Poisson Distribution

Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

a. no more than three customers arrive?

To find this, you add the probabilities for 0, 1, 2, and 3 customer arrivals:

Calculate each probability using the formula: $P(X = k) = e^{-\lambda}\lambda^{k}/k!$

Add them together to get the total probability of having up to 3 customers.

= 0.0817

b. at least two customers arrive?

This involves subtracting the probability of getting fewer than 2 customers (0 or 1 customer) from 1:

Calculate the probability for 0 and 1 customer using the same formula.

Subtract their sum from 1 to find the probability of getting 2 or more customers.

= .9927

c. exactly five customers arrive?

Simply apply the formula for the Poisson distribution for k=5

Use the Poisson formula to calculate $P(X=5)$

= .1277

<u>Discrete Uniform Distribution</u>

A box contains five keys, only one of which will open a lock. Keys are randomly selected and tried, one at a time, until the lock is opened (keys that do not work are discarded before another is tried). Let Y be the number of the trial on which the lock is opened.

a Find the probability function for Y .

The scenario is like a mini experiment repeated until success: trying keys on a lock until one works. Since there's only one correct key among five, each trial where a key is tried is independent of the others. The probability function P(Y=k), represents the probability that the correct key is found on the k-th trial. Given the setup, where each key is equally likely to be the correct one, and incorrect keys are not retried, the probability function is simply:

$P(Y=k) = \frac{1}{5}$

For each k = 1, 2, 3, 4, 5, because each key has an equal chance of being tried at each step until the lock opens.

b Give the corresponding distribution function.

The distribution function $F(y)$ tells us the probability that the lock has been opened by the y-th trial or before. It accumulates probabilities from the first trial up to the y-th trial. Mathematically, it's calculated as:

$$F(y) = P(Y \leq y) = \sum_{k=1}^{\lfloor y \rfloor} P(Y = k)$$

This cumulative total increases by ⅕ for each possible value of k from 1 to y, as long as y does not exceed the total number of keys (5). After 5, F(y), equals 1, meaning the lock is certain to be opened.

c What is P(Y < 3)? P(Y ≤ 3)? P(Y = 3)?

- P(Y<3): This asks for the probability that the lock opens in fewer than 3 trials. Since the lock can either open on the first or the second trial, you sum P(y = 1) and P(y=2), both ⅕ .
- P(y<=3): This considers up to the third trial inclusive. Hence, it's P(y = 1) + P(y = 2) + P(y = 3), all ⅕ each
- P(y = 3): This is straightforward—the probability of opening exactly on the third trial, which is ⅕.

d If Y is a continuous random variable, we argued that, for all $-\infty < a < \infty$, $P(Y = a) = 0$.

In continuous random variable probability at a point P(Y=a)=0 for any specific value "a". In this problem, Y is a discrete random variable, and we calculated specific probabilities for the discrete values 1,2,3,4 and 5. These probabilities are not zero, as they are associated with discrete values, not continuous ones.

Expected value of continuous random variable

4.29 The temperature Y at which a thermostatically controlled switch turns on has probability density function given by

$$f(y) = \begin{cases} 1/2, & 59 \le y \le 61, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $E(Y)$ and $V(Y)$.

Given: f(y) = ½, 59<= y <= 61
0, otherwise

E(Y) = the integral from negative infinity to infinity (x * f(x)dx)

$$= \int_{59}^{61} x \times \frac{1}{2} dx$$

= ¼ x 240 = 60

And then, you plug in the variance formula

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$$

Var(Y) = 0.3333

Continuous Variables and Their Probability Distributions

Upon studying low bids for shipping contracts, a microcomputer manufacturing company finds that intrastate contracts have low bids that are uniformly distributed between 20 and 25, in units of thousands of dollars. Find the probability that the low bid on the next intrastate shipping Contract

a is below $22,000.

Here's how we find the probability of a bid being below $22,000:

1. Understand the range: Bids are uniform between $20,000 and $25,000, so each price within this range is equally likely.
2. Set up the fraction: For a uniform distribution, the probability is the fraction of the range below $22,000.

3. Calculate the fraction: The fraction is (22−20)/(25−20), which represents the portion of the range from \$20,000 to \$22,000.
4. Result: This fraction simplifies to 0.4, meaning there's a 40% chance the bid will be below \$22,000.

b is in excess of \$24,000

To find the probability of a bid exceeding \$24,000:

1. Probability till \$24,000: Calculate the probability for all bids up to \$24,000 using the same method as above, which is (24−20)/(25−20).
2. Subtract from total: Since the total probability for any outcome is 100%, subtract the calculated probability from 1 (or 100%) to get the probability of the bid being more than \$24,000.
3. Result: After subtraction, you get a result of 0.2, so there's a 20% chance the bid will be over \$24,000.

Exponential Distribution

Historical evidence indicates that times between fatal accidents on scheduled American domestic passenger flights have an approximately exponential distribution. Assume that the mean time between accidents is 44 days.

a. If one of the accidents occurred on July 1 of a randomly selected year in the study period, what is the probability that another accident occurred that same month?

- If we let X denote the times between fatal accidents on scheduled American domestic passenger flights, then according to the question, X~exp(144). Thus, the probability distribution function will be: f(x)=1/44e^−x/44, x>0
- One of the accidents occurred on July 1 of a randomly selected year in the study period, and we have to find the probability that another accident occurred that same month. Since July has 31 days, another accident could happen on any day from July 1 to July 31, therefore the required probability is P(X≤31). This probability can be computed as:

$$P(X \leq 31) = \int_0^{31} \frac{1}{44} e^{-\frac{x}{44}}\, dx$$

= 0.5057

b. What is the variance of the times between accidents?

- Since the mean time is 44 days, the variance of the times between accidents is: Var(X)=(44)^2=1,936

<u>Multivariate Probability Distributions</u>

Three balanced coins are tossed independently. One of the variables of interest is Y1, the number of heads. Let Y2 denote the amount of money won on a side bet in the following manner. If the first head occurs on the first toss, you win \$1. If the first head occurs on toss 2 or on toss 3 you win \$2 or \$3, respectively. If no heads appear, you lose \$1 (that is, win −\$1).

a Find the joint probability function for Y1 and Y2.

To find the joint probability function for Y1 and Y2, consider all possible outcomes of the three coin tosses and their corresponding winnings.

Let H represent a head, and T represent a tail. The possible outcomes for Y1 (number of heads) are 0,1,2,or 3, and the corresponding outcomes for Y2 (amount of money won) are −1,\$1,\$2, or \$3, respectively.

Y1 = 0
Y2 = -1
outcome : TTT = ⅛

Y1 = 1
Y2 = \$1
Outcome: HTT, THT, TTH = ⅜

Y1 = 2
Y2 = \$2
Outcome: HHT, HTH, THH = ⅜

Y1 = 3
Y2 = \$3
Outcome:HHH = ⅛

Y1	Y2	P(Y1,Y2)
0	-1	1/8
1	1\$	3/8
2	2\$	3/8
3	3\$	1/8

b What is the probability that fewer than three heads will occur and you will win $1 or less? [That is, find F(2, 1).]

- $F(2,1) = P(Y1 < 3$ and $Y2 <= \$1)$
  $= ½$
  So, the probability that fewer than three heads will occur and win $1or less (F(2,1)) is 12 or 0.5.