MATH/CSCI 387

Homework 2

Due Thursday, February 13

Practice exercises from the book

1.29, 1.46, 2.1, 2.4, 2.5, 2.6, 2.9, 2.10, 2.15

Problems

1. For each of the following languages, prove either that it is regular or that it is not regular. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w \mid w \text{ contains an equal number of 0s and 1s}\}$
   (b) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1, \text{ and } y \text{ contains at least } k \text{ 1s}\}$
   (c) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1, \text{ and } y \text{ contains at most } k \text{ 1s}\}$
2. Consider the language $L = \{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.
   (a) Show that L is not regular.
   (b) Show that L does not look irregular as far as the pumping lemma goes. That is, give a pumping length p and show that L satisfies the conditions of the pumping lemma.
   (c) Explain why the two things you've shown above do not contradict.
3. For each of the following languages, give a CFG that generates the language. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w \mid w \text{ contains at least three 1s}\}$.
   (b) $L = \{w \mid w \text{ has odd length and its middle symbol is 0}\}$.
   (c) $L = \{0^m 1^n \mid m \neq n\}$.
   (d) $L = \{0^i 1^j 0^k \mid j > i + k\}$.
4. Draw the state diagram of a PDA that accepts each of the following languages. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{0^m 1^n \mid m \neq n\}$.
   (b) $L = \{w \mid w \text{ contains more 0s than 1s}\}$.

Bonus problems

1. Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a set A of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from A in base k (with no leading zeros). For example, if $A = \{3, 5\}$ then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a k-symbol alphabet. Give a set A for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).

2. Let G be the following CFG:

$$S \to \mathrm{a}S\mathrm{b} \mid \mathrm{b}Y \mid Y\mathrm{a}$$
$$Y \to \mathrm{b}Y \mid \mathrm{a}Y \mid \epsilon$$

Give a simple English description of the language of G. Use this description to give a CFG that recognizes the complement of that language.

3. $L = \{xy \mid |x| = |y| \text{ and } x \neq y\}$.
   (a) Give a CFG that generates the language L.
   (b) Give a PDA that accepts the language L.