Operational Research (LINMA 2491) - 2022-2023
Homework/Project instructions

Friday March 31, 2023 (updated on Wednesday April 5)

1 General instructions

- Due date: Friday May 19, 2023, 23:59pm
- The assignment can be done alone or in a team of 2. You can discuss with your classmates, but the report and code (that will be checked) must be the personal work of the (team of) student(s) submitting the work for this project.
- The language for the project is either Julia with the modeling package JuMP and bindings to the CPLEX or GUROBI solver, or Python with CPLEX or GUROBI via their Python API. (Julia with JuMP is recommended.)
- The code of your implementation, tests, etc, must be delivered with the report.
- Don't forget to properly reference any source you use.
- Delivering the report and code via a Jupyter Julia or Python Notebook is preferred (see for example https://github.com/JuliaLang/IJulia.jl and the short Julia notebook `lot_sizing_FL_reformulation_example.ipynb` on Moodle for a quick start).

2 Small Benders decomposition example with feasibility cuts

Consider the MIP

$$\min_{y,x} 0.1y + 1x \tag{1}$$
$$y + x \geq 10 \tag{2}$$
$$-2y + x \geq 6 \tag{3}$$
$$-x \geq -9 \tag{4}$$
$$y, x \geq 0,\ y \in \mathbb{Z} \tag{5}$$

1. What is the master problem of a Benders decomposition applied to this problem, and what is the Benders subproblem in its dual form (here max...) ?
2. Show that for the candidate master solution $(y, x_0) = (0, 0)$, the Benders subproblem (dual form) is unbounded and give a corresponding extreme ray. Write the corresponding feasibility cut. Give an interpretation of this constraint on y in view of the constraints (2) and (4). Are there other feasibility cuts ?
3. Add to the master problem all possible optimality cuts. Hint: Analyse the feasible set of the (dual) Benders subproblem and identify its vertices.
4. Draw the optimality cuts (in the space (y, x_0)) and give an interpretation.

3 Solving the farmer's problem example via the L-shaped method

Consider the stochastic optimization program of the farmer's problem given on slide 12 of the slide deck "Course 5 - Introduction to Stochastic Programming via examples".

1. Solve the problem via the L-shaped method.
2. Report on the optimality and feasibility cuts generated when solving the problem. Are there feasibility cuts ? Can we have anticipated this in view of the structure of the initial problem (problem in extensive form) ?

4 Solving a capacity expansion planning problem via the L-shaped method

Consider the capacity expansion planning problem described in Section 1.2 of the course textbook (Anthony Papavasiliou, Stochastic Dual Dynamic Programming, manuscript, see the file "textbook.pdf" on Moodle) and on slide 9 of the course on Performance of Stochastic Programming Solutions.

1. Implement the extensive form of the corresponding stochastic optimization problem (see the textbook page 10 for the general formulation), solve it directly and verify that you obtain the same solution as in the notes.
2. Now, re-solve the extensive form program assuming that you cannot use demand response, i.e. the last technology "DR" is not available, and report the results.
3. Do we expect to have to generate feasibility cuts if we solve this new problem (without allowing for demand response) via the L-shaped method ? Why ?
4. Solve the problem via the L-shaped method and report on the feasibility and optimality cuts generated during the resolution.