Automation

Laboratory Exercise 1

CONTINUOUS-TIME LINEAR TIME-INVARIANT (LTI) SYSTEMS – DESCRIPTION, ANALYSIS AND CONTROL DESIGN

Assume a single-input single-output (SISO) LTI system given by the differential equation:

$$a_2 \cdot y''(t) + a_1 \cdot y'(t) + a_0 \cdot y(t) = b_0 \cdot u(t)$$

where $u(t)$ and $y(t)$ is the input and output, respectively.

Replace the general constants a_2, a_1, a_0, b_0 with the values according to your individual assignment and perform the following tasks:

1. Write the transfer function. Determine the zeros, poles, order and relative order. Then decide if the system is stable/unstable; underdamped/overdamped/critically damped; and minimum phase/non-minimum phase.
2. Calculate the step response and the impulse response functions (analytically, i.e. "manually") and plot the graphs of both calculated functions. Then plot the step and impulse responses by using the MATLAB commands and compare the obtained results.
3. Express the sinusoidal transfer function (frequency response function) and plot the Nyquist diagram (Nyquist plot) on the basis of the analytical calculations (via "table of values"). Then plot the Nyquist diagram by using the MATLAB command and compare the obtained results. Moreover, plot also the Bode diagrams (Bode plots) – just in MATLAB is enough.
4. Choose 2 arbitrary controller design methods (including the application of a stability criterion) and design 2 continuous-time controllers that ensures the stable feedback control system and reference tracking. Perform a simulation for each controller in the MATLAB+SIMULINK environment. (Consider a step-wise reference signal with step change from 1 to 2 in the half of simulation time).

(SISO).

$$a_2 \cdot y''(t) + a_1 \cdot y'(t) + a_0 \cdot y(t) = b_0 \cdot u(t)$$

Where:

$a_\circ = 3$
$a_1 = 5$
$a_2 = 2$
$b_\circ = 4$
$y_{(0)} = 0$
$y'_{(0)} = 0$

1. Transfer Function:

$$\mathcal{L}\,[2y''_{(t)} + 5y'_{(t)} + 3y_{(t)}] = \mathcal{L}[4\,u_{(t)}]$$

- $2 * \mathcal{L}[y''] + 5 * \mathcal{L}[y'] + 3 * \mathcal{L}[y] = 4 * \mathcal{L}[u]$
- $2 * \left[S^2\,\mathcal{L}(y) - \cancel{y_{(0)}} - \cancel{y'_{(0)}}\right] + 5 * \left[S\,\mathcal{L}(y) - \cancel{y_{(0)}}\right] + 3 * [\mathcal{L}(y)] = 4 * [\mathcal{L}(u)]$
- $2\,S^2 * Y(s) + 5\,S * Y(s) + 3 * Y(s) = 4 * \mu(s)$
- $(2\,S^2 + 5\,S + 3) * Y(s) = 4 * \mu(S)$
- $G(s) = \frac{Y(s)}{\mu(s)} = \frac{4}{2S^2+5S+3}$

Poles:

$2S^2 + 5S + 3 = (2S + 3) * (s + 1)$

$S \neq -\frac{3}{2}$ & $S \neq -1$

Zeros: constant.

Transfer Function order is: 2
Relative order is: 2
All Poles are negative in the left-hand side of the S-plane → System is stable.
All Poles are real numbers and different → System is overdamped.
No Zeros in the right-hand side of the S-plane → System is minimum Phase.

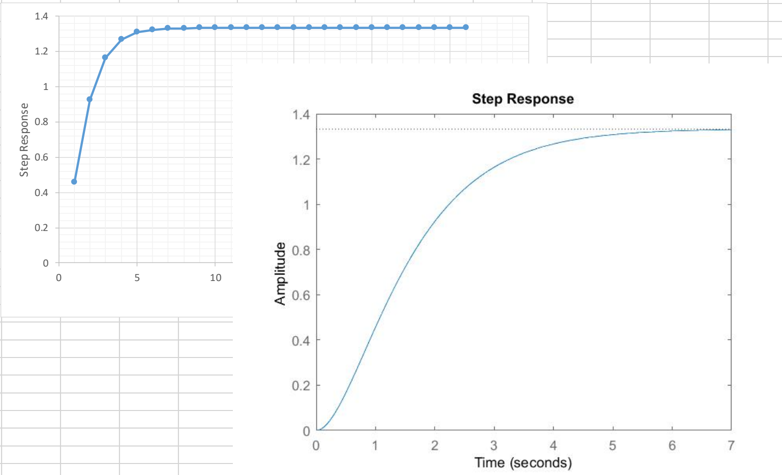
2. Step Response:

$$\mathcal{L}^{-1}\left[\frac{G(s)}{S}\right]=\mathcal{L}^{-1}\left[\frac{4}{S*(2S+3)*(S+1)}\right]=\mathcal{L}^{-1}\left[\frac{A}{S}+\frac{B}{(2S+3)}+\frac{C}{(S+1)}\right]$$

$$\mathcal{L}^{-1}\left[\frac{\frac{4}{3}}{S}+\frac{\frac{16}{3}}{(2S+3)}+\frac{-4}{(S+1)}\right]=\frac{16}{6}e^{-1.5t}-4e^{-t}+\frac{4}{3}$$

$$\therefore\quad Y(t)=\frac{16}{6}e^{-1.5t}-4e^{-t}+\frac{4}{3}$$

t	$Y(t)=\frac{16}{6}e^{-1.5t}-4e^{-t}+\frac{4}{3}$
1	0.456829329
2	0.924757716
3	1.163809051
4	1.266680784
5	1.307856437
6	1.323747417
7	1.329759236
8	1.332007867
9	1.33284335
10	1.333152549
11	1.333266709
12	1.333308797
13	1.333324301
14	1.333330009
15	1.33333211
16	1.333332883
17	1.333333168
18	1.333333272
19	1.333333311
20	1.333333325
21	1.33333333
22	1.333333332
23	1.333333333
24	1.333333333
25	1.333333333
26	1.333333333

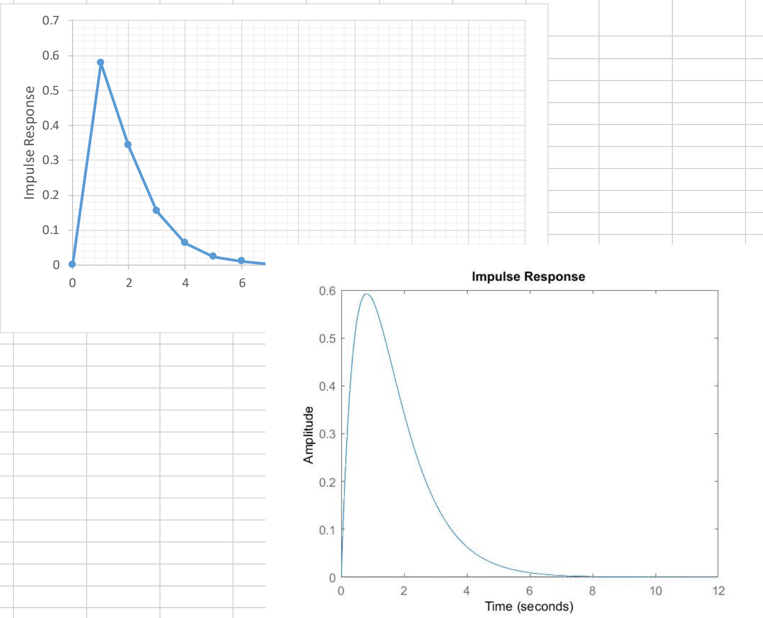


Impulse Response:

$$\mathcal{L}^{-1}[G(s)]=\mathcal{L}^{-1}\left[\frac{4}{2S^2+5S+3}\right]=\mathcal{L}^{-1}\left[\frac{A}{(2S+3)}+\frac{B}{(S+1)}\right]=\mathcal{L}^{-1}\left[\frac{-8}{(2S+3)}+\frac{4}{(S+1)}\right]$$

$$H(t)=4e^{-t}-4e^{-1.5t}$$

t	$H(t)=4e^{-t}-4e^{-1.5t}$
0	0
1	0.578997124
2	0.342192859
3	0.154712287
4	0.063347547
5	0.024739451
6	0.009421369
7	0.003537382
8	0.001317274
9	0.000488155
10	0.000180376
11	6.65338E-05
12	2.45159E-05
13	9.02772E-06
14	3.32308E-06



3. Frequency Response:

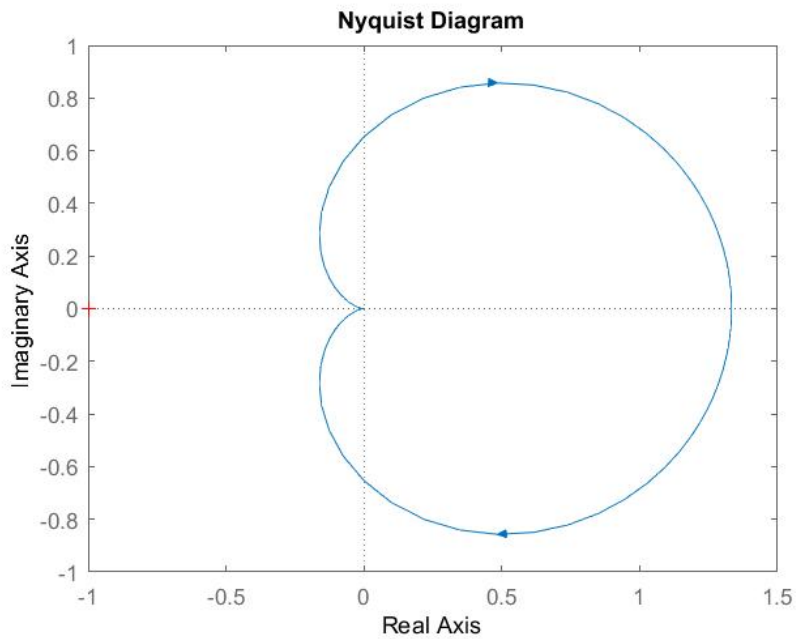
$$H(i\omega) = \frac{4}{2*(i\omega)^2 + 5*(i\omega) + 3}$$

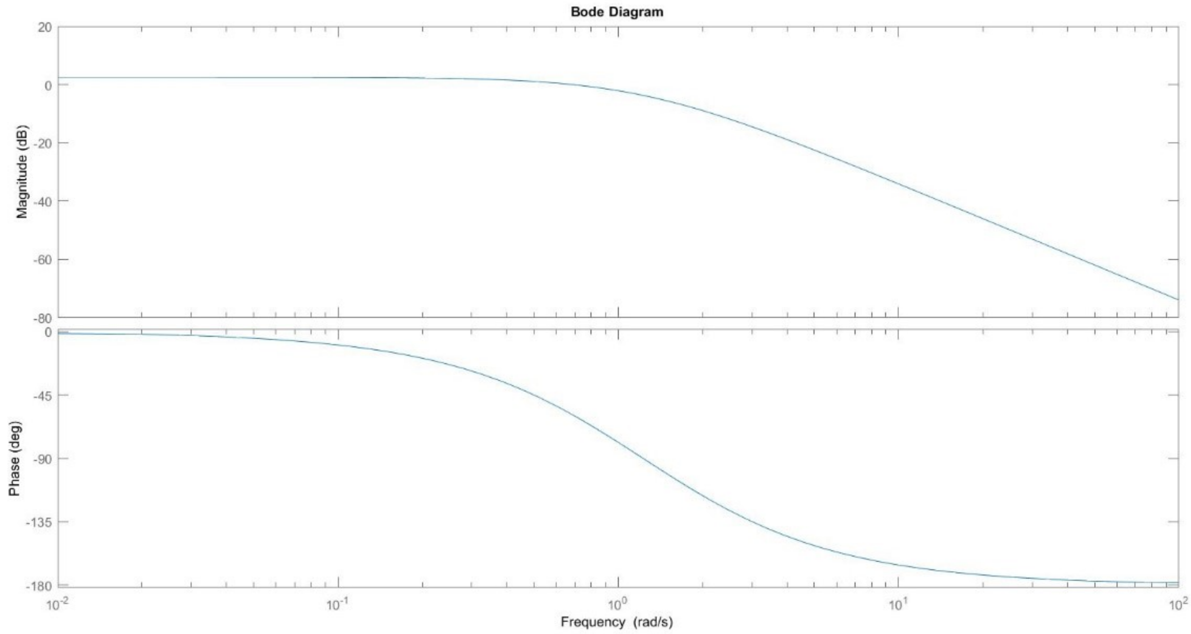
$$H(i\omega) = \frac{4}{2*(i\omega)^2 + 5*(i\omega) + 3} = \frac{4}{-2\omega^2 + 5*(i\omega) + 3} * \frac{-2\omega^2 - 5\,(i\omega) + 3}{-2\omega^2 - 5\,(i\omega) + 3}$$

$$H(i\omega) = \frac{-8\omega^2 - 20(i\omega) + 12}{4\omega^4 + 13\omega^2 + 9} = \left[\frac{-8\omega^2 + 12}{4\omega^4 + 13\omega^2 + 9}\right] - \left[\frac{20(\omega)}{4\omega^4 + 13\omega^2 + 9}\right] i$$

$$\text{Amplitude} = |\text{H}(\text{i}\omega)| = \sqrt{\left[\frac{-8\omega^2+12}{4\omega^4+13\omega^2+9}\right]^2 + \left[\frac{20(\omega)}{4\omega^4+13\omega^2+9}\right]}$$

$$Phase(\phi) = arctan\frac{Imaginary}{Real} = \arctan\left[\frac{\left[\frac{20(\omega)}{4\omega^4 + 13\omega^2 + 9}\right]}{\left[\frac{-8\omega^2 + 12}{4\omega^4 + 13\omega^2 + 9}\right]}\right]$$





4. PI- Controller setting on the basis of the step response (Overdamped):

Stability criterion (feedback closed loop)

PI controller: TF [C(s)] $C(s) = \frac{q1S+q0}{S}$

$1 + C(s) * G(s) = 0$

$[G(s)poles * C(s)poles] + [G(s)Zeros * C(s)Zeros] = 0)$

$S * (2S^2 + 5S + 3) + 4 * (q1S + q0) = 2S^3 + 5S^2 + (3 + 4q1)S + 4q0 = 0$

To guarantee feedback closed loop system stability it's necessary for the following term to not change Signs. <u>(Routh-Hurwitz Criterion)</u>

$$\frac{(15 + 20q1) - (8q0)}{5} > 0$$

$$q1 > \frac{8q0 - 15}{20}$$

$4q0 > 0$

$q0 > 0$

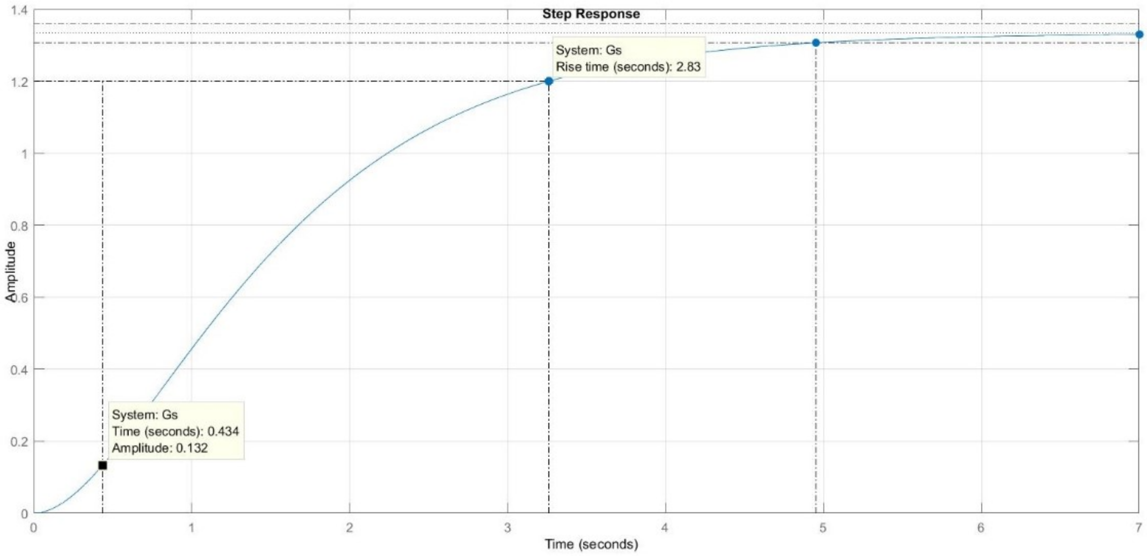
Take (integrator gain) q0 = 0.5
Then, (Proportion gain) q1 > 0.55

S^3	2	(3+4q1)
S^2	5	4q0
S^1	$\frac{(15 + 20q1) - (8q0)}{5}$	0
S^0	4q0	0

Form Step Response
Tu= 0.434 seconds
Tn= 2.396 seconds

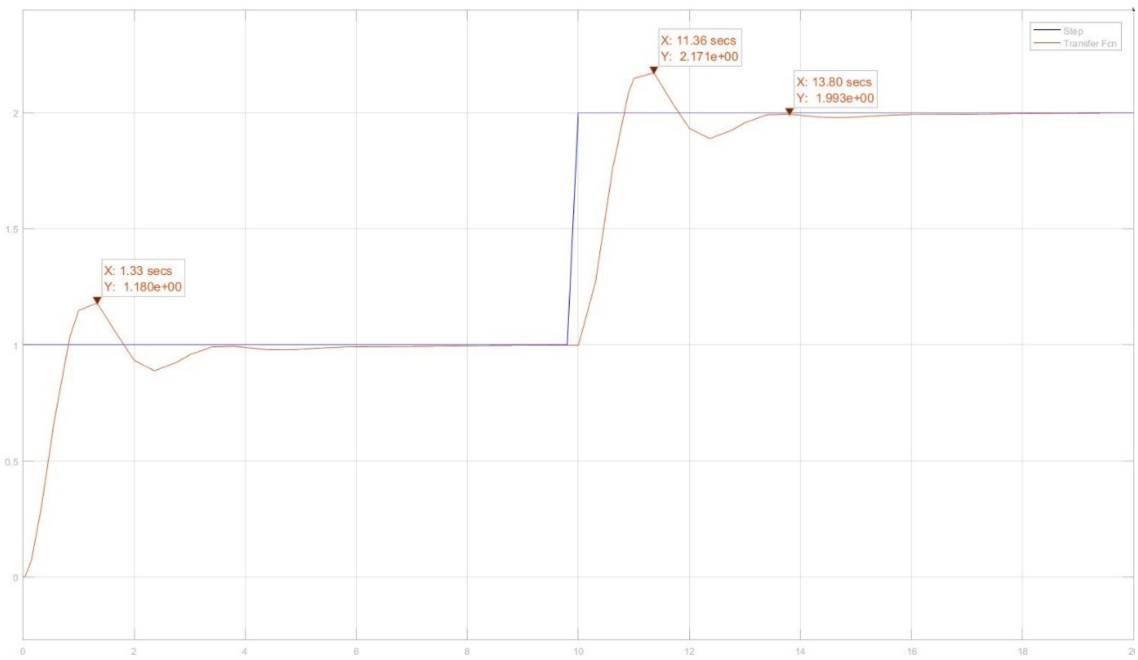
$\gamma = \frac{Tn}{Tu} = \frac{2.396}{0.434} = 5.52$ $\boldsymbol{Kr} = 0.9 * \gamma * \frac{1}{K} = 0.9 * 5.52 * \frac{3}{4} = \mathbf{3.72}$

$\mathbf{T_I} = 3.5 * Tu = 3.5 * 0.434 = \mathbf{1.51}$



Results:

System is stable.
Overshoot= 17%



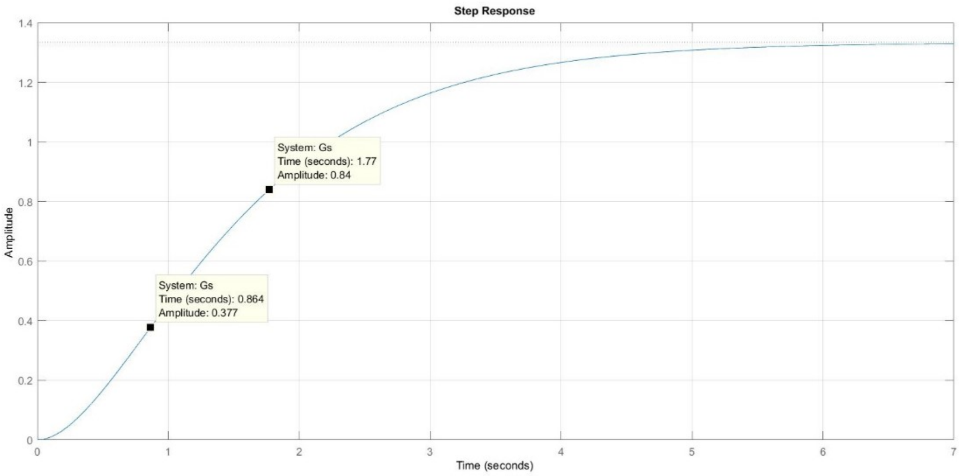
5. PI-Controller (Cohen-Coon Method):

T1@ 0.283 K= 0.86 second
T2@ 0.631 K= 1.77 second

$$T = \frac{3}{2}(t2 - t1) = 1.36$$

$$\theta = t2 - T = .41$$

$$r = \frac{\theta}{T} = 0.3$$



$$\boldsymbol{Kr} = \frac{1}{K*r}\left(0.9 + \frac{r}{12}\right) = \mathbf{2.31}$$

$$\boldsymbol{TI} = \frac{30+3r}{9+20r} * \theta = \mathbf{0.84}$$

Results:

Stable system
Overshoot = 0.4%