MATH/CSCI 387

Homework 7

Due Tuesday, April 8

Practice exercises from the book

7.44, 8.8, 8.9, 8.16

Problems

1. This problem is based on the *Minecraft* solitaire computer game. (If you haven't played it before, it's easy to find online.) In the game there is a grid, and some of the grid squares are chosen as mines. When a player clicks on a grid square, they immediately lose if it is a mine. If it is not, a number is revealed indicating how many of the adjacent squares have mines. You win if you uncover all the non-mine squares.

   We are going to generalize this game, so that instead of playing on a grid you play on an arbitrary undirected graph, with nodes being mines (or not) and numbers indicating how many adjacent nodes are mines. Then, instead of playing the game, we're going to ask a question about it. Say we get a partially-revealed graph, where some nodes have numbers indicating the number of adjacent mines, and other nodes have no numbers. We want to know if the partially revealed graph is *consistent*, meaning we want to know whether there exists some subset of the unrevealed nodes that could be mines that are consistent with the numbers on the already-revealed nodes.

   Formulate this problem as a formal language and prove that it is NP-complete.

2. The game Gomoku is played by two players on an $n \times n$ board. The players alternate placing pieces, with one placing red and the other placing blue. (The pieces must be placed on open spaces.) The winner is the first player to achieve a line of 5 consecutive markers (in a row, column, or diagonal). A *position* consists of a description of what stones are on the board and whose turn it is. Let $GOMOKU$ be the set of positions from which red can force a win. Show that $GOMOKU \in$ PSPACE.

3. Show that PSPACE is closed under the star operation.

Bonus problems

1. Let $EQ_{NFA} = \{\langle N, M\rangle \mid M \text{ and } N \text{ are NFAs with the same language}\}$. Show that this language is in PSPACE.