

# MATH/CSCI 387

## Homework 4

Due Thursday, March 6

### Practice exercises from the book

4.11, 4.12, 5.1, 5.10, 5.12

### Problems

1. Let  $NONHALT = \{\langle M, w \rangle \mid M \text{ will not halt on input } w\}$ . Is  $NONHALT$  decidable? Prove your answer.
2. Let  $L = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$ . Is  $L$  decidable? Prove your answer. (We use  $w^R$  as the book does to indicate the reverse of  $w$ .)
3. Let  $L = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$ . Is  $L$  decidable? Prove your answer.
4. Let  $THREE = \{\langle M \rangle \mid M \text{ is a Turing machine that accepts exactly 3 strings}\}$ . Is  $THREE$  decidable? Prove your answer.
5. Consider the question of determining whether a given Turing machine  $M$  on a given input  $w$  ever tries to move its head left when it's already pointing at the left-most cell on the tape. Formulate this problem as a language. Show that this language is undecidable.

### Bonus problems

1. Prove that a language  $C$  is recognizable if and only if there exists a decidable language  $D$  such that  $C = \{x \mid \exists y \text{ such that } xy \in D\}$ .
2. Write a program (in any programming language) that prints its own code to the screen. The program cannot interact with the file that contains the program. (It can't, for example, read or copy the file that contains its code.) Don't look up hints on how to do this.
3. Let  $S = \{\langle M \rangle \mid M \text{ is a Turing machine } L(M) = \{\langle M \rangle\}\}$ . Show that neither  $S$  nor  $\bar{S}$  is a recognizable language.