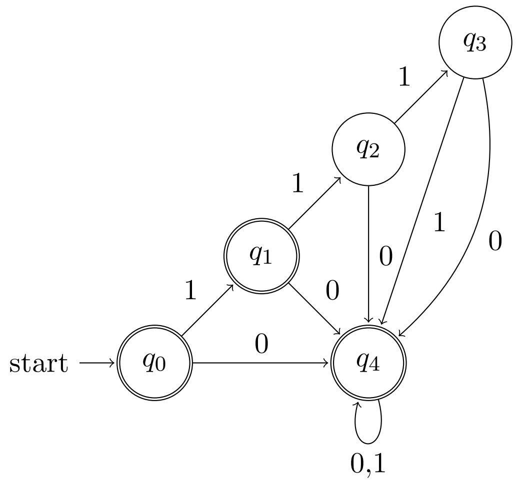
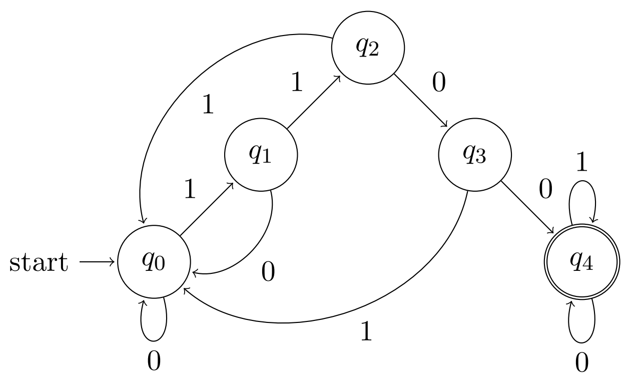
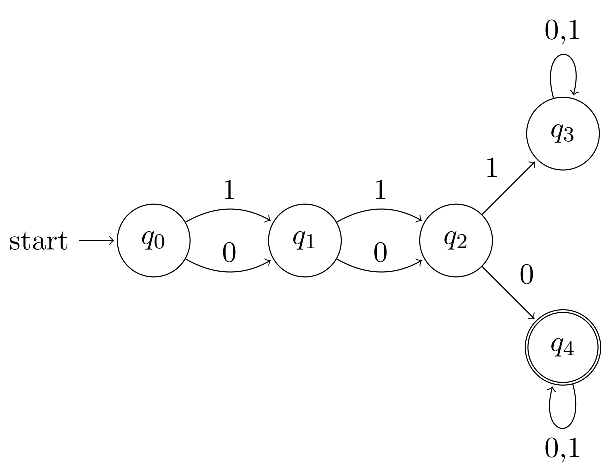
Bram Schuijff
CSCI 387, §S01/02
Homework 1
February 6th, 2025 by 11:59pm



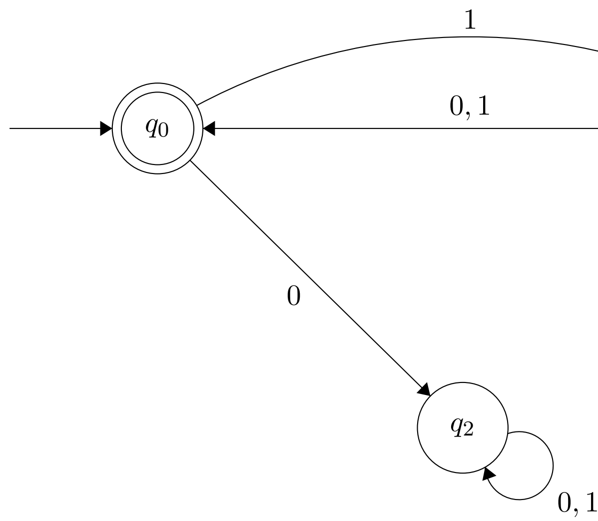
1. (a)



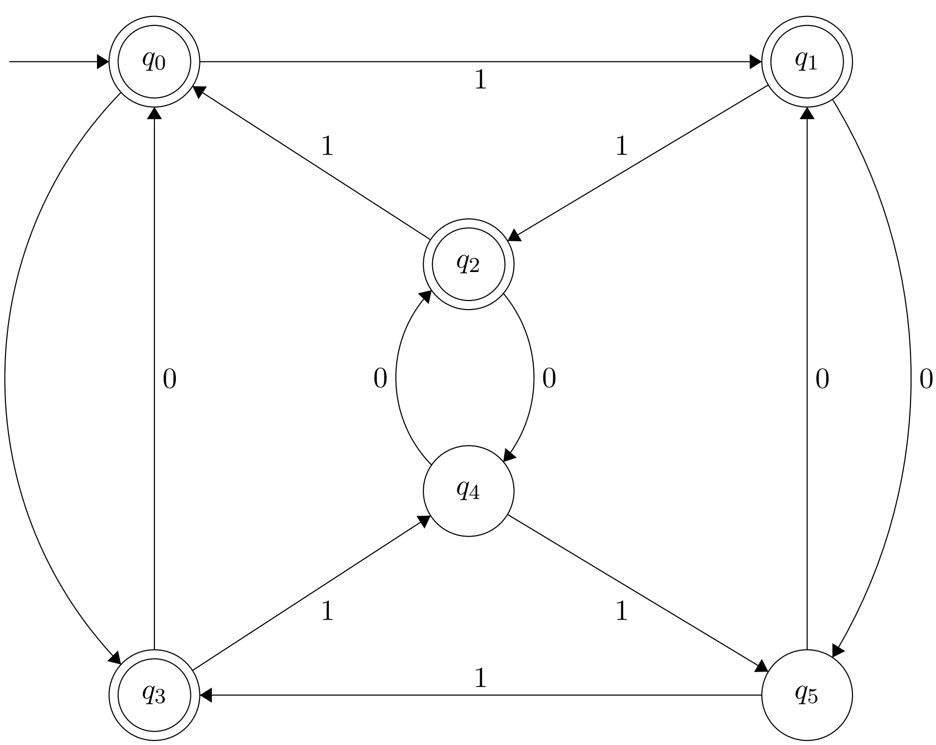
(b)



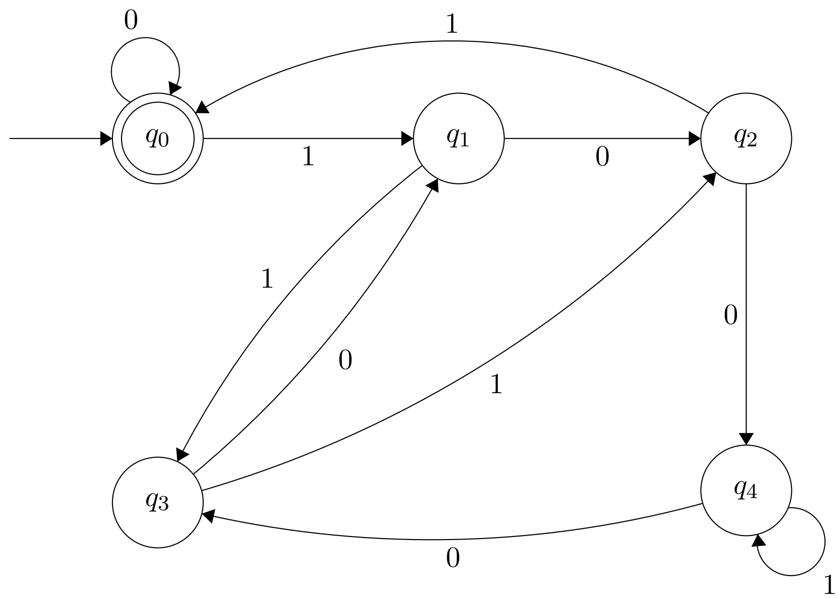
(c)



(d)

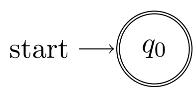


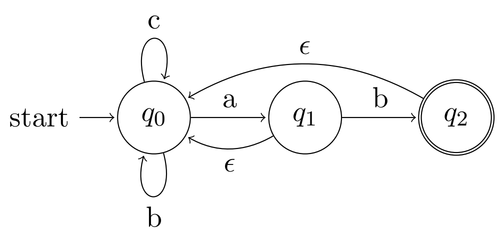
(e)


q_0
q_1
q_2
q_3
q_4
0
1
1
0
1
0
0
1
0
1

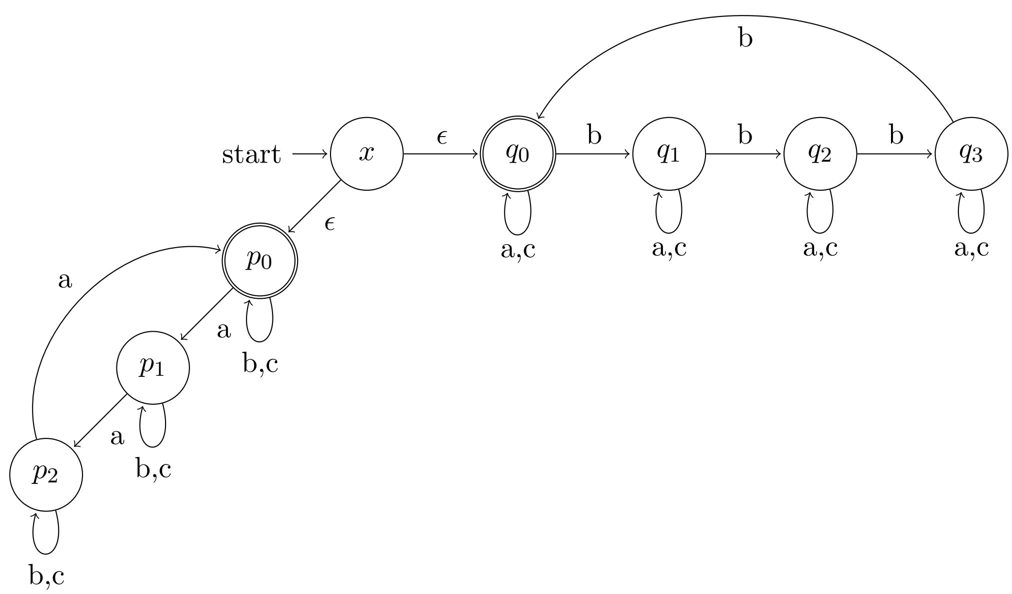
(f)

2. (a)

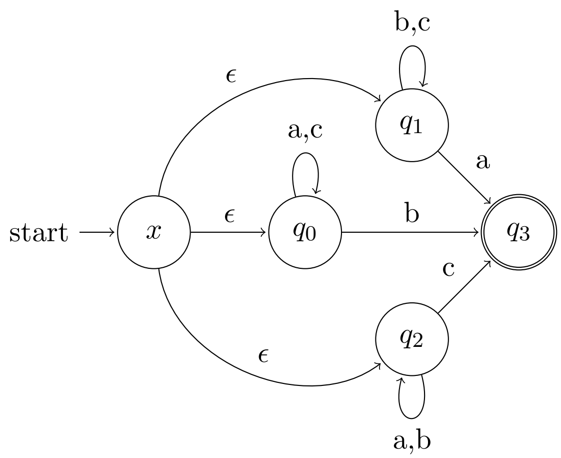

start
q_0


start
q_0
q_1
q_2
c
ϵ
a
b
ϵ
b

(b)


start
x
ϵ
q_0
q_1
q_2
q_3
b
b
b
b
a,c
a,c
a,c
a,c
ϵ
p_0
p_1
p_2
a
a
a
b,c
b,c
b,c

(c)



(d)

3. (a) The regular expression for this statement is
$$L = \Sigma^5\Sigma^*$$

   (b) The regular expression for this statement is
$$L = 1 \cup 1\Sigma^*$$

   (c) The regular expression for this statement is
$$L = (0 \cup \epsilon)(1 \cup 10 \cup 101 \cup 110)^*0^*$$
This should produce the desired behavior as it is not possible to have more than 2 0s with a 1 afterwards.

4. *Proof:* Let A, B be regular languages. Then, there exist DFAs for A, B that we will refer to as A', B'. To prove that $A \cap B$ is a regular language, we can prove that there exists a DFA accepting $A \cap B$ which we will denote as $(A \cap B)'$. We will proceed by construction. First, we say that $Q_{A\cap B} = Q_A \times Q_B$. This is guaranteed to be finite because Q_A and Q_B are finite. Σ remains unchanged as both A and B need to have the same alphabet for their intersection to be well-defined. We define $\delta_{A\cap B}$ as follows:
$$\delta_{A\cap B} : Q_{A\cap B} \times \Sigma \to Q_{A\cap B}, \qquad \delta_{A\cap B}((q_A, q_B), \sigma) = (\delta_A(q_A, \sigma), \delta_B(q_B, \sigma))$$
where δ_A, δ_B are the transition functions of A', B' respectively. Because these functions are individually well-defined on their domains, $\delta_{A\cap B}$ is also well-defined. We then let $q_{0,A\cap B} = (q_{0,A}, q_{0,B})$ where these are the start states of A', B' respectively. Finally,
$$F_{A\cap B} = \{(q_a, q_b) \in Q_A \times Q_B \mid q_a \in F_A, q_b \in F_B\}$$
In conclusion, we define
$$(A \cap B)' = (Q_{A\cap B}, \Sigma, \delta_{A\cap B}, q_{0,A\cap B}, F_{A\cap B})$$
This is a well-defined DFA because A and B are regular languages and as such we conclude that $A \cap B$ is a regular language.