicella F, Sicca F (2014), Classification of autism spectrum disorder using supervised learning of brain connectivity measures extracted from synchrostates. J Neural Eng, 11:4, p.046019
- [23] Dijkstra HA (2006), The ENSO phenomenon: theory and mechanisms. Adv. Geosci. 6, pp.3–15
- [24] Landsea CW, Knaff JA (2000), How Much Skill Was There in Forecasting the Very Strong 1997–98 El Niño? Bull. Amer. Meteor. Soc. 81, pp.2107–2119
- [25] Kalnay E, Kanamitsu M, Kistler R, Collins W, Deaven D, Gandin L, Iredell M, Saha S, White G, Woollen J, Zhu Y, Leetmaa A, Reynolds B, Chelliah M, Ebisuzaki W, Higgins W, Janowiak J, Mo C, Ropelewski K, Wang C, Dennis (1996), The NCEP/NCAR 40-Year Reanalysis Project. Bulletin of the American Meteorological Society
- [26] Huang B, Thorne PW, Banzon VF, Boyer T, Chepurin G, Lawrimore JH, Menne MJ, Smith TM, Vose RS, Zhang H-M (2017), Extended reconstructed sea surface temperature, version 5: upgrades, validations, and intercomparisons. J Clim 30:20, pp.8179–8205
- [27] Radebach A, Donner RV, Runge J, Donges JF, Kurths J (2013), Disentangling different types of El Niño episodes by evolving climate network analysis. Phys Rev E Stat Nonlin Soft Matter Phys, 88:5, p.052807
- [28] Hochreiter S, Schmidhuber J (1997), Long short-term memory. Neural Comput. 9:8, pp.1735–1780
- [29] Gers FA, Eck D, Schmidhuber J (2002), Applying LSTM to Time Series Predictable Through Time-Window Approaches. In: Tagliaferri R, Marinaro M (eds), Neural Nets WIRN Vietri-01, London, Springer London, 2002, pp.193–200
- [30] Sutskever I, Vinyals O, Le QV (2014), Sequence to Sequence Learning with Neural Networks. arXiv:1409.3215

- [31] Donges JF, Zou Y, Marwan N, Kurths J (2009), Complex networks in climate dynamics: Comparing linear and nonlinear network construction methods. Eur. Phys. J. Spec. Top. 174:1, pp.157–179
- [32] Donges JF, Heitzig J, Beronov B, Wiedermann M, Runge J, Feng QY, Tupikina L, Stolbova V, Donner RV, Marwan N, Dijkstra HA, Kurths J (2015), Unified functional network and nonlinear time series analysis for complex systems science: The pyunicorn package. Chaos 25:11, p.113101
- [33] Costa LdaF, Rodrigues FA, Travieso G, Villas Boas PR (2007), Characterization of complex networks: A survey of measurements. Adv Phys, 56:1, pp.167–242