

N. 4

$$d = -\text{sig}(a_n) \|a_n\|_2$$

$$v = a_1 + \text{sig}(a_n) \|a_n\|_2 e_1 = \begin{pmatrix} a_{n1} + \text{sig}(a_n) \|a_n\|_2 \\ a_{n2} \\ \vdots \\ a_{nn} \end{pmatrix}$$

$$= \begin{pmatrix} a_{n1} + (-d) \\ a_{n2} \\ \vdots \\ a_{nn} \end{pmatrix}$$

$$\Rightarrow v_1 = a_{n1} - d$$

$$\|v\|_2^2 = \sum_{i=1}^n v_i^2 = \sum_{i=1}^n (a_{ni} + \text{sig}(a_n) \|a_n\|_2 e_i)^2$$

$$(a_{n1} - d)^2 = a_{n1}^2 - d^2$$

$$= a_{n1}^2$$

$$= (a_{n1} - d)^2 + \sum_{i=2}^n (a_{ni})^2$$

$$= (a_{n1} - d)^2 + \|a_n\|_2^2 - a_{n1}^2$$

$$= a_{n1}^2 - 2a_{n1}d + d^2 + \|a_n\|_2^2 - a_{n1}^2$$

$$= -2a_{n1}d + \|a_n\|_2^2 + \|a_n\|_2^2$$

$$= 2(-a_{n1}d + \|a_n\|_2^2)$$

$$= 2(-(a_{n1}d - \|a_n\|_2^2))$$

$$= -2v_1d$$

qed //

$$v_1 \cdot d = (a_{n1} - d) \cdot d$$

$$= a_{n1}d - d^2$$

$$= a_{n1}d - \|a_n\|_2^2$$