

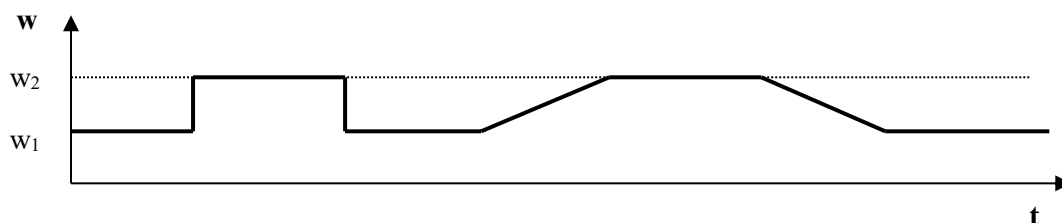
Automation

Laboratory Exercise 2

IDENTIFICATION AND DISCRETE-TIME CONTROL OF A HEATING SYSTEM

- 1) Connect the control loop with a real heating system and familiarize yourself with the monitoring and control software "WCONTROL".
- 2) Measure the step response of the controlled system when changing the manipulated variable (input to the controlled system) by 20% (the system is supposed to be linear). Recommended area of the measurement is from 50% to 70% of the maximum power. Archive the obtained step response with a suitable period.
- 3) Approximate the measured step response (by using the least squares method) and construct a continuous time mathematical model (transfer function) of the controlled system, specify the parameters including the physical units.
- 4) Compare the step response of the obtained (identified) model with the measured characteristic (in one chart).
- 5) Compute discrete-time transfer function (Z-model) of the system under assumption of the zero-order hold. Select the sampling period so that the transient process will be covered by 8-14 samples.
- 6) Specify the differential equation for calculated discrete-time transfer function.
- 7) Perform the calculation of discrete-time step response and compare it with the continuous-time one.
- 8) Design a discrete-time PID controller by using Desired Model Method (formerly known as Inverse Dynamics Method) for control behavior without overshoot (choose appropriate size of sampling period). Perform the real control experiment.
- 9) Simulate the control experiment from the previous point in Matlab + Simulink environment and compare the simulation result with the real measurement.

For points 8 and 9 assume the following general shape of the reference signal w (typically $w_2 - w_1 = 20^\circ\text{C}$):



1. From measured data graph, it has been assumed that the heater behavior could be described with second order differential equation. Represented in the S-plane with the following equation:

$$G(s) = \frac{K}{(T1 * S + 1)(T2 * S + 1)}$$

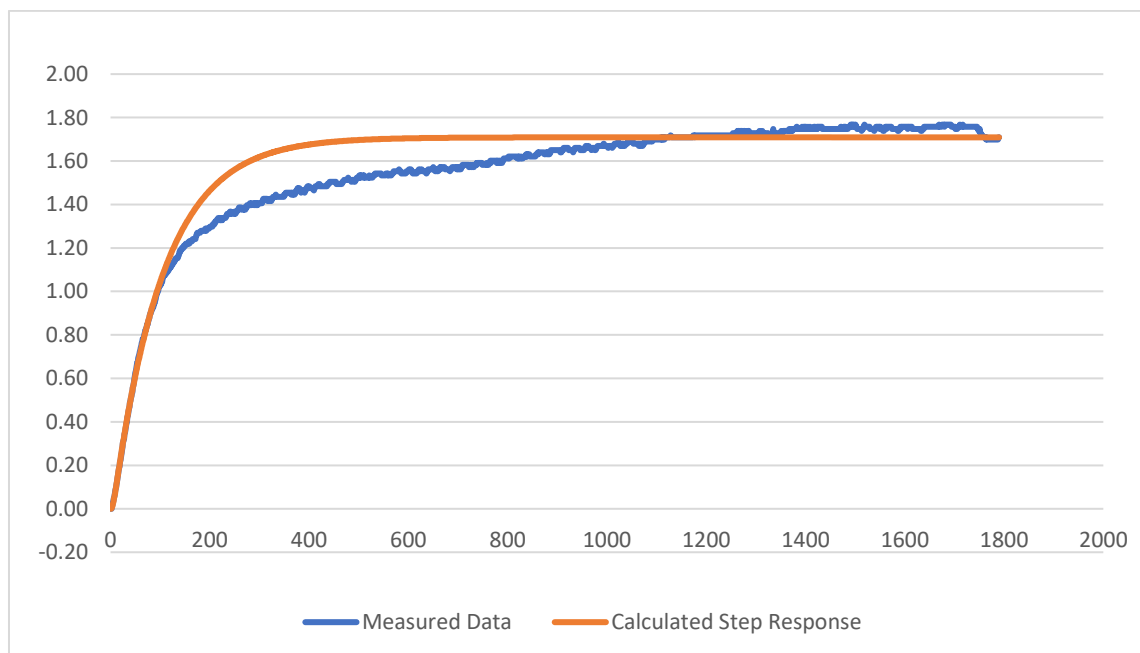
- getting the step response using inverse laplace transform:

$$H(t) = \mathcal{L}^{-1} \left[\frac{G(s)}{s} \right] = \left[\frac{K}{s(T1 * s + 1)(T2 * s + 1)} \right]$$

$$H(t) = K * \left[1 + \frac{T1}{T2 - T1} * e^{\frac{-t}{T1}} + \frac{T2}{T2 - T1} * e^{\frac{-t}{T2}} \right]$$

-Using LSM to get values (K, T1, T2) and plug it in the step response equation H(t), then compare the results with the acquired data:

T1	100.0554279
T2	4.488164455
K	1.709



2. Z-Model:

Assumptions:

- Zero Relative shift
- Zero-order hold.

$$T(\text{Sampling time}) = \frac{\text{Readings count}}{\text{restricted number of samples}} = \frac{100}{8} = 12.5 \text{ seconds}$$

$$G(s) = \frac{1.7}{(100S + 1)(4.5S + 1)} = \frac{1.7}{450 * S^2 + 104.5 * S + 1}$$

- Using Z-transform table

$$G(z) = \frac{0.13 * Z + 0.05}{Z^2 - 0.94 * Z + 0.05} = \frac{Y(z)}{U(z)}$$

3. Difference Equation:

- Applying inverse Z-transform:

$$Y(z) * [Z^2 - 0.94 * Z + 0.05] = U(z) * [0.13 Z + 0.05]$$

$$\frac{Y(z) * [Z^2 - 0.94 * Z + 0.05]}{Z^2} = \frac{U(z) * [0.13 Z + 0.05]}{Z^2}$$

$$Y(Z^{-1}) * [1 - 0.94 * Z^{-1} + 0.05 * Z^{-2}] = U(Z^{-1}) * [0.13 * Z^{-1} + 0.05 * Z^{-2}]$$

$$Z^{-1}[Y(Z^{-1}) * [1 - 0.94 * Z^{-1} + 0.05 * Z^{-2}]] = Z^{-1}[U(Z^{-1}) * [0.13 * Z^{-1} + 0.05 * Z^{-2}]]$$

$$y(k) - 0.94 * y(k - 1) + 0.05 * y(k - 2) = 0.13 * u(k - 1) + 0.05 * u(k - 2)$$

$$\mathbf{y(k) = 0.94 * y(k - 1) - 0.05 * y(k - 2) + 0.13 * u(k - 1) + 0.05 * u(k - 2)}$$

4. Desired Model Method:

$$G(z) = r0 * \left[1 + \frac{T}{TI} \frac{Z}{Z-1} + \frac{TD}{T} \frac{Z-1}{Z} \right]$$

$$Tw(\text{feedback time constant}) = \frac{T}{0.286} = \frac{12.5}{0.286} = 44 \text{ seconds}$$

$$C1 = e^{\frac{-T}{T1}} = e^{\frac{-12.5}{100}} = 0.88$$

$$C2 = e^{\frac{-T}{T2}} = e^{\frac{-12.5}{4.5}} = 0.06$$

$$Cw = e^{\frac{-T}{Tw}} = e^{\frac{-12.5}{44}} = 0.75$$

$$T1 = 100$$

$$T2 = 4.5$$

$$K = 1.7$$

$$r0 (\text{proportional}) = \frac{(1 - Cw)T1}{KT} = \frac{(1 - 0.75) * 100}{1.7 * 12.5} = 1.17$$

$$TI (\text{integral}) = \frac{C1 + C2 - 2C1C2}{1 - C1 - C2 + C1C2} * T = \frac{0.88 + 0.06 - (2 * 0.88 * 0.06)}{1 - 0.88 - 0.06 + (0.88 * 0.06)} * 12.5 = 92.46$$

$$TD (\text{Derivative}) = \frac{C1C2}{C1 + C2 - 2C1C2} * T = \frac{0.88 * 0.06}{0.88 + 0.06 - (2 * 0.88 * 0.06)} * 12.5 = 0.79$$

$$q0 = r0 \left[1 + \frac{T}{T1} + \frac{TD}{T} \right] = 1.17 * \left[1 + \frac{12.5}{100} + \frac{0.79}{12.5} \right] = 1.39$$

$$q1 = -r0 \left[1 + \frac{2TD}{T} \right] = -1.17 * \left[1 + \frac{2 * 0.79}{12.5} \right] = -1.31$$

$$q2 = r0 \left[\frac{TD}{T} \right] = 1.17 * \left[\frac{0.79}{12.5} \right] = 0.073$$

$$Gr(z) = \frac{q0 + q1Z^{-1} + q2Z^{-2}}{1 - Z^{-1}} = \frac{1.39 - 1.31Z^{-1} + 0.07Z^{-2}}{1 - Z^{-1}}$$

- Using T(Sample Time)= 1 second, results shows;
No overshoot, and good reference tracking.

