Numerik HA 10

(1) b) $x_1 = 0, x_2 = 1$ $\quad$ $p(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3$

$y_1 = 1, y_2 = z_1 = z_2 = 0$ $\quad$ $p'(x) = p_1 + 2p_2 x + 3p_3 x^2$

I $\quad p(0) = 1: \quad \underline{p_0 = 1}$

II $\quad p'(0) = 0: \quad \underline{p_1 = 0}$

III $\quad p(1) = 0: \quad p_0 + p_1 + p_2 + p_3 = 0 \iff p_2 + p_3 = -1$

IV $\quad p'(1) = 0: \quad p_1 + 2p_2 + 3p_3 = 0 \iff 2p_2 + 3p_3 = 0$

IV $- 2$III $= \quad p_3 = 2$

$p_3 = 2$ in III

$p_2 + 2 = -1 \iff p_2 = -3$

$$\underline{\underline{p(x) = 1 - 3x^2 + 2x^3}}$$

$z_1 = 1, y_1 = y_2 = z_2 = 0$

I $\quad p(0) = 0: \quad p_0 = 0$

II $\quad p'(0) = 1: \quad p_1 = 1$

III $\quad p(1) = 0: \quad p_0 + p_1 + p_2 + p_3 = 0 \iff p_2 + p_3 = -1$

IV $\quad p'(1) = 0: \quad p_1 + 2p_2 + 3p_3 = 0 \iff 2p_2 + 3p_3 = -1$

IV $- 2$III $= \quad p_3 = 1$

$p_3 = 1$ in III

$p_2 + 1 = -1 \iff p_2 = -2$

$$\underline{\underline{p(x) = x - 2x^2 + x^3}}$$

(2)

a)

$$s_3(x) = a_i + b_i x + c_i x^2 + d_i x^3$$

$$s_3'(x) = b_i + 2c_i x + 3d_i x^2$$

$$s_3''(x) = 2c_i + 6d_i x$$

$s_3(-1) = 16$: $a_0 - b_0 + c_0 - d_0 = 16$

$s_3(0) = 8$: $a_1 = 8$

$s_3(2) = 16$: $a_1 + 2b_1 + 4c_1 + 8d_1 = 16$

$s_3(0)_- = s_3(0)_+$: $a_0 = a_1 \Rightarrow a_0 = 8$

$s_3'(0)_- = s_3'(0)_+$: $b_0 = b_1$

$s_3''(0)_- = s_3''(0)_+$: $2c_0 = 2c_1 \Leftrightarrow c_0 = c_1$

$s_3''(-1) = 0$: $2c_0 - 6d_0 = 0$

$s_3''(2) = 0$: $2c_1 + 12d_1 = 0$

b)

i	0	1	2
x_i	-1	0	2
y_i	16	8	16
h_i	1	2	–

$$A = \left(2(2+1)\right) = (6)$$

$$\gamma = \left(6 \cdot \left(\frac{16-8}{2} - \frac{8-16}{1}\right)\right) = \left(6 \cdot (4-(-8))\right) = (72)$$

$6\beta_1 = 72 \Leftrightarrow \underline{\beta_1 = 12}$

$$\alpha_1 = \frac{16-8}{2} - \frac{1}{3}12 \cdot 2 - \frac{1}{6} \cdot 0 \cdot 2 = 4 - \frac{24}{3} = -4$$

i	0	1	2
β_i	0	12	0
α_i	0	-4	0

$$s_3(x) = \begin{cases} 16 + \frac{12}{6} \cdot (x+1)^3 = 16 + 2(x+1)^3 & \text{für } x < 0 \\ 8 - 4x + \frac{12}{2}x^2 + \frac{-12}{12}x^3 = 8 - 4x + 6x^2 - x^3 & \text{für } x \geq 0 \end{cases}$$