MATH/CSCI 387

Homework 5

Due Tuesday, March 18

Practice exercises from the book

7.1, 7.7, 7.9, 7.13, 7.11, 7.19, 7.20

Problems

1. Take $\mathit{MODEXP} = \{\langle a, b, c, p\rangle \mid a, b, c$ and p are binary integers and $a^b \equiv c \pmod{p}\}$.

   (a) Show that $\mathit{MODEXP} \in P$. (Hint: The obvious algorithm doesn't run in polynomial time. Try it first where b is a power of 2.)

   (b) Say that instead we dropped the reduction modulo p. That is, say $\mathit{EXPONENT} = \{\langle a, b, c\rangle \mid a, b$ and c are binary integers and $a^b = c\}$. Would the obvious modification of your algorithm in (a) still be polynomial time? Why or why not?

2. Show that P is closed under union, concatenation, and complement.

3. Show that P is closed under the star operation. (This is hard. You will want to use dynamic programming, doing something that is in some ways similar to what we did in class to show that context-free languages were in P.)

4. Let *3COLOR* be the set of 3-colorable graphs. That is, $3COLOR = \{\langle G\rangle \mid G$ is a graph, and we can assign each node of G one of three colors such that no two nodes of G that are connected by an edge have the same color)$\}$.

   (a) Show that $\mathit{3COLOR} \in \mathrm{NP}$ by giving a polynomial time nondeterministic Turing machine for it.

   (b) Show that $\mathit{3COLOR} \in \mathrm{NP}$ by giving a polynomial time verifier for it.

5. Given an undirected graph, a set of nodes in the graph is *independent* if no two nodes in the set are connected by an edge. Let $\mathit{IND} = \{\langle G, k\rangle \mid G$ is a graph, k is an integer, and there is an independent set of size k in $G\}$.

   (a) Show that $\mathit{3COLOR} \in \mathrm{NP}$ by giving a polynomial time nondeterministic Turing machine for it.

   (b) Show that $\mathit{3COLOR} \in \mathrm{NP}$ by giving a polynomial time verifier for it.