Numerik HA 12

3b) $$\int_1^5 \frac{1}{x}\,dx = [\ln(x)]_1^5 = \ln(5) - \ln(1) = \ln(5) \approx 1{,}609438$$

$$T = \frac{5-1}{2} \cdot (f(1) + f(5)) = 2 \cdot \left(\frac{6}{5}\right) = \frac{12}{5}$$

$$\tilde{T} = \frac{3-1}{2} \cdot (f(1) + f(3)) + \frac{5-3}{2} \cdot (f(3) + f(5))$$

$$= \frac{4}{3} + \left(\frac{1}{3} + \frac{1}{5}\right) = \frac{20}{15} + \frac{8}{15} = \frac{28}{15}$$

$$\tilde{F} = \frac{1}{15} |\tilde{T} - T| = \frac{1}{15} \left|\frac{28}{15} - \frac{12}{5}\right| = \frac{1}{15} \cdot \left|\frac{28}{15} - \frac{36}{15}\right| = \frac{1}{15} \cdot \frac{8}{15}$$

$$= \frac{8}{225} = 0{,}03\overline{5}$$

$$T_l = \frac{3-1}{2} (f(1) + f(3)) = \frac{4}{3}$$

$$\tilde{T}_l = \frac{2-1}{2} (f(1) + f(2)) + \frac{3-2}{2} (f(2) + f(3))$$

$$= \frac{1}{2} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{5}{6} = \frac{3}{4} + \frac{5}{12} = \frac{14}{12} = \frac{7}{6}$$

$$\tilde{F}_l = \frac{1}{15} \cdot |T_l - \tilde{T}| = \frac{1}{15} \cdot \left|\frac{7}{6} - \frac{8}{6}\right| = \frac{1}{15} \cdot \frac{1}{6} = \frac{1}{90} = 0{,}0\overline{1} < 0{,}02$$

$$T_r = \frac{5-3}{2} \cdot (f(3) + f(5)) = \frac{8}{15}$$

$$\tilde{T}_r = \frac{4-3}{2} (f(3) + f(4)) + \frac{5-4}{2} (f(4) + f(5))$$

$$= \frac{1}{2} \cdot \frac{7}{12} + \frac{1}{2} \cdot \frac{9}{20} = \frac{7}{24} + \frac{9}{40} = \frac{70}{240} + \frac{54}{240} = \frac{124}{240} = \frac{62}{120}$$

$$\tilde{F}_r = \frac{1}{15} |\tilde{T}_r - T_r| = \frac{1}{15} \cdot \left|\frac{31}{60} - \frac{8}{15}\right| = \frac{1}{15} \cdot \left|\frac{31}{60} - \frac{32}{60}\right| \qquad = \frac{31}{60}$$

$$= \frac{1}{15} \cdot \frac{1}{60} = \frac{1}{900} = 0{,}00\overline{1} < 0{,}02$$

Also:

$$\int_1^5 \frac{1}{x}\,dx \approx \tilde{T}_l + \tilde{T}_r = \frac{7}{6} + \frac{31}{60} = \frac{101}{60} = \underline{\underline{1{,}68\overline{3}}}$$