Nr. 3

$$A = \begin{pmatrix} 5 & 7 & -17 \\ 10 & -1 & -19 \\ 10 & 20 & 5 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 5 \\ 10 \\ 10 \end{pmatrix} \qquad \|a_1\|_2 = \sqrt{25 + 100 + 100} = \sqrt{225} = 15$$

$$a_{11} = 5 > 0$$

$$v_1 = a_1 + 15 e_1 = \begin{pmatrix} 20 \\ 10 \\ 10 \end{pmatrix} \qquad \|v_1\|_2^2 = 400 + 100 + 100 = 600$$

$$Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{300} \begin{pmatrix} 400 & 200 & 200 \\ 200 & 100 & 100 \\ 200 & 100 & 100 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 & -2 & -2 \\ -2 & 2 & -1 \\ -2 & -1 & 2 \end{pmatrix}$$

$$\tilde{A}_1 = Q_1 A = \begin{pmatrix} -\frac{5}{3} - \frac{20}{3} = \frac{20}{3} & -\frac{7}{3} + \frac{2}{3} - \frac{40}{3} & \frac{17}{3} + \frac{38}{3} - \frac{18}{3} \\ 0 & -\frac{14}{3} - \frac{2}{3} - \frac{20}{3} & \frac{34}{3} - \frac{38}{3} - \frac{6}{3} \\ 0 & -\frac{14}{3} + \frac{1}{3} + \frac{40}{3} & \frac{34}{3} + \frac{19}{3} + \frac{19}{3} \end{pmatrix} = \begin{pmatrix} -15 & -15 & 15 \\ 0 & -12 & -3 \\ 0 & 9 & 21 \end{pmatrix}$$

$$A_1 = \left(\begin{array}{c|cc} 20 & -15 & 15 \\ 10 & -12 & -3 \\ 10 & 9 & 21 \end{array}\right) \qquad \begin{pmatrix} -15 \\ * \\ * \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -12 \\ 9 \end{pmatrix} \qquad \|a_2\|_2 = \sqrt{144 + 81} = \sqrt{225} = 15$$

$$a_{22} = -12 < 0$$

$$v_2 = a_2 - 15 e_1 = \begin{pmatrix} -27 \\ 9 \end{pmatrix} \qquad \|v_2\|^2 = 729 + 81 = 810$$

$$\tilde{Q}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{405} \begin{pmatrix} 729 & -243 \\ -243 & 81 \end{pmatrix} = \frac{1}{405} \begin{pmatrix} -324 & 243 \\ 243 & 324 \end{pmatrix} \qquad Q_2 = \frac{1}{405} \begin{pmatrix} 405 & 0 & 0 \\ 0 & -324 & 243 \\ 0 & 243 & 324 \end{pmatrix}$$

$$\tilde{A}_2 = Q_2 \tilde{A}_1 = \begin{pmatrix} -15 & 0 & 0 \\ 0 & 15 & 15 \\ 0 & 0 & 15 \end{pmatrix} \qquad A_2 = \left(\begin{array}{c|cc} 20 & 0 & 0 \\ 10 & -27 & 15 \\ 10 & 9 & 15 \end{array}\right) \qquad \begin{pmatrix} -15 \\ 15 \\ * \end{pmatrix}$$

$$\Rightarrow A_3 = \left(\begin{array}{c|cc} 20 & 0 & 0 \\ 10 & -27 & 15 \\ 10 & 9 & * \end{array}\right) \qquad \begin{pmatrix} -15 \\ 15 \\ 15 \end{pmatrix}$$

$$\Rightarrow Q = (Q_2 Q_1)^T \qquad R = \begin{pmatrix} -15 & 0 & 0 \\ 0 & 15 & 15 \\ 0 & 0 & 15 \end{pmatrix}$$

$$= \frac{1}{1215} \begin{pmatrix} -405 & -810 & -810 \\ 162 & -891 & 810 \\ -162 & -810 & 891 \end{pmatrix}^T$$

$$= \frac{1}{1215} \begin{pmatrix} -405 & 162 & -162 \\ -810 & -891 & -810 \\ -810 & 810 & 891 \end{pmatrix}$$