Numerik HA 07

1

$$A = \begin{pmatrix} 10 & 15 & 20 \\ 15 & -50 & 25 \\ 20 & 25 & -75 \end{pmatrix}$$

eleminiere a_{31}:

$$\bar{s}_1 = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

$$c = \frac{15}{\sqrt{15^2+20^2}} = \frac{15}{25} = \frac{3}{5} \qquad s = \frac{20}{25} = \frac{4}{5}$$

$$\tilde{Q}_1 = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \qquad Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$H = Q_1 A Q_1^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \cdot \begin{pmatrix} 10 & 15 & 20 \\ 15 & -50 & 25 \\ 20 & 25 & -75 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & -\frac{4}{5} \\ 0 & \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \cdot \begin{pmatrix} 10 & -7 & 0 \\ 15 & -50 & 55 \\ 20 & 75 & 65 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -7 & 0 \\ 25 & 30 & 85 \\ 0 & 85 & -5 \end{pmatrix}$$

4

$$f(x) = x + \ln(x) - 2 \qquad x \in X = [1,2]$$

$$f'(x) = \frac{1}{x} + 1$$

a)

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k + \ln(x_k) - 2}{\frac{1}{x_k} + 1}$$

$$= \frac{x_k \cdot \left(\frac{1}{x_k} + 1\right)}{\frac{1}{x_k} + 1} - \frac{x_k + \ln(x_k) - 2}{\frac{1}{x_k} + 1}$$

$$= \frac{1 + x_k - x_k - \ln(x_k) + 2}{\frac{1}{x_k} + 1} = \frac{-\ln(x_k) + 3}{\frac{1}{x_k} + 1}$$

$$\phi(y) = \frac{\ln(y) -}{\frac{1}{y} + 1}$$

$\phi(X) \subseteq X$, denn für $y \in [1,2]$

$$\phi(y) = \frac{-\ln(y) + 3}{\frac{1}{y} + 1} \leq \frac{-\ln(2) + 3}{\frac{1}{2} + 1} \approx 1{,}53$$

$$\phi(y) \geq \frac{-\ln(1) + 3}{\frac{1}{1} + 1} = \frac{3}{2}$$

$\phi'(y) =$

b) $\alpha = \frac{1}{4}$

$x_0 = 1$

$x_1 = 1{,}5$

$$\frac{\alpha^k}{1-\alpha} \cdot \| 1{,}5 - 1 \| = \frac{\frac{1}{4^k}}{\frac{3}{4}} \cdot \frac{1}{2} = \frac{\frac{1}{4^k}}{\frac{3}{2}} = \frac{2}{3 \cdot 4^k} \leq 10^{-6}$$

$$\Leftrightarrow \frac{2}{3 \cdot 4^k} \leq 10^{-6}$$

$$\Leftrightarrow \frac{2}{10^{-6}} \leq 3 \cdot 4^k$$

$$\Leftrightarrow 6 \cdot 10^6 \leq 4^k$$

$$\Leftrightarrow \log_4(6 \cdot 10^6) \leq k$$

$$\Leftrightarrow k \geq 11{,}26 \Rightarrow \underline{\underline{k = 12}}$$