$$C = AB = \begin{array}{c} \\ m \\ \\ \end{array} \underbrace{\begin{bmatrix} A(1,:) \\ A(2,:) \\ \vdots \\ A(m,:) \end{bmatrix}}_{p} \underbrace{\left[\begin{array}{c|c|c|c} B(:,1) & B(:,2) & \cdots & B(:,n) \end{array}\right]}_{n} \begin{array}{c} \\ p \\ \\ \end{array}$$

$$c_{ji} = A(j,:)\,B(:,i)$$

$$C(:,i) = \begin{bmatrix} A(1,:)\,B(:,i) \\ \vdots \\ A(m,:)\,B(:,i) \end{bmatrix} = A \cdot B(:,i)$$

column

$$= \sum_k A(:,k)\, b_{ji}$$

$$C = \left[\begin{array}{c|c|c} C(:,1) & \cdots & C(:,n) \end{array}\right]$$

$$= \left[\begin{array}{c|c|c} \sum_k A(:,k)\, b_{k1} & \cdots & \sum_k A(:,k)\, b_{kn} \end{array}\right]$$

$$= \sum_k A(:,k) \left[\begin{array}{c|c|c} b_{k1} & \cdots & b_{kn} \end{array}\right] = \sum_k A(:,k)\, B(k,:)$$

$$\begin{pmatrix} 3 & -1 & 2 \\ 6 & -1 & 5 \\ 0 & -1 & 1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Forward)

$$\begin{bmatrix} 3 & -1 & 2 & 1 \\ 6 & -1 & 5 & 2 \\ 0 & -1 & 1 & 3 \end{bmatrix}$$

back) $x_3 = \frac{3}{2}$

$x_2 = -(3 - x_3) = -\frac{3}{2}$

$x_1 = \frac{1}{6}(2 + x_2 - 5x_3)$

$= -\frac{7}{6}$

pivot

$$\begin{bmatrix} 6 & -1 & 5 & 2 \\ 3 & -1 & 2 & 1 \\ 0 & -1 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 & 5 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -1 & 1 & 3 \end{bmatrix} = E_{21}\left(\tfrac{1}{2}\right)[A ; b]$$

pivot

$$\begin{bmatrix} 6 & -1 & 5 & 2 \\ 0 & -1 & 1 & 3 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -1 & 5 & 2 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & -1 & -\frac{3}{2} \end{bmatrix} = E_{32}\left(\tfrac{1}{2}\right)E_{21}\left(\tfrac{1}{2}\right)[A ; b]$$

$= [U ; y]$

$$E_m \cdots E_1 A = I.$$

$$\Leftrightarrow E_m \cdots E_1 = A^{-1}$$

$$\Leftrightarrow E_m \cdots E_1 \cdot I = A^{-1}$$

$$A \cdot adj(A) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$$

$$\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix} \begin{bmatrix} A_{i1} \\ A_{i2} \\ \vdots \\ A_{in} \end{bmatrix}$$

$$= a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in} = det(A)$$

$$\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix} \begin{bmatrix} A_{j1} \\ A_{j2} \\ \vdots \\ A_{jn} \end{bmatrix} \quad (i \neq j)$$

$= a_{i1}A_{j1} + a_{i2}A_{j2} + \cdots + a_{in}A_{jn}$: A 의 j^{th} row를 i^{th} row로 대체한 matrix 의 det.

$$= det \begin{array}{r} \text{ith row} \rightarrow \\ \\ \text{jth row} \rightarrow \\ \\ \end{array} \begin{bmatrix} & & \vdots & \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ & & \vdots & \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ & & \vdots & \end{bmatrix} = 0$$

$$\Rightarrow A \cdot adj(A) = det(A) I$$

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