$$\left.\begin{aligned} f(x) &= \sqrt{x} \\ f'(x) &= \frac{1}{2}\, x^{-1/2} \\ f''(x) &= -\frac{1}{4}\, x^{-3/2} = \left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right) x^{-3/2} \\ f'''(x) &= \frac{3}{8}\, x^{-5/2} = \left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right) x^{-5/2} \end{aligned}\right\} f^{(n)}(x) = \prod_{i=0}^{n-1}\left(\frac{1}{2}-i\right) \; x^{\frac{1}{2}-n}$$

$$\frac{a_i}{a_{i-1}} = \frac{f^{(i)}(x_0)\,(x-x_0)^i}{i!} \cdot \frac{(i-1)!}{f^{(i-1)}(x_0)\,(x-x_0)^{i-1}} = \frac{f^{(i)}(x_0)\,(x-x_0)}{i\, f^{(i-1)}(x_0)}$$

$$= \frac{\prod_{j=0}^{i-1}\left(\frac{1}{2}-j\right)\, x_0^{\frac{1}{2}-i}\;(x-x_0)}{\prod_{j=0}^{i-2}\left(\frac{1}{2}-j\right)\, x_0^{\frac{3}{2}-i}\;\; i}$$

$$= \frac{\left(\frac{3}{2}-i\right) x_0^{1/2-i}\,(x-x_0)}{i\, x_0^{3/2-i}}$$

$$= \left(\frac{3}{2i}-1\right) x_0^{-1}\,(x-x_0)$$

$$= \left(\frac{x}{x_0}-1\right)\left(\frac{3}{2i}-1\right)$$

$$\Rightarrow \quad \frac{a_i}{a_{i-1}} = \left(\frac{x}{x_0}-1\right)\left(\frac{3}{2i}-1\right)$$

$$\Leftrightarrow \quad a_i = \left(\frac{x}{x_0}-1\right)\left(\frac{3}{2i}-1\right) a_{i-1} \quad //$$