

STAT 463

Assignment 1

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Question 1

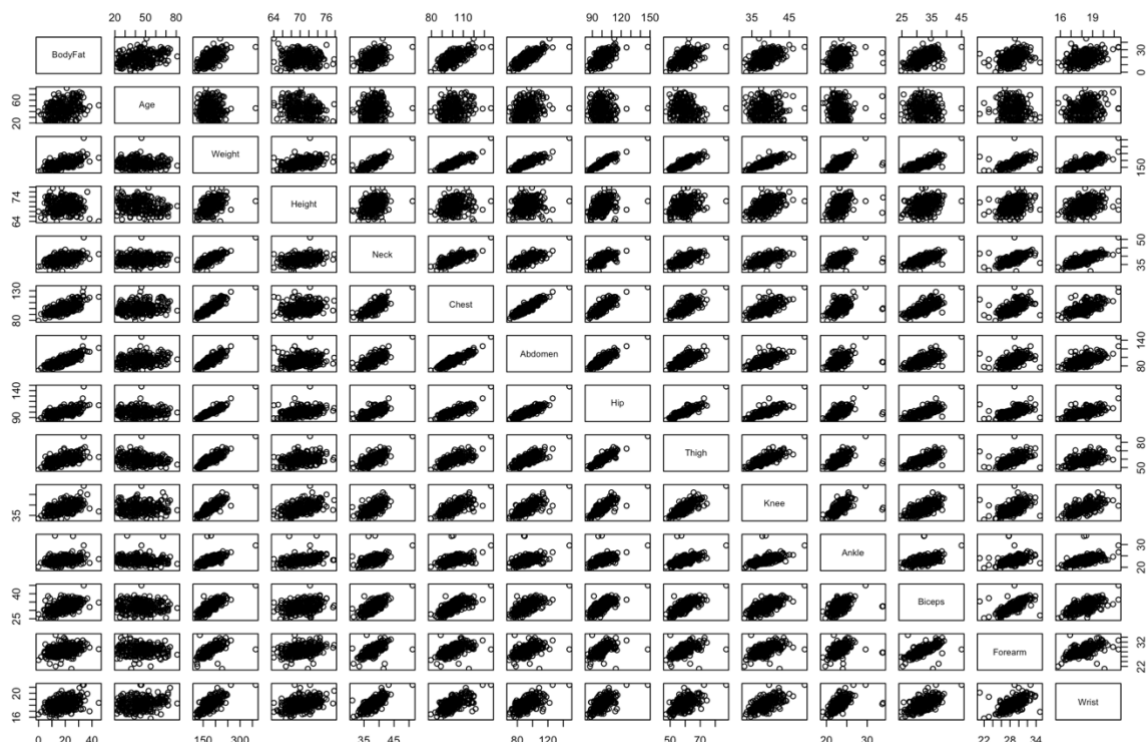
a. # Find out what measures are well correlated with body fat:

```
> round(cor(bodyfat[, -1]), 2)
```

| | BodyFat | Age | Weight | Height | Neck | Chest | Abdomen | Hip | Thigh | Knee | Ankle | Biceps | Forearm | Wrist |
|---------|---------|-------|--------|--------|------|-------|---------|-------|-------|------|-------|--------|---------|-------|
| BodyFat | 1.00 | 0.29 | 0.61 | -0.02 | 0.49 | 0.70 | 0.81 | 0.63 | 0.56 | 0.51 | 0.27 | 0.49 | 0.36 | 0.35 |
| Age | 0.29 | 1.00 | -0.01 | -0.25 | 0.11 | 0.18 | 0.23 | -0.05 | -0.20 | 0.02 | -0.11 | -0.04 | -0.09 | 0.21 |
| Weight | 0.61 | -0.01 | 1.00 | 0.49 | 0.83 | 0.89 | 0.89 | 0.94 | 0.87 | 0.85 | 0.61 | 0.80 | 0.63 | 0.73 |
| Height | -0.02 | -0.25 | 0.49 | 1.00 | 0.32 | 0.23 | 0.19 | 0.37 | 0.34 | 0.50 | 0.39 | 0.32 | 0.32 | 0.40 |
| Neck | 0.49 | 0.11 | 0.83 | 0.32 | 1.00 | 0.78 | 0.75 | 0.73 | 0.70 | 0.67 | 0.48 | 0.73 | 0.62 | 0.74 |
| Chest | 0.70 | 0.18 | 0.89 | 0.23 | 0.78 | 1.00 | 0.92 | 0.83 | 0.73 | 0.72 | 0.48 | 0.73 | 0.58 | 0.66 |
| Abdomen | 0.81 | 0.23 | 0.89 | 0.19 | 0.75 | 0.92 | 1.00 | 0.87 | 0.77 | 0.74 | 0.45 | 0.68 | 0.50 | 0.62 |
| Hip | 0.63 | -0.05 | 0.94 | 0.37 | 0.73 | 0.83 | 0.87 | 1.00 | 0.90 | 0.82 | 0.56 | 0.74 | 0.55 | 0.63 |
| Thigh | 0.56 | -0.20 | 0.87 | 0.34 | 0.70 | 0.73 | 0.77 | 0.90 | 1.00 | 0.80 | 0.54 | 0.76 | 0.57 | 0.56 |
| Knee | 0.51 | 0.02 | 0.85 | 0.50 | 0.67 | 0.72 | 0.74 | 0.82 | 0.80 | 1.00 | 0.61 | 0.68 | 0.56 | 0.66 |
| Ankle | 0.27 | -0.11 | 0.61 | 0.39 | 0.48 | 0.48 | 0.45 | 0.56 | 0.54 | 0.61 | 1.00 | 0.48 | 0.42 | 0.57 |
| Biceps | 0.49 | -0.04 | 0.80 | 0.32 | 0.73 | 0.73 | 0.68 | 0.74 | 0.76 | 0.68 | 0.48 | 1.00 | 0.68 | 0.63 |
| Forearm | 0.36 | -0.09 | 0.63 | 0.32 | 0.62 | 0.58 | 0.50 | 0.55 | 0.57 | 0.56 | 0.42 | 0.68 | 1.00 | 0.59 |
| Wrist | 0.35 | 0.21 | 0.73 | 0.40 | 0.74 | 0.66 | 0.62 | 0.63 | 0.56 | 0.66 | 0.57 | 0.63 | 0.59 | 1.00 |

Matrix plot – Person factor left out

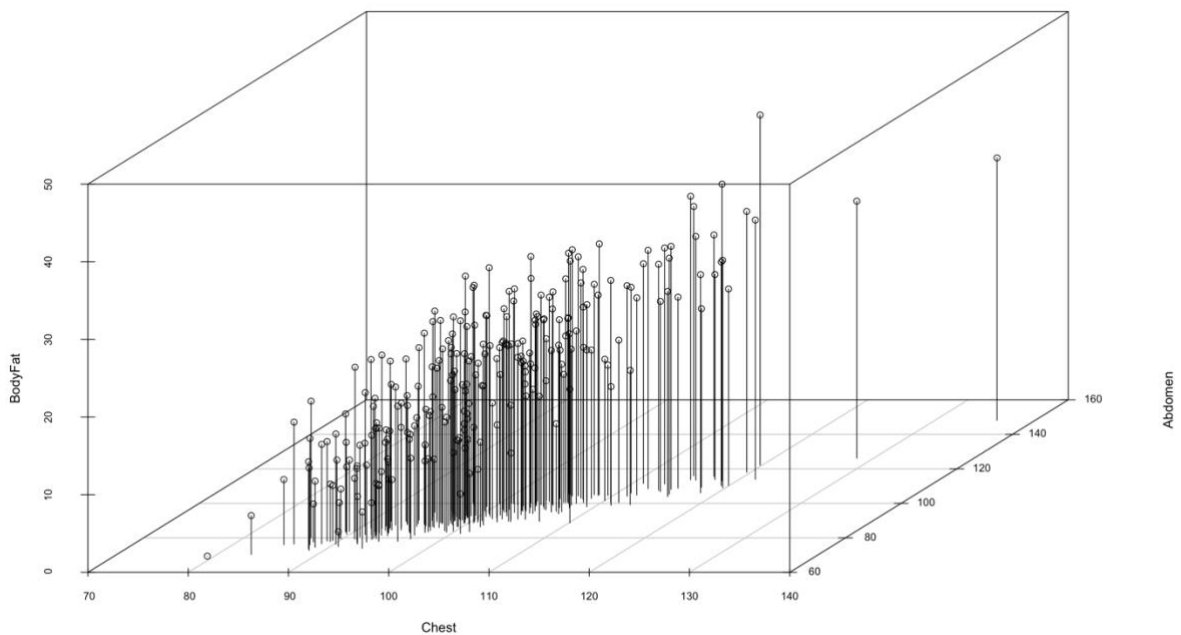
```
> pairs(bodyfat[, -1])
```



b. The two most useful measures of body fat : **Chest & Abdomen**

c. #Creat a sensible 3-D plot with “Chest”, “Abdomen” and “BodyFat”

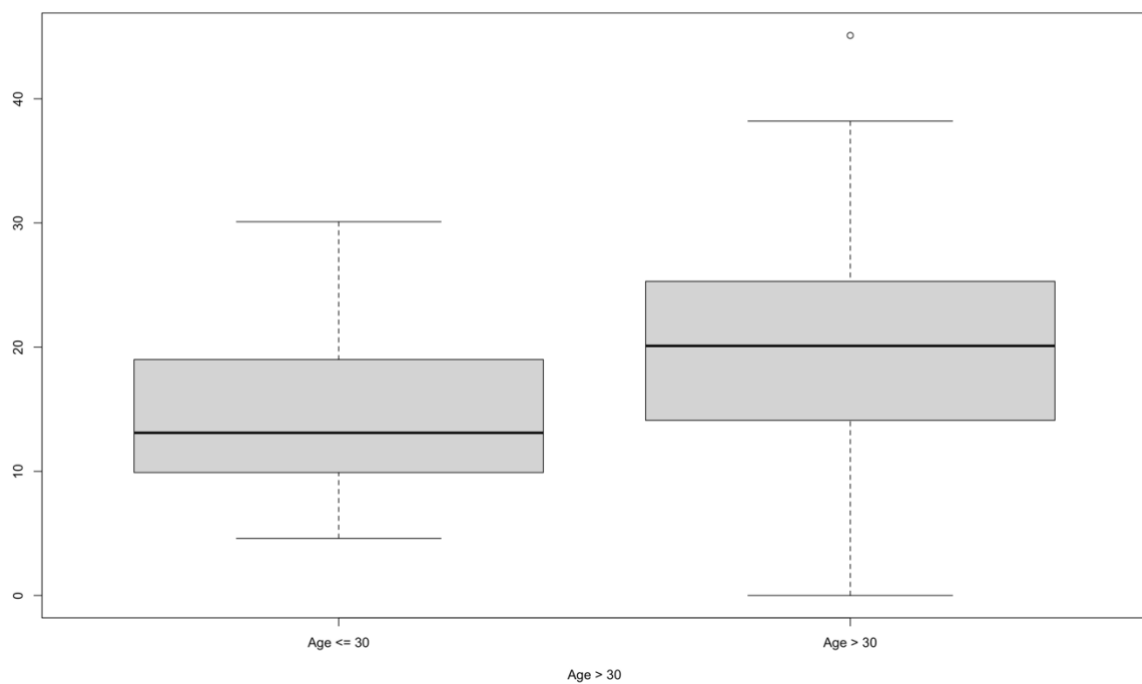
```
> library(scatterplot3d)
> with(bodyfat, scatterplot3d(Chest, Abdomen, BodyFat, type = "h"))
```



“Chest”, “Abdomen” and “BodyFat” are factors that are quite well linear correlated.

- d. # Group the data into two age-groups(≤ 30 & >30), compare body fat between the two age-groups

```
> with(bodyfat, boxplot(BodyFat ~ Age > 30, names = c("Age <= 30", "Age > 30")))
```



Compared to people aged more than 30, people aged less than 30 are with lower body fat.

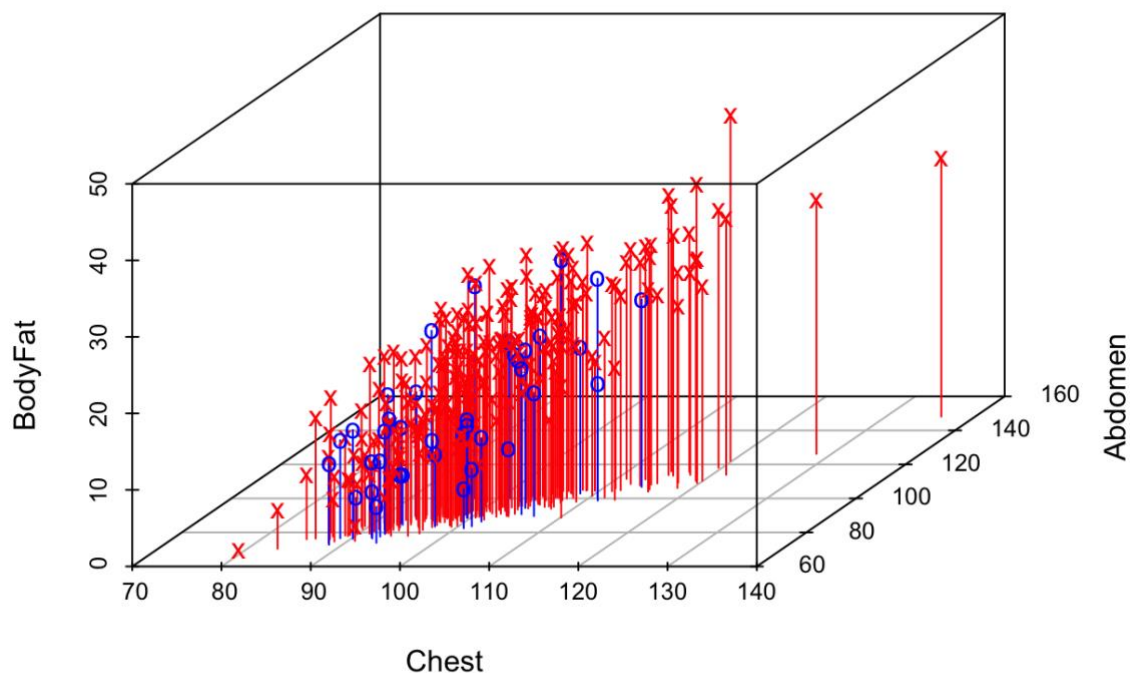
e. #Repeat the analysis for the two groups separately

```
> round(cor(bodyfat[bodyfat$Age > 30, -1]), 2)
```

| | BodyFat | Age | Weight | Height | Neck | Chest | Abdomen | Hip | Thigh | Knee | Ankle | Biceps | Forearm | Wrist |
|---------|---------|-------|--------|--------|------|-------|---------|-------|-------|------|-------|--------|---------|-------|
| BodyFat | 1.00 | 0.20 | 0.65 | 0.04 | 0.53 | 0.72 | 0.81 | 0.67 | 0.61 | 0.51 | 0.29 | 0.53 | 0.43 | 0.37 |
| Age | 0.20 | 1.00 | -0.02 | -0.22 | 0.09 | 0.15 | 0.17 | -0.04 | -0.20 | 0.03 | -0.06 | -0.04 | -0.04 | 0.27 |
| Weight | 0.65 | -0.02 | 1.00 | 0.50 | 0.84 | 0.91 | 0.91 | 0.94 | 0.87 | 0.85 | 0.60 | 0.80 | 0.65 | 0.72 |
| Height | 0.04 | -0.22 | 0.50 | 1.00 | 0.35 | 0.27 | 0.24 | 0.38 | 0.34 | 0.54 | 0.38 | 0.33 | 0.34 | 0.40 |
| Neck | 0.53 | 0.09 | 0.84 | 0.35 | 1.00 | 0.79 | 0.77 | 0.74 | 0.71 | 0.68 | 0.48 | 0.74 | 0.63 | 0.74 |
| Chest | 0.72 | 0.15 | 0.91 | 0.27 | 0.79 | 1.00 | 0.93 | 0.84 | 0.74 | 0.72 | 0.49 | 0.74 | 0.62 | 0.66 |
| Abdomen | 0.81 | 0.17 | 0.91 | 0.24 | 0.77 | 0.93 | 1.00 | 0.89 | 0.79 | 0.73 | 0.46 | 0.70 | 0.55 | 0.63 |
| Hip | 0.67 | -0.04 | 0.94 | 0.38 | 0.74 | 0.84 | 0.89 | 1.00 | 0.89 | 0.82 | 0.54 | 0.73 | 0.56 | 0.62 |
| Thigh | 0.61 | -0.20 | 0.87 | 0.34 | 0.71 | 0.74 | 0.79 | 0.89 | 1.00 | 0.79 | 0.51 | 0.75 | 0.58 | 0.54 |
| Knee | 0.51 | 0.03 | 0.85 | 0.54 | 0.68 | 0.72 | 0.73 | 0.82 | 0.79 | 1.00 | 0.60 | 0.67 | 0.59 | 0.67 |
| Ankle | 0.29 | -0.06 | 0.60 | 0.38 | 0.48 | 0.49 | 0.46 | 0.54 | 0.51 | 0.60 | 1.00 | 0.48 | 0.43 | 0.56 |
| Biceps | 0.53 | -0.04 | 0.80 | 0.33 | 0.74 | 0.74 | 0.70 | 0.73 | 0.75 | 0.67 | 0.48 | 1.00 | 0.71 | 0.63 |
| Forearm | 0.43 | -0.04 | 0.65 | 0.34 | 0.63 | 0.62 | 0.55 | 0.56 | 0.58 | 0.59 | 0.43 | 0.71 | 1.00 | 0.59 |
| Wrist | 0.37 | 0.27 | 0.72 | 0.40 | 0.74 | 0.66 | 0.63 | 0.62 | 0.54 | 0.67 | 0.56 | 0.63 | 0.59 | 1.00 |

The two most useful measures of body fat are still : **Chest & Abdomen**

```
> with(bodyfat, scatterplot3d(Chest, Abdomen, BodyFat, type = "h", color = ifelse(Age > 30, "red", "blue"), pch = ifelse(Age <= 30, "o", "x")))
```



“Chest”, “Abdomen” and “BodyFat” are factors that are quite well linear correlated. People aged more than 30(red) are with higher body fat while people aged below 30 have less body fat but more variability.

Question 2 & 3

Question 2.

$$\Delta |A| = 6 \times 3 - (-1)(2) = 18 + 2 = 20$$

$$\Delta A^{-1} = \frac{1}{6 \times 3 - (-1)2} \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{20} & \frac{1}{20} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$$

$$\Delta 0 = \left| \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \Rightarrow 0 = \begin{vmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix}$$

$$\Rightarrow (\lambda-6)(\lambda-3)+2=0 \Rightarrow \lambda^2-9\lambda+20=0 \Rightarrow (\lambda-4)(\lambda-5)=0 \Rightarrow \lambda=4, 5$$

$$\begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{cases} 6x_1 - x_2 = 4x_1 \\ 2x_1 + 3x_2 = 4x_2 \end{cases} \Rightarrow \begin{matrix} \textcircled{1} & \textcircled{2} \\ 2x_1 = x_2 \end{matrix} \Rightarrow e_1 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 5 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{cases} 6x_1 - x_2 = 5x_1 \\ 2x_1 + 3x_2 = 5x_2 \end{cases} \Rightarrow \begin{matrix} \textcircled{1} & \textcircled{2} \\ x_1 = x_2 \end{matrix} \Rightarrow e_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Question 3.

$$\Delta |B| = 2 \times (0+4) - (-3)(10+1) + 1 \times 8 = 8 + 33 + 8 = 49$$

$$\Delta B^{-1} = \frac{1}{49} \begin{bmatrix} 4 & -(-15-4) & (9-1) \\ -((10+1)) & 10-1 & -(-2-2) \\ 8-(-8+3) & 0-(-6) & \end{bmatrix} = \begin{bmatrix} \frac{4}{49} & \frac{19}{49} & \frac{8}{49} \\ -\frac{11}{49} & \frac{9}{49} & \frac{4}{49} \\ \frac{8}{49} & -\frac{11}{49} & \frac{6}{49} \end{bmatrix}$$

$$\Delta 0 = \left| \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = \begin{vmatrix} 2-\lambda & -3 & 1 \\ 2 & -\lambda & -1 \\ 1 & 4 & 5-\lambda \end{vmatrix}$$

$$0 = (2-\lambda)(-5\lambda+\lambda^2+4) - (-3)(10-2\lambda+1) + (8+\lambda)$$

$$0 = -10\lambda + 2\lambda^2 + 8 + 5\lambda^2 - 2\lambda^3 - 4\lambda + 30 - 6\lambda + 3 + 8 + \lambda$$

$$0 = -\lambda^3 + 7\lambda^2 - 19\lambda + 46 \Rightarrow \lambda^3 - 7\lambda^2 + 19\lambda - 46 = 0 \Rightarrow \lambda = ?$$

Then the same way as Question 2 to work out e_1, e_2, e_3 .