MATH/CSCI 387

Homework 1

Due Thursday, February 6

Practice exercises from the book

1.1, 1.2, 1.3, 1.6, 1.7, 1.11, 1.13, 1.14, 1.18, 1.20

Problems

1. For each of the following languages, give a DFA that recognizes the language. In all cases $\Sigma = \{0, 1\}$.

   (a) $L = \{w \mid w \text{ is any string other than 11 or 111}\}$

   (b) $L = \{w \mid w \text{ contains the substring 1100}\}$

   (c) $L = \{w \mid w \text{ has length at least 3 and has 0 for the third symbol}\}$

   (d) $L = \{w \mid w \text{ has a 1 in every odd position}\}$

   (e) $L = \{w \mid w \text{ contains a multiple of 3 1s or an even number of 0s}\}$

   (f) $L = \{w \mid w\text{, when thought of as a binary number, is a multiple of 5 }\}$ (We consider leading zeros irrelevant, so 0110 and 110 both represent the number 6. You can treat ϵ as representing 0 and accept it or treat it as an invalid input and reject it, whichever you prefer.)

2. For each of the following languages, give a NFA that recognizes the language using no more than the listed number of states. In all cases $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$.

   (a) $L = \{\epsilon\}$, 1 state

   (b) $L = \{w \mid w \text{ ends in } \texttt{ab}\}$, 3 states

   (c) $L = \{w \mid w \text{ contains a multiple of 3 } \texttt{a}\text{'s or a multiple of 4 } \texttt{b}\text{'s}\}$, 8 states

   (d) $L = \{w \mid w \text{ ends in the first occurrence of some symbol}\}$, 5 states

3. For each of the following languages, give a regular expression that represents the language. In all cases $\Sigma = \{0, 1\}$.

   (a) $L = \{w \mid |w| \leq 5\}$

   (b) $L = \{w \mid \text{ every odd position of } w \text{ is a } 1\}$

   (c) $L = \{w \mid w \text{ does not contain the substring 001}\}$

4. Show that the class of regular languages is closed under intersection. That is, if A and B are both regular languages, then so is $A \cap B$. (Note: A language is *regular* if it is the language of some DFA/NFA.)

Bonus problems

1. In class we showed that any n-state NFA can be converted to a 2^n-state DFA. Show that this bound is roughly tight. Specifically, show that for every n there exists a language that can be recognized with an $(n+1)$-state NFA but cannot be recognized by a DFA with fewer than 2^n states.

2. Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A and B are regular, then A/B is regular.