MATH/CSCI 387

Homework 3

Due Thursday, February 27

Practice exercises from the book

2.30, 2.42, 3.9, 3.15, 3.16, 3.22

Problems

1. Consider the language A above consisting of all strings (with alphabet $\{0, 1\}$) that contain exactly twice as many 0s as 1s (in any order).
   (a) Write an English-language implementation-level description of a Turing machine for this language. (For a good example of what this looks like, consider M_2 in Example 3.7 on page 143 of Sipser, or M_3 in example 3.11 on page 146.)
   (b) Draw the state diagram of the Turing Machine described above. (Some choices of answers for the previous part, while correct, will make this very hard. Feel free to alter your answer so that this part is easier, but make sure that this machine operates in a way consistent with the description above.)
   (c) Write the computation history of your machine on the string 01011.
2. 5-in-1 Turing machines are Turing machines that are allowed to write up to 5 symbols in any box (and have a transition function that takes into account the presence of up to 5 symbols). Show that 5-in-1 Turing machines are equal in power to regular Turing machines.
3. Show that the class of Turing-recognizable languages is closed under concatenation.
4. Consider the language A, where

   $A = \{\langle M \rangle \mid M$ is a DFA with alphabet $\{0, 1\}$ and M does not accept any string containing more than three 0s$\}$.

   Prove that A decidable.

Bonus problems

1. With $\Sigma = \{0, 1\}$, take the language $L = \{0^i 1^j \mid i \neq j \text{ and } 2i \neq j\}$. Prove that L is context free or that it is not.
2. Consider a Turing machine that cannot write over its input. That is, whatever length of tape that the input is on cannot be changed, but can be read as normal (and the rest of the tape can be changed as normal). Show that Turing machines of this type can recognize only regular languages.