Foundations of Computational Math 1 Fall 2023

Programming assignment 4

Your task is to implement, analyze, and empirically test and demonstrate the capabilities of the method discussed in the class:

- bisection
- Newton's method
- Fixed point method

Test 1

Determine the roots and the various intervals of convergence for the methods on the following problem:

$$f(x) = xe^{-x} - 0.06064$$

(1) Plot $f(x)$ (and you will know the number of roots).

(2) Bisection:

- Assume we don't know the roots. Choose your own initial $[a, b]$'s and describe how you choose intervals.
- Collect the size of the interval $|b_k - a_k|$ at each step and log-plot the results. Discuss the results.

(3) Newton's method:

- Test $x_0 = 1$. Describe and discuss the results. Your code should detect this situation.
- Test $x_0 = 0.99$. Show and discuss the results.
- Can you choose better x_0's? For each root, choose two different x_0's, show and discuss the results.

(4) Fixed point method

- Design your own fixed point iteration.
- Show and discuss the results.

Note: All of your choices should be different from the conditions given in Correctness Test.

Test 2

Determine the roots and the various intervals of convergence for the methods on the following problem:

$$g(x) = x^3 - x - 6$$

(1) Plot $g(x)$ (and you will see $x = 2$ is the root).

(2) Bisection:

- Assume we don't know the root. Choose your own initial $[a, b]$ and describe how you choose this interval.
- Collect error $|x - x_k|$ at each step and log-plot them.
- (Extra 5 points) Demonstrate the convergence order (or speed of convergence, see the convergence theorem). If the accuracy is $|x_k - x| < 10^{-6}$ and the initial interval is $[-5, 10]$, how many iteration steps do you expect? How this value compare to your actual iteration steps (i.e. is the actual result consistent with the theory)?

(3) Newton's method:

- Test $x_0 = \frac{1}{\sqrt{3}}$. Describe and analyze the results. Your code should detect this situation.
- Test $x_0 = 0.57735$. Show and discuss the results.
- Can you choose a better x_0? Show and discuss the results.
- (Extra 5 points) Design the test and demonstrate quadratic convergence.

(4) Fixed point method

- Design your own fixed point iteration.
- Show and discuss the results.

Correctness Test

For $f(x)$ in **Test 1**:

1. For bisection method:
   - use initial interval: $[1, 13]$
   - use stop condition: size of the interval is less than 10^{-6}
   - plot all iterations (i.e. $f(x_k)$ vs x_k for all k).
   - report the iteration steps.
2. For Newton's method:
   - use $x_0 = 2$
   - use stop condition: $|f(x_k)| < 10^{-6}$
   - plot all iterations (i.e. $f(x_k)$ vs x_k for all k).
   - report the iteration steps.
3. For fixed point method:
   - use $x_0 = 2$
   - use $x_{k+1} = \phi(x_k)$ where
   $$\phi(x) = xe^{-x} + x - 0.06064$$
   - use stop condition: $|f(x_k)| < 10^{-6}$
   - plot all iterations (i.e. $f(x_k)$ vs x_k for all k).
   - report the iteration steps.

In your report:

The information collected should be displayed in an appropriate organized manner to support your answers to all the questions above. You are expected to show and analyze the results for each method. Discussions on comparison of three methods are also expected.

Submission of Results

Expected results comprise:

- A document describing your solutions as prescribed in the notes on writing up a programming solution posted on the Canvas.
- The source code, makefiles, and instructions on how to compile and execute your code including the Math Department machine used, if applicable.
- Code documentation should be included in each routine. (You don't need to paste your code in the writing report).
- All text files that do not contain code or makefiles must be PDF files. **Do not send Microsoft word files of any type.**

These results should be submitted by 11:59 PM on the due date. Submission of results is to be done via Canvas.