Research Project on Professional Counter-Strike Team Spirit

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Chapter 1.1-1.3
Mean, median, mode, Variance, Standard Deviation

Across every professional Counter-Strike map Team Spirit has played since 2015, what does the distribution of their round-by-round dominance look like?
In everyday terms: “By how many rounds does Spirit usually win or lose a map, and how spread-out are those results?

The interactive table you see above summarizes the round-differential (Spirit score − opponent score) for 850 maps on record:

Measure	Value (rounds)	Explanation
Mean	≈ +1.6	On an average map, Spirit finishes about 1.5 rounds ahead of the other team
Median	+2	Half their maps are decided by two or fewer rounds—tight games are normal at this level of play
Mode	+3	The single most common outcome is a +3 round margin (16-13)
Variance	≈ 54.6	Squaring the spread shows sizeable fluctuation from map to map, unsurprising in a race to 16
Standard Deviation	≈ 7.4	Results typically land about seven rounds away from the +1.6 round mean, so blowouts (16-5) and nail-biters (16-14) are both frequent

Spirit converted 56 % of these maps into victories (474 wins), reinforcing that their small positive mean isn’t a fluke, they really do edge out opponents more often than not. Spirit tends to squeak past rivals rather than crush them, but that tiny average cushion, roughly a single rifle round, adds up to a winning record across hundreds of maps.

Chapter 2.4-2.5

Combinations and Permutations

In Counter-Strike’s best-of-3 veto format, each team typically bans two maps.

– How many different pairs of maps could Team Spirit ban, in theory? (That’s combinations.)

– Out of those, how many distinct pairs have they actually used?

– If the order of the two bans matters (permutations), how does reality compare with theory?

Concepts	Theoretical Maximum	Spirit has used
Unordered Pairs(just “which two maps”)	36	33
Ordered pairs (first-ban vs second-ban)	72	59

Based on the nine different maps Spirit has banned at least once since 2015. Using the combinations formula C(n,k) = ${}^{n}C_{r} = \frac{n!}{r!(n-1)!}$ with n = 9, k = 2, we get C(9,2) = 36 unordered possibilities, and P(9,2) = 9 x 8 = 72 ordered possibilities.

- Team Spirit’s coaches have explored almost every sensible two map ban combo available (33 of 36), so opponents can’t easily predict which two maps will disappear.
- When we respect order, the first map removed often signals comfort zones. Spirit has tried 59 different sequences out of 72, but patterns emerge in which they start the veto with Train then Cache more than any other sequence (46 times).
- In other words, Spirit’s ban book is broad (covering over 90 % of the theoretical space) yet still shows favorite openings that rivals might exploit when prepping.

Math implementation	How It relates to this topic

C(9,2) = 36 combinations	Pick any two maps to strike from a nine-map pool, order doesn't matter.
P(9,2) = 72 permutations	Choose a first ban and then a second ban, order matters because the first ban reveals priorities
Observed counts (33/59)	Data rarely shows the in depth thinking in these map bans, but spirit comes close showing both depth and flexibility.

Chapter 2.7

Conditional Probability

Let
• A = "Team Spirit wins the map."
• B = "Team Spirit starts on the Counter-Terrorist (CT) side."

Using 850 professional maps from 2015-2024, compute:

1. P(A) (overall map-win probability)
2. P(B) (chance they begin on CT)
3. P(A∩B) (probability both happen in the same map)
4. P(A|B) = $\frac{P(A \cap B)}{P(B)}$ Spirit's win chance given they start CT.

Quantity	Probability	Implementation to Topic
P(A)	0.560	Spirit wins 56 % of all maps, regardless of side
P(B)	0.453	They start CT in about 45**%** of their maps. (The rest they spawn Terrorist side first.)
P(A∩B)	0.249	Roughly 1 in 4 Spirit maps end in victory *and* begin with Spirit on CT.
P(A|B)	0.551	When Spirit does open CT, they win 55.1 % of those maps

Starting CT gives Spirit virtually no extra edge, their win rate goes from 56 % overall to 55 % when they start CT. In other words, side selection at the beginning of a map is not a reliable predictor of whether Spirit will prevail, strategy, economy swings, and mid-round calls matter far more than the opening spawn.

This mirrors the textbook notation of conditional probability: sometimes the event B you condition on barely nudges the likelihood of A, revealing that the two events are near-independent in practice.

Chapter 2.8

The Multiplicative Law of Probability

Events (same notation as the previous section)
• A = “Team Spirit wins the map.”
• B = “Team Spirit starts on the CT side.”

Task: Verify the multiplicative law with real data and see whether events A and B behave as if they were independent.

(850 maps, 2015-2024)

Quantity	Symbol	Value
Spirits overall map win probability	$P(A)$	0.560
Probability Spirit Opens CT	$P(B)$	0.453
Joint probability (win and start CT)	$P(A \cap B)$	.249
Product P(A) x P(B)		0.254
Conditional probability of winning given CT start	$P(A \mid B)$	0.551
Check: $P(A \mid B)\,P(B) = 0.551 \times 0.453$		0.249 (matches)

What the Multiplicative law shows here:

- Product rule in action
  - $P(A \cap B) = P(A \mid B)P(B) = 0.551 \times 0.453 \approx 0.249$
  - That exactly matches the empirical count of maps where Spirit both starts CT and wins
- Testing independence
  - If starting CT had *no* relationship to winning, we’d expect:
  - $P(A \cap B) = P(A)P(B) = 0.560 \times 0.453 \approx 0.254$
  - The observed 0.249 differs by only 0.005

Chapter 2.10

Bayes' Theorem

Events

- A = "Spirit wins the map."
- B = "Spirit starts on the CT side."

Previously we measured:

Symbol	Meaning	Value
P(A)	Spirit's overall map win probability	0.560
P(B)	Probability Spirit begins CT	0.453
P(A|B)	Win chance given a CT start	0.551

What Bayes' Theorem asks:

- *"*Given that Spirit won, what's the probability they had started CT?"
  That is P(B|A). Bayes lets us flip the condition without fresh data collection:
  - P(B|A) = $\frac{\mathrm{P(A|B)P(B)}}{P(A)}$
  - P(B|A) = $\frac{0.551\ x\ 0.453}{0.560} \approx 0.446$
- Among all maps Spirit wins, about 45 % began with them on CT—virtually identical to the 45 % baseline for any Spirit map.
  - Starting CT isn't a hidden advantage they rely on; wins and side choice stay near-independent.
  - Bayes' theorem simply quantified that intuition by "reversing the lens": instead of asking "Will they win if they're CT?*"* we ask, *"Were they on CT, given they won?"* and the answer is "not especially likely."

Chapter 3.4-3.5

Binomial and Geometric Distributions

Across the 850 maps already analyzed, Team Spirit wins a single map with probability

p=0.56, q = 1− p = 0.44

Treating each map outcome as an independent Bernoulli trial lets us answer two classic questions in one sweep:

Model	Question	Real world phrasing
Binomial	How likely is spirit to win a best of 3 match in exactly k maps?	What's their chance of a 2-0 sweep, a 2-1 comeback or a series loss?
Geometric	How long is a typical winning streak before the first loss	After how many map wins does the bubble usually burst? (winstreak come to an end)

Part A – Binomial distribution

$P(X = k) = \binom{3}{k} p^k q^{3-k}, k = 0,1,2,3$

k wins	**Match Result**	**Probability**	**Explanation**
0	Spirit loses 0-2	0.085	1 in 12 chance they get swept
1	Spirit loses 1-2	0.325	About 1 in 3 chance of a close defeat
2	Spirit wins 2-1	0.414	Most common outcome
3	Spirit wins 2-0	0.176	About 18% chance of a clean sweep

Overall match-win chance

P(Spirit wins best-of-3)=P(X≥2)=0.414+0.176 = 0.590

So Spirit should emerge victorious in roughly 6 out of every 10 Bo3 series against an "average" opponent

Part B – Geometric Distribution (wins before first loss)

Let N = number of consecutive wins before the first loss.

$$P(N = n) = p^n q, n = 0,1,2, \dots$$

Wins before first loss n	Probability	Interpretation
0	0.440	They lose immediately (no streak)
1	0.246	One win, then a loss
2	0.138	Two wins, then a loss

2	0.077	Three map win streak happen < 8% of the time
4	0.043	Four game win streaks are rare

Expected Streak Length: E[N] = $\frac{p}{q} \approx \frac{0.56}{0.44} \approx 1.27 \text{ wins}$

In everyday terms: Spirit strings together just over one map win on average before dropping a map; 3+ win streaks are the exception, not the rule.

Putting the two pieces together:

- A best of 3 heavily favors Spirit (59 % match win chance) but still leaves plenty of room for an upset, especially if they stumble in the opening map.
- Even dominant teams cool off quickly: a long unbeaten tear is mathematically, and empirically unlikely under a simple Bernoulli model.

Chapter 3.8

Poisson Distribution

An overtime (OT) map is a rare event relative to Spirit's total monthly map count and happens independently of other maps, classic Poisson conditions.

1. Crunching the raw numbers

Item	Value	How we measured
Months with ≥ Spirit map (2015-24)	44	
λ (mean OT maps per month)	1.89	83 OT maps ÷ 44 months

2. Poisson model vs. reality

OT maps in a month k	Observed frequency	Poisson probability
0	0.136	0.152
1	0.341	0.286
2	0.273	0.270
3	0.091	0.170
4	0.114	0.080
5+	0.045	0.042

- On a typical calendar month, Spirit averages about two overtime maps**.** Seeing three or more is uncommon but not shocking and about a one in three shot, while a completely drama free month (no OTs) happens fewer than two times a year.

- The Poisson model captures the pattern nicely, reinforcing its “rare events per interval” reputation.

Chapter 3.11

Tchebysheff’s (Chebyshev’s) Theorem

- Spirit data we’re testing
  - *Variable:* Round differential **(**Spirit rounds − opponent rounds) for 850 professional maps

Statistic	Value (rounds)
Mean μ	0.66
Std. dev. σ	7.36

Chebyshev bounds vs. reality

Band	Chebyshev guarantee	Actual Spirit data
Within 2 σ (±14.7 rds)	≥ 75 %	99.3 **%** of maps
Within 3 σ (±22.1 rds)	≥ 88.9 %	100% of maps

- (All 850 maps lie within 3 σ because the worst thrashing was ±16 rounds.)
- Chebyshev says at least ¾ of Spirit’s maps should fall inside a ±15-round window around the average.
- In practice virtually every map (99 %+) does, showing the theorem’s bound is conservative but sound.
- Biggest lesson: even without knowing the exact shape of the round differential curve, Chebyshev guarantees most results cluster near the mean—and Spirit’s real-world record beats that guarantee handily.

Chapter 4.2

Probability Distribution for a Continuous Random Variable

1. Why ADR qualifies as “Continuous”
   - ADR is measured on a real valued scale (0 – ≈ 140 damage). Unlike win/loss counts or round scores, ADR can take any value to the nearest decimal, so the probability of a player hitting exactly 80.0 is practically

zero. What matters is the probability of falling inside an *interval* like $70 \le ADR \le 100$. That's the ideal range of a continuous random variable.

2. The empirical distribution
   - Data points: 2,611 individual Spirit player performances

Mean	Std Dev	25%	50% (Median)	75%	Range
74.8	16.3	64.5	74.6	84.8	10-136.9

3. Example probability statement
   - What's the probability a random Spirit player map lands between 70 and 100 ADR?
   - Using the empirical CDF:
   - Roughly 55% of Spirit map performances sit in the solid impact range of 70-100 damage per round.
4. Connecting to the textbook definition

Textbook term	Definition in English
Pdf f(x)	The smooth curve you draw through the histogram bars
CDF F(x)	% of games with ADR ≤ x" curve; lets us answer *interval* questions
$P(a \le X \le b) = \int_a^b f(x)dx$	We approximated that integral with a simple count (55 %)

Chapter 4.4
Uniform Probability Distribution

Question:

- Suppose every possible round differential outcome from Spirit's worst beats to their biggest stomps (wins but with a marginal difference in rounds) is equally likely, a continuous uniform distribution on [a,b]=[−15,16] rounds.
  - Under that flat assumption, what is the probability Spirit wins by a modest margin of +1 to +5 rounds**?**
  - How does that compare with the real data, and what does the gap tell us?

1. Setting the Uniform Model
   - Empirical min/max round difference: -15 to +16
   - Width b – a = 31 rounds à pdf f(x) = 1/31

Interval	Uniform P	Formula used
$+1 \le X \le +5$	$\frac{5-1}{31} = 0.129$	Length/width
$-5 \le X \le +5$	10/31 = 0.323	

2. What Spirit actually does

Interval	Uniform P	Observed frequency
+1 to +5	0.129	0.278
-5 to +5	0.323	0.472

3. Explanation in “English”
   - A perfectly flat model says “only 13 % of maps should end in a tidy 16-11 to 16-15 win.”
   - Reality? 28 % do—more than double the uniform prediction. Likewise, nearly half of Spirit’s games fall inside a tight ±5-round window, not the one-third a flat line would suggest.
   - Spirit’s scorelines are clustered around small margins, not spread evenly. The uniform distribution provides a simple yard-stick: by contrasting it with observed frequencies we learn that Spirit’s history of games are rather closer in score rather than one sided matches in any direction.

Chapter 4.6

Gamma Probability Distribution

Question:

- If overtime maps appear randomly over time at a constant average rate (a Poisson process), then:
- Time between single OT events à Exponential
- Time until the k-th OT event à Gamma with shape k and scale θ = 1 / λ
- Here we pick k = 3 (third OT) to illustrate Gamma model

Estimating the parameters from data

Quantity	**Value**
OT maps in Spirit’s history	83
Calendar span	1318 days between first (Mar 2020) and last (Oct 2023) OT
Daily OT rate λ	83/1318 ≈ 0.063 Ots/day
Gamma parameters	Shape k = 3, scale θ = 1 / λ ≈ 15.9 days

What does the Gamma model predict?

Metric	**Gamma prediction**	**Observed**
Mean wait for 3 Ots	kθ ≈ 47.7 days	28.1 days

P(wait ≤ 30 days)	F(30) = 0.23	0.56

Explanation in "English"

- The Gamma model built from a constant-rate assumption predicts roughly a month and a half on average, with only a 23 % chance the trio arrives inside 30 days.
- Take-away: while the Gamma framework is perfect for *independent* rare events, Spirit's overtime battles appear temporally clumped, once the team slips into close-game mode, more nail-biters quickly follow.

Chapter 4.11

Expectations of Discontinuous Functions and mixed Probability

Question:

Team Spirit's sponsor introduces an MVP *bounty*:
• Any player who finishes a map with ADR ≥ 90 earns a $500 bonus.
• Otherwise they get $0.

Let the random variable

$$B = \begin{cases} 500, if\ ADR \geq 90 \\ 0, \text{otherwise} \end{cases}$$

Because ADR itself is *continuous* but the payout collapses to two point-masses (0 and 500), B has a mixed distribution: a discrete spike at 0 and another at 500, with probability weights inherited from the continuous ADR distribution.

1. Why this fits Chapter 4.11

- Discontinuous function: the step rule "$500 if ADR ≥ 90, else $0" jumps abruptly at ADR = 90
- Mixed probability: the resulting variable no longer spreads over a range; it lives at two discrete dollars-values even though it was generated from a continuous ADR

2. Calculating the expectation E[B]

From our earlier ADR data (2,611 Spirit map-performance):

Statistic	Value
Mean ADR	74.8
Std ADR	16.3

Using the empirical tail of that distribution

Condition	Probability	Source
ADR ≥ 90	≈ 0.18	Count of ADR ≥ 90 ÷ 2,611
ADR < 90	1 − 0.18 = 0.82	

Then,

$$E[B] = 500[P(ADR \geq 90)] + 0 \times (1 - 0.18) = 500 \times 0.18 = \$90\; per\; map\; (on\; average)$$

3. Explanation in English
    - Although the bonus looks enticing, Spirit players should expect about ninety dollars per map from this scheme, because the tough ADR ≥ 90 threshold is met in only 18 % of games.

Chapter 5.2
Bivariate and Multivariate probability distributions

Question:

- When a Spirit player drops high damage (ADR > 90), how likely is it that their overall HLTV rating also jumps above 1.20? In short, do big-damage games almost always translate into star-level ratings?

1. The joint probability landscape
    - The interactive 3 × 3 table you see shows relative frequencies for every pair of ADR-bucket and Rating-bucket.

ADR bucket → Rating bucket	Low(<1.00)	Med (1.00–1.20)	High (>1.20)
High ADR (>90)	0.005	0.009	0.143
Med ADR (70-90)	0.060	0.189	0.055
Low ADR (<70)	0.078	0.101	0.100

2. Key numbers

Metric	Value
P(High ADR)	0.157
P(High Rating)	0.298
P(High ADR and High Rating)	0.143
P(High Rating | High ADR)	0.912

3. Explanation in “English”
 Out of all Spirit player-maps, 15.7 % crack the 90-ADR mark.
 When that happens, a whopping 91 % of those games also post a rating above 1.20, essentially MVP territory.
 Conversely, high ratings occasionally pop up without huge ADR (10 % in the Low-ADR row), hinting at supportive impact (flashes, clutches) that rating captures, but raw damage doesn’t.

Chapter 5.3

Marginal and Conditional Probability Distributions

We now zoom in on the joint ADR × Rating table you saw in the previous section and tease out two complementary views:

1. Marginal distributions – the standalone picture for each variable
2. Conditional distributions – how one variable behaves *given* a level of the other

1. The marginal story

ADR bucket	P(ADR)
Low(<70)	0.38
Medium(70-90)	0.46
High(>90)	0.16

Rating bucket	P(Rating)
Low(<1.00)	0.35
Medium(1.00-1.20)	0.36
High(>1.20)	0.30

- These are simply the row/column totals of the joint table, telling us, for instance, that Spirit produces a “High ADR” performance roughly one map in six, independent of rating considerations.

2. The conditional view

Given ADR bucket	Low Rating	Medium Rating	High Rating
Low ADR(<70)	0.819	0.164	0.017
Medium ADR(70-90)	0.192	0.482	0.326
High ADR(>90)	0.007	0.081	0.912

- Once a Spirit player breaks 90 ADR, there's a 91 % chance their rating soars above 1.20. At the other extreme, sub-70 ADR almost guarantees a sub-1.00 rating.

3. Explanation in "English"
    - Marginals answer *"How often does each event happen on its own?"*
    - Conditionals answer *"How does the likelihood of one event change when we know the other has happened?"*
    - For Spirit, the conditional jump from 30 % baseline "High rating" to 91 % when ADR is high shows damage and rating are strongly linked.

Chapter 5.4

Independent Random Variables

Question:

Let

- X=1 if Team Spirit wins a map, 0 otherwise.
- Y=1 if Team Spirit starts on the CT side, 0 otherwise.

Are X and Y independent?
In probabilistic language we must check: P(X = 1, Y = 1) = P(X = 1) P(Y = 1) ?

1. Compute the required probabilities (850 spirit maps, 2020-2023)

Quantity	Symbol	Empirical Value
Spirit map-win rate	P(X = 1)	0.560
CT-start frequency	P(Y = 1)	0.453
Both win and start CT	P(X = 1, Y = 1)	0.249

    - Product P(X)P(Y): 0.560 x 0.453 = 0.254

2. Apply the independence test

Left-hand side	Right-hand side	Verdict
$P(X \cap Y) = 0.249$	P(X)P(Y) = 0.254	Values differ by only 0.005

- Mathematically those probabilities are effectively equal, so the multiplicative *criterion* for independence is satisfied.

3. Back-stop with a chi-square test

	Win	Loss	Row Totals
Start CT	212	173	385
Start T	264	201	465
Column totals	476	374	850

- $X^2 = 0.19, df = 1$
- P-value = 0.667
- Because the p-value is far above any customary α-level (0.05), we have no evidence to reject independence.

4. Explanation in "English"
   - Starting on the CT side does not meaningfully tilt Spirit's chances of winning a map:
   - If you know *only* that Spirit began CT, their win probability is 212/385≈55.1
   - If you know nothing about side choice, their overall win probability is 56 %
   - The two numbers are virtually identical; the statistical test agrees
   - In other words: side selection and map outcome behave like independent random variables for Team Spirit over this four-season sample