

# MATH/CSCI 387

## Homework 6

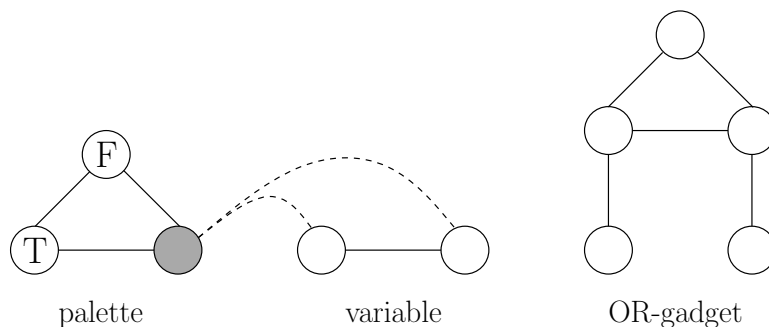
Due Tuesday, April 1

### Practice exercises from the book

7.22, 7.25, 7.31, 7.34, 7.38

### Problems

1. Let  $DOUBLESAT = \{\langle \phi \rangle \mid \phi \text{ is a boolean formula with at least two different satisfying assignments}\}$ . Show that this language is NP-complete.
2. Show that if  $P = NP$ , then every language in  $P$ , except for  $\emptyset$  and  $\Sigma^*$ , is NP-complete. Why do we need to make exceptions for  $\emptyset$  and  $\Sigma^*$ ?
3. A *coloring* of a graph is an assignment of way of assigning a color to each vertex so that no two adjacent vertices have the same color. Take  $3COLOR = \{\langle G \rangle \mid G \text{ has a coloring that uses only three colors}\}$ . Show that  $3COLOR$  is NP-complete. (Hint: Use the following subgraphs.)



4. Let  $SET-SPLITTING = \{\langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \dots, C_k\} \text{ is a collection of subsets of } S \text{ such that each element of } S \text{ can be colored red or blue so that each } C_i \text{ has at least one element of each color}\}$ . Show that this language is NP-complete.
5. Considering the following problem. You have a list of final exams and a list of students. Each student is taking some specified subset of the final exams. Each final exam is a single time slot in length. The problem is to determine if these finals can be scheduled in only  $h$  time slots so that no student has two simultaneous exams. Formulate this problem as a language and show that it is NP-complete.