A Review of Recent Developments in Simultaneous Localization and Mapping

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***Abstract*—Simultaneous Localization and Mapping (SLAM) problem has been an active area of research in robotics for more than a decade. Many fundamental and practical aspects of SLAM have been addressed and some impressive practical solutions have been demonstrated. The aim of this paper is to provide a review of the current state of the research on feature based SLAM, in particular to examine the current understanding of the fundamental properties of the SLAM problem and associated issues with the view to consolidate recent achievements.**

***Index Terms*—Extended Kalman Filter, Extended Information Filter, Observability, Optimization, SLAM**

I. Introduction

Robotic navigation, particularly when an external location reference such as a global positioning system (GPS) is not available, requires the robot to be able to build a map of the unknown environment in real-time and simultaneously work out its own location within the map. Robust solutions to the "Simultaneous Localization and Mapping (SLAM)" problem, therefore, underpins successful robot deployment in many application domains such as urban search and rescue, underground mining, underwater surveillance and planetary exploration.

The essential problem in SLAM is to estimate robot location and the map of the environment, typically represented by a set of geometric features, as measurements are gathered from a sensor as the robot moves through the environment. In the late 90's it was recognised that the probabilistic solution to the combined localization and mapping problem converges as the robot continues to gather information about its environment. Three essential properties of the estimation theoretic solution to the SLAM problem in a linear Gaussian setting [1] was demonstrated in 2001. The linear assumption was subsequently relaxed in [2] and one of the fundamental limitations of using extended Kalman filters (EKF) in solving SLAM, namely the possibility of resulting in overconfident estimates, that has been earlier observed by a number of researchers [3] was theoretically demonstrated. Despite many advances by a number of research teams around the world (see [4] and references therein), this limitation, together with its quadratic computational complexity associated with the presence of a dense covariance matrix [5], makes it impractical to use EKF to solve large-scale SLAM problems [4]. Although extended Information filter (EIF) based algorithms can overcome the computational issues to some extent [6][7][8][9] the issue of inconsistency remained unresolved.

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Recently, the availability of large memory spaces has made it possible to store all observations gathered by the robot and adopt a nonlinear optimization framework to solve the SLAM problem. In this scenario, an objective function based on Maximum Likelihood (ML) is optimized [10], leading to more robust and consistent SLAM solutions. Although the number of variables to be estimated is significantly higher as all the robot poses from where the measurements were taken are now part of the state, the information matrix is sparse. Hence the problem can be solved efficiently for scenarios consisting of a few thousand of robot to feature observations [10]. In addition to the brief summary mentioned above, vast interest in SLAM among the robotics community has resulted in many publications exploring different aspects of this complex problem. Aim of this paper is to review and consolidate recent work on the fundamental properties of feature-based SLAM and discusses some of the future challenges.

The paper is organized as follows. Section II restates the basic SLAM problem to facilitate the discussion in the subsequent sections. Section III examines the observability of the different versions of SLAM problems while Section IV discusses issues associated with the convergence of SLAM. Influence of the process and sensor characteristics are dealt with in Section V. Consistency and information exploitation are discussed in Sections VI and VII. Section VIII briefly discusses the computational efficiency. In Section IX, some open questions on the hidden structure of SLAM are posed. Section X concludes the paper.

II. The Statement of the SLAM Problem

SLAM is "the process of building a map of an unknown environment while concurrently generating an estimate for the location of an autonomous vehicle".

In its most fundamental form, only information available to the SLAM process are from sensors on-board the vehicle that observe the vehicle egomotion and the environment. The environment is assumed to consists of "features" that are stationery. The system state at time k consists of the location of robot $X_r(k)$ and the feature description $X_f(k) = X_f$. That is

$$X(k) = \begin{bmatrix} X_r(k) \\ X_f(k) \end{bmatrix} = \begin{bmatrix} X_r(k) \\ X_f \end{bmatrix}. \quad (1)$$

A process model that relates the state at time $k+1$ with the state at time k is available and is usually described by

$$\begin{array}{rcl} X_r(k+1) & = & f(X_r(k), u(k), v(k)) \\ X_f(k+1) & = & X_f(k) \end{array} \quad (2)$$

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where $u(k)$ is the control at time k and $v(k)$ is the process noise at time k.

The observation at time k is a function of $X(k)$ given by

$$z(k) = h(X_r(k), X_f(k), w(k)) = h(X_r(k), X_f, w(k)) \quad (3)$$

where $w(k)$ is the observation noise at time k.

The solution to the SLAM problem aims to obtain an estimate of $X(k)$ using the information gathered from the sensors from the beginning up to time k.

Many variations to this basic problem occur under practical scenarios. External information such as the vehicle location with respect to some global coordinate frame may be intermittently available, for example, through a GPS receiver. Known parts of the environment, for example, a landmark with a known global location, may be observed from time to time. The process model may be unreliable or not available at all. Presence of these variations can have a significant influence on the fundamental properties of the SLAM problem.

III. Observability

The obvious first question to ask is whether the SLAM problem is solvable. That is, whether the information available is sufficient to estimate the current state $X(k)$. From the control theoretic point of view, observability is the ability to uniquely compute the system state from a sequence of control actions and observations. The system state for SLAM consists of two parts: (a) the state of the map, which in fact does not change and (b) the location of the robot, which evolves in time as described by the process model, starting from some initial value. As observations available to SLAM only relate the robot location to the map of the unknown environment, initial location of the robot can not be obtained using these observations. The fundamental version of SLAM described in Section II from this perspective is not observable. Results from a number of publications that use techniques from control theory confirm this intuitive result [11]. Alternatively, from the estimation theoretic point of view, observability condition is evaluated by examining the Fisher information matrix (FIM) [12]. The fact that the FIM is singular in case of SLAM has also been recently demonstrated [13].

Much of the SLAM literature assumes either that there is some knowledge about the initial robot location available or fixes the coordinate frame of the map to the initial pose of the robot. The latter is equivalent to assuming perfect knowledge of the initial robot location and is almost universally used in practical demonstrations of SLAM. Therefore, a more appropriate question on observability is whether the system state can be uniquely determined if a part of the initial state, i.e. the initial location of the robot, is available. This question can not be answered by applying the formulae available from control theory without modifying the system equations for SLAM to include the initial pose of the robot. The same is true for the standard approaches to computing the FIM, which assume that the initial information content is zero. This is clearly incorrect if some knowledge of the initial robot state is available.

The special character of the basic SLAM problem lends itself to a relatively straightforward way to analyze its observability in the scenario that the initial robot state is known, at least to some known precision. Evolution of the robot state in time is governed by a process model, which is only a function of the robot state and is independent of the state of the map. Thus, if the process model and measurements of the appropriate control inputs are available, the robot state is observable. That is, the current robot state can be directly computed using the knowledge of the initial robot state and a sequence of control inputs, albeit at increasing uncertainty if process and control noise is present. When the robot state is known, SLAM problem reduces to a mapping problem. Therefore, the conditions for the observability of the SLAM problem are similar to that of the mapping problem, which is clearly observable even with only one unknown feature, when a range and bearing sensor is available to observe the environment.

Extensive analysis of the observability of stationary or maneuvering targets using stationery or moving observers is available in the target tracking literature. It has been demonstrated that with bearing-only and range-only sensors, specific observer or target maneuvers are required for observability [14][15]. It is also clear that SLAM is observable even when the features are “dynamic”.

Once a map with respect to some local frame is available, what further information (in the form of globally known feature locations or robot locations) is necessary to make SLAM observable with respect to a global coordinate frame is also a relatively straightforward question, related to the constraints required to anchor a rigid body in space.

When a process model is not available, the observability problem becomes more interesting. Computer vision literature addresses this for bearing-only sensors (cameras) through projective geometry.

IV. Convergence

The fact that SLAM problem is observable if the initial state of the robot is known implies that an appropriate technique can be used to estimate how the system state $X(k)$ evolves as a function of time. A fundamental question to ask is whether the uncertainty of the estimated state converges to a finite value, given a sufficient number of observations or amount of time. This is the convergence problem.

As there is information flow from the observations to the feature states (observability condition), and that noise is not injected to feature states through the process model (stationary features), feature location uncertainty will monotonically decreases during SLAM. A number of researchers confirm this fact for both the linear case [1] and the nonlinear case [2] in their work on EKF/EIF based SLAM algorithms.

Lower bounds to the uncertainty of the system states for some specific instances are also available but achieving these require controlling the robot to move in a specific manner. What is practically more realistic is to ask how the robot motion or sensing could be controlled in order to achieve a certain desired accuracy of the robot location and the map.

This is the active SLAM problem, for which some solutions are available [16][17][18][19] but much more work is perhaps needed to answer questions such as how to obtain a map with given accuracy in minimum time, or how to maximize the coverage with a fixed time horizon and a required map quality.

V. Sensor and Process Models

Even when noise is assumed to be Gaussian, sensor bias in proprioceptive and exteroceptive sensors can have a significant influence on the SLAM solution. Changes to the character of the process model, for example, when driving through different terrain, is also likely to have similar impact. Algorithms for on-line estimation of sensor biases are beginning to appear [20][21][22]. The key question of observability when the SLAM state is augmented with bias parameters has also received some attention. Theoretical problems posed when the noises are not zero-mean as well as practical solutions to this problem, perhaps through on-line calibration or use of external information such as known landmarks, calibration templates remain interesting challenges.

Virtually all SLAM literature assumes process models with additive noise of the form

$$X_r(k+1) \quad = \quad f(X_r(k), u(k)) + v(k) \tag{4}$$

where $u(k)$ is the control at time k and $v(k)$ is the process noise at time k.

In all practical robots, there is noise injection through a non-linear function of the control variable. When the equations are linearized during the estimation, this distinction does not cause any significant issues. However, if a given SLAM algorithm exploits the structure of the process model to achieve computational advantages, a practitioner using this algorithm must modify it to suit the practical application at hand or be aware of the consequences of the underlying assumption. Furthermore, for high-speed vehicles, even the localization problem requires careful treatment of the process noise [23]. When SLAM is deployed in high-speed, real-life scenarios, this can become a major issue, thus will require some attention.

VI. Consistency

A solution to a dynamic estimation problem is said to be consistent if the estimate is unbiased and the estimated covariance matrix matches the real mean square error [12]. It has now become clear that both the EKF based solution as well as the particle filter based solution to SLAM can produce inconsistent estimates.

In EKF based SLAM, even when the system noises are zero-mean Gaussian, inconsistency can occur unless the relevant Jacobians are evaluated at the true system state, which is obviously not available. The extent of inconsistency is related to the uncertainty of the state estimate, in particular to the robot orientation uncertainty. In particle filter based SLAM, the more the number of particles used, the less the extent of the inconsistency [3]. While the potential for estimator inconsistency was known for some time [24], this became an obvious problem only when researchers began using more complex geometric features, for example using line features in large scale environments [25].

In recent years there has been a growing interest on the SLAM consistency issue among the research community. It has become apparent that the overconfident estimate for a SLAM algorithm is due to the fact that the estimation process is fed with incorrect information as a result of Jacobian of observation/odometry functions with respect to the same feature/pose gets evaluated at different feature/pose location estimates [2][26].

While estimator consistency is scientifically important, whether inconsistency can be tolerated in practice depends on the intended application of the SLAM output. For example, in the case of an urban search and rescue scenario where the purpose of the map is purely for human interpretation, what is of importance is whether the topological structure of the map is correct. Indeed in the RoboCup Rescue competition which is one of the few "real-life" SLAM applications, variations of map referenced iterative closet point algorithm using laser scans are the most popular SLAM algorithms used, although they almost certainly produce inconsistent estimates.

While it is unlikely that consistency can be guaranteed as the SLAM problem is inherently non-linear, what is perhaps important is to compare the tendency for inconsistency in different algorithms. Examining the potential of using active control to keep the robot pose uncertainty within bounds to maintain consistency may also prove to be fruitful in practice.

VII. Information Exploitation

In point feature based SLAM, only a small fraction of information available from popular sensors such as laser range finders is exploited. Much of the data that do not correspond to the features represented in the map are discarded. Alternatively, a number of SLAM algorithms that does not use an explicit representation of a map, have recently emerged. Here, the complete robot trajectory, as opposed to the last robot pose, is present in the state vector [27]. More recently a family of successful SLAM algorithms that exploits complete laser scans and estimate the robot trajectory without explicitly generating a map has emerged [28][29]. Equivalent versions of fastSLAM that exploits complete laser scans have also been presented [30].

In this context, it is important to examine the two distinct steps in which information from an exteroceptive sensor is fused into the SLAM state vector. Information gathered from a reading of the sensor is first used to provide a description of the environment just being observed, resulting in some information capture. When the data association process recognizes the fact that some elements of the current observation is related to part of the map that has already been observed in the past, information capture through the refinement of the map state and the robot pose takes place. Clearly, if the map is not represented in the state vector, the update is only partial and there is some information loss. On the other hand, using the complete scan for computing the incremental motion of the robot makes use of a significantly higher fraction of the information available from a sensor in comparison to an algorithm that uses only few of the observations that are associated with point features in the environment.

However, not having an environment description in the state vector may cause some information loss. An evaluation of pose only SLAM algorithm as compared with feature based SLAM is performed in terms of information loss [31]. This work asks how much accuracy or consistency is compromised for the efficiency in tree-based network optimizer (TORO) [28] or pose only SLAM. It is shown that while information is indeed lost, overall gain in terms of number of sensor observations may be an important factor, explaining the practical success of pose only SLAM algorithms when laser scans are available.

A number of more direct methods for incorporating a larger fraction of the information gathered by the sensors, for example by modeling the environment with more complex geometric primitives such as lines [25] or B-splines [32] have recently been reported. While promising, some issues related to the consistency of the solution have been observed for both these scenarios and require further work [33]. Modeling the region surrounding a point feature using a shape model in a coordinate frame attached to the feature [34] is a hybrid strategy where the point feature map can be updated while the shape models are only used to increase the information content of the observation. A fully parameterized environment representation which captures all the information available in an observation, able to be updated in a statistically consistent manner, yet is computationally tractable remains an interesting challenge [35].

VIII. Computational Efficiency

A review paper on SLAM is not complete without any mention of the computational efficiency, or recent work on large loop closure. An increasingly large array of algorithms that make use of a wide range of mathematical techniques to achieve impressive results are becoming available [10][29][36][37]. Although this paper does not aim to examine these important aspects of SLAM, it must be said that all algorithms have their own advantages and disadvantages but direct comparisons are yet to become commonplace in the SLAM literature. It is clearly the time to follow the standard scientific practice and create a set of benchmark problems and datasets, such as those available in the optimization literature. While some progress in this direction has been made by individual researchers making datasets available, such as the Radish dataset repository and papers that are being published in the International Journal of Robotics Research, much remains to be done.

IX. Some Hidden Structure in SLAM Problems

It is the general consensus that the optimization based approach can provide more accurate and consistent solutions to the feature based SLAM. In contrast to the filter based method, optimization based approaches such as smoothing and mapping (SAM) [38] estimate the complete robot trajectory and the map.

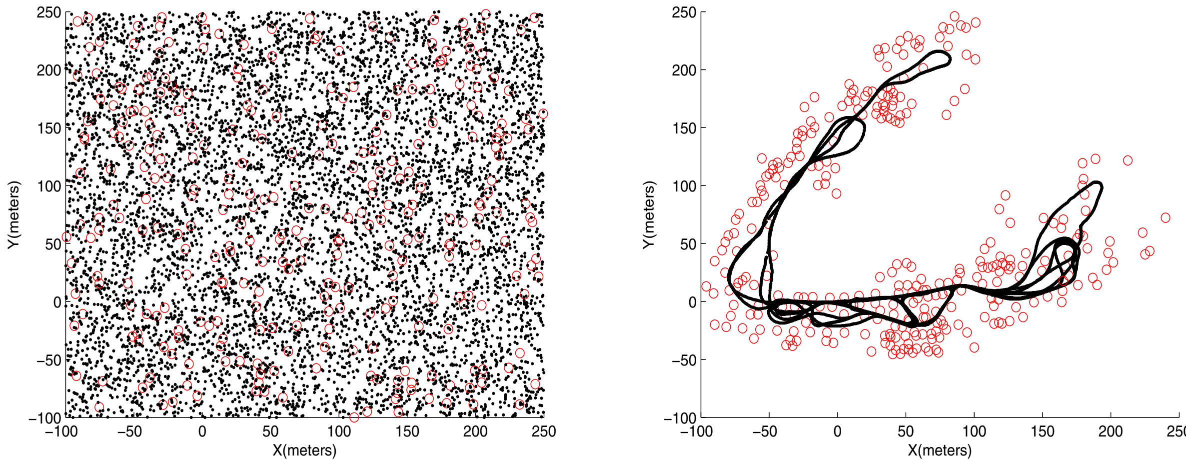
Despite a number of optimization based SLAM algorithms have been developed in the last few years, solving the SLAM as an optimization problem is in general a difficult task as the problem is non-linear and non-convex with a large search space. Most of the existing optimization based SLAM algorithms are based on local optimizers such as Gauss-Newton or Levenberg-Marquardt that starts from an initial guess to the robot poses and the map. Traditionally, the initial guess to the solution is obtained through odometry. In real scenarios, odometry information may not be available. Furthermore, even if the odometry information is available, it could potentially be very noisy and a significant error can be accumulated especially for long trajectories. Moreover, if the initial guess is not accurate enough, the algorithm may be trapped into a local minimum.

Recent research in optimization based SLAM has demonstrated some surprising results. Olson [39] uses stochastic gradient descent (SGD) algorithm to solve the pose only SLAM problem by addressing each constraint individually in a sequence. In general, such a strategy would only work if the problem does not have local minima. However, it is evident from [28][39] that the pose only SLAM can converge to the correct solution most of the time even if it starts from a poor initial guess, provided that the covariance matrix of the relative pose information is near spherical. This is unexpected and therefore indicates the presence of some special structure that makes this happen.

Some recent work [40], demonstrates a similar phenomenon for the feature based SLAM problem. Contrary to the obvious expectation that a simple Gauss-Newton algorithm will be trapped into a local minimum, it has been shown using the popular Victoria Park dataset [5], that the algorithm converges to the global minimum, 80% of the time if all the covariance matrices are set to identity matrices, with random initial guesses to the 6898 vehicle poses and the 299 feature positions (Fig. 1). Furthermore, the final result is very close to the true solution obtained using the correct covariance matrices. Fig. 2 shows the results from different SLAM algorithms using the same dataset. Fig. 2(a) shows the compressed filter SLAM result by ACFR in [5]. Fig. 2(b) shows the result of exactly sparse extended Information filter (ESEIF) algorithm in [8]. Fig. 2(c) shows the decoupled SLAM (D-SLAM) implementation in [9]. Fig. 2(d) shows the result of incremental smoothing and mapping (iSAM) in [10]. Published results are slightly different from each other in terms of the number of features, the vehicle trajectory estimate, and the uncertainty estimate about the map [5][8][9][10]. This is likely due to the use of different noise levels, model characteristics, and data associations.

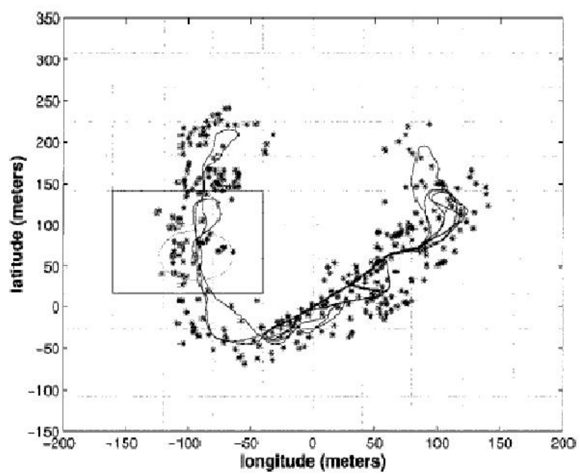
It was shown in [40] that the same simple Gauss-Newton algorithm converges to the global minimum when zeros are used as the initial guesses for the DLR Spatial-Cognition dataset, another popular dataset used by SLAM researchers (available at http://www.sfbtr8.spatial-cognition.de/insidedataassociation/data.html). Still, the convergence to the global minima occurs only when the covariance matrices are set to identity matrices. Interestingly, for the DLR dataset, the Gauss-Newton algorithm converges to a local minimum when a random initial guess is used.

These results raise some interesting questions: How far

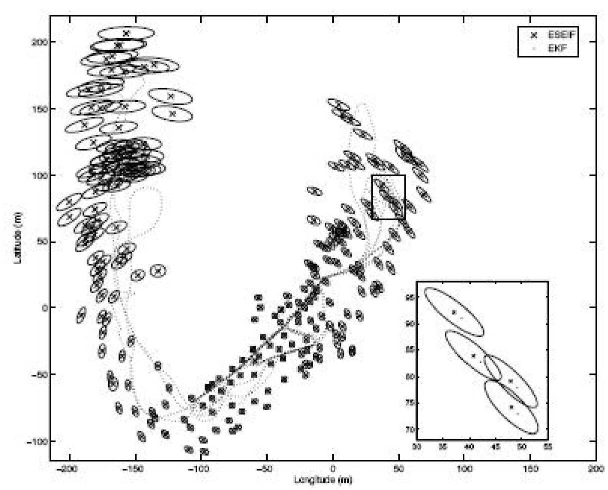


(a) (b)

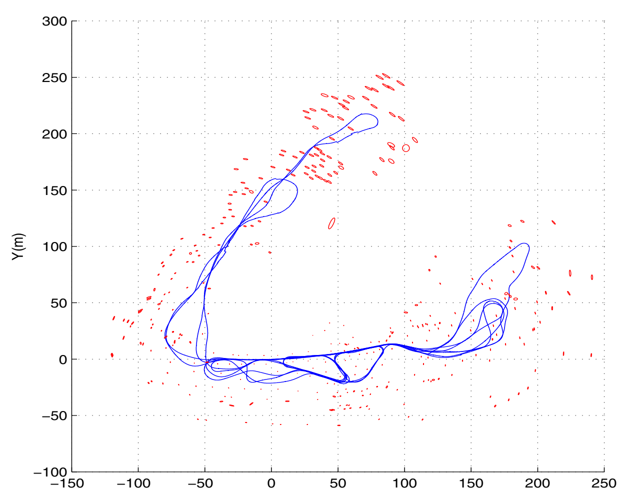
Fig. 1. (a) Random values assigned to 6898 vehicle poses and 299 feature positions as initial guess to a Gauss-Newton algorithm; (b) The algorithm converges to the global minimum which is very close to the ground truth.



(a) Map and robot trajectory by Compressed filter, the figure is from [5]



(b) Map and robot trajectory from ESEIF, figure is from [8]



(c) Map and robot trajectory from D-SLAM; the figure is from [9]

(d) Map and robot trajectory from SAM, the figure is from [10]

Fig. 2. Different SLAM implementation results using Victoria Park dataset

is SLAM from a convex optimization problem? Do local minima exist near the global solution to the SLAM problem? Since an incorrect location estimate may lead to disastrous consequences in many applications of autonomous systems, it is important to address these questions before these simple approaches can be confidently deployed in practical scenarios.

X. Conclusion

While the SLAM community has made great strides in the past few years, there are some further insights to be gained and some open questions that require answers, even on the fundamental aspects of SLAM. Moving towards some standards for examining and evaluating SLAM algorithms will also be essential to make recent outcomes of SLAM research useful to a practitioner.

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