Online Multi-Agent Path Finding: New Results - Supplementary Materials

Jonathan Morag, Roni Stern, Ariel Felner, Dor Atzmon, Eli Boyarski

Ben-Gurion University of the Negev
{moragj, sternron}@post.bgu.ac.il, felner@bgu.ac.il, {dorat, boyarske}@post.bgu.ac.il

Observation 1. If $\Delta(P^+) > \Delta(P)$, it holds that for every minimal-length path for the new agent there exists a conflict with the paths of the other agents in any $\Pi_{sn}(P)$.

Proof. By contradiction, assume that $\Delta(P^+) > \Delta(P)$ but there exists a minimal length path π_i^* for the new agent a_i that does not conflict with any path in $\Pi_{sn}(P)$. Therefore, $\Pi_{sn}(P) \cup \{\pi_i^*\}$ is a valid and optimal solution for P^+, with the cost

$$SOC(\Pi_{sn}(P^+)) = SOC(\Pi_{sn}(P)) + |\pi_i^*|. \tag{1}$$

$\Pi_{or}(P)$ is an optimal solution for the group of agents in P, so with the addition of agent a_i in P^+,

$$SOC(\Pi_{or}(P^+)) \geq SOC(\Pi_{or}(P)) + |\pi_i^*|. \tag{2}$$

Subtracting 2 from 1, we get

$$\Delta(P^+) = SOC(\Pi_{sn}(P^+)) - SOC(\Pi_{or}(P^+)) \leq$$
$$SOC(\Pi_{sn}(P)) + |\pi_i^*| - SOC(\Pi_{or}(P)) - |\pi_i^*| =$$
$$SOC(\Pi_{sn}(P)) - SOC(\Pi_{or}(P)) = \Delta(P).$$

Contradicting the assumption that

$$\Delta(P^+) > \Delta(P).$$

□

Observation 2. If $\Delta(P^+) > \Delta(P)$, it holds that for every snapshot-optimal solution $\Pi_{sn}(P^+)$ there exists at least one agent whose plan was made longer. For an old agent it means its new path is longer than its old path ($\Pi_{sn}(P)$). For a new agent a_i, it would mean its path is longer than π_i^*.

Proof. By contradiction, assume $\Delta(P^+) > \Delta(P)$ and

$$SOC(\Pi_{sn}(P^+)) = SOC(\Pi_{sn}(P)) + |\pi_i^*|. \tag{1}$$

$\Pi_{or}(P)$ is an optimal solution for the group of agents in P, so with the addition of agent a_i in P^+,

$$SOC(\Pi_{or}(P^+)) \geq SOC(\Pi_{or}(P)) + |\pi_i^*|. \tag{2}$$

By isolating $|\pi_i^*|$ in both 1 and 2, we get

$$SOC(\Pi_{or}(P^+)) - SOC(\Pi_{or}(P)) \geq |\pi_i^*|$$
$$= SOC(\Pi_{sn}(P^+)) - SOC(\Pi_{sn}(P)).$$

Meaning,

$$SOC(\Pi_{sn}(P)) - SOC(\Pi_{or}(P)) \geq$$
$$SOC(\Pi_{sn}(P^+)) - SOC(\Pi_{or}(P^+)).$$

Contradicting the assumption that

$$\Delta(P^+) > \Delta(P).$$

□

Observation 3. If $\Delta(P^+) > \Delta(P)$ then for every oracle-optimal solution $\Pi_{or}(P^+)$ and snapshot-optimal solution $\Pi_{sn}(P)$, it holds that $\Pi_{sn}(P)$ is not a prefix of $\Pi_{or}(P^+)$ up to time step t^+.

Proof. By contradiction, assume $\Delta(P^+) > \Delta(P)$, and exists an oracle-optimal solution $\Pi_{or}(P^+)$ and a snapshot-optimal solution $\Pi_{sn}(P)$ such that $\Pi_{sn}(P)$ is a prefix of $\Pi_{or}(P^+)$ up to t^+. $\Pi_{or}(P^+)$ can be split into two partial solutions: $\Pi_{or}(P^+)[0 : t^+ - 1]$ and $\Pi_{or}(P^+)[t^+ : \infty]$. Because $\Pi_{sn}(P)$ is a prefix of $\Pi_{or}(P^+)$,

$$SOC(\Pi_{or}(P^+)[0 : t^+ - 1]) = SOC(\Pi_{sn}(P)[0 : t^+ - 1]).$$

By definition,

$$SOC(\Pi_{or}(P^+)) \leq SOC(\Pi_{sn}(P^+)).$$

So,

$$SOC(\Pi_{or}(P^+)[0 : t^+ - 1]) + SOC(\Pi_{or}(P^+)[t^+ : \infty]) \leq$$
$$SOC(\Pi_{sn}(P)[0 : t^+ - 1]) + SOC(\Pi_{sn}(P^+))[t^+ : \infty].$$

Subtracting $SOC(\Pi_{sn}(P)[0 : t^+ - 1])$ from both sides,

$$SOC(\Pi_{or}(P^+)[t^+ : \infty]) \leq SOC(\Pi_{sn}(P^+))[t^+ : \infty].$$

But $\Pi_{or}(P^+)$ is a prefix of $\Pi_{sn}(P)$ up to t^+, so by the definition of snapshot-optimal,

$$SOC(\Pi_{or}(P^+)[t^+ : \infty]) \geq SOC(\Pi_{sn}(P^+))[t^+ : \infty].$$

Therefore,

$$SOC(\Pi_{or}(P^+)[t^+ : \infty]) = SOC(\Pi_{sn}(P^+))[t^+ : \infty]$$

and

$$SOC(\Pi_{sn}(P^+))[t^+ : \infty] - SOC(\Pi_{or}(P^+)[t^+ : \infty]) =$$
$$\Delta(P^+) = 0.$$

By definition, $\Delta(P) \geq 0$, so, contrary to the assumption that $\Delta(P^+) > \Delta(P)$,

$$\Delta(P^+) \leq \Delta(P).$$

□

Map	Dist	λ	#	OO	SO	RS	0Δ	$\mu\Delta$	$M\Delta$
empty -16-16	R	0.05	50	565	565	565	100	-	-
		0.3	50	567	568	568	82	0.35	0.86
		0.6	45	561	562	564	69	0.33	1.10
		1	40	567	569	574	68	0.66	2.49
	U	0.05	50	550	550	550	96	0.18	0.19
		0.3	50	550	551	552	66	0.23	0.57
		0.6	49	551	552	554	35	0.30	0.70
		1	47	549	550	553	28	0.34	0.95
lak-303d	R	0.05	39	9,879	9,879	9,880	100	-	-
		0.3	19	8,791	8,791	8,793	95	0.02	0.02
		0.6	15	8,329	8,329	8,330	100	-	-
		1	14	8,976	8,976	8,977	93	0.01	0.01
	U	0.05	16	11,972	11,972	11,975	100	-	-
		0.3	0	-	-	-		-	-
		0.6	0	-	-	-		-	-
		1	0	-	-	-		-	-
maze -32-32 -2	R	0.05	43	2,299	2,299	2,302	95	0.12	0.21
		0.3	25	2,120	2,120	2,122	88	0.10	0.16
		0.6	15	2,050	2,050	2,053	87	0.10	0.16
		1	10	2,014	2,014	2,014	100	-	-
	U	0.05	44	2,523	2,523	2,526	95	0.06	0.09
		0.3	1	2,135	2,135	2,153	100	-	-
		0.6	0	-	-	-		-	-
		1	0	-	-	-		-	-
random -32-32 -10	R	0.05	49	1,299	1,299	1,299	100	-	-
		0.3	48	1,289	1,289	1,290	81	0.10	0.31
		0.6	47	1,284	1,284	1,287	74	0.17	0.39
		1	40	1,257	1,257	1,260	78	0.16	0.26
	U	0.05	48	1,244	1,244	1,244	85	0.09	0.16
		0.3	48	1,245	1,245	1,246	69	0.13	0.24
		0.6	44	1,247	1,248	1,250	43	0.16	0.39
		1	38	1,240	1,241	1,244	42	0.13	0.27
room -32-32 -4	R	0.05	49	1,203	1,203	1,204	92	0.10	0.19
		0.3	34	1,103	1,103	1,107	82	0.27	0.46
		0.6	28	1,022	1,022	1,025	82	0.26	0.70
		1	18	1,064	1,065	1,067	83	0.52	0.79
	U	0.05	50	1,301	1,302	1,303	74	0.14	0.26
		0.3	42	1,294	1,296	1,303	43	0.20	0.47
		0.6	12	1,201	1,203	1,213	42	0.24	0.52
		1	4	1,214	1,216	1,230	75	0.71	0.71
ware-house -10-20 -10-2-2	R	0.05	50	5,238	5,238	5,238	96	0.04	0.04
		0.3	49	5,184	5,184	5,185	94	0.13	0.36
		0.6	48	5,249	5,250	5,250	94	0.08	0.19
		1	44	5,207	5,208	5,208	98	0.16	0.16
	U	0.05	50	5,311	5,311	5,311	98	0.02	0.02
		0.3	42	5,266	5,267	5,268	83	0.03	0.06
		0.6	41	5,292	5,293	5,294	93	0.03	0.04
		1	40	5,295	5,295	5,296	80	0.03	0.04

Table 1: Comparison of the different algorithms